

# Peirce's convergence theory of truth redux

## *Revisitação da teoria convergente da verdade de Peirce*

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**Abstract:** Peirce's convergence theory of truth is an intuitive and reasonable account of truth. In its most general sense, it links truth to the results of inquiry. In accord with the pragmatic maxim, Peirce realized that the practical consequences of true claims are that they tend to bring inquiries to fruition and settle opinion. Nonetheless, Peirce's theory of truth is much maligned and misunderstood. It is argued here that once it is understood that the convergence theory is an inference and generalization from the remarkable mathematical theorem known as the Law of Large Numbers, and how that theorem provides mathematical certainty to induction as the core of scientific method, many of the problems go away. Part of the misunderstanding is also due to the fact that Peirce had three different senses of convergence, which many commentators mix up or misinterpret. With this understanding, it will be shown that, contrary to the analyses by Cheryl Misak and Christopher Hookway, Peirce argues that persistent inquiry by good methods is not merely a regulative ideal for attaining truth—an intellectual hope—but a mathematically proven possibility that has already resulted in established truths.

**Keywords:** Convergence theory of truth. Peirce.

**Resumo:** *A teoria convergente da verdade de Peirce é uma abordagem intuitiva e razoável da verdade. No seu sentido mais geral, vincula a verdade aos resultados da investigação. De acordo com a máxima pragmática, Peirce percebeu que as consequências práticas de afirmações verdadeiras são aquelas que tendem a trazer as investigações à fruição e opinião estabelecida. No entanto, a teoria da verdade de Peirce é muitas vezes difamada e mal-entendida. É argumentado aqui que uma vez que se entende que a teoria convergente é uma inferência e generalização oriunda do notável teorema matemático conhecido como a Lei dos Grandes Números e como o dito teorema fornece certeza matemática à indução como cerne do método científico, muitos dos problemas desaparecem. Parte deste mal-entendido também se deve ao fato de que Peirce tinha três sentidos distintos de convergência, os quais muitos comentadores confundem ou interpretam erroneamente. Com este entendimento, será mostrado que ao contrário das análises de Cheryl Misak e Christopher Hookway, Peirce argumenta que investigação persistente por bons métodos não é apenas um ideal regulador para atingir a verdade – uma esperança intelectual – mas uma possibilidade comprovada matematicamente que já resultou em verdades estabelecidas.*

**Palavras-Chave:** *Teoria Convergente da verdade. Peirce.*

## 1 Introduction

Peirce's convergence theory of truth is an intuitive and reasonable account of truth. In its most general sense, it links truth to the results of inquiry and, in typical pragmatic fashion, defines truth in terms of what it does, rather than what it is. Peirce realized that the properties of true claims is that they tend to bring inquiries to fruition and settle opinion. Thus, when the results of inquiries done with good methods achieve a high level of agreement among competent inquirers, that is a good indication that the opinion considered in the inquiry is true. This describes fairly well what goes on in the history of science, which Peirce, among many others, think is the exemplar of inquiry.

Nonetheless, Peirce's theory of truth is much maligned and misunderstood. Peirce tended to over-dramatize some of the early formulations in terms of opinions that were "fated" and results of inquiries that were "destined", and he had to backtrack on some of these claims. But, once it is understood that the convergence theory is an inference and generalization from the remarkable mathematical theorem known as the Law of Large Numbers, and how that theorem provides mathematical certainty to induction as the core of scientific method, many of the problems go away.

The Law of Large Numbers is an intuitive, well-established theorem at the foundation of all statistical theory. The law was first proved by Jacob Bernoulli in 1713, expanded by Siméon Poisson in 1837, followed by refinements in the 20<sup>th</sup> century by Emil Borel (1909), among others. Simply stated, it expresses a very intuitive concept, namely, that if an inquiry is attempting to determine the proportion of a variable in a population, as the number of samples of that population increases, the mean of those samples will approach the true proportion of that variable in the population. Said more conveniently, as the samples increase, the mean of the samples will converge to the true value in the population. Or said a bit more cautiously, as the samples increase, the difference between the mean value of the sample and true value of the variable in the population will diminish, that is, will *approximate* to the true value. As Jacob Bernoulli noted in a letter to Gottfried Leibniz that "even the stupidest man knows by some instinct of nature *per se* and by no previous instruction, that the greater the number of confirming observations, the surer the conjecture" (BERNOULLI, 2005, ch. 4). The French mathematician, Émile Borel, proved the Strong Law of Large Numbers in 1909, which pertains specifically to inductive sampling. Whereas the weak law of Bernoulli and Poisson cannot guarantee that, even after a large number of samples, the mean of the samples will continue to converge toward the mean of the population without aberrations, Borel's strong law does show this. As Peirce explains this simply:

As we go on drawing inference after inference of the given kind, during the first ten or hundred cases, the ratio of successes may be expected to show some considerable fluctuations; but when

we come into the thousands and millions, these fluctuations become less and less; and if we continue long enough the ratio will approximate towards a fixed limit (CP 2.650, 1878).

