Developing competencies to teach exponential and logarithmic functions using GeoGebra from a holistic approach

INÉS Mª GÓMEZ-CHACÓN[1]
NURIA JOGLAR PRIETO[2]

Resumo

A integração da tecnologia nas aulas de Matemática é uma questão complexa que tem de ser abordada numa perspectiva holística que leva em conta as diferentes componentes inter-relacionadas. Neste artigo, propomos o estudo do desenvolvimento de quatro componentes (cognitivo, didático, técnico e afetivo), e de suas interações, trabalhando com vários grupos de futuros professores durante dois anos e a realização de um estudo de caso com os alunos selecionados para análises mais profundas . A estratégia de pesquisa se enquadra dentro de um Design Based Research (DBR). Como parte da metodologia utilizada na formação de professores pré-serviço, multimídia cenários de aprendizagem para ajudar os formandos a aprender a ensinar funções exponenciais e logarítmicas com GeoGebra são desenvolvidas e implementadas.

Palavras chave: *Instrução de Professores de Matemática (Ensino Superior); Ensino de funções exponenciais e logarítmicas utilizando softwares de Geometria Dinâmica; Aprendizagem baseada em competências.*

Abstract

Integrating technology into Math lessons is a complex issue that has to be addressed from a holistic viewpoint that takes into account different interrelated components. In this article, we propose the study of the development of four components (cognitive, didactic, technical and affective), and of their interactions, working with several groups of prospective teachers during 2 years and conducting a case study with selected students for deeper analyses. The research strategy is framed inside a Design- Based Research (DBR). As part of the methodology used in the pre-service teacher training, multimedia learning scenarios to help the trainees learn how to teach exponential and logarithmic functions with GeoGebra are developed and implemented.

***Keywords**: Mathematics Teacher Instruction (Higher Education); Teaching exponential and logarithmic functions using Dynamic Geometry software; Competency-based learning.*

Introduction

In recent decades, many advances in science and engineering have been determined by the development and improvement of powerful numerical and symbolic computational software such as MatLab or Mathematica. Although such tools are also used in education, especially in universities, nowadays there are many simpler tools specifically

[1] Facultad de Ciencias Matemáticas, Universidad Complutense de Madrid – email: igomezchacon@mat.ucm.es
[2] Ingeniería Técnica en Informática de Sistemas, Universidad Complutense de Madrid - email: nuria.joglar@ajz.ucm.es

designed as educational software for the teaching and learning of Mathematics at all educational levels. Those distributed under a public license are especially relevant. Free accessibility of software allows teachers and students to really incorporate the software into their mathematical *toolbox*. This is the case of *GeoGebra* (http://www.geogebra.org/cms/), a free and multi-platform dynamic mathematics software for all levels of education that brings together geometry, algebra, calculus and statistics in one user friendly interactive application (HOHENWARTER & JONES, 2007; HOHENWARTER & LAVICZA, 2007; BÖHM, 2008).

However, this progress in technology development is not being accompanied yet by an appropriate and rigorous training of the teachers who are to use those tools with their students (CUBAN, KIRKPATRICK & PECK, 2001; HOYLES *et al*, 2006; RUTHVEN AND HENNESSEY, 2002; WALLACE, 2004). The role of the teacher changes when using technology in the classroom, she/he has to become a *technology mediator* and a new teaching contract with the students needs to be established. Integrating technology into Math lessons is a complex issue (MONAGHAN, 2004). Hence, prospective teachers clearly require new technological and didactic skills (ARTIGUE, 2002; TROUCHE, 2005). Accordingly, a serious review of the current strategies for pre-service (and in-service) training of teachers in this recent complex domain is called for. These ideas are already being observed by several researchers in Mathematics Education (GÓMEZ-CHACÓN & JOGLAR, 2008; PREINER, 2008; ANDRESEN & MISFELDT, 2009; GOOS, 2009; JONES *et al*. 2009; BENNINSON & GOOS, 2010).

In this article, we will present some findings on the use of multimedia learning scenarios to help undergraduate students (prospective Mathematics teachers) learn how to teach exponential/logarithmic functions with GeoGebra. This work is part of a larger study, called ESCEMMAT and funded by the Universidad Complutense de Madrid in 2007, where a multidisciplinary research team, composed of secondary education Mathematics teachers together with researchers in Education, in Mathematics, and in Mathematics Education, worked together during two academic years designing, developing and implementing, in website format, multimedia learning scenarios. One of the main objectives of the ESCEMMAT project was that the participating students (prospective secondary school Math teachers) acquire and improve the skills needed to teach functions through a technology-based Mathematics lesson by following a project-based learning model. Documentation where the research methodology and the theoretical frameworks were discussed, as well as videos showing some of the participants and expert secondary school teachers performances in real classrooms, were included in the scenarios.

By working with our scenarios, we not only carried out a study on the competencies needed to teach Mathematics by using technology, but we also analyzed the strategic knowledge that arises in the proposed complex learning situations in order to establish some control mechanisms over them. Within the two frameworks of *competency-based learning* and *strategic knowledge*, the prospective teachers must understand how to use the instruments in order to influence the cognitive and mathematical activity of their students.

Competency-based learning models aim to develop both generic and specific skills in the students, with the purpose of training them on scientific and technical knowledge (NISS, 2003; VILLA & POBLETE, 2007; JENSEN, 2007; ORTIZ, RICO & CASTRO, 2007; FERNÁNDEZ *et al*, 2010). These models pursue the development of the student's ability to apply this knowledge in various complex contexts, integrating the knowledge with their own attitudes and values and therefore growing their own way of acting personally and professionally. From this point of view, in our analyses we have considered four different domains for the classification of competencies and we have adopted a three-perspective approach (holistic view) to teacher training in technological contexts (see Section 2 below). Within each domain at least three different levels are defined and are controlled by three indicators. The notion of *strategic knowledge* in learning situations involving technology has previously been applied by several authors (BAIRRAL, GIMENEZ & TOGASHI, 2001; GÓMEZ-CHACÓN & JOGLAR, 2008; GÓMEZ-CHACÓN & KUZNIAK, 2010).

