

About affect and meta-affect in mathematics education: an interview with Gerald A. Goldin

Sobre afeto e meta-afeto na educação matemática: uma entrevista com Gerald A.
Goldin

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Abstract

Researcher Gerald A. Goldin presents a trajectory in mathematics education dating back to the early 1970s. His works position him as an influential researcher, contributing to investigations in regard to systems of representation and affect related to the teaching and learning of mathematics. Our primary interest in having a dialogue with Goldin was to record his considerations about the Affective Domain, with an emphasis on meta-affect, as well as to gather his views on current and future directions for research in the field.

Keywords: *Affective Domain, meta-affect, discrete mathematics, affective architecture, conation.*

Resumo

O pesquisador Gerald A. Goldin apresenta uma trajetória na educação matemática que remonta aos primeiros anos da década de 1970. Seus trabalhos o colocam como um pesquisador influente, contribuindo com investigações sobre sistemas de representação e sobre afetos no ensino e aprendizagem de matemática. Nosso interesse principal no diálogo com Goldin foi o de registrar suas considerações sobre o Domínio Afetivo, com ênfase no meta-afeto, bem como coletar suas opiniões sobre o atual e futuro caminhos para as pesquisas na área.

Palavras-chave: *Domínio Afetivo, meta-afeto, matemática discreta, arquitetura afetiva, conação.*

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Introduction

Gerald A. Goldin has been active in Mathematics Education for almost 50 years and has been an influential name in this field, impacting generations of new researchers and research groups around the world, especially with his works discussing concepts about representation systems and affect in teaching and learning math.

His career in the field began in the early 1970s, after obtaining his Ph.D. and postdoctoral research in theoretical quantum physics. The initial investigations he undertook sought to elucidate two main sets of ideas: the use of artificial intelligence (AI) research techniques, aimed at characterizing the structure of mathematical problems and the behavior related to their resolution, and the influence, in behavior, of the characteristics of the tasks related to problem solving.

In the 1980s, Goldin went deeper into the study and analysis of representations and systems of representation and how they might apply to mathematics and mathematical cognition (GOLDIN, 1987, 1988). In the following decades, with Valerie DeBellis, at first, he not only expanded Doug McLeod's (1989) initial ideas about Affective Domain, incorporating values as a component of that domain, but brought the notion of meta-affect to the field (DEBELLIS, 1996, DEBELIS; GOLDIN, 1997).

From the 2000s onwards, in addition to expanding his research and publications on these themes, he also began to develop the concept of affective structure (GOLDIN 2000, 2002; DEBELLIS and GOLDIN, 2006) and the relationship of affect to discrete mathematics (GOLDIN, 2004, 2010, 2018), published about the structure of beliefs (GOLDIN, 2002; GOLDIN; RÖSKEN; TÖRNER, 2009; GOLDIN *et al.*, 2011) and, more recently, has taken up the theme of the conation in mathematical engagement (GOLDIN, 2019), without failing to publish in the field of physics and astronomy. In 2010, the Mediterranean Journal for Research in Mathematics Education published an entire number as a tribute to their work, highlighting and reinforcing the enormous respect that the researcher and his production have in academia (GAGATISIS, 2010).

Because of his importance in the field of affect in mathematics education, we are motivated to interview him by having a conversation in a room on the Rutgers University's New Brunswick campus (New Jersey, USA) where the researcher is a Distinguished Professor, member of the Faculty of Graduate Studies in Mathematics, Faculty of Graduate Studies in Physics and Faculty of Graduate Studies in Education, associate of the Center for Mathematics, Science and Education in Informatics and

permanent member of the Center for Discrete Mathematics and Science of Theoretical Computing (DIMACS).

This text presents the full interview held in the late afternoon of May 2019. Our main interest in the dialogue with Gerald Goldin was to record his considerations on affect in mathematics education, with an emphasis on meta-affect, a variable still little studied, as well as to gather his views on the current and future paths for research in the area. We believe that the publication of this interview also in Portuguese can contribute to disseminate knowledge about the Affective Domain in Mathematical Education in Lusophone countries.