Peirce gives an intuitive example of this in a rather long passage, concerning a hypothesis about the distance of an observed fire:

Let the second man, having seen the fire, ask "Would you say, now, that that fire was about three miles away?" This virtually suggests that if the first man or any other man will fill his purse, and take ship, and go to Westminster, and break into the houses of Parliament, and bring away the standard yard, and lay it down repeatedly on the ground from where the two stand to where the fire is, and utter the cardinal numbers in their order as the successive layings down proceed, or if he will perform any other experiment virtually amounting to that, then the last number uttered might be 5280, and if it should prove to be a number near to that, he might not be surprised. Extensive experience leads us to expect that if an experiment virtually amounting to that were tried a hundred times, different numbers would be obtained which would cluster about one of them, and that among a million trials the clustering would be still more marked, according to a law well-known to mathematicians. It is possible, no doubt, that if our experience were still more extensive, we should find that if the experiment were tried, say, more than a billion times, then a new phenomenon would emerge and the oftener it was tried the less marked might grow the clustering. Our hope, however, in endeavoring to make a measurement extremely precise, is that there is a certain value toward which the resultant of all the experiments would approximate more and more, without limitation. Having that hope [...] whenever we endeavor to state the distance, all that we aim at is to state as nearly as possible what that ultimate result of experience would be. We do not aim at anything quite beyond experience, but only at the limiting result toward which all experience will approximate,—or, at any rate, would approximate, were the inquiry to be prosecuted without cessation (CP 8.112, 1900).

The "law well known to mathematicians" to which Peirce refers is of course the Law of Large Numbers.

Since, as Peirce argues, all ampliative reasoning is from sampling, or inference from part to whole, and reasoning is the means by which truth is attained, then truth rests on such sampling and, thus, on the Law of Large Numbers (CP 6.40, 1892; CP 5.346, 1868; CP 5.352, 1868), which he sometimes called "the law of high numbers" (CP 7.221, 1901). "All positive reasoning," as Peirce says, is "of the nature of judging the proportion of something in a whole collection by the proportion found in a sample" (CP 1.141, 1897). "Judging of the statistical composition of a whole lot from a sample is judging by a method which will be right on the average in the long run..." (CP 1.93, 1896).

Given this, it seems odd to say, as Cheryl Misak does, that scholars should “stay clear of identifying his account of inductive inference with his account of truth” (MISAK, 1991, p. 119). If anything, the convergence theory of truth falls out of induction. Peirce sees the scientific method as the best method for “fixing belief” in the long run and, as Peirce sees it, abduction, deduction and induction are the core reasoning processes involved in such a method. Induction plays the primary role of detecting any error in a hypothesis and, in that regard, it is self-correcting.

Nor must we lose sight of the constant tendency of the inductive process to correct itself. This is of its essence. This is the marvel of it. The probability of its conclusion only consists in the fact that if the true value of the ratio sought has not been reached, an extension of the inductive process will lead to a closer approximation” (CP 2.279).

Thus, induction would be the best means for assuaging doubt. Peirce makes this link between induction and his theory of truth very clear. He argues that induction, as a “method persistently applied to the problem must in the long run produce a convergence (though irregular) to the truth; for the truth of a theory consists very large in this, that every perceptual deduction from it is verified” (CP 2.775, 1902).

As Peirce argues, since inductions test hypotheses primarily through methods of sampling to observe predicted or expected outcomes, and such a method is *mathematically certain* thanks to the Law of Large Numbers, sampling over time *would*, in principle, sort out true from false hypotheses. However, whether inquiries *actually* succeed in sorting out true from false hypotheses is a matter of historical contingency, and there are some matters which inquiries cannot possibly solve. Nonetheless, many inquiries have resulted in “established truths,” where continued results of inquiry are stable, and agreement of opinion is manifest. Because of the mathematical certainty of induction, and the existence of established truths, there is good reason to *hope* that future inquiries will resolve in true claims over time. Given this linkage of possibility, hope and actuality, Peirce often uses formulations of the convergence theory of truth that expresses one or the other of these ideas, and this adds to some of the confusion and complaints about his theory.

Because of this reasoning from mathematical certainty as proving possibility that inquiry will approximate to the truth, the existence of established truths, and a justified hope for future inquiries, the *complete* convergence theory of truth has three modalities. Sometimes it is expressed in the *subjunctive* modality, which emphasizes the *possibility* that inquiry *would* converge to the truth. Peirce claims that, in fact, many inquiries have been successful and have converged on the truth, so that the claim that inquiries *will* in the long run converge to the truth is a plausible empirical claim. In that sense, the convergence theory is often expressed in an indicative modality. Given the proof of possibility and existing established truths, there is a justification for the *hope* than any inquiry *will* converge to the truth, so that inquirers may act *as if* that were so—which serves as a regulative ideal, in Kant’s sense, for any future inquiries.