Some researchers such as LABORDE (2001), TAPAN (2006) and LAGRANGE (2009) have indicated that, in general, undergraduate students (prospective teachers) lack the knowledge required to teach mathematics integrating educational software in the classroom. This problem is in fact related to two different aspects. On the one hand, the prospective Mathematics teachers are unaware of the development of mathematical notions in teaching situations, and on the other, they have serious technical difficulties in using the software in a learning situation. These difficulties make it necessary to integrate specific work concerning the use of educational software into pre-service Mathematics teacher training. The ESCEMMAT project was also conceived as a pilot experience at our university (*Universidad Complutense de Madrid*, Spain) aimed at finding guidelines for the curriculum design of the new Master in Education, required to become a secondary education Mathematics teacher within the *European Higher Education Area*.

The work presented in this paper is based on the following assumption: when a university undergraduate student (prospective Mathematics teacher at Secondary Education) uses educational software to solve Math problems, she/he does not spontaneously generate a method for integrating software into her/his future experiences as a teacher.

In this article some initial answers to the following three questions, stated as a point of departure for our research, will be provided:

- What competencies are required for undergraduate students in Math Education programs to teach exponential/logarithmic functions with GeoGebra? Which of them are most needed in this situation? Which of them are harder to acquire by the prospective teacher during pre-service training?
- Which factors stimulate or inhibit the development of the skills of a prospective teacher to teach the exponential/logarithmic functions with technology?
- What components (cognitive, didactic, technical or affective) are reflected in the practice of these undergraduate students? What is the logic behind the behavior of a student when she/he acquires the strategic knowledge for teaching with technology?

In a first approximation to answer these questions, we carefully analyzed the general results of the larger ESCEMMAT project. This initial global analysis, conducted with 118 students divided in four groups from two consecutive academic years, indicated the existence of three different student profiles according to the level of competencies in each of the domains considered. With this information in mind and in order to conduct a more exhaustive analysis to find more precise answers to these questions, we selected one of the groups from the second year (14 students) for a case study and, besides, we chose a particular student who resulted to belong to the highest profile to make a deeper study.

The rest of the paper is organized as follows. First we describe the theoretical frameworks for our research. Then the research methodologies used in the project ESCEMMAT and more specifically in the analyses conducted in this article are presented. A section on the results of all the analyses is presented afterwards including some answers to the questions formulated above. Finally the first conclusions of our work and some suggestions for future studies are presented.

1. Theoretical frameworks

In recent years some researchers in Math Education have been making an effort to understand teachers' activities in technology-based Mathematics lessons (MONAGHAN, 2004; GOOS, 2010). Following Saxe's model, Monaghan outlines the vision of teachers' practices in technological environments "as a whole". He considers the interrelations among tools, conventions, social interactions and prior understandings in the activities where unexpected emergent goals appear and pass away.

We agree with this author in the importance to propose a holistic way to examine (pre-service and in-service) teachers' practice in technological contexts. Under this perspective, we consider different interrelated aspects and levels:

- Prior understandings of the subjects: mathematical knowledge, pedagogical knowledge, institutional knowledge, attitudes, personal beliefs and professional identity position.
- Artefacts: a computer with Math Educational Software (GeoGebra).
- Social interactions and performance in the classroom in technology-based Mathematics lessons (institutional component).

In particular, we will try to provide pre-service teacher training models for teacher professional development in technological contexts. With this aim in mind, we proposed in this work a *competency-based learning model* for training undergraduate students (prospective Mathematics teachers) to develop Mathematics lessons in technological contexts. More precisely, to determine and to classify the competencies required to teach Mathematics with digital technology we consider the combination of two theoretical frameworks: *the ergonomic and didactic approach* (ROBERT & ROGALSKI, 2002) and *the instrumental approach* (RABARDEL, 1999; ARTIGUE, 2002; LAGRANGE, 2009) adding also an approximation less covered by these approaches: the *emotional dimension* of subject (GÓMEZ-CHACÓN & HAINES, 2008; GÓMEZ-CHACÓN, 2010a). This combination of approaches will produce a classification of competencies to develop in our scenarios following the holistic view.

To be able to answer the questions stated in the Introduction above, we designed two multimedia scenarios to work with undergraduate Mathematics students who were specializing in Mathematics Education and who wanted to become secondary education teachers. Activities involving basic functions and their graphs were implemented using Math educational software (Derive at first, and GeoGebra more exhaustively later).

In general young students do not begin working with the notion of function until they get to middle school (12-13 years), and this is one of the most complicated concepts for them to acquire. Some experts in Math Education have argued that an adequate understanding of the concept of "function" requires the ability to move from its algebraic form of representation to its geometrical form of representation and vice versa (VINNER, et. al. 1983; ELIA et al., 2007). Therefore, it is essential to coordinate information across representational systems and to connect the visualization of the graph of the function with its algebraic equation and also with tables of inputs and outputs and this is where GeoGebra can make a big impact (TALL, 2009; BÖHM, 2008; HOHENWARTER *et al*, 2008).

In particular, exponential and logarithmic functions are key mathematical concepts that play central roles in advanced mathematics. We also agree with the conclusions of recent studies showing that the learning of these functions causes serious difficulties for students. In particular, we appreciate the Engbersen's overview -based on textbook analysis- about the Hungarian, Austrian and Dutch situation (ENGBERSEN, 2009). In this paper we try to provide some ideas on how to embed this topic in pre-service teacher practice from a more *realistic* perspective (HAINES, 1991) involving Mathematical modelling using GeoGebra. More precisely and for the deeper analyses mentioned in the Introduction, a sub-group of prospective teachers were asked, by their professor from the *Methods in Math Education* course, to explain exponential and logarithmic functions for high-school students at 11^{th} grade with GeoGebra and taking advantage of the two views GeoGebra provides and being able to give graphical, numerical and algebraic approaches. (by the time we proposed the activity, 2007, dynamic worksheets in GeoGebra were not available yet). To do so, they were given a real context problem regarding *sonority of iPods* with precise and clear instructions to provide the mathematical modelling and to solve the questions proposed. The precise statement of the activity was as follows:

> MP3 abuse can advance in 30 years hearing loss. Young people who now routinely use these players may begin to show symptoms of hearing loss 30 years earlier than their parents. Experts no longer recommend its use for more than an hour away and, at most, at 70% of

> its volume capacity. Injuries caused by these devices are slow and irreversible, so it is unknown precisely how it may affect the population. Discos, concerts and movies also can affect hearing. Clubs (with peaks of up to 124 db), a rock concerts (120 dB) or go to the movies are other common leisure habits among young people and short-term can affect their hearing. (20 minutes, Local Free Newspaper 31.10.2007).
>
> Instructions: Model the sonority function of the MP3 and try to answer the following question: If an iPod with stereo speakers is playing at 90 dB, how many decibels produce two iPods?

