On using the ATD: Some clarifications and comments

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Abstract. The main purpose of this opening lecture is to clarify and comment on a number of aspects of the genesis, structure and use of the ATD that I believe deserve clarification. The current weight of common opinion in didactics, in fact, has sometimes led either to the forgetting of certain key elements of the anthropological theory of the didactic, or to a somewhat limited understanding of what it implies. The most visible effect of this phenomenon is the fact that the theoretical and practical use of the theory does not always seem optimal. In the following, I have therefore tried to identify and cope with a selection of these difficulties of reception, that I felt it was possible to address in a meaningful way in the context of this presentation.

1. An introductory remark

In this opening lecture, I shall address a few thorny points involved in the use of the ATD as well as in its development and reception. The *anthropological theory of the didactic* is a complex construct and whoever intends to use it should be reminded that it cannot be altered at will, in a carefree, happy-go-lucky manner. As in any scientific endeavour, it is critical to pay close attention to the words we use and to grasp their intended meaning as well as we can. But before I go into this, I invite you to consider the following quote from William P. Byers’s book, *How Mathematicians Think: Using Ambiguity, Contradiction, and Paradox to Create Mathematics* (2010):

> Normally, the development of mathematics is reconstructed as a rational flow from assumptions to conclusions. In this reconstruction, the problematic is avoided, deleted, or at best minimized. What is radical about the approach in this book is the assertion that creativity and understanding arise out of the problematic, out of situations I am calling “ambiguous.” Logic abhors the ambiguous, the paradoxical, and especially the contradictory, but the creative mathematician welcomes such problematic situations because they raise the question, “What is going on here?” Thus the problematic signals a situation that is worth investigating. The problematic is a potential source of new mathematics. How a person responds to the problematic tells you a great deal about them. Does the problematic pose a challenge or is it a threat to be avoided? It is the answer to this question, not raw intelligence, that determines who will become the successful researcher or, for that matter, the successful student. (p. 6)

I suggest that we keep this in mind and apply it to *didactics* itself.

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2. The didactic

The first expression I’d like to mention is a relatively new one: it is that of the didactic continent. About the word “continent” I shall make no remark, except that it is derived from Latin terra continens, which means “continuous land” or “land in one piece”. Here, the expression “didactic continent”, in which “continent” is, of course, used in a metaphoric way, refers to a twofold reality.

First of all, the didactic continent in a given society comprises the whole of the didactic present within that society. The didactic is the set of all the didactic acts by which any instance—an instance being here either a person or an institution—tries to help some instance to learn something that we traditionally call the didactic stake involved in the situation. When the two instances are one and the same, we say that the didactic act is autodidactic.

Let me stress that what precedes is a definition, within the ATD, of the word “autodidactic”. I take advantage of this definition to emphasise a key principle in the building-up of a scientific theory and therefore of the ATD: the free, though orderly, use of words. This is what I call the Humpty Dumpty principle. Allow me to recall to you this famous quote from Lewis Carroll’s Through the looking-class, and What Alice Found There (1871): “When I use a word,’ Humpty Dumpty said, in rather a scornful tone, ‘it means just what I choose it to mean—neither more nor less.’”

We speak of “the didactic” just as one speaks of “the political”, “the religious”, “the sexual”, etc. Although it is a vital necessity for them, all societies tend to repress the didactic as if it were an insuperable flaw. They therefore try to hide it in selected, isolated places—namely, schools and classrooms. By contrast, didacticians, who by definition study the didactic, must allow themselves to look for it wherever it occurs in society, not only where society says it is.

3. Didactics

The second constituent part of the didactic continent is didactics, which can aptly be defined—as a first approximation—as the science of the didactic. As you can see, I do not refer here to the “didactics of mathematics” or to the “didactics of English as a foreign language”, or even to the “didactics of foreign languages”, etc. I simply refer to didactics—full stop. As you certainly know, there is a lot of debate going on, and a lot of posturing going on, too, about whether one can rely on such a general notion. In what follows, I shall however leave aside the usual argument for and against the so-called “existence”—or “nonexistence”—of didactics and take quite another view of all the squabbling going on about these conflicting perspectives.

As any scientific theory, the ATD provides models that are research tools for those who use this theory. What I call “Didactics” is first and foremost just such a model. It leads one to regard all the research done by the whole diversity of didacticians throughout the didactic continent as potential parts of a great whole simply called didactics. Didactics is thus made up of all the contributions, not necessarily immediately compatible with one another, that result from the more or less disconnected, or even conflicting, study of the didactic, wherever such a study takes place.

This model must be understood to be a vantage point from which one can best observe the didacticians’ doings and sayings. At the current stage of development of the didactic continent, it allows one to say that the people inhabiting this continent often ignore each other or are
suspicious of one another. In fact, some of them seem to live in a state of permanent armed peace.

The model that the ATD affords, in this respect, can be made slightly more realistic by introducing in it the new though age-old concept of territory. This word comes from the Latin territorium, “land around a town, domain, district”, which in turn is supposed by some experts to be derived from the Latin verb terrere “to frighten” (a fact of which the words terror and terrible are subtle reminders). One etymology dictionary (Harper, 2011-2015) thus aptly concludes that “territorium would mean ‘a place from which people are warned off.’” Be that as it may, one can observe that, to this day, a number of didacticians seem to behave like “territorial animals”. Didacticians from two supposedly distinct territories—say, the didactics of mathematics and the didactics of physics—generally agree that they should not mix with each other, for the sake of their respective, particular pursuit. In the ATD, this traditional defensive attitude is not taken for granted. It is regarded as a problematic fact, that awaits explanation but whose perpetuation is not mandatory.