## 2 Three Senses of Convergence

Peirce also employs three different senses of convergence, which often get mixed up in commentator's analysis and can cause misunderstandings. The first is convergence understood as *approximation* to the truth. The second is convergence as the *destined result* of inquiry, that all paths of inquiry eventually lead to the same result. The third sense is in terms of a convergence of opinion, a *consensus* among inquirers as to whether the results of inquiry are sufficient to count the opinion as true.

In the first sense, Peirce argues that inquiries using the scientific method, specifically induction, *approximate* to the truth. This should be considered as the most fundamental sense of convergence, since it makes the other two possible. Some commentators, such as Willard Quine, think this amounts to a verisimilitude theory of truth. However, this is not the case. Such theories claim that hypotheses or theories can be ranked in terms of more truth-likeness than others. It may also imply that, as inquiry proceeds over time, hypotheses and theories are progressive in this sense. As Quine argues, "... there is a faulty use of numerical analogy in speaking of a limit of theories, since the notion of limit depends on that of 'nearer than' which is defined for numbers and not for theories" (1960, p. 23).

Verisimilitude is difficult to defend. Formulating a criterion for deciding whether one theory is truer than another, as Karl Popper (1963) attempted, is notoriously problematic, since it can't decide among false theories which is closer to the truth. Verisimilitude also lends itself to the idea of linear progression in science when, as Thomas Kuhn showed, it is often punctuated by paradigm shifts—something which Peirce also suggested (KUHN, 1962; CP 6.17, 1891).

However, Quine's account is a serious misunderstanding of Peirce. As Peirce formulates it, approximation is not approximation in the sense that hypotheses or theories come closer to the truth, but for *the method of their testing* that does, namely, induction. All of the textual references to "approximation to the truth" are in the context of induction. As an avowed frequentist and anti-Bayesian (which Peirce calls the "doctrine of inverse chances"), he would never assign probabilities to hypotheses:

The theory here proposed does not assign any probability to the inductive or hypothetic conclusion, in the sense of undertaking to say how frequently *that conclusion* would be found true [...]. The theory here presented only says how frequently, in this universe, the special form of induction of hypothesis would lead us right. The probability given by this theory is in every way different—in meaning, numerical value, and form—from that of those who would apply to ampliative inference the doctrine of inverse chances (CP 2.748, 1878).

As Peirce explains, "the validity of an inductive argument consists [...] in the fact that it pursues a method which, if duly persisted in, must, in the very nature of things, lead to a result indefinitely approximating to the truth in the long run" (CP 2.781, 1902; see also CP 5.170, 1903; CP 6.40, 1892). Peirce also formulates this in semiotic terms:

An Induction is a method of forming Dicot Symbols concerning

a definite question, of which method the Interpretant does not represent that from true premisses it will yield approximately true results in the majority of instances in the long run of experience, but does represent that if this method be persisted in, it will in the long run yield the truth, or an indefinite approximation to the truth, in regard to every question (CP 2.269, c. 1897).

Thus, it is quite possible, *à la* Kuhn, that induction could throw out a whole line of hypotheses and theories if inductive testing warrants it. As will be shown, approximation will mean different things, depending on whether the type of induction involved is, on Peirce's terms, *quantitative* or *qualitative*.

In her spirited defense of Peirce against Quine, Cheryl Misak's misses this point, mostly due to the fact that she adopts the consensus sense of convergence, rather than Peirce's considered sense of it as approximation. She argues that because Peirce thought of consensus of inquirers as the core of his theory, and that consensus is not approximation to a limit, that Peirce avoids Quine's criticism. But there's no reason one couldn't think of consensus as a kind of convergence that approaches a limit (1991, p. 122). As Peirce says in this context of inquiry, differences of belief over time "[...] become indefinitely small" (R 408, p. 147, c. 1893).

For Peirce, verisimilitude, instead, is a consideration in abduction, not induction, as one factor, including plausibility and probability, in determining whether a particular hypothesis is worth testing (CP 2.662, 1878). He defines verisimilitude as "that kind of recommendation of a proposition which consists in evidence which is insufficient because there is not enough of it, but which will amount to proof if that evidence which is not yet examined continues to be of the same virtue as that already examined..." (CP 8.224, c. 1910).