The competencies proposed here as necessary for the teachers to teach exponential/logarithmic functions with GeoGebra will be classified following the frameworks described above. This classification was also the starting point for the preparation of questionnaires for the global and local analyses described in Section 4. An interrelated analysis of all the factors, putting a special weight to the affective variable, is also included in Section 3 below.

Strategic knowledge is understood as the knowledge on the variables (cognitive, didactic, technical and affective) which make it possible to control the teaching situations. Within these frameworks, the student teachers must understand how to use the instruments in order to influence the cognitive and mathematical activity of the user.

This study identify some students' behaviors relative to this strategic knowledge which can be used as examples to show during teachers training especially on: the schemes used to solve a mathematical problem with software and schemes used to analyze and construct teaching situations with software. Under this perspective, training scenarios that closely link the technical elements on the software with the elements of mathematics education are necessary.

To finish this section, we include a description of the four different competencies we have chosen articulated from the holistic view mentioned above. Within each domain at least three different levels are defined which in turn are controlled by three indicators.

- *Competency 1.* Mathematical content and problem-solving (prior understanding: mathematical knowledge).
- *Competency 2.* Curriculum organization, instructional planning and mathematical modeling (prior understanding: pedagogical and institutional knowledge).

- *Competency 3.* Use of resources and materials (artefacts: focus on a computer with GeoGebra; instrumental genesis).
- *Competency 4.* Personal and affective component (attitudes, personal emotions, beliefs, professional identity position, social practices in the classroom, interpersonal skills and creativity).

In the deeper analyses, we have decided to concentrate on the competencies 3 and 4 which turned to be more critical in our settings. Hence, we will only include the domain levels (proficiency) and indicators for these two, although we will also make general comments concerning the first two competencies, especially regarding modeling skills in Competency 2 as part of the first global analyses in subsection 3.1.

Competency 3: Use of resources and materials.

We have distinguished three levels of proficiency:

- Properly manage and create files / documents with a scientific word processor and with dynamic mathematics software (eg MS-Word, MathType and GeoGebra). Surfing the Internet (search for material) and manage reference texts or articles.
- Edit text documents of certain complexity (eg MS-Word). Properly use a scientific word processor (eg. MathType). Edit GeoGebra files and manage basic commands: graph of a function, intercepts of graphs. Using sliders. Copy and paste a snapshot from GeoGebra to the text editor.
- Manipulate and interpret complex graphs in 2 dimensions with GeoGebra. Change logarithm base with GeoGebra[3]. Mathematically model real problems with GeoGebra. Learn to find several different solutions to a given problem with GeoGebra. Interpret the results.

Competency 4: Personal and affective component

Prospective teachers have the opportunity to explore and analyze their personal beliefs, assumptions and attitudes about teaching and learning during pre-service training sessions. Teacher students must be able to connect their emotions with what they know about Mathematics contents, pedagogy, school discourses, personal histories and curriculum (GÓMEZ-CHACÓN, 2010b).

We have distinguished four levels of proficiency:

- Teaching and learning of Mathematics in technological environments: motivation for computer use, attitudes toward the interaction between Mathematics and computers, mathematical beliefs, vocational mathematical professional identity position.

[3] All participants were working with GeoGebra 2.0 since GeoGebra 3.0 was released in the Spring 2008.

- Express their own ideas in a structured and intelligible way both verbally and in writing. Establish dialogues with peers, teachers and students.
- Convey conviction and security. Adequately adapt to situations and contexts (known formal and colloquial registers). Be timely. Transmit relevant information.
- Connect easily with the audience capturing their attention with colorful and interesting examples. Make the difficult contents look easy (without falling into the trivialization). Adapt appropriately to the audience or reader. Convey enthusiasm for the subject explained. Listen and be patient with responses / reactions from the audience.

2. Research Methodology

This section reports the research methodology of the general project ESCEMMAT, as well as a detailed description of the aspects concerning the methodology of the pre-service teacher training situation and the teaching situation in real or simulated high school classrooms.

In order to study which competencies are required by a prospective teacher to teach Mathematics with technology, we designed and developed multimedia scenarios for two concrete situations in this complex domain within the ESCEMMAT project. The scenarios were designed as training curriculum material for the master in the Math Education program at our university. In the first scenario, linear functions were taught using Derive or GeoGebra, and in the second scenario exponential and logarithmic functions were taught using GeoGebra. To create, implement and validate these scenarios, we considered a *Design-based research methodology* where design experiments were developed as a way to carry out formative research in order to test and review educational designs based on principles derived from prior research (DBR Study Guide, Design-based research collective retrieved 2010). This methodology has an action-research oriented perspective which allows the continuous refinement of the proposed model where researchers work together with educators in a multidisciplinary team. Once the prospective teacher has received the basic formative course (pre-service teacher training situation), she/he had to give a lecture corresponding to a given topic from one of the scenarios in a real or simulated high school classroom (teaching situation).

2.1. General methodology of the ESCEMMAT project

The research strategy followed in the ESCEMMAT project is framed within *Design-Based Research* (DBR). DBR is an important methodology for understanding how, when, and why educational innovations work in practice and aims to uncover the relationships between educational theory, designed artefact, and practice (COBB *et al*, 2003; COLLINS *et al*, 2004). In recent years, a new paradigm has emerged for engaging in theoretical research in realistic learning settings. Design experimentation is an inter-disciplinary approach that acknowledges the fundamentally applied nature of educational research. The following characteristics of DBR have proven to be useful in our research. The theoretical development is closely interrelated with learning designs and teaching designs. The design is conducted in real-world settings. DBR allows working from different theories and multiple perspectives and also enables the collaboration of researchers and teachers from various backgrounds. DBR has an action-research perspective and allows flexible designs. DBR might be considered an extreme example of flexible designs since iterations of stages and theory is a central issue and it does not put an emphasis on aggregated results.

The design research approach we follow working with the scenarios has a cyclic character: each design consists of research cycles in which theoretical experiments and teaching experiments alternate. A macro cycle of design research consists of three phases: the preliminary design phase, the teaching experimental phase, and the phase of retrospective analysis (see Figure 1). The DRB methodology involves an intervention design where the final product is the result of incorporating to the initial approach the findings from the different stages of the cycle. It also incorporates formative evaluation and redesigning stages. The figure below may be illustrative of this process.