Interview with Gerald A. Goldin

***Interviewer:** Professor Goldin, on behalf of the Post-Graduate Studies in Mathematics Education, of the Pontifical Catholic University of São Paulo, would like to thank you for the kindness to agree to this interview.*

***Goldin:** It's a pleasure.*

***Interviewer:** Your trajectory began with a distinguished undergraduate degree in Chemistry and Physics at Harvard University, followed by a PhD in Theoretical Physics at Princeton. At one point, you went through mathematics education. How did these universes begin to talk and what sparked your interest in mathematics education?*

***Goldin:** Okay. So, it's actually a complicated history. I received my PhD from Princeton in theoretical physics. I finished my work in 1968 although, officially, my degree was awarded in January 1969. Nevertheless in September 1968 I began a postdoctoral position at the University of Pennsylvania in physics. I was doing work in theoretical physics. At this time (the year was 1968), there was a lot of social activism and concern. In particular, the war in Vietnam was going on and it was affecting people I knew. It was, for me, something very wrong that my country was doing. I was involved in the protest movement. This protest movement had followed very shortly after the major civil rights movement to end segregation in the American South. So it was a time of social activism. In my work, I began to feel that what I was doing was becoming very specialized, and I was becoming an expert in a very narrow field that was isolated from the social issues of the time. In addition, a lot of the funding for physics research was coming from the government, and much of this was motivated by wanting to develop weapons, especially atomic weapons. I didn't want to participate in this; but many people I knew, who were in physics, were more favorably disposed toward it because it was the source of funding for research in physics.*

In 1968, there was also a change in outlook. There was at that time, before 1968, a great demand for new PhDs in mathematics and physics. They were in demand to enter top universities. But there had been a wave of people, in my generation, talented kids, who were encouraged to go into the sciences. This was, in fact, because of the Cold War and the rivalry with the Soviet Union and the arms race. By the time 1968 arrived, there were many more people applying for positions than

there were openings. So, I was very lucky to get the postdoctoral position that I had, because people in my class, even from top universities like Princeton, with excellent credentials, were having trouble finding jobs in the academic world.

I had this postdoctoral position but it was just for two years, and then the question was what I should do at that point. I had an offer of a tenure-track academic position in Cleveland, Ohio, but by then I was feeling I should somehow broaden out intellectually and professionally. I had the opportunity to take a fellowship to study the learning of mathematics, and I did.

I had always been very interested in learning and education. My parents were both teachers. My father was a mathematics teacher in high school, and my mother taught English in high school. And so, I grew up in an “education family”. I had the feeling that to study the learning of mathematics would really enable me to build on what I understood in the sciences because, I was skilled in mathematics. There were also things I wanted to understand in cognitive psychology.

At that point in my life, much of my undergraduate work became very relevant. At Harvard, although I was a major in chemistry and physics, I received a liberal arts degree, bachelor of arts in these fields. That meant I had studied some psychology, had studied some philosophy, some epistemology, some considerable English literature and art and even music. I mean, I had a broad education. And when I entered the field of mathematics education, all of a sudden these other studies, which had always fascinated me and which were mostly irrelevant to a physicist, became very relevant to me in education. So, I thought this was a great thing to do. So, I took this fellowship, I studied on fellowship for a year and then I was offered a position on the faculty at the University of Pennsylvania in the Graduate School of Education to teach in mathematics education. And that's how I entered the field. But I never stopped doing scientific work. My whole life I've done both. This led to a lot of unexpected difficulties which is, perhaps, not any longer answering your question. That is, there was a consequence. It was not easy to do both mathematical physics and education, and it was very difficult to make the transition.

Interviewer: *So, how did these interests contribute to the construction of your first researches in mathematics education?*

Goldin: *Because I came from physics, I was at first very focused on the question of measurement. How do we measure things? How can we... if we want to understand learning, how can we measure learning? If we want to understand problem solving, how can we measure problem solving processes? My first interest was in the psychology of problem solving because, for me, this was what I did as a mathematician and physicist to solve problems, but I had not thought much about its psychology. How did I solve the problems? I just did this the best I could, and solved the problems.*

But once I became interested in the psychology of problem solving, trying to make explicit the processes involved in problem solving, I realized that not only would that be interesting to mathematics education but also, maybe, I would become a better problem solver in physics.