Whereas the first sense of convergence focuses on the *method* of inquiry, the second focuses on the *results of inquiry*. It is articulated in Peirce's review of Berkeley's works, and proposes that any inquiry on the same matter, over time, should reach the same conclusion, no matter the inquirer. Peirce also gives a version of this earlier in *How to Make Our Ideas Clear*:

One man may investigate the velocity of light by studying the transits of Venus and the aberration of the stars; another by the oppositions of Mars and the eclipses of Jupiter's satellites; a third by the method of Fizeau; a fourth by that of Foucault; a fifth by the motions of the curves of Lissajoux; a sixth, a seventh, an eighth, and a ninth, may follow the different methods of comparing the measures of statical and dynamical electricity. They may at first obtain different results, but, as each perfects his method and his processes, the results are found to move steadily together toward a destined centre. So with all scientific research. Different minds may set out with the most antagonistic view, but the progress of investigation carries them by a force outside of themselves to one and the same conclusion (CP 5.407, 1878).

This sense of convergence is similar to what Robert Merton calls multiple

discoveries, the case where similar discoveries are made by scientists working independently of one another. The most famous case is the contemporaneous discovery of the calculus by Newton and Leibniz. He argued that rather being the exception, this turns out to be a very common pattern:

The pages of the history of science record thousands of instances of similar discoveries having been made by scientists working independently of one another. Sometimes the discoveries are simultaneous or almost so; sometimes a scientist will make anew a discovery which, unknown to him, somebody else had made years before. Such occurrences suggest that discoveries become virtually inevitable when prerequisite kinds of knowledge and tools accumulate in man's cultural store and when the attention of an appreciable number of investigators becomes. Not only does this account for the many cases of contemporaneous but independent discoveries of similar matters, such as the calculus, but it covers the more mundane claim that independent repetitions of the same experiment with similar results tend to confirm a hypothesis over time (1971, p. 371).

This sense of convergence also fits the more common practice of experimental replication of results. The very nature of scientific experimentation makes it reproducible and, therefore, any qualified scientist could, in principle, reproduce the results of an experiment (or not), independently of the original investigator. Replicating experiments is the surest way to detect error or fraud, as the famous case of Pons and Fleishmann's theory of cold fusion showed (GILET and THANUKOS, 2018).

More broadly, Peirce suggests in this sense of convergence that there is a kind of drift in human inquiries that leads them to the same conclusions concerning the same matter. All paths of inquiry on the same matter converge on the same destination:

[...] human opinion universally tends in the long run to a definite form, which is the truth. Let any human being have enough information and exert enough thought upon any question, and the result will be that he will arrive at a certain definite conclusion, which is the same that any other mind will reach under sufficiently favorable circumstances (CP 8.12, 1871).

This idea has a parallel with the theory of convergent evolution, as proposed by Simon Conway Morris (2003) and Richard Dawkins (1996), among others. Like Peirce, Morris defines it simply as the tendency for evolutionary mechanisms to arrive at similar solutions to similar problems (2003, p. xii). Convergent evolution occurs when unrelated species nonetheless evolve similar functional solutions that are nicely adaptive. Eye and wing evolution are thought to be classic examples of such convergences. According to M. Land and D. Nilsson (2002), eyes have evolved independently in as many as a 100 cases. The camera eye developed independently in cephalopods such as the squid, mammals, and cnidarians, such as box jellies. Morris provides many examples of convergence as he says, "on the ground, above

the ground, below the ground,” sufficient to claim that “convergence is pervasive” (2003, p. 134).

Peirce states this version of convergence somewhat dramatically, but recognizes its qualifications nonetheless. He notes in the passages quoted that inquirers would need “enough information” and sufficient “exertion of thought.” But, of course, the fact is that some people are better thinkers, scientifically trained, knowledgeable in a certain discipline and, therefore, more *competent* than others in this regard. After all, it was Leibniz and Newton that both discovered the calculus independent of one another, not just anyone. It should be added that employing good methods of inquiry would also be a factor. Christopher Hookway proposes an account of Peirce’s convergence theory of truth in this sense of convergence with some of these qualifications:

If it is true that p, then anyone who inquired into the question whether p long enough and well enough (using good methods of inquiry) would eventually reach a stable belief that p which would not be disturbed by further evidence or investigation (2002, p. 49).

Notice that Hookway expresses this version subjunctively, as a ‘would be’ rather than indicatively as a ‘will be’ as Peirce does here. The distinction between Peirce’s subjunctive and indicative versions of the convergence theory become an important consideration as already noted.

The third sense of convergence is expressed as convergence in agreement of investigators’ opinions about some hypothesis or theory. The classic source of this sense of convergence is found in *How to Make Our Ideas Clear*: “The opinion which is fated to be ultimately agreed to by all who investigate, is what we mean by the truth [...]” (CP 5.407, 1877). This is often more positively stated as “consensus” and “unanimity” of belief (CP 6.610, 1893), the “settlement of opinion” or the “fixation of belief.” (CP 8.41, 1885). As he says succinctly, “[...] human inquiries [...] tend toward the settlement of disputes and ultimate agreement in definite conclusions [...]” (CP 8.41, 1885). In sum, “[...] there is a general **drift** in the history of human thought which will lead it to one general agreement, one catholic consent” (CP 8.12, 1871). Said more negatively, the differences among beliefs on a matter of inquiry “[...] become indefinitely small” (R 408, p. 147, c. 1893). Peirce writes optimistically that there is “[...] to every question a true answer, a final conclusion, to which the opinion of every man is constantly gravitating.” Even if an individual may die before the truth is reached “[...] there is a definite opinion to which the mind of man is, on the whole and in the long run, tending” (CP 8.12, 1871).