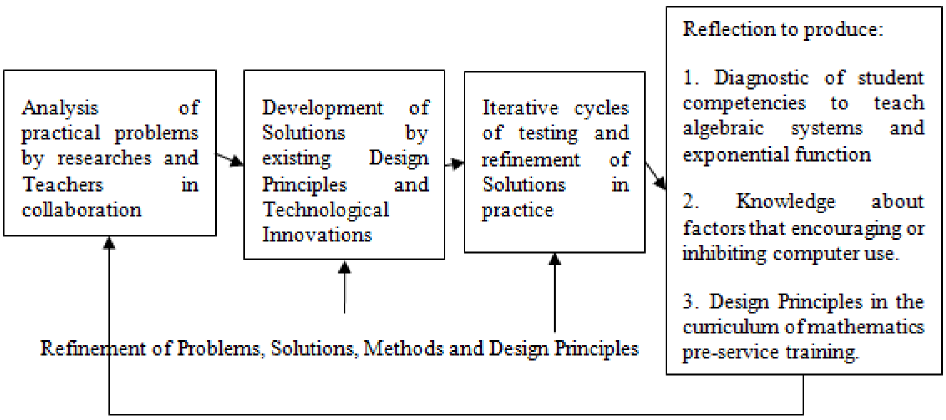

FIGURE 1. Schema of the research strategy with DBR methodology.
Source: Adapted from http://projects.coe.uga.edu/dbr/FAQ.html retrieved December 2010.

In our study it was also important to develop a learning trajectory which has to be tangible in the instructional activities.

Based on collaboration among researchers and practitioners, the ESCEMMAT team consisted of an interdisciplinary group of 6 researchers (mathematicians, researchers in mathematics education and in research methods in education), 6 secondary school teachers from 3 different secondary schools, and 3 external evaluators. As we have mentioned above, the overall study was conducted with 118 student teachers belonging to four class groups. Each group of students was followed over a period of 6 months and the students' practicum experience was carry out in parallel in most cases during 6 months in secondary schools.

For presentation purposes, firstly, we have considered the total group of 118 student teachers for the initial global analyses, and secondly, we have selected a more specific sub-group (14 students belonging to a group class) for further analyses of competencies and their interactions. The reason for choosing this group as a case study group is that they had been following a graduate training program and they had also been doing mathematical modelling activities involving functions more intensively than the general group. This subgroup is then considered as a methodological case study. Therefore, their cases offered us a deeper understanding of the rationale behind the behavior of the student teacher when she/he acquires the strategic knowledge to learn to teach with technology, identifying factors that inhibit or stimulate the development of skills too.

To be able to provide more precise answers to the three questions stated in the introduction of this paper as guiding our research, we also conducted a more exhaustive study and description of a particular student. The criteria for the selection of this concrete student were determined by their vocational-professional status to teaching, their conceptions of mathematics and their attitudes toward technology.

For the overall initial analyses and according to the aspects underlined in the theoretical framework about of the dual approach, we have considered the results of a variety of activities offered to students such as written assignments and oral presentations about mathematics problems and modeling activities; teaching practices (video recordings, grills and diagnostic evaluation of competencies); interviews containing their reflections about the teaching of Mathematics; scripts for a lecture including teaching schedules; reports of teaching practices; and computer applications developed (applets and webquests).

As an example we have studied the activity of Mathematics student teachers in the special situation of integration of the software GeoGebra in a high school classroom. In this case, dialectic between "productive activity" and "constructive activity" is introduced to specify that the teacher has to contribute to the activity of the school, but she/he also transforms herself/himself in the long-term. Setting this dialectic one can take into account both ends: students' learning in the classroom and the restrictions determined by factors beyond the classroom (institutional, social and personal). In practice, this means that in our research we had to study the skills related to different components (cognitive, didactic, technical and affective) as well as to establish times and levels (local and global) in the analyses within the theoretical frameworks described.

To finish this subsection, we just want to mention that for the evaluation and validation of data obtained from the case studies, we have used different sources, such as questionnaires, interviews, video recordings of classes, classroom observations and triangulation of perspectives (three judges). The team of judges was composed of researchers in Math Education and secondary school experienced teachers outside our group.

2.2. Methodology of the training situation and the teaching situation

As we have mentioned earlier in this section, the scenarios are designed as training curriculum material. Therefore, it is important to note that two types of contexts have to be differentiated: the *training situation*, where the trainer (university professor) and the trainees (prospective Mathematics teachers) appear, and the *teaching situation*, where the student teacher works interacting with high-school students (see Figure 2). In these situations different levels of activity and practice are set for the trainer and the prospective teachers (see Figure 3).

FIGURE 2: Performances in real secondary school classrooms.

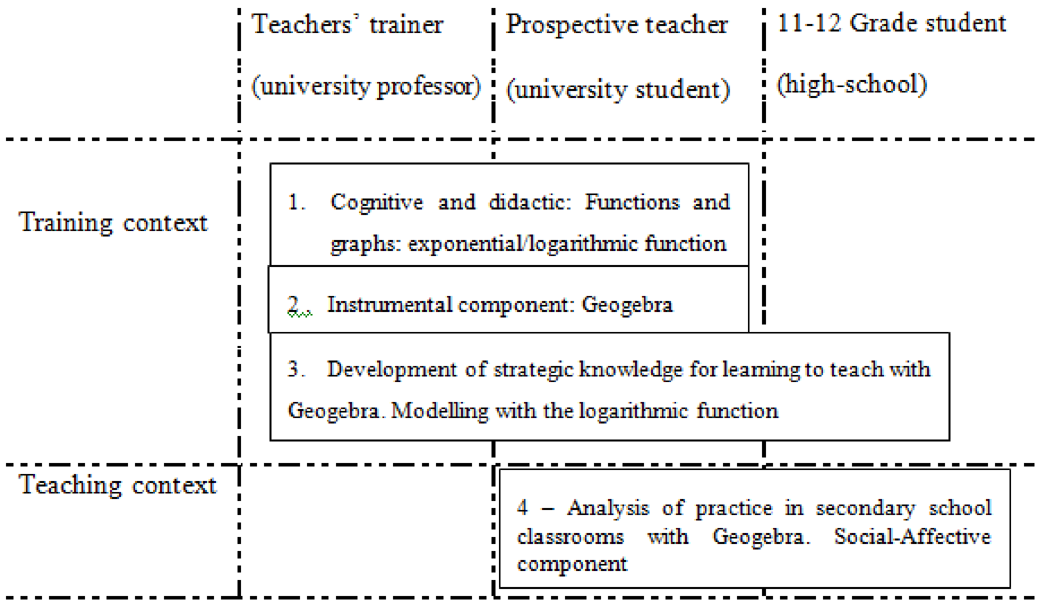

FIGURE 3: Training and teaching context in the scenario.