4. Relations to an object

In thinking about another future for didactics, two basic notions of the ATD will turn out to be essential: the notion of praxeology and that of condition. But before coming to grips with them, I shall dwell for a moment on a much overlooked notion that in fact predated the concept of praxeology and in very truth even nurtured its emergence: the relation of a person $x$ to an object $o$, denoted by $R(x, o)$ and also called the personal relation of $x$ to $o$. This notion extends to institutions or, more exactly, to any position $p$ in any institution $I$, in which case we write it $R(p, o)$. This relation to $o$ is the institutional relation to $o$ for persons occupying position $p$ in $I$. We can thus consider the relation of a student $x$ to the object $o = \sqrt{2}$.

When the person $x$ occupies the student position $p = s$ in a certain classroom $C$, the personal relation of $x$ to $o$, $R(x, o)$, should appear to conform to the institutional relation $R_C(s, o)$ for people occupying position $s$ in classroom $C$, a fact that can be written as follows: $R(x, o) \cong R_C(s, o)$. Of course, it is often the case that $R(x, o)$ does not conform to $R_C(s, o)$. More generally, when a person $x$ occupies the position $p$ in an institution $I$, if the personal relation of $x$ to $o$ fails to conform to the institutional relation $R(p, o)$, we say that $x$ is a “bad subject” of $I$ in the position $p$ as concerns the object $o$. It should be kept in mind that the institutional relation $R(p, o)$ is always an artificial construct and that no real person $x$ exists whose personal relation to $o$ can be fully identical to it: the equality $R(x, o) = R(p, o)$ cannot take place and $R(p, o)$ must best be construed as the “personal” relation of a mere cardboard cutout, the “perfect” subject of $I$ in position $p$ being always a fictional character.

Given two institutional positions $(I, p)$ and $(J, q)$, we can also consider the comparison between $R(p, o)$ and $R(q, o)$, which leads to the conclusion that we have, or not, $R(p, o) \cong R(q, o)$. In the former case, that is when $R(p, o)$ is found to conform to $R(q, o)$, we say that $R(p, o)$ and $R(q, o)$ are congruent. In the following, I will express the fact that $R(x, o)$ has some set of properties by conjuring up an imaginary relation $R$ to $o$ supposedly endowed with exactly those properties, with which $R(x, o)$ is held to be congruent, that is to say to which it is assumed to conform: $R(x, o) \cong R$. Again, this applies to institutional relations $R(p, o)$.
I have not yet defined the notion of an object. Let us say that an object \( o \) exists for a person \( x \) if the relation \( R(x, o) \) is nonempty: \( R(x, o) \neq \emptyset \). In the same way, we shall say that an object \( o \) exists for the position \( p \) in the institution \( I \) if \( R(p, o) \neq \emptyset \). We then define an object \( o \) to be anything—any “entity”, material or not—that exists for at least one person or one institutional position. At this point, however, I have not yet clarified what the relation to an object \( o \) is made of. The personal relation of \( x \) to \( o \) is the system of all the ways in which \( x \) relates to \( o \), be it in terms of knowledge, “know-how”, beliefs, expectations, feelings, fantasies or daydreams. Taking the word in a broad sense—remember the Humpty Dumpty principle!—, I shall say that \( R(x, o) \) describes what \( x \) knows about \( o \), or that it collects together all the knowledge \( x \) has about \( o \). I shall say that \( x \) knows the objet \( o \) if \( R(x, o) \neq \emptyset \), that is, if \( o \) exists for \( x \). Of course these definitions relating to persons extend to institutional positions.

Since we are supposed to juggle between English and two Romance languages, I would like to quote here, once again, from the etymology dictionary I referred to earlier. Its author first posits that to know derives “from Proto-Germanic *knew-.” Then he stresses the following, which I believe is of interest to us as a matter of clarification:

Once widespread in Germanic, this form is now retained only in English, where however it has widespread application, covering meanings that require two or more verbs in other languages (such as German wissen, kennen, erkennen and in part können; French connaître, savoir; Latin novisse, cognoscere [...]. The Anglo-Saxons used two distinct words for this, witan (see wît) and cnawan.

In French, I shall use connaître—“\( x \) connaît \( o \)” or “\( x \) ne connaît pas \( o \)”— and, in Spanish, we shall likewise say “\( x \) conoce (a) \( o \)” and “\( x \) no conoce (a) \( o \)”.

5. Cognition and the cognitive

Let me now introduce another fundamental dimension of human societies: the dimension of the cognitive. Personal relations and institutional relations—or “positional” relations—are the stuff that the cognitive is made of. In the wake of this definition, one can define the cognitive universe \( U(x) \) of a person \( x \) or the cognitive universe \( U(p) \) of a position \( p \) in an institution \( I \), written \( U(x) = \{(o, R(x, o)) / R(x, o) \neq \emptyset \}\) and \( U(p) = \{(o, R(p, o)) / R(p, o) \neq \emptyset \}\).

Roughly speaking, the cognitive is the hugely extended set made up of what persons and institutions know about this or that object, while, as we know, the didactic is the substantial, more or less surreptitious set of processes by which persons and institutions have come to know this or that object the way they know it or could come to know this or that object in such or such a way. While the analysis of didactic processes belongs to didactic analysis, the analysis of cognitive universes pertains to cognitive analysis.