This sense of convergence is the source of many of the misconceptions about Peirce’s theory of truth. It should be noted that Peirce is articulating here, in *How to Make Our Ideas Clear*, the *meaning* of truth in accord with the pragmatic maxim, that the clarification of the concept of truth is best accomplished in terms of its practical consequences, the principal one being that it would lead to a consensus of opinion.

Some scholars, such as Cheryl Misak, use this as the principal sense of convergence in her explication of Peirce’s convergence theory of truth (MISAK,

1991, p. ix). She ignores the sense of it as approximation to the truth and in fact, surprisingly, claims that “Peirce’s account of truth does not follow from his account of induction” (1991, p. 119). That is somewhat true since it does not follow directly from induction but, as argued here, it does follow from the Law Of Large Numbers, which grounds induction.

The sense of convergence as consensus of opinion needs careful unpacking in a way that is consistent with the other senses of convergence, particularly the sense of approximation to the truth. There are a number of matters to consider in this account. First, in terms of the wording, notice that Peirce does not use the term ‘belief’ in this formulation, only ‘opinion’—and there is a reason for that. Peirce is careful to distinguish the two:

[...] belief, that is, the adoption of a proposition as a *kléma es aei* [a possession for ever] [...] has no place in science at all. We *believe* the proposition we are ready to act upon. *Full belief* is willingness to act upon that proposition in vital crises, *opinion* is willingness to act upon it in relatively insignificant affairs. But pure science has nothing at all to do with *action*. The propositions it accepts, it merely writes in the list of premises it proposes to use [...]. Its accepted propositions, therefore, are but opinions at most... (CP 1.635, 1898).

Both Misak and Hookway formulate Peirce’s convergence theory in terms of belief of the inquirers rather than opinion, and this can lead to some confusion about the relation between inquiry and the consensus of opinion, as will be explained (MISAK, 1991, p. ix; HOOKWAY, 2002, p. 49).

Second, given what he means by opinion, what does he mean by the “fated” opinion? Something that is fated is an inevitable outcome of action or thought. “Fate means merely that which is sure to come true, and can no how be avoided” (CP 5.407 n1, 1878). It suggests in this context that whoever inquires into the matter will come to the same opinion. This is consistent with the second version of convergence. Given his realism, Peirce is certainly not a constructionist, and wouldn’t say that something is true merely because inquirers agree to it. Thus, there must be a link between the result of the inquiry (through inductive testing) and the agreement to the opinion expressed in the hypothesis being tested. As Peirce says, those opinions are “alone [...] the result of investigation carried sufficiently far [...]” (CP 5.408, 1878). Since the results of inquiry (by good methods of induction) approximate to the truth, then the claim would appear to be that opinions should converge as the results of inquiry approximate more and more to the truth. The reason for holding the opinion is the result of the inquiry. The results of the inquiry must make the opinion inevitable.

Both opinions and beliefs are illocutionary, but have different illocutionary force, as Peirce explains it. As illocutionary, they are attitudes toward certain propositions. In Peirce’s terms, holding to an opinion about a certain proposition is to have sufficient lack of doubt about it, enough to claim it as a premise in further investigations (CP 1.635, 1898). That lack of doubt is gained through the results of inductive testing, specifically through its ability to approximate to the truth. So

presumably, the results have so closely approximated to the truth as to make holding the opinion compelling. The result of the inquiry will serve as a reason for accepting the opinion, but reasons act differently than causes, and it's quite possible that there are good reasons to accept an opinion, but the opinion is not accepted for reasons other than good scientific reasons. Peirce notes this as well. There are people who, despite evidence, are unwilling to reverse their beliefs, although he suggests that the characteristic of changing one's belief in light of remarkable evidence marks the characteristic of a "sane" man (R 673: 11, 1911).