In the training context, assuming that the university student already knew the materials related to the didactic of Calculus and the topic of the exponential/logarithmic function from previous courses, two types of problems were proposed. University students were supposed to solve them and also to try to generate similar activities for high-school students in order to be used in a real classroom as a practice session. Activities that simply required students to follow basic routines were called mechanical activities. Problems that required conceptual and procedural knowledge were called modelling activities. The answers for the second type of activities, not only required a constructive process considering different alternatives to arrive to the solution, but they also required conceptual knowledge and its applications, being able to transfer this knowledge to the graphic and symbolic categories.

3. Results

As we have mentioned earlier in this article, in this section we are going to present two types of initial results. Firstly, we will state general results about the four competencies analyzed through the variety of activities carried out with all the 118 participants in the ESCEMMAT project (2006-08). This initial general analysis led us to the detection of three student profiles in the big group of prospective teachers according to the different levels they presented in the four competencies worked with them in the pre-service training sessions from the "Methods in Math Education" course. With this overall analysis, some answers to the first question proposed in the Introduction are provided. Secondly, we will include a more detailed analysis for competencies 3 and 4 based on the modelling activity with the logarithmic function that a subgroup composed of 14 students solved in 2008 as a written homework assignment for their Methods in Math Education course. This case study will also throw some light on Question 2 from Section 1.

To finish this section, we will include the detailed analysis of a particular student trying to answer Question 3 from the Introduction. This student was invited to give a real lecture (following his script previously commented by his professor) to a group of 11th grade students at a public high school. His lecture was recorded and evaluated, using the questionnaires mentioned in Section 3, by three independent external judges. After carefully examine these evaluations, a summary of the assigned values at each item will be included too.

The results described in this section have confirmed our initial hypothesis: when a university undergraduate student (prospective Mathematics teacher at Secondary Education) uses educational software to solve Math problems, she/he does not spontaneously generate a method for integrating software into her/his future experiences as a teacher. And moreover, the study shows that student teachers have little knowledge of the teaching process, they are unaware of the development of modelling mathematical notions in teaching situations and they have difficulties using software in a learning situation.

3.1. Global analyses: student profiles

In the comprehensive study of the four groups of students during the two academic years (2006-2008) we have detected three profiles among the students according to different levels we have observed in the four competencies originally stated in our theoretical framework. In addition to analyzing the levels in the four prominent components (cognitive, didactic, technical and affective), we also looked for the qualitative description of the operational coherence between the different components and try to detect if students showed flexibility to move from one frame to another (algebraic, geometric, numerical and instrumental), while training with the selected software.

As a consequence of this first global analysis, we were able to answer the first question posed in the introduction to this article: all four competencies are required for undergraduate students in Math Education programs to teach functions with GeoGebra. Competencies 1 and 2 are in general easier to acquire following basic courses on Calculus and Methods on Mathematics Education, but Competencies 3 and 4 are shown weaker in general in the prospective teachers, so a more comprehensive work needs to be done with the students in order to help them master these components. Within the component 2, the mathematical modelling turned to be the most difficult to acquire by the future teachers (more than 50%). In their papers, students show little ability to pass from the real context to the mathematical model, and after solving the mathematical problem, they also have difficulties to translate their mathematical solutions to the initial real context. They also have in general very little creativity to introduce new modelling situations.

A brief description of the three profiles detected as a result of the global analyses is given below.

- Profile A: vocational mathematical professional identity position, good knowledge of mathematical contents regarding basic functions and their graphs. Students in this profile have also acquired an appropriate methodology to plan and propose activities to their pupils. Students in this profile show great skill in using the math software to illustrate the solutions to the problems in their written homework assignments. They have shown a good integration of the Mathematics educational software in the classroom too (in a simulated situation or in a real situation at a high school). They are able to mathematically model the initial situation posed by their trainer. Besides, students in this profile are very creative and they show skills to independently design activities in real contexts in which mathematical modelling requires the use of functions and their graphs. They are careful and precise writing their assignments. If we focus on their attitudes towards the use of technology for learning mathematics, theses students have high values for all variables: mathematics confidence, motivation, engagement and interaction between computers and mathematics and they do not show anxiety when they do mathematics. Concerning the affective component of their belief about mathematics, they regarded mathematics to consist in problem-solving and exploring new ideas, they had a dynamic and instrumental conception of mathematics. Their “mathematic styles” were approximately 80% geometric and 20% algebraic. They turned to be very communicative with their classmates and self-confident. Besides, students had confidence in their personal curiosity and creativity.

- Profile B: vocational mathematical professional identity position, good knowledge of mathematical contents regarding basic functions and their graphs. Students classified in this profile are lacking instrumental competencies to integrate the mathematics educational software in the classroom (in a simulated situation or in a real situation at a high school), even though they show certain ability in using the software to illustrate their solutions to the problems from written homework assignments. They have been able to mathematically model the initial situation posed by their professor at the university but only with some help from her. They actively participate in the process, but they did not become involved in the process: they did not take their own decision. Students in this profile, have not been very creative, showing difficulties to design coherent activities in real contexts where the use of basic functions and their graphs are required. They are also careful and precise writing their assignments. If we focus on the personal and affective dimension, regarding for instance their attitudes towards learning mathematics with technology, they are highly motivated, confident and engaged in mathematics, who have, however, low confidence in computer use and who do not show in the questionnaires good interactions between mathematics and computer. This group has high competence in understanding problem solving. They have a dynamic vision of mathematics as a humanistic domain. However, in contrast with the profile A, their anxiety index is higher than the group average. Their "mathematical styles" were balance.

- Profile C: lack of career guidance, poor knowledge of mathematical contents regarding basic functions and their graphs, lack of technical expertise and poor management of software, especially when illustrating problems in the classroom (in a simulated situation or in a real situation at a high school) but also when using the software to try to solve the proposed problems in the homework assignments. Students belonging to this profile are not very careful in presenting their work. Sometimes they even make mistakes in spelling and writing. In this group conflict between the professional choices and the mathematical professional identities as teachers is detected. They believe that they have a good capacity for understanding mathematics and they show an absolutist/positivist vision of Mathematics as a science. They show high anxiety when working with the computer.