So that contributed to my research focus on problem solving.

I focused on models for problem solving. I was very intrigued by the efforts to create artificial intelligence models for human problem solving, to understand problem solving as taking place in a “problem space”, to bring abstract ideas, structural ideas (because, in mathematics, structures are so fundamental) to the study of the psychology of problem solving. And again, I found a lot of difficulties in trying to

map ideas and perspectives and values from the sciences and mathematics to the work in education.

Interviewer: *The research on affect for a long time was linked to the use of questionnaires and quantitative methods and also related to the study of attitudes. Now, from the point of view of methodology, could you say where the research on affect is going?*

Goldin: *Well, there are really two questions here: where is it going and where should it be going. These may not be the same question. A big investment has been made in the use of questionnaires to measure attitudes, to measure beliefs, to measure even the tendency to have certain emotional feelings in connection with mathematics, or in connection with mathematical success or failure, in connection with testing and so on. So I think that that approach is going to continue for some time.*

But we learn different things from the qualitative, research from a in-depth close and look at what students experience emotionally as they solve problems, what their desires are as they are solving problems, and how these change from moment to moment, and how they are influenced by the teacher, and how they are influenced by the success or failure in the problem, and especially how they are influenced by interactions with others in group problem solving, for example. We find that the phenomena are much more complex than can be captured with survey questions. So, I think that this qualitative research has added a dimension to the study of affect which is much needed to be able to look at the complexity.

For me, there are two ingredients to this complexity which are not well captured by the survey research. One ingredient has to do with the representational nature of affect. Psychologically, the emotions that we feel encode information and have meaning to us. And that information, that meaning, is not simply emotional. That meaning may be cognitive, may have to do with the situation and may have to do with how the person is in relation to the situation. So, that dimension, that semantic, if I can use that term, semantic aspect of emotions is very important.

A survey, often, asks about emotional feelings, but doesn't necessarily capture what information is encoded in those feelings. That information is different in different situations.

For example, if I am feeling angry, I could be angry because I have to work in mathematics when I prefer to be going to the movie. I may be angry because I made some careless mistake in mathematics and I'm blaming myself because I should have gotten it right and I didn't check my work. I may be angry at the teacher because the teacher doesn't appreciate what my ability is or the teacher thinks I'm not so good, but I think I'm much better than that. I may be angry with my father because he thinks I should be working harder than I'm actually working. I may be angry because I lost \$5 from my wallet this morning and I don't know what happened to it. It had nothing to do with the mathematics I'm doing, but I'm doing the mathematics in an angry mood. What the anger is encoding for me may be very different in all those situations. So, if you are asking me in the survey "do you feel angry when you do mathematics?", I remember, yeah, I did feel angry last week when I did mathematics, so I say, yes. I don't see that this is getting at the complexity of the meaning of the emotion.

The other dimension is what I call meta-affect. I don't think that's a new idea, the idea that emotions about emotions transform the experience of emotion. In literature, we see this all the time. We have a feeling and then we have a feeling

about the feeling. An example from a creative writing workshop I participate in was the expression, "He looked at her through stupid tears". Stupid tears. Tears refer to sadness. Stupid tears refer to an emotion about the sadness. I'm angry with myself for having these tears. I have evaluated that I'm stupid for feeling sad, but I feel sad anyway, I can't help it, I'm frustrated with myself. All of this complexity of emotion just in the phrase "stupid tears". It is meta-affective. It's not enough to say, okay, which of these emotions did you feel at that moment? Sadness, anger, you know, I have to choose. You know, I was angry about the sadness. In another situation, I might feel joyful about that anger. I might say, yeah, I'm angry and at last I have the right to be angry because I never let myself express this anger before and now, I am furious at this. Boy, oh, I feel good.