Didactics per se has no interest in the cognitive in itself. It is concerned with the processes by which the cognitive has been formed or could be formed or reformed. Its interests are thus twofold: didactics focuses on the way persons and institutions’ relations to objects have come to be what they are and on the way they could change to conform to predefined relations. Didactic analysis has therefore two components. One of these components is well known to all of us: it aims to examine if and how a given set of didactic acts can bring about a given change in the personal relations \( R(x, o) \) to an object \( o \) in persons \( x \) of a certain kind. This is forward didactic analysis.
Forward analysis has been at the heart of didactic research since the start. By contrast, the other component of didactic analysis, I mean, backward didactic analysis, remains much overlooked, although it has much to say to the didactician. In backward analysis, the key question is: Given an observed personal or institutional relation $R$, how is it that this relation is there, and why? How and why did it come to be? What exactly is its cognitive and didactic history? Why is it what it is, and not something else? This last question pertains to ecological analysis, which raises questions about why things are there or are not there. A famous example of such a question arises in the history of art with a doctoral dissertation devoted to explaining the absence of landscape paintings in... catacombs (Gombrich, 1966, p. 107).

Let me give two quick examples relating to the cognitive. Many people “know” the object $o = \sqrt{2}$ insofar as they know that this number satisfies the inequality $o > 0$ and the equality $o^2 = 2$. But they most often miss the following easy sequence of implications:

\[
\begin{align*}
\quad \quad o^3 &= 2 & \iff & o^3 - 1 &= 1 \\
\iff & (o - 1)(o^2 + o + 1) &= 1 \\
\iff & o &= 1 + \frac{1}{1 + o}
\end{align*}
\]

We thus arrive at a fixed-point equality—a “model” of $o$—useful to generate approximate values of $\sqrt{2}$. Starting from 1, for instance, we get the following sequence of approximations of $\sqrt{2}$: $1 \mapsto 1.5 \mapsto 1.4 \mapsto 1.41666667 \mapsto 1.4137931 \mapsto 1.41428571 \mapsto 1.41420118 \mapsto 1.41421569 \mapsto 1.4142132 \mapsto 1.41421362 \mapsto 1.41421355 \mapsto 1.41421356 \mapsto \ldots$ (An electronic calculator gives: $\sqrt{2} = 1.414213562373\ldots$) Now, how can we explain that such a simple mathematical fact remains unknown to so many people with a high school mathematics education? Conversely, if you meet someone who does know about it, what episodes in this person’s didactic biography can explain this seeming cognitive “anomaly”?

My second example refers to the English words teach and teacher. Most people involved in education around the world “know” them. More particularly, many Spanish-speaking people know that to teach is enseñar in Spanish. They all know, of course, that the Spanish verb enseñar also means to show. Many French-speaking people know that to show is montrer in French. Very few of them, at least today, know that the verb montrer—that is, to show—used to mean “to teach” too, as exemplified by this quote from the entry “Montrer” in the most celebrated dictionary of the French language, Émile Littré’s Dictionnaire de la langue française (1863-1877): “Enseigner. Montrer les langues, la grammaire, les mathématiques. Montrer à écrire.” Now it seems that few educationists around the world know what my favourite etymology dictionary divulges: teach comes from Old English tæcan [ˈtæːtʃən], which means “to show, point out, declare, demonstrate” and also “to give instruction, train, assign, direct; warn; persuade”. Consequently, in the late 13th century, the noun teacher was used in the sense of “index finger” or “pointer finger”. Therefore, in all three languages, a teacher is metonymised as one who shows, points to the object $o$ to be “learnt”. This, of course, should remind us that “pointing to things” is an immemorial, basic didactic act. The general question that arises here is: why do people know this and are ignorant of that? This question, that falls under backward didactic analysis, entails the question “How can this state of things change or be changed?”, which belongs to forward didactic analysis.
6. Becoming a didactician

We should now take stock of where we are and where we want to go. I will emphasise essentially one aspect of the model developed up to this point. As you can see, this model is in no way dependent on which area in the didactic continent a researcher in didactics decides to study. It should enable a didactician to study equally well—or equally badly—any area of the didactic continent: everywhere the researcher will have to conduct didactic analyses, and cognitive analyses—which, let me remind you, are means to an end. In the present state of the model provided by the ATD, all this boils down to using the notions of person, institution, object, relation and didactic act, which make up a rather restricted set of tools, uninfluenced by any “disciplinary” commitment. Wherever the didactician decides to settle down on the didactic continent, these tools should be made good use of.

In relation with this fact, I have created a kind of more or less fictional character whom I call the gyrovague didactician. The word gyrovague comes from Latin gyrovagus, formed from gyro- “circle” and vagus “wandering”. It was used to describe wandering or itinerant monks, who had no fixed residence. These non-sedentary monks were accused “of indulging their passions and cravings” (Gyrovagues, n.d.) and much was done to “discipline” them, but in vain; so that eventually the Council of Chalcedon (451) prohibited their practice. By contrast, the gyrovague didactician is regarded here as a “positive” figure, that allows us to raise relevant questions about the didactic continent, to help us escape automatic, unthoughtful behaviour.

The main question in this respect is: What does a didactician need to know in order to do research in a given area of the didactic continent? The usual answer is both unnecessary and insufficient. In metaphoric terms, this answer can be reduced to this: in order to know an area of the didactic continent in an appropriate way, it is necessary and almost sufficient to have been raised and educated in this area, to be, so to speak, an autochthonous inhabitant of the area, not someone from another part of the didactic continent or from outside this continent. Concretely speaking, a didactician who claims to do research in an area regarded as belonging to some academic discipline is expected to have been trained as a teacher in that discipline and, more exactly, as a secondary school teacher in that discipline. This is in my mind, let me repeat it, neither necessary nor sufficient. The problem thus raised is an old one, to which the ATD gives a new solution.

The tenet that a didactician must have been educated as a teacher in the discipline whose teaching and learning are considered is not corporatism pure and simple. It rests upon the false idea that, in order to know how some people live, one needs to be, or have been, one of them. This is a belief contrary to the scientific ethos, which claims that, to study birds, you don’t have to be a bird yourself, even if you become close to being one of them—here, I think of The Lecturer in Robert Altman’s movie, Brewster McCloud (1970). Therefore, the only realistic question is: what should a didactician do—and learn—to study the didactic specific to an area to which he or she is not native? To answer this question, we shall have to go deeper into the model of the didactic offered by the ATD.