To finish this subsection, a table (Table 1) with a more comprehensive analysis of the four competencies and the interrelation among them in the subgroup of 14 students (case study) is included.

The data presented in Table 1 show that positive attitude toward technology is closely related to students' conceptions of mathematics and its position in relation to their future as professional teachers (what we are naming "personal dimension of the subject"). Also, we note that it is important to take into account the interactions between technology management, knowledge of mathematics and modelling. In this smaller group, the difficulties of coordination between the mathematics modelling and the use of GeoGebra were even more pronounced than in the general group. All of the 14 students admitted that they had to receive help from their trainer to be able to translate the real problem into a mathematical model. It is also worth mentioning that around 80% of this group did not give complete instructions on how to organize the activity with their high school students and all the guiding questions that they wrote for their students were posed in a closed form.

Table 1. Holistic assessment of the group relative to several dimensions.

Student	VMPIP	MB	CM	CMIA	ERTS	C1	C2	C3	C4	TDG	MPC	EV
S1	C	mb3	2.87	2.27	2	1 [45]	1 [80]	1 [65]	1	3	0 [55]	D
S2	A	mb1	3.62	3.36	1	2/3 [82]	3 [90]	1 [55]	3 [87]	1_3_7	3 [45]	A
S3	A	mb1	3.75	3.45	1	3 [90]	3 [77]	2/3 [8]	3 [8]	3_5	3_4 [80]	C
S4	A	mb2	3.87	3.54	1	2 [75]	2 [70]	3 [80]	2 [75]	6	[95]	C
S5	A	mb1	3.25	2.90	3	2 [78]	2/3 [85]	2 [68]	3 [80]	0_3_5_7	3_4 [80]	A
S6	A	mb2	3.5	3.18	1	1 [34]	2 [65]	1 [55]	1 [65]	1_3_5	1_3 [60]	A
S7	C	mb3	3	3	3	2 [77]	2 [70]	1 [60]	21 [55]	1_5	3_4 [80]	A
S8	C	mb2	3.37	3.36	5	1 [35]	1 [50]	2/3 [70]	2 [70]	5	1_3_4 [50]	D
S9	C	mb3	2.62	2.72	2	2 [66]	2 [65]	1 [55]	2 [55]	1_3_5	3_5 [60]	D
S10	A	mb1	3	2.72	3	3 [88]	3 [80]	2 [75]	3 [95]	1_3_4	3-4 [80]	C
S11	A	mb1	3.12	3.54	1	3 [95]	3 [80]	3 [95]	3 [90]	3_5	[95]	C
S12	B	mb3	3.62	3.09	2	1 [30]	1 [40]	1 [30]	1 [40]	1_3_5	0_1_3 [35]	D
S13	C	mb3	4.25	3.81	1	1 [35]	1 [40]	1 [40]	1 [40]	0_3	0_1_3 [40]	D
S14	A	mb2	3.87	3.45	1	3 [85]	3 [80]	3 [95]	3 [78]	6	[95]	C

VMPIP: Vocational Mathematical Professional Identity Position l: A (Yes), B (No), C (lack of career guidance)

VMPIP: Vocational Mathematical Professional Identity Position l: A (Yes), B (No), C (lack of career guidance)

MB: Mathematical Beliefs: Humanistic view (mb1), Instrumental and dynamic science (mb2), Absolutist/positivist science abstract knowledge (mb3)

CM: Computer motivation (mean, scores from 1 to 5)

CMIA: Computer-Mathematics Interaction Attitude (mean, scores from 1 to 5)

ERTS: Emotional reaction toward technology in situation: 1 (positive), 2 (negative), depends on activities (3)

C1: Competency 1. Mathematical content and problem-solving: dimensions, scores.

C2: Competency 2. Curriculum organization, instructional planning and mathematical modeling: dimensions, scores.

C3: Competency 3. Use of resources and materials**:** dimensions, scores.

C4: Competency 4. Personal and affective domain (personal beliefs, social practices in the classroom, creativity, interpersonal skills): dimensions, scores.

TDG: Types of difficulties in learning mathematics with GeoGebra: Problem understanding (0), Instrumental genesis (1), Affective block with the artefact (2), Mental blocks in the global control of the different geneses of modelling work (3); To spend time- inversion (waste time) (4); Processes of solving problem with the artefact (5); There is not any register of difficulties by the student (6)

MC: Modelling Process Competencies: (0) Perceived reality of physical model, (1) Formulation of task, (2) Systematization, (3) Mathematization, (4) Mathematical analysis, (5) Interpretation/ evaluation, (6) Validation

EV: Evolution of teacher students in the design modelling activities with computer: Final Results (Description (D): when the student teacher participated in the process taking an external viewpoint; Argumentation (A): when the student-teacher participated actively in the process, Contribution (C): when the student-teacher, resides participating, became evolved in the process by taking her own decision.

3.2 Deeper analyses: competencies 3 and 4

As we have mentioned above in the introduction of this section, to go deeper in our analysis of competencies 3 and 4, we decided to select a sub-group of 14 students during the second year of the ESCEMMAT project. This group was proposed the modelling activity involving logarithmic functions in the context of studying the sonority of mp3 devices described before.

Students were supposed to solve this activity twice, at first without the use of technology and afterwards using GeoGebra. They were also asked to give instructions on how to plan a real lesson with high school students covering this activity with them. In addition to modelling the writing task, students completed questionnaires on how they were feeling at the time of solving the proposed activity without the use of technology and with GeoGebra and about how they would think their alleged pupils would feel when receiving this lesson.

Competency 3

As an example, we mention now some of the tasks that students had to carry on with the help of GeoGebra as part of the modelling activity prescribed by their trainer. We indicate in each item the difficulties encountered by the students.

- Understanding the definition of natural logarithm together with its graphical interpretation. Representation of the graphs. Using reflections to check this equivalence (as inverse functions). These require the student to be competent in basic commands such as representation of points, representation of lines, representation of general graphs, intersections, reflections, zoom and sliders. Even though all of these functionalities are very intuitively displayed in GeoGebra, only around 60% of the students included all of them in their constructions. Here the geometric power of GeoGebra was felt by the all of students. It was surprising (and somehow contradictory) to find that around 15% of the students were suspicious about the graphs provided by GeoGebra and they argued that the ones “they had made by hand were more accurate”. They all claimed that working in groups had been very helpful in this regards, they spontaneously created groups to prepare the activities outside the classroom (ANDRESEN & MISFELDT, 2009).