So, yes, we classify anger on a scale as a negative emotion, highly negative, but in this context, it is not highly negative, it is highly positive. Why is it highly positive? Because the emotions about the anger are emotions of joy. The anger is experienced joyfully. So, in every emotion, we can have situations in which the emotions about the emotions transform the emotions.

In what I wrote, I think you read it, I spoke about enjoying the fear of a ride on a roller coaster. The experience to fear joyfully. In mathematics and in problem solving, a pivotal emotion is the emotion of frustration. Frustration is typically seen as a negative emotion, but in problem solving the very definition of a problem involves some moment of impasse, of being unable to reach the goal that the problem has posed and the more we want to reach the goal the more frustrating it is that we're unable to reach it. But if we knew how to reach it, it wouldn't be a problem. Then we would be simply doing a routine exercise and not a mathematical problem. So, the very fact of doing mathematics and solving problems means experiencing this emotion of frustration. The question is not how to avoid frustration or whether is good to have frustration. The question is how do we feel about the frustration, do we experience the frustration positively or negatively. This is a meta-affective question. For me, these two things: (1) meta-affect and (2) the meanings of the affect, the representations, are aspects that we get to when we look at the in-depth qualitative research. It is very difficult even to design a survey that can get to them.

Interviewer: How do you analyze the issue of educational research results taking many years to reach teacher education and therefore the classroom?

Goldin: I think that the biggest obstacle has to do with the ways in which educational policies are made. We have a situation today (it hasn't always been quite so bad), but in the United States, at least, the situation is that policymakers are very insulated from educational research. Not because they have to be, but because they want to be. The policies are being determined based on political goals and the educational research is then... cherry-picked. That is, "look for the research that supports my political call". And the funding is granted to do research that supports the political goals. So, it's always easy to find people who say they have this political objective. The research is not really used. It's only used as a kind of... what should I say?... justification for something that policymakers want to do in any case. The research that seems to contradict the value of what they want to do, they prefer not to know about.

In teacher education, there are many pressures that are faced by teacher education programs and these pressures have to do with, again, political goals. So, in New Jersey, there are political goals that have led to certain testing processes for new

teachers. They have to submit portfolios, lessons in video, and so forth. So, the teacher education programs have to reorient themselves to accommodate such demands. This is, more or less, the opposite of what the research says would be best teacher education. Is it the best teacher education to train teachers how to make a nice video of themselves, or is it the best teacher education to give teachers the ability to teach really well? They're different things. The research does not say that making nice videos of themselves is a really important skill, but the policymakers impose it. Then the teacher education program doesn't have the room to bring in what the research says would be best.

For decades, all recommendations of research have said that expertise in mathematics is very important to be a good mathematics teacher, at the level of kindergarten through 8th grade. And there have been recommendations, based on research, that teachers who are certified to teach elementary school, which typically in the U.S means having the ability to teach up to 8th grade, should have a foundation of four courses in mathematics with specific content that is spelled out that is appropriate for this grade level range. Almost nowhere, in the United States, do such requirements exist. It doesn't exist at Rutgers. At Rutgers, what it required for is, approximately, one course, not four, and even the one is a bit questionable. After all, since I'm involved with teacher education at Rutgers, why can't I say we should implement the findings of research? And this is not new research. This is research that is 50 years old and has never changed in its conclusion. Unfortunately, policymakers demand other priorities of the program and there is no room in the program to meet such a recommendation. So, again, the policies are in the way of using good research to improve the education of teachers. If I wanted to have a course in affect and motivation and learning mathematics, I can't do it because there isn't room in the program and there is no mandated policy that says I should have it. So, it doesn't exist.

Interviewer: Beliefs are a component of the Affective Domain and are no longer a "hidden variable" in mathematics education research, is that right? And the other components of affect, how do you think they are considered in research in mathematics education?

Goldin: I do agree that beliefs are no longer a hidden variable... there's a book with that title. So, certainly, the idea of paying attention to beliefs is very important, is quite accepted today. I think that attitude is one of the components and emotion is one of the components and, in the paper I wrote with my former student Valerie DeBellis, we argue that values should be distinguished from beliefs and regarded it as a major component of affect. These components also involve cognition, they also involve what I call conation, that is desire and will, but they certainly involve affect.