There are plenty of cases of quite competent and reputable scientists, who for all sorts of reasons, rejected hypotheses that, as far as can be determined, are settled as true among scientists in the long run. To use an example from Peirce's own time, Louis Agassiz, the greatest biologist of his generation, rejected Darwin's theory, although his students did not—but they also believed that Lamarckian evolution was part of the picture, as did Peirce. Even when, in the 1880s, August Weismann's "germ plasm" theory disproved the claim that acquired traits could be inherited, Peirce still believed that Lamarckian forms of evolution were part of biological evolution (CP 1.105, 1903; CP 6.298, 1893). Louis Agassiz was a deeply religious man, who was vested in the notion of intelligent design and what is now called creationism, and this could be a good explanation of why he refused to agree to Darwin's theory. Why people will adopt a theory or not is not a direct causal relation from the evidence, but an ethical stance. If the premises are true and the inference valid, then one ought to accept the conclusion. If there is sufficient inductive evidence for a claim, then one ought not to doubt that claim. This is why Peirce counts logic as a normative science, since it is how people *ought* to reason, not how they in fact do.

However, over the course of some 150 years, there has been enough inquiries and modifications of Darwin's theory to result in a near catholic agreement among scientists in the relevant disciplines that evolution of the Darwinian type explains biological evolution as opposed to Lamarckian types and that, certainly, evolution is the superior theory to creationism. So, even though there may be resistance to new theories that prove true in the long run, inquirers do seem to drift to an opinion that is "fated" to be. Thus, Peirce is wise in the *How to Make Our Ideas Clear* formulation to put in the proviso that the agreement is the "ultimate" agreement. In this regard, he often calls this ultimate agreement the *final* opinion which, unfortunately, also causes some misunderstandings.

Bertrand Russell interpreted the "final" opinion as the last opinion of the last inquirers on earth (1939). As Peirce explains, it could be for a number of "perverse" reasons that inquiries into certain propositions are never completed, and it's quite possible that the last people on earth are mistaken about their opinions (CP 5.408, 1878). 'Final' opinion can mean the very last opinion, whatever that might be, or it can mean, one that there is no longer any reason to doubt, and further inquiries no longer are warranted since they prove nothing different. Russell thinks of it as the former, but Peirce thinks of it as the latter.

A third consideration in the *How to Make Our Ideas Clear* formulation raises issues of who qualifies as an inquirer, and which agreement among inquirers counts? Since Peirce is using the language of opinion rather than belief, it can be assumed that the inquirers are scientists, inquirers competent in performing the inquiry—not

just any inquirers. As Peirce says, "... in science a question is not regarded as settled or its solution as certain until all intelligent and informed doubt has ceased and all *competent* persons have come to a catholic agreement..." (CP 1.32, 1903, *emphasis added*). He continues,

The man of science attaches positive value to the opinion of every man as competent as himself, so that he cannot but have a doubt of a conclusion which he would adopt were it not that a competent man opposes it; but on the other hand, he will regard a sufficient divergence from the convictions of the great body of scientific men as tending of itself to argue incompetence, and he will generally attach little weight to the opinions of men who have long been dead and were ignorant of much that has been since discovered which bears upon the question in hand.

In this context, he contrasts the "man of science" with "metaphysicians", such that "fifty" of them, "each holding opinions that no one of the other forty-nine can admit, will nevertheless generally regard their fifty opposite opinions as more certain than that the sun will rise tomorrow" (CP 1.32, 1903).

Despite the fact that inquiry into the theory of evolution has gone as fruitfully as it could reasonably go, even though there is a near consensus among scientists in the relevant disciplines about the theory of evolution, only 60% of Americans believe in the theory (GROSS, 2015). It's the case that 97% of scientists agree that climate change is real and that human activity is a large cause of it, while only 70% of the U.S. population does (MEYER, 2019). But, how many of the American populace have really inquired into the matter of evolution? In that case, it's not clear how many of the 60% agree on the basis of the authority of science, rather than their own investigations and, conversely, how many of the 40% are stuck in beliefs in creationism or some other alternative, based on religious authority. Did Peirce mean to say that the agreement to an opinion is among those who inquire, not everyone? It would appear so: "the opinion which is fated to be ultimately agreed to *by all who investigate*, is what we mean by the truth [...]" (CP 5.407, 1878, *emphasis added*). Even if there are some among the general population who make inquiries into these issues, are they sufficiently competent to assess the evidence provided by creationists or climate change-deniers?

### 3 Three formulations of the convergence theory of truth

Peirce came to realize that there were three important questions in regard to truth: Is it possible for inquiry to converge to the truth? If possible, will inquiries converge to the truth? If possible, what can be hoped for through inquiry? These questions are somewhat reminiscent of Kant's fundamental questions of philosophy: What can I know, what ought I to do, what can I hope for, and what is man? (p. 29, 1800). The convergence theory of truth answers each of Peirce's three questions, expressed in different modalities: subjunctively, regulatively and indicatively. Peirce claims at different times and different places that inquiry, specifically by means of induction, *would* converge to the truth in the long run. He also claims that inquiries, if persisted in with good methods, *will* converge to the truth in the long run and, in fact, have

already done so. Finally, he often claims that because convergence to the truth is possible and inquiries have so succeeded in many cases, then inquirers have a justified hope that any of their inquiries will converge to the truth—so they should *suppose* that it will succeed, and act *as if* their inquiries will be successful.