- Implementation of changes of log base since GeoGebra only considered natural logarithms at the time the study was conducted. It is necessary to mention here, that all participants were working with the last version of GeoGebra 2.0 since GeoGebra 3.0 was released in the Spring of 2008. Examples with base 2 and 10 were considered. All students commented this aspect in their papers: they had previously been advised by their trainer.

- Exploration and verification of properties of logarithms and exponentials with GeoGebra. At this point the lack of symbolic capabilities of GeoGebra 2.0 was a determining factor. Students could only check the properties using concrete numerical examples or graphs of functions of x. Around 65% of the group used trial and error to experiment and look for conjectures.

Competency 4

To identify what factors inhibit or stimulate the development of skills of a prospective teacher to teach the logarithmic functions with GeoGebra, they were asked about their motivations, emotional reactions and attitudes towards the use of GeoGebra while solving the proposed modelling assignment. About 50% of the subgroup stated having experienced positive emotional reactions towards using GeoGebra for learning mathematics compared with 20% who stated that they felt insecure and frustrated when solving the task with GeoGebra. The remaining 30% declared that their emotions were varied depending on the type of operation they had to perform.

Regarding the variations in students' motivation to using computers to solve math problems, we have found that over 64% of students show a high interest in computers and found that learning by using them is enjoyable and entertaining (much more than without the use of technology). Besides, they acknowledge that the use of computers gives them more freedom for experimenting with new ideas. A small percentage of the group recognized that experimenting with GeoGebra allowed them to make conjectures and try to provide possible proofs. This statement shows a bit of how they think of the interaction between software and Mathematics. However, 50% of the students in this group believe that the computer restricts their freedom to do math because they do not know how to handle certain operations that are needed: they are blocked to perform further by not having enough GeoGebra usage expertise. Moreover, all found that using GeoGebra was very convenient to help them make connections between algebraic and geometric thinking.

We have found, by analyzing their responses to these questionnaires, that their expectation of success in achieving good interaction between mathematics attitude and GeoGebra software is lower than the motivation for the use of technology.

In summary, the students' beliefs about themselves as future teachers (professional identity), in particular their motivation, emotional reaction, attitudes and task perception, are highly relevant to the way they deal with their present university computer classroom environment and to the future practice of their profession. Together with the professional identity, the view of mathematics is a key element that influences the identification and selection of values to teach mathematics in the technological environment. Students with a clear professional identity as a teacher, based on experience, attribute more values to computer mathematics learning.

Luis: a case study from Profile A

As we have already stated in the Introduction, we are trying to understand how the logic behind the behavior of a student when she/he acquires the strategic knowledge for teaching with technology works. So, we end this section by presenting the comprehensive study of a student who belongs to the highest detected profile. We have analyzed various activities undertaken by this student: personal interview, where he also answers the questionnaires included in subsection 4.2; solving proposed writing activities and exercises with GeoGebra; designing real context activities for high school students that involve exponential and logarithmic functions and providing solutions using GeoGebra and student's performance with the script of this last activity in a classroom with 15 students in grade 11^{th} classroom at a public high school. In fact he was the author of the activity asking for modelling the loudness of MP3 devices which was used with the sub-group the second year.

The average results assigned to the student's performance in the real situation by the three independent reviewers through the questionnaires, combined with his professor's appreciations from the rest of the activities, are summarized in frame 1 below. All items were designed from 1 to 5, 5 meaning the best performance.

In this case we note that the cognitive, pedagogical, technical and personal components are not independent, but they are supported by logical and coherent hierarchies (research Question 3). Luis was chosen as a case study subject because of the clear interaction between his values and his professional identity as teacher and the technological professional development. He reported positive professional experience in formal education at the university and at his secondary school, although he had not acquired any classroom experience as a secondary school teacher yet. He showed a full command of the subject.

Frame 1. Luis' case: competencies and dimensions.

Competency 1. Mathematical content and problem-solving		
Dimension 1	Basic knowledge of Mathematics	4
Dimension 2	Basic knowledge about functions and their graphs	3.3
Dimension 3	Basic knowledge about exponential and logarithmic functions	3.8
Competency 2. Curriculum organization, instructional planning and mathematical modelling		
Dimension 1	Instructional planning: organization	3.8
Dimension 2	Development of teaching and learning process: didactical transposition	3.7
Dimension 3	Mathematical modelling	4
Competency 3. Use of resources and materials		
Dimension 1	Appropriate use of Internet	4.5
Dimension 2	Use of GeoGebra to solve problems	4
Dimension 3	Use of GeoGebra in the classroom and in modelling activities	3.8
Competency 4. Personal and affective component		
Dimension 1	Relationships with Mathematics and technology	3.9
Dimension 2	Verbal and written communication	3.4
Dimension 3	Conviction and security	4.2
Dimension 4	Relationship with audience, presentation and enthusiasm	4.1

Regarding research Question 2 about factors that stimulate or inhibit the development of the skills of a prospective teacher to teach the exponential/logarithmic functions with technology, this case highlights of the positive results from the emotional component and cognitive component of belief about mathematical and technical knowledge and computer. In the personal interview this student indicated:

Luis – The mathematical activity is related to reality and modelling of the structures of nature.

Luis emphasizes the need to learn to teach (stress in the processes of communication) as a priority in his personal and professional development. In his desire to be a teacher points out the importance of the teacher as mediator.

Luis – To become a teacher is my first choice. I like the programme at the university. I like making people better understand the world of mathematics, which is so "black" and complicated for the majority. Since, I understand the great influence that teachers have over our lives, and I would not like to contribute poorly in the education of a person, especially in the stages of primary school and secondary school, I will make sure to become a good teacher.

For the professional development Luis considers the integration of manipulative materials and technological resources as a key aspect.

Luis – We must know what will be the best method to communicate mathematics, considering that this concept has changed over time, from the dogmatic method in which the student is a container where the teacher puts his wisdom, to the heuristic method ... Mathematics is related to physical facts, and make use manipulative and technological resources as we saw in the practice of the logarithmic function is essential.