How they are being considered, unfortunately, is still at a very superficial level. People look for correlations: the correlation between the positive attitude and mathematical achievement as measured through test scores, or something like this. And the answer is there's some correlation and it accounts for a relatively modest amount of the variance and that's it. So, doing more studies like that, for me, doesn't have a lot of value in adding to the research base. We already know that most of the variance is not coming from something as straightforward as attitude measured by questionnaire. I think that we are beginning to understand more about the complexity of mathematical affect, to take account of all of these components at the level of what happens in a fine-grained study of affect; to take account of how affect

occurs in the moment, how the emotions occur, how the meta-emotions occur, the meta-affect, what the meanings are of these emotions, how they are influenced by attitudes and beliefs and how, in turn, they influence beliefs. The idea that emotional experience stabilizes beliefs, so that to change beliefs becomes an uncomfortable and even a painful thing, is a very important part of our understanding. I think that understanding in research is growing very slowly.

Interviewer: *With DeBellis you have introduced the term "meta-affect". Can meta-affect be understood as an "umbrella" term for affect in mathematics? In general terms, how is it possible to explain meta-affect and its importance in mathematics education?*

Goldin: *I think meta-affect is a bit of a generalization of the idea of meta-emotion, which was a term that has been used previously. But I don't see it as an umbrella term. I see it as describing a very particular kind of thing or really two particular kinds of things. One is affect about affect, or affect about cognition about affect, or affect about something that comes back to affect. And that affect may be emotional, and may involve beliefs. I can have beliefs about beliefs. I can have beliefs about emotions and so forth. All of this, to me, is meta-affect. The other aspect of meta-affect has to do with regulation, how we regulate our affect? How do we, in the moment, regulate our emotions? What do we do with our anger? What do we do with our boredom? What do we do with our frustration? How do we keep it from impeding productive mathematical activity? How do we regulate our attitudes and our beliefs? These are important meta-affective questions and mostly I don't think they've been studied very much in mathematics education research. So, the importance, I think, is enormous.*

I think that so much of what we see... the prevalence of math anxiety in the culture has to do with meta-affect, has to do with the experience of affect negatively when the same affect could have been experienced positively. Frustration can be experienced negatively or positively. Other feelings that occur during mathematical activity: satisfaction, elation, joy, positive emotions or boredom or negative emotions... there are ways of regulating, there are ways of having meta-affect about these feelings. How do we regulate boredom? A bright student is in a mathematics class and is very bored with the slow pace of activity. How is this regulated? Is the boredom experienced in a way that leads to "okay, so I'm going to think about a much more interesting mathematical question" or is the boredom regulated in a way of, "I hate this class". These are meta-affective responses to an emotion of boredom. So, I think this is really of central importance in mathematics education and deserves a lot more attention.

Interviewer: *Okay. Returning to the subject of "... no longer a hidden variable ...", when we observe the impact of the term "metacognition", with thousands of researches about it and becoming very widespread, it is also possible to consider the "meta-affect" as a "no longer hidden" knowledge? What kind of expectation would be desirable or what can we expect regarding the impact of this variable in the coming years? In Brazil, to get an idea, the term does not appear in any publication title.*

Goldin: *And very few publication titles in America either. So, I'm not sure it's right to say meta-affect is a "no longer hidden variable", it may still be a hidden variable with just a little bit of attention given to it. I think that what would be desirable, would be for the impact to take the form of changing how we see the affective goals*

of mathematics education. Very often, the goal is now seen as people having positive emotions about mathematics. That's the way the goal is defined. But this leads to counterproductive activity, because when children experience negative emotions, the teacher tries to stop the negative emotions by helping the child not to feel frustrated, by helping the child not to feel upset, not to feel annoyed, not to feel angry, not to feel fear, not to feel any negative emotions, only to feel satisfaction, only to feel good. And, for me, this is counterproductive, not productive, because the far more enduring sense of enjoyment of mathematics comes from having experienced difficulties, having experienced challenges, having been very frustrated, even angry, even upset... and having nevertheless succeeded, having been able to persist.