7. The scale of conditions

To do so, we need complementary tools of analysis. The first of them is the notion of didactic system, that is, the coming together of a didactic stake \( \star \), a set \( X \) of persons \( x \) supposed
to study ♥, and a set Y of persons y supposed to help the x’s to study ♥. Such a didactic system is denoted by S(X; Y; ♥). As always in the ATD, this notion is very general. Of course you can observe didactic systems in classrooms, where, as a rule, Y is a singleton whose unique element is the teacher. But when a person x asks for help from another person y in using the newly arrived coffee-machine, provided y agrees to help x, there comes to life a short-lived but full-fledged didactic system S(x; y; ♥), where ♥ is the art of getting coffee from that damned coffee machine. When a student x tackles some homework ♥, we observe a didactic system that can be written as S(x; ∅; ♥) and is termed autodidactic. If x looks for help from his or her mother y, the new didactic system thus formed will be written S(x; y; ♥). Let me add that, here and now, we are wallowing in a didactic system formed around the didactic stake at the heart of our meeting, I mean the ATD, and of which you are the x’s and I am a transient y. And of course, when a researcher x or a research team X study a research question ♥, maybe under the supervision of some “head of research” y, they form a didactic system too.

A great number of didactic acts occur in didactic systems. But the didactic analysis of the functioning of such a system shows that the didactic acts carried out within it do not depend only on the system’s basic constituents, X, Y, and ♥. They are jointly determined by a host of conditions that didactic analysis must identify. The study of these conditions has led to a diagram called the *scale of levels of didactic codeterminacy*:

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Humankind   ↓↑
Civilisation ↓↑
Society    ↓↑
School     ↓↑
Pedagogy   ↓↑
Didactic system
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Each level is the place from where a number of conditions that may influence what goes on in didactic systems seem to proceed. Note that, save for the highest level, that of humankind, the name of each level should be understood in the plural: there are and there have been many didactic systems, many pedagogies, many schools, many societies, and many civilisations.

Let me add, too, that the double arrows between levels simply mean that the conditions at a given level may affect all the other levels (by transitivity). Moreover, a condition is called a *constraint* relative to a position p if a person in position p cannot as such delete or modify this condition. A condition created by a teacher is generally a constraint for this teacher’s students. A condition created by the school’s headmaster is a constraint for the school’s teachers. This applies to any institution, be it a family, a scientific conference, a health care institution, or what have you.

Starting from the level of didactic systems and going upward from bottom to top, we meet first of all with the level of *pedagogy*, which is the art of guiding the student x towards the didactic stake ♥, stopping short of guiding the student’s *study* of it. As always in the ATD,
pedagogy has a universal ring about it: in a given school, it consists of all the conditions—of
time and place, of didactic opportunities and cognitive comfort, etc.—that shape the way of the
student to the study of the didactic stake.

A pedagogy supposes the third level of the scale, that of the school. A school is essentially
what the Greek skhole, which originally meant leisure, evolved to mean: a place where it is
legitimate to turn away from the ordinary, immediate concerns of daily life to devote one’s time
to studious leisure or seemingly otiose study. Unless they are stillborn, all didactic systems
come to life in the womb of some school. For us here and now, the school is the 5th
international congress on the ATD. In the case of the coffee machine, it is the place where people work,
accepted to allow for such small interactions around the coffee machine. As for the homework
example, it is expected that the student’s family and home will of necessity function as a school
of its own, whether it be a “good school” or not.

The higher levels are those of society, civilisation, and humankind. They remind us that the
didactic observable in didactic systems does not occur in a social vacuum. Here is a first
example. Imagine a society in which most people take it for granted that some people have a
liking for study while the others have no such propensity, looked at as a deeply entrenched
personality trait. The active presence of this “theoretical” view will certainly affect the range of
pedagogic acts performed in any school in relation to any possible school subject. Imagine,
likewise, that some subject matter is thought of generally as “something for the few”, and that,
therefore, most people consider it normal to be repelled at the perspective of having to confront
it. (Here you can think of mathematics in a vast array of societies, ancient or contemporary.)
Imagine now a society in which the matter under study is seen as not needed because everyone
is seen as already equipped with the means to do without it. Here I think of the learning of
English in France and also of the learning of all foreign languages in English-speaking
countries—to say nothing of Spanish-speaking communities. This leads towards the notion of
civilisation.

Historically, “civilisation” is quite a difficult word. So let me invoke Humpty Dumpty to say
that a civilisation is a type of societies with similar (types of) institutions and cognitive
universes—relating for example to compulsory schooling, the praise of knowledge and study,
gender equality, equal treatment of rich and poor, indifference to differences, or their opposites.
In practice, most of us, I presume, are imbued with the idea that Spain and France and England,
or any country whatever, are different societies with at times very different ways of living. To a
large extent, however, these societies do belong in the same civilisation: many of their
institutions are very similar, and this is what the civilisation level signifies.