He indicates he likes mathematics and he likes solving problems looking for different solutions. He thinks that using the computer is amusing. He feels pleased and at ease when he uses the computer to work on mathematics, and he finds it easier than learning mathematics through the traditional methods.

The scenarios proposed (ESCEMMAT) as training activity at university and the teaching practices in secondary school are a source of experiences and opportunities, helping the student in the interactions between modelling, technology and mathematical content. Luis highlights the effect in his personal and professional identity.

Luis – This experience has made me mature, both in the personal and the professional dimension. I think that more weight to the usage of technologies in the training of secondary student teachers should be given, and not just the management and development of relevant skills, but a strategic knowledge for *teaching in the classroom. I had difficulties to choose realistic modelling problems to foster mathematical concepts with GeoGebra. And also another difficult for me was how to guide students. So in addition, I need to improve my competencies to create and design problems with technology and to go deeper in the integration of mathematical concept through technology*.

Conclusions

Integrating technology into Math lessons is a complex issue that has to be addressed from a holistic view that takes into consideration different interrelated components. In this article, we have proposed the study of the development of four components (cognitive, didactic, technical and affective), and the generation of their interactions, working with several groups of prospective teachers during 2 years and conducting a case study with a selected group of them for deeper analyses. In particular, we have presented some findings on the use of multimedia learning scenarios to help undergraduate students (prospective Mathematics teachers) learn how to teach exponential/logarithmic functions with GeoGebra.

As a consequence of a first overall analysis with more than 100 prospective teachers, we are able to provide an answer to the first question posed in the introduction of this article: the four competencies are required for undergraduate students in Math Education programs to teach functions with GeoGebra. Competencies 1 (cognitive component) and 2 (didactic component) are in general easier to acquire for the prospective teacher following regular courses on Methods on Mathematics Education, but Competencies 3 (instrumental component) and 4 (personal-affective component) are shown to be weaker in general in the prospective teachers, so a more comprehensive work needs to be done with the students in order to help them master these components. This first conclusion has made us consider a deeper analysis with a sub-group of 14 students (class group case study) who were singled out for more extensive monitoring. Within the component 2, the mathematical modelling turned to be the most difficult to acquire by the future teachers (more than 50%), so we also included the modelling issue in the deeper studies. About 50% of the subgroup stated having experienced positive emotional reactions towards using GeoGebra for learning mathematics compared with 20% who stated that they felt insecure and frustrated when solving the learning tasks with GeoGebra. The remaining 30% declared that their emotions were varied depending on the type of operation they had to perform.

Regarding the variations in students' motivation to using computers to solve math problems, we have found that over 64% of students show a high interest in computers and found that learning by using them is enjoyable and entertaining (much more than without the use of technology). Besides, they acknowledge that the use of computers gives them more freedom for experimenting with new ideas and make conjectures. However, we underline that a small percentage of this group recognized the capacity of GeoGebra to proof and establish formal demonstration.

Our findings are consistent with results of other researches on the educational uses of technology in which the significance of personal component (motivation and vocation towards the professional dimension, attitudes and beliefs) (FORGASZ, 2006; BENNINSON & GOOS, 2010) and the difficulties that students encounter when they have to mathematical model from realistic situations are highlighted.

Analysis of GeoGebra incorporated modeling tasks deriving from the written assignments made by the participants, showed different interrelations among mathematical content, technology, and modelling competencies. In particular, the study highlights the need to consider the transitions between those phases in the modelling processes in which the students have acquired less competencies and have more difficulties. Students' difficulties and mental blocks appear in the articulation of the links between mathematical modelling competencies (in our case exponential and logarithmic functions) and use the technology when solving these modelling problems.

Our analysis has shown that in order to successfully teach Mathematics in technological contexts it is necessary to master all the competencies established in the theoretical framework. This expertise has turned to be essential to improve the strategic knowledge of learning how to teach with technology with a focus on the transitions between phases in the modelling process and instrumental genesis with the software (GALBRAITH *et al*, 2007).

The project on learning exponential/logarithmic functions using GeoGebra showed that the technical skills required to use computers to solve a given task, were closely related to a conceptual understanding of the mathematics behind the technique. Developing computer techniques and instrumentation schemes proved to be difficult for the teacher students. Overcoming these difficulties often involved a conceptual development that was relevant for our objectives. The joint development of technical skills and conceptual understanding is seen as a support for an instrumental approach to using ICT tools.

Another important aspect of the use of technology is the level of confidence, motivation and easiness that teacher students believe they possess in accessing and using technology. The existence of qualitatively different levels of confidence, motivation and expertise (see the three profiles described in Section 4) clearly proves the need for introducing mathematical modelling with GeoGebra at pre-service teacher training.

To conclude, from the holistic perspective chosen, we can say that the teaching experiment helped prospective teachers develop their professional knowledge, in particular *strategic knowledge*, to teach by means of technology in three areas: their conceptions about using interactive geometry software (ICT) in mathematics teaching, the impact of future working methodologies (realistic situations) on them as future teacher, and the development of their professional identity. The teacher students' responses show aspects of this latter development, especially as they assume new perspectives and values that they relate to their future professional role. The development of a professional identity as a mathematics teacher involves, among many other things, the biographical process of establishing a personal relation with ICT and the development of perspectives about the mathematics teacher's role regarding this technology (strategic knowledge). Many of the participants claim that they have developed new perspectives on how learning takes places, emphasizing discovery learning versus transmissive teaching. These experiences led participants to anticipate their future role as teachers and to relate this knowledge to what they think is happening in schools. These statements indicate aspects of pre-service teacher's biographical identity defining processes, involving transactions between inherited and envisioned identities, as they reflect upon past ideas and conceptions and start showing appreciations regarding what their future work as mathematics teachers will be.

With respect to the design research methodology, we consider that the teaching experiment (scenarios) has been a useful instrument in all phases of design research. During the design phase, it is the theoretically grounded vision of the learning process that is specified for concrete instructional activities. During the teaching experiments, it is the Competency-based learning models, guide observations and data collection that offer a framework for educational decisions during the teaching experiments.

In the light of our findings, we have identified the need for a larger study increasing the duration of the stages, the size of the population and its homogeneity, as well as the number of addressed topics, in order to achieve a better validation of the proposed model. This would also be crucial for the complex task of thoroughly specifying the levels, the descriptors and the indicators of all the competencies involved in teaching and learning Mathematics with technology.

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