#### 4 The possibility of convergence to the truth

The Law of Large Numbers answers the first question of possibility. If an inquiry *were* to use the method of induction, and sampling *were* to be sufficiently persisted in, then the result *would be* an approximation to the truth over time. The Law of Large Numbers as a mathematical proof is hypothetical and, thus, formulated as a subjunctive—it shows what *would* follow by necessity from the premises, not that premises will actually happen, since there may be countless contingencies that would prevent them from happening. In this sense, it is a proof of possibility, and not a matter of prediction. Peirce makes this clear in his Carnegie Foundation application in 1902, that the method of induction, upon which the truth-getting aspect of the scientific method rests, is *mathematically certain*. He writes in the application that,

It will be shown to be mathematically impossible that induction indefinitely persisted in should ultimately lead to a false conclusion in any case whatsoever, whether there be any definite probability or not, whether there be any real universe or not, whether the universe be presided over by a malign power bent upon making inductions go wrong or not. Such things might prevent inductions from being drawn, but they could not make them go ultimately wrong if they were rightly conducted and sufficiently persisted in" (L75, 1902, p. 268-270, Draft D).

The mathematical proof that Peirce refers to here, of course, is the well-established Law of Large Numbers.

To give a somewhat simplified, fanciful example of the difference between the subjunctive, *would be* claims of mathematical proofs, and the factual, *will be* claims of historical actuality, consider an inquiry into the square footage of an enormously large rectangular space. To make this a little fun, suppose that the area is heaven as described in *Revelations*. Suppose that heaven-bound engineers are assigned to its measurement. Given the size, engineers could spend an eternity measuring its dimensions. Suppose the only way to actually confirm the square footage is to measure each dimension of the area centimeter by centimeter, a laborious process that, combined with the number of independent measurements needed to get an accurate measurement would require a big chunk of eternity. *Would* such an inquiry be successful in the long run?

It would certainly be possible, since the calculation of the area is mathematically certain, and we would only need to figure the length and width to find the square footage by inductive sampling, and a very large number of samples of measurements would get a close approximation to the actual length and width of the area. Is there reason to *hope* that it will be successful? Yes, because it is possible. *Will* it be

successful? It may or may not. Assuming that the heavenly engineers have all the time in the world, and have nothing to prevent them from accomplishing the task, it is inductively certain that the inquiry would converge toward a certain number. But, perhaps they petition to stop the project since it is spoiling their stay in heaven, or suppose that they are now directed to another project, or there could be just a number of things that prevent the successful accomplishment of the task.

It should be stressed that the convergence theory of truth is *mathematically* certain, although not *absolutely* certain. True to his fallibilism, Peirce insists that there can be no absolute certainty. "The only safety is to say that man is incapable of absolute certainty" (CP 7.108, c. 1910). Although "mathematical certainty is not absolute certainty" (CP 4.478, c. 1903), it is of a higher level of certainty than that achieved by any empirical (positive) science. Mathematical certainty rests on the idea of the certainty of hypotheticals, captured by Benjamin Peirce's famous definition of mathematics as "that which draws necessary conclusions," that is, draws necessary conclusions from hypotheses or postulates (CP 3.558, 1898). The Law of Large Numbers provides mathematical proof for the best method of attaining a convergence toward truth in inquiry—namely, induction, as part of the three-part reasoning process in science, including abduction and deduction.

Because of the mathematical certainty that the Law of Large Numbers provides for inductive sampling, Peirce claims that many scientific hypotheses are already *inductively certain* and work as "established truths," such that inquirers are prepared to act upon these hypotheses with what he calls a *practical certainty*. In other words, there are degrees of certainty. Although induction is mathematically certain to approximate toward the truth, it only provides, as such, approximations. But these approximations are often so believable, that people will act on them with practical certainty, *as if they were true*. It is this practical certainty, the fact that science does deliver the epistemic goods and demonstrations of success, that buoys the continued practice of inquiry. These notions of certainty, as paradoxical as it sounds, work hand in hand with Peirce's fallibilism. By the fact that fallibilists do not hold anything absolutely certain, but allow anything to be questioned and continued to be questioned, they become more certain of what they do not continue to doubt. Put somewhat differently, if inquirers cannot, through rigorous tests, find any error in the hypothesis, sufficient to reject it, then there is all the more reason not to doubt it.