What I want to stress especially here is a “trait” that belongs to the highest level of the scale
of didactic codeterminacy, that of humankind. This largely-shared trait is our propensity to see
ourselves as different, a disposition that I call localism—a word meaning more generally
“preference for one’s own area or region.” Things are as they are and not otherwise because
they are “different” and, in truth, incomparable. It is highly likely that our localism is an
inheritance that has come down to us from the most remote past. If we are to believe the
anthropologist Claude Lévi-Strauss (1952), this attitude of human beings has indeed a history
that spans millennia:
This attitude of mind, which excludes “savages” (or any people one may choose to regard as savages) from human kind, is precisely the attitude most strikingly characteristic of those same savages. We know, in fact, that the concept of humanity as covering all forms of the human species, irrespective of race or civilization, came into being very late in history and is by no means widespread. Even where it seems strongest, there is no certainty — as recent history proves — that it is safe from the dangers of misunderstanding or retrogression. So far as great sections of the human species have been concerned, however, and for tens of thousands of years, there seems to have been no hint of any such idea. Humanity is confined to the borders of the tribe, the linguistic group, or even, in some instances, to the village, so that many so-called primitive peoples describe themselves as “the men” (or sometimes — though hardly more discreetly — as “the good”, “the excellent”, “the well-achieved”), thus implying that the other tribes, groups or villages have no part in the human virtues or even in human nature, but that their members are, at best, “bad”, “wicked”, “ground-monkeys”, or “lousy eggs”. (pp. 11-12)

How does this affect us? Essentially by enticing us to remain unswervingly “attached” to the local area of the didactic continent where we “grew up” as didacticians. It is not entirely irrelevant to note that, according to Harper (2011-2015) and Ayto (1990), the word “attach” reached English via Old French ataçier, an alteration of earlier Old French estachier, to fasten with a stake—estaque in Old French. Stake and estaque both come from a hypothetical Germanic word *stakon (whose cognates are Old Norse stiaki, Danish stage, Old Frisian stak, Middle Dutch stake, Dutch staak, German stake), also borrowed in Spanish and Catalán (estacá) and in Italian (stacca), and borrowed back in English as attach in the 18th century. Stakes can also be used to mark off territory. In what follows I will try to suggest that the localist stance can have dire consequences for the healthy development of didactics.

8. A general model and its localizations

Up to now, the model expounded is innocent of collusion with any particular area of the didactic continent. One point, however, must be emphasised: if its generality is unrestricted, so that it applies in principle to any part of the didactic continent, it involves many parameters that always take definite values when the model is put to work. Of course, this is particularly so with the didactic stake ♥. Although the model propounded so far is always used in specific situations and contexts, it is a general model whose scope extends to the entire didactic continent. This notwithstanding, its full reception is hampered by the deep-rooted localist attitude I already mentioned.

A number of factors contribute to keep a tight rein on the development of didactics. Paradoxically, one such factor derives from what is considered central to science since its inception, if I may say so: the existence of an integrated and active community of researchers, which serves as what I will call an “archschool” (like archangel, archpriest, archenemy, etc.), that is, a school of schools, under which, as a rule, all schools in the community are subsumed. In his Dictionary of Word Origins, Ayto writes: “The Greek prefix arkh- was based on the noun archos ‘chief, ruler’, a derivative of the verb arkhin ‘begin, rule’…” Then this author appropriately notes that “its use has gradually extended from ‘highest in status’ and ‘first of its kind’ to the ultimate—and usually the worst—of its kind’ as in archtraitor.” As the Latin phrase goes, Corruptio optimi pessima, the corruption of the best is the worst. When the community of
researchers working in a given area of the didactic continent becomes too limited, because the area of specialisation being researched has been vigilantly fenced off from the rest of the didactic continent, then the scientific debate in this community declines as the dialectic of media and milieus (in the ATD’s vocabulary) tends to go round and round in circles, while a more or less undisputable orthodoxy is forced upon the community.

Living in a narrowed community may lead its members to mistake the picture of the world built-up by this community for the world itself. It is precisely what seems to happen all too often on the didactic continent, where some didacticians are prone to equate the didactic with what is said of it by “their” didactics, in their journals and seminars. The “areal” community then becomes a quasi-closed system living on its own, with little renewed interaction with the didactic. Moreover, the dividing up of the field of didactics into an indefinite number of “disciplinary” subfields (didactics of mathematics, of physics, of history, of English, of French, etc.) is pushed further on to take into account changing conditions. In this respect, the differentiation based on subject matter is but a particular case of a general tendency to distinguish between as many subfields of didactics, allegedly capable of existing on their own, as there are significantly different sub-areas of the didactic continent. By contrast, the solution advanced by the ATD is much more in keeping with the lex parsimoniae, the “law of parsimony” dear to William of Ockham (c. 1287–1347): researchers in didactics who tackle this or that area of the didactic continent must take into account all the conditions that are reasonably suspected to weigh significantly on the didactic under analysis.

Allow me to give a quick example relating to the teaching of French. There are certainly differences between teaching French in France to native French speakers or to students newly arrived in France. And both cases are different from teaching French to U.S.-born students who live permanently in the U.S. or to Canadian students learning French in a minority situation. A localist didactician may therefore want to distinguish between many didactics of the French language, each of them “very different” from the others. On the contrary, the unperturbed didactician, who considers the didactic continent as a whole, will try to identify the main conditions that determine what the didactic is, in each and every case.

My example is about teaching French in France and teaching French abroad, for example in the U.S. In the former case, it is felt there is little need to teach the “right” pronunciation of French words, simply because the students live in a French-speaking environment. In the latter case, it is the teacher’s duty to show them how to pronounce even basic words. The situation is very different because the teacher has to decide which pronunciation is right, an ordeal that the teacher of French in France generally avoids. Of course there are traditional rules, but most French people decidedly ignore them. In an online course of French for English-speaking people, the Illinois-born author Laura K. Lawless (2016) introduces the traditional rule that governs the pronunciation of ai and ais. In je t’aime (I love you), je serais (I would be), je donnais (I was giving), anglais (English), frais (fresh, cool), and lait (milk), the letter combination ai is pronounced like è (as in “bed”). In j’ai (I have), je serai (I will be), and je donnai (I gave), ai is pronounced like é (as in fiancée). Be that as it may, almost all the French persons I know fail to respect this rule, or even have never heard of it! However, when analysing the teaching of the French language, the unperplexed didactician will simply note that, in the case of teaching French “abroad”, the teacher is compelled by a constraint that can
be almost completely ignored on the French soil—a fact that can have far-reaching consequences, to the extent of changing the contents taught.