The goal should be framed meta-affectively. We want students to be able to experience a spectrum of emotions, both those normally considered positive and normally considered negative in the context of mathematics, and to acquire the experience of having felt these emotions positively, having had these emotions lead to outcomes that have satisfaction; to be encouraged, to take a greater satisfaction after having experienced a negative path, to value the mistakes... and so forth. This is a different way to frame goals, affective goals, in mathematics education.

You know, in art, where art appreciation is something which is very highly valued to say that we should teach children to think, to appreciate only art that is pretty would be very silly, right? It's the art that depicts complexity, sometimes depicts struggle, sometimes depicts suffering, sometimes depicts beauty and sometimes depicts ugliness and sometimes depicts contrast between beauty and ugliness. And this is the art that we learn to appreciate and then we understand something about art.

So, to teach mathematics in a way that says let's give the kids only positive experiences with it is like trying to teach art appreciation by saying let's only say you should look at pretty pictures. This is not art appreciation. It is not doing mathematics to do it in a way in which everything comes easily. So that's my perspective on what our affective goals should be in mathematics. I believe are in the beauty of mathematics. It's a fundamental motivator for mathematics, is to experience that beauty. But you don't experience beauty when you haven't seen ugliness. It doesn't come without struggle.

Interviewer: You have published articles that relate discrete mathematics to affect. Could you explain the importance of the relationship between these two aspects and also the meta-affect in learning mathematics?

Goldin: Well, I pointed to a couple of things in discrete mathematics. I offer what I call affective affordances, good opportunities to approach what I consider to be great affective goals in mathematics. One of these affordances is that discrete mathematics is not typically part of the standard curriculum. So, if we are able to spend some time with it, we're not bound by the objective of teaching the students to pass tests on it. At least, this is true in the high school. The other affordance that it offers is that in many situations the elementary examples of discrete mathematics are approachable. They have very few prerequisites, they can be a subject to be explored, they offer a lot of room for discovery, for detection of patterns, and then for generalization from these patterns, and they also have applicability to easily developed real life situations.

For example, whether it be elections, or whether it be finding the shortest route on a map, or creating a network that is connected in certain ways, these can be easily

represented to students and made easily understandable to kids even at an early age. So, there's an opportunity to explore something and discover patterns that are not standard ones, and to draw generalizations and inferences from these... and, yes, to experience frustrations and various blind alleys and so forth, but then to take a lot of satisfaction on what's been discovered. That, for me, is an advantage of discrete mathematics at this point. However, if we made discrete mathematics part of the standardized curriculum it would lose most of those advantages. It would then become a subject for which we had to teach certain rules and certain formulas that students had to learn in order to apply them and get the right answer on the test, and this would destroy the affordance. So, again, it's a situation where policies are in place that are quite contradictory of the best research. So, let's see, the meta-affect... I think I mentioned enough about that.

Interviewer: Can you explain what Affective Architecture is and why do you consider it central to future research in mathematics education?

Goldin: Well, the idea of affective architecture, I use it as sort of parallel to what some call the architecture of cognition. That is, how are we as human being structured affectively? How do our emotions actually function so as to encode information? How do our emotions function in relation to each other? I suppose, ultimately, this has to do with the brain and how we're wired.

The answers may be somewhat different for different people. Certainly, there is a lot of variability in people's propensity to feel certain emotions. Sometimes, some people are very empathetic with other people and others are less so and I suppose that sociopaths are not at all empathetic with others. So, there are a lot of differences, and I don't know whether fundamental research on affective architecture is going to have big implications for mathematics education.