*Inductive certainty* is something that has a likelihood of 1 which, technically, statisticians call "almost certainly" (CP 6.474, 1908). But, Peirce says, "of course there is a difference between probability 1 and absolute certainty" (CP 7.214, 1901). *Practical certainty* is the certainty found when induction has gathered together a sufficient amount of evidence about a belief or hypothesis such that people are willing to act on that belief. "But the kind of reasoning which creates likelihoods by virtue of observations may render a likelihood practically certain—as certain as that a stone let loose from the clutch will, under circumstances not obviously exceptional, fall to the ground—and this conclusion may be that under a certain general condition, easily verified, a certain actuality will be probable, that is to say, will come to pass once in so often in the long run" (CP 2.664, 1878). Would anyone in their right mind be willing to lay underneath a two ton block held up by a crane, and let it be released on the chance that, this time, the law of gravity will fail? Peirce

concludes that “[...] practically, we know that questions do generally get settled in time, when they come to be scientifically investigated; and this is practically and pragmatically enough” (CP 5.494, c. 1906).

The Law of Large Numbers makes persistent sampling of induction mathematically certain. However, as is well-known, Peirce divides induction into three types: crude, quantitative and qualitative, each of which has degrees of strength in confirming a hypothesis. So does the Law of Large Numbers serve as a basis for all three types? Hypotheses based on crude induction approximate to the truth on the fact that no observable consequences has yet refuted it (CP 2.756-757, 1905). As examples, Peirce notes that the earth has always turned on its axis so far about once every 24 hours, from which we conclude that it will do so in the future. In every case, every human being has been born of a woman (CP 8.237, c. 1910).

*Quantitative induction* is the strongest form of induction, in that it predesignates a certain outcome and uses a “fair” sample in order to ascertain whether the proportion of the variable in the sample is the proportion in the target population, thus it is most aligned with the Law of Large Numbers (CP 2.758, 1905). Such a type of induction approximates to the truth in the following way:

[...] when we say that a certain ratio will have a certain value [V] in “the long run,” we refer to the *probability-limit* of an endless succession of fractional values; that is, to the only possible value from 0 to  $\infty$ , inclusive, about which the values of the endless succession will never cease to oscillate; so that, no matter what place in the succession you may choose, there will follow both values above the probability-limit and values below it; which if V be any other possible value from 0 to  $\infty$  [...] there will be some place in the succession beyond which all the values [...] will agree, either in all being greater than V, or else in all being less (CP 2.758, 1905).

In other words, by the Law of Large Numbers, sufficient fair sampling will tend to show one of two things: that the mean of the sample approximates the prediction of the ratio of the variables, or, it refutes it by exhibiting a mean way below or above the predicted ratio (the “probability-limit”).

Peirce calls the third type of induction, *qualitative*. It consists in deducing as many conditional predictions from the hypothesis as practicable. This method of induction approximates to the truth by putting to the test as many as these conditional predictions as possible. It then “goes on to judge of the combined value of the evidence, and to decide whether the hypothesis should be regarded as proved, or as well on the way toward being proved, or as unworthy of further attention, or whether it ought to receive a definite modification [...]” (CP 2.759, 1905). As Peirce explains:

Induction takes place when the reasoner already holds a theory more or less problematically (ranging from a pure interrogative apprehension to a strong leaning mixed with ever so little doubt); and having reflected that if that theory be true, then under certain conditions certain phenomena ought to appear

[...], proceeds to experiment, that is, to realize those conditions and watch for the predicted phenomena. Upon their appearance he accepts the theory with a modality which recognizes it provisionally as approximately true. The logical warrant for this is that this method persistently applied to the problem must in the long run produce a convergence (though irregular) to the truth; for the truth of a theory consists very largely in this, that every perceptual deduction from it is verified. (CP 2.775, c. 1902).

## 5 The proof that inquiries do converge to the truth

But, if convergence to the truth is possible, is there any evidence that inquiry has actually succeeded? That is, *will* inquiry result in truth, and how will it be known that it has succeeded? This might be called Peirce's *halting problem*, somewhat analogous to Turing's halting problem in computer programming—which is undecidable (1936-37). Will a computer program lapse into an infinite loop or will it end? If a program halts in a relatively short period of time, no problem, but if programs can possibly run indefinitely, it would seem impossible to decide whether it will halt or not, since the waiting period would be indefinite. The problem with the convergence theory is even more severe, since even if there is an approximation to the truth, or a convergence of belief at some point about a claim, it's always possible that the claim will prove to be false or anomalous.

Using Kurt Gödel's strategy for a proof of incompleteness, Turing showed that it would be impossible to devise a program that could decide whether all programs would halt or not. Suppose, via *reduction ad absurdum*, there is a program that can decide whether any program will halt or run indefinitely. The decider program will always halt, whether it detects a program that will halt or run indefinitely, giving an output of 'yes' for the former, and 'no' for the latter. Now, suppose there is a complementary program that, takes in as input, the output of the decider program. However, if the output of the decider program is 'yes, halts', it runs indefinitely; and if the output of the decider program is 'no-runs indefinitely', the complementary program halts. Now suppose the complementary program is expanded so as to include the decider program and the complementary program's reaction to the decider program's output. Could the decider program