What should we do then? Let me invite you to engage in a thought experiment which, in my view, is the founding experiment of the didactician’s condition. Imagine a didactician, denoted by the Greek letter $\xi$ who wants to study and research some area $\mathfrak{d}$ of the didactic continent. Let us simplify a bit the experiment by assuming that all didactic stakes $\heartsuit$ fall under one and the same academic discipline $\mathfrak{d}$. Now suppose that $\xi$ knows very little about the area $\mathfrak{d}$. This means in particular that $\xi$ hardly knows $\mathfrak{d}$, which may be mathematics, or French, or English, or Spanish, or carpentry, for instance. Remember that the goal of $\xi$ is not to become a teacher of $\mathfrak{d}$—although, at some point in time, $\xi$ can choose, for scientific or personal reasons, to get hired as a teacher of $\mathfrak{d}$. What will $\xi$ learn as the experiment proceeds?

This would-be didactician of $\mathfrak{a}$ will sooner or later discover that, from the point of view of research, and more precisely of the didactics of $\mathfrak{a}$, no didactic area and in particular no subject matter $\mathfrak{d}$ can be mastered “perfectly”, even in the long run, because, as the research develops, it encompasses new aspects of $\mathfrak{a}$ and $\mathfrak{d}$ relevant to the didactician. Being aware of this crude fact is central to research, whereas “pure” teachers can safely ignore it. Mark Twain (1884/1979) once judiciously remarked, “We could use up two Eternities in learning all that is to be learned about our own world and the thousands of nations that have arisen and flourished and vanished from it.” He added, self-deprecatingly, “mathematics alone would occupy me eight million years”.

It is my belief that, to stay in good health both cognitively and didactically, a didactician should now and again consent to study, possibly as part of the research being conducted, some field of knowledge $\mathfrak{d}$ until then personally unexplored. Such a repeated experience seems necessary to get rid of, or at least to protect oneself against, the illusion of mastery and the illusion of transparency that often affect teachers’ self-image. If this is achieved in order to become capable of studying the teaching of some subject matter $\mathfrak{d}$ previously unknown to the researcher, or even to some research team, then the didactician will discover that, quite soon, some relevant research results will follow, provided one adapts the questions chosen for study to the “cognitive equipment” progressively made available. Suppose for example you never studied Latin and, however, decide to start studying the teaching of Latin to beginners—which implies that you start studying Latin at about the same pace. One can imagine that your research will rapidly elicit some unexpected conclusions about how, and to what extent, ordinary didactic acts, performed either by the teacher or by the students, are transformed to adapt to the specificities of Latin.

When one looks obliquely at the teaching of an academic discipline $\mathfrak{d}$ that has been centre stage for quite a long time, the impression prevails that the “noosphere” around it has developed into a terrific fortress and citadel, through the accumulation of an awe-inspiring body of knowledge about $\mathfrak{d}$ and its teaching. However such a plethora of knowledge and doctrines is its own enemy because its content obsolesces, it seems, at an ever-increasing pace and falls rapidly into oblivion. In fact, any school system seems to be a system with little or no memory, a kind of markovian system that continually erases its past. The first consequence of this for the “incipient” didactician is that the reality one has to cope with is less impressive than one imagines. Paradoxically, it will be the task of the didactician to restore at least part of the
forgotten past whenever this can shed light on the present and the future of the didactic continent.

Can we be equal to the didactician’s task? The relative mastery of the subject matter is only one requirement among many. In all cases, when trying to didactically analyse a situation or institution, we have to pinpoint the conditions of all levels that are constraints for some actors of the situation or institution and to identify the conditions that can be “freely” created by the same token. In such a quest for relevant conditions and constraints, we shall rely, up to a point, on the work done by “experts” on the distinct levels (of pedagogy, school, society, etc.) that make up the scale of didactic codeterminacy. Generally, however, the questions raised by didactic analysis (about pedagogies, schools, societies, etc.) have not been taken into account by “licensed” experts because they are out of scope of their specialty. As a consequence, we can rarely find ready-made answers that would pleasantly await us. Therefore, didactics research legitimately encompasses all the levels of the scale, provided the questions studied stem from didactic analysis. Of course this cannot be achieved by a single person: this is where the notion of a “collective intellectual” may be useful to renew the idea of a “scientific community”. Because we are, here and now, such a collective intellectual, I will not pursue this point here.

9. The theory of praxeologies

Something is missing in the model presented until now: the key notion of a praxeology. Let me first stress—the following remark applies essentially to French-speaking didacticians—that I speak of a praxeology, not of “praxeology”, defined as the alleged “science of human action and conduct” as fantasised by some nineteenth-century authors. As you all know, a praxeology is formally defined as the system, more or less integrated, formed by four components: a type of tasks $T$; a technique, denoted by the Greek letter $\tau$ (tau), which is a way of performing tasks of type $T$; a technology, denoted by the Greek (small) letter $\theta$ (theta), which is a “rational discourse” on the technique $\tau$, that purports to justify it, to legitimate it, and to make it intelligible; last but not least, a theory, denoted by the Greek (capital) letter $\Theta$ (theta), that help generate, legitimate and justify the technology $\theta$. A praxeology $\mathfrak{p}$ (“Weierstrass p”) can therefore be written as a 4-tuple: $\mathfrak{p} = [T / \tau / \theta / \Theta]$. The name “praxeology” as used in the ATD reminds us that a praxeology is the union of a praxis part $\Pi = [T / \tau]$ and a logos part $\Lambda = [\theta / \Theta]$, which we can write as follows: $\mathfrak{p} = [T / \tau / \theta / \Theta] = [T / \tau] \oplus [\theta / \Theta] = \Pi \oplus \Lambda$.