But even an old idea from psychology of emotion, the idea of defense mechanisms which goes back to Freudian psychology and psychoanalysis, has implications. The idea that people create defense mechanisms to conceal, even from themselves, the meanings behind their emotions is (assuming this idea is correct) part of our architecture as human beings. So, this is one of the things we do. We have built up structures and defenses. So, when a student has certain attitudes about mathematics and we ask how they came about, if the attitudes are negative ones, they probably came about through a whole series of defenses the student constructed. Some of those defenses may take the form of beliefs. "I'm not very good in mathematics so I can't be expected to do it or I'm just not a mathematical person". Some of those defenses may take the form of avoidance mechanisms and so forth and taken together... we say this person has a low attitude score with regard to mathematics. But when we tried to unpack it, we have to look at the architecture.

Interviewer: What do you see most significant in the findings of research on affect in mathematics today and in what direction or directions would it be interesting for the area to go?

Goldin: So, in the findings today... I think what is most significant is coming out of the qualitative research, the more fine-grained analysis, the way in which emotions shift very quickly in the moment from one emotion to another emotion, which apparently it doesn't really contradict but seems to contradict some other findings of the survey research that would suggest that attitudes are very stable, and so if somebody feels negatively about mathematics that's not likely to change much. But in the moment, it changes a lot. So, I think that tension between what we observe in

the moment and what we observe, you know, in the long-term traits is a very important finding. A study is going on, that I'm participating on the advisory board, for looking at student engagement in mathematics. Affect is certainly an important component here. And we see students describing the changes in their affect over the course of a year, studying mathematics, getting very different affective responses and different levels of engagement at different points in the year depending on what subject in mathematics comes up, depending on how successful they are, and so forth, and exploring those more rapidly changing relationships. I think the importance of doing that is emerging as a significant research finding.

I have written a paper or two about conation and, for me, that's a very interesting direction for the research to go, to bring in the conative dimension and to connect that with the affective domain and to look of at interactions between these domains in the context of mathematics education. We could get at fundamental motivations for doing mathematics, maybe, and get to the point of changing our approach, to develop these fundamental motivations in a way that would help to create a new generation that no longer is suffering from math aversion and math anxiety.

Interviewer: *Can you explain a little bit about conation?*

Goldin: *Well, conation has to do with human needs, desires, what we will to happen. It is about the reasons, why we do what we do. So, there are many needs that take the form of feelings, but they're not emotions exactly. So primitive needs like hunger and thirst are feelings, but they're not emotions. Something like "I need to belong" may lead to loneliness, which is sometimes called an emotion; but what emotion is it? Sadness is involved, but more than a sadness. It is a sadness in connection with something that is needed, the need to belong or the need for intimacy or the need for human connection. So, if we will bring this thinking into mathematics education, it raises the question how does mathematics meet fundamental human needs? Now, I'm not talking about the idea that, for example, mathematics helps us to do biological research and we have a need for safety from disease and so the biological research is going to cure the diseases and so on. I'm talking about needs in the moment that the 5th grader is experiencing or the 6th grader or the high school student. What needs of theirs are being met now and through the learning of mathematics? And, unfortunately, right now, the answer is, "very few". Hardly at all. Mathematics is not experienced that way and I think we could learn to teach mathematics in a way that it does meet the needs in the moment, so that people experience it as meeting needs for connection, needs for understanding, needs for power, needs for beauty, among others. These needs are very universal and very fundamental. And by teaching mathematics in a way that meets them, we can offer students an experience of mathematics quite different from what it has been up to now, and that's my hope.*

Interviewer: *Really appreciate being able to talk to you, Professor Goldin, and thank you once again. Would you have any final comments?*

Goldin: *No, I think you asked a lot of good questions and I hope this is helpful. I would say, you know, my own affect in relation of mathematics is very positive, obviously, but it's took a long time to develop and when I was a young student I experienced a lot of frustration in mathematics because it was not being taught for understanding, I did not experience it as meeting my needs, but I had the advantage that my father was a mathematics teacher. I found ways, not always at school, more outside of the school, through which it could satisfy me. And I did a lot of*

independent study when I was in high school which I found to be doing more to meet my needs than simply following the curriculum. So, some of this is influenced by my own experience.

Interviewer: Thank you very much.

Goldin: Thank you.

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