A core tenet of the ATD is that all human actions can be modelled as a sequence of tasks $t_1, t_2, ..., t_n$ of types $T_1, T_2, ..., T_n$, performed thanks to a sequence of corresponding praxeologies $\mathfrak{p}_1, \mathfrak{p}_2, ..., \mathfrak{p}_n$. Historically, the notion of praxeology in the ATD was the answer to this question: Where does a personal relation to an object $o$ arise from? The answer is: this relation results from the use of the object $o$ in all the praxeologies involving $o$ in one way or another that the person has had to deal with. Once again, the same applies to institutional, that is positional, relations to objects. It is to be emphasised that whenever anyone does anything, a praxeology, not only a technique, is drawn up upon. In other words the person resorts not only to a technique, but also to a technology and a theory, which praxeological analysis has to make clear.

It is an obvious consequence of the praxeological model that the ATD remains untouched by the distinctive features of any subject matter whatsoever. You need a praxeology to solve a quadratic equation, to brush your teeth, to write a sonnet, to welcome a friend, to read a
newspaper, to sing a song, to make an omelette, to hammer a nail, to prepare tea, to call somebody names, etc. Didactic stakes ♥ are either praxeological complexes or parts of them: they can be “ingredients” of mathematical praxeologies, of writing praxeologies (in this or that language), of computer programming praxeologies, of driving praxeologies, of cooking praxeologies, etc. Each of these “praxeological entities” is called a work—in French une œuvre, in Spanish, una obra. Note that I speak of a work, not necessarily a “work of art” or a work of literature. A work can be a “work of mathematics”, a “work of carpentry”, etc. More generally, a work is any reality created by human beings with a view to achieving some praxeological function. Consequently, there is no reason to consider it as “noble”, “praiseworthy”, or “commendable”. For example, the work consisting in the well-known technique for blowing one’s nose on a football field—not in a classroom—may be found uncouth, although it is useful to the unabashed football player—works are always useful, at least in some situations. A didactic stake ♥ is a work W. The didactic system S(X; Y; ♥) can therefore be written as S(X; Y; W).

The study of a praxeological entity entails didactic praxeologies, which implement the “didactic acts” I mentioned earlier. Likewise, what is usually called “methodology” in science is conceptualised in the ATD as the study of the research praxeologies on which you may draw when you “do research”. Praxeological analysis, which is a hard part of the ATD, therefore applies most notably 1) to didactic stakes, 2) to didactic organisations, and 3) to research methods. It is through praxeological analysis that the peculiarities of any praxeological entity whatsoever are revealed. Without it, didactic analysis essentially boils down to pedagogical analysis. (which has its own relevance).

10. The Herbartian schema

At the present stage of its development, the ATD affords a model of what the study of a work W can consist of. In this perspective, a crucial category of works is that of questions. A question Q is indeed a work—created purposefully by human beings. Harper (2011-2015) reminds us of the following history of the word “question”:

early 13c., “philosophical or theological problem;” early 14c. as “utterance meant to elicit an answer or discussion,” also as “a difficulty, a doubt,” from Anglo-French question, Old French question “question, difficulty, problem; legal inquest, interrogation, torture,” from Latin quaest ionem (nominative quaesitio) “a seeking, a questioning, inquiry, examining, judicial investigation,” noun of action from past participle stem of quae rere “ask, seek” (see query (v.)).

The praxeological function that a question assumes is to trigger or rekindle an inquiry into something. How can we describe what happens when a student x or a class X studies a question Q under the supervision of Y? Or when a researcher ξ or a research team Ξ studies a question Q, possibly under the supervision of a head of research ζ (zeta) or a collective of supervisors Z?

The model introduced so far is first enriched with the following formal description called the reduced Herbartian schema: S(X; Y; Q) → A. (Beware! Here, the adjective herbartian, which refers to the German philosopher and pedagogue Johann Friedrich Herbart (1776-1841), is something of a misnomer.) Here, A is the answer to the question Q that the didactic system (or the research system) is expected to produce. It is usual to write the answer A with a heart ♥ in superscript: S(X; Y; Q) → A♥, a gentle reminder of the fact that, henceforth, this answer will be
“at the heart” of the didactic system, of which it will allegedly be—at least for some time—the “authorised” answer to question \( Q \). It is not entirely irrelevant to learn a bit more about the word \textit{answer} used here (Harper, 2011-2015):

Old English \textit{andswar} “an answer, a reply,” from \textit{and-} “against” (see \textit{ante}) + -\textit{svaru} “affirmation,” from \textit{swerian} “to swear” (see \textit{sware}), suggesting an original sense of “make a sworn statement rebutting a charge.” A common Germanic compound (cognates: Old Saxon \textit{antswor}, Old Norse \textit{ansvar}, Old Frisian \textit{ondser}, Danish and Swedish \textit{ansvar}), implying a Proto-Germanic *\textit{andswara}-. Meaning “a reply to a question,” the main modern sense, was present in Old English.

Meaning “solution of a problem” is from c. 1300.

Of course, an answer should be thought of as a work, that is, a praxeological entity, and not as a mere sentence.

The next step in building up our model is the introduction of the didactic or research milieu, \( M \), which is the (fuzzy) set of material and immaterial tools that the students or the researchers gather in order to carry out their inquiry into question \( Q \). The reduced Herbartian schema then becomes the \textit{semi-developed} Herbartian schema: \[ S(X; Y; Q) \hookrightarrow M \hookrightarrow A^\circ. \] Here, in a more or less disorderly fashion, the didactic system is seen to create the milieu \( M \) and to produce the answer \( A^\circ \) by drawing upon the milieu \( M \). In the quest for an answer \( A \) to the question \( Q \), three main components stick out. The first is the search—in the literature and, in particular, on the Internet—for existing answers offered by other persons or institutions. Such answers are usually denoted by \( A^\circ \), where the letter \( A \) is followed by a small rhombus, which can be read “a diamond”—the rhombus being regarded generically as denoting the “hallmark” of some institution or person. A teacher (in direct instruction) or a textbook or a webpage are thus institutions that, de facto, “hallmark” their answers to the questions they tackle. At his stage, the milieu \( M \) is therefore to be written thus: \( M = \{ A_1^\circ, A_2^\circ, \ldots, A_m^\circ, \ldots \} \).

To draw upon the answers \( A_i^\circ \) (1 \( \leq \) \( i \) \( \leq \) \( m \)), the didactic system has recourse to works of various kinds, like theories, experiments, historiographical narratives, etc. Therefore the milieu is now to be written: \( M = \{ A_1, A_2, \ldots, A_m, W_{m+1}, W_{m+2}, \ldots, W_n, \ldots \} \). To use these works, the student as well as the researcher needs to study them. What does it mean to study a work \( W \) which is not itself a question? Such a study consists in studying a number of \textit{questions} \( Q \) about the work under study. Thus the study of any work boils down to the study of \textit{questions}. The set of questions \( Q \) depends on the inquiry being conducted: as a general rule, they may differ according to the generating question \( Q \) and the way the inquiry into it proceeds. Much more generally, every single item in the milieu is bound to raise questions that, up to a point, the didactic system will have to study. So that the milieu \( M \) takes on the following appearance: \( M = \{ A_1, A_2, \ldots, A_m, W_{m+1}, W_{m+2}, \ldots, W_n, Q_{m+1}, Q_{m+2}, \ldots, Q_n \} \).

11. A remark on methodology

Among the works a didactic system may rely on to arrive at the answer hoped for, there are works which are crucial to empirical research: \textit{collections of data} of various kinds, which I shall denote by the letter \( D \). The milieu can now be written thus: \( M = \{ A_1^\circ, A_2^\circ, \ldots, A_m^\circ, W_{m+1}, W_{m+2}, \ldots, W_n, Q_{m+1}, Q_{m+2}, \ldots, Q_n, D_{p+1}, D_{p+2}, \ldots, D_q \} \). Of course the questions \( Q_k \) (\( n + 1 \leq k \leq p \)) do not refer only to the answers \( A_i \) and the works \( W \); they may as well refer to the data collections \( D_l \) (\( p + 1 \leq l \leq q \)). In this respect, I wish I could have given a broader presentation of the model. As time

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escapes me, with the minutes getting shorter and shorter, I shall restrict my account to two, related, issues.

What is usually called “experiments” in the methodological literature is regarded in the ATD as particular cases of what I call—with no relation to medicine—clinical observation, in which the observer “leans over” to examine critically anything of interest that can be observed (the Greek verb klinein, from which “clinical” and “lean” derive, meant “to cause to slope, slant, incline”). More precisely, I shall distinguish three main kinds of data collection. The first follows the principle of the Paleolithic hunter-gatherer, in which case you content yo

In practice, the didactician will often be tempted to explore the scale of didactic codeterminacy not higher than the level of pedagogy, and to ignore more or less completely the highest levels. The consequences of such a timorous attitude are manifold. One of them is that students and researchers tend to forget that, as a rule, there is no free and easy access to the didactic and its determining conditions.

The difficulty is not so much to get access to at least one favourable position. For example, many didacticians are teachers and so have the possibility to observe other teachers’ classrooms, so to speak by contiguity. Similar arrangements can be drawn upon by “ordinary” students. The main problem is that the use of only one observation position leaves a dead space, where significant but unseen events may occur. While it may be the fantasy of some teachers—and, by continuity, of some didacticians—to “see” all that can occur in a classroom, we should remember that no observation position, taken alone, is really “panoptic”.

There exist a classical argument raised against the collecting of data according to the “hunter-gatherer mores”. Here is a typical plea against “desultory collecting” (Whitley & Kite, 2013):

In laboratory study, researchers have complete control over the situations in which they collect their data. Field researchers can choose their research settings and sometimes can control the specific circumstances under which they collect their data. Internet researchers, however, have
very little control over the data collection environment (Reips, 2000). Internet participants may be at home, at work, at school, or in a public library; they may be alone or with others; they may be in a tranquil setting or one replete with distractions. From a technological perspective, variations in the computer hardware and software used by research participants can cause variations in the fidelity of graphics, color, and sound reproduction. Users may need plug-ins to view animated graphics or sound and, even if these can be downloaded for free, users may be unwilling or unable to do so (Fraley, 2007; Plous, 2000). All these factors can affect the accuracy of the data collected... (p. 509)

In truth, all data collecting techniques raise one and the same problem: from the point of view of the ATD, we are never sure to identify the relevant conditions and constraints (bearing on the researcher, the “research participants”, the witnesses, etc.) under which the responses are elicited and reported. This would be the starting point for a more detailed presentation of what the ATD offers in terms of methodology or research praxeologies.

12. A conclusive note

Let me conclude by emphasising that, besides trying to unite the didactic continent, the ATD sets forth a unitary approach to the student’s as well as the researcher’s activity. The sketch of a model presented here must be seen as paving the way for a didactic theory of inquiry, that is to say a didactic theory of study and research, on which many of you are currently working. Allow me to remind you that parts of this theory already exist, even if some of them seem to be more or less neglected, such as the model of didactic moments, which is a tool for questioning the study of any possible work. But this is enough for me today. Thank you all for being here and doing the work you do!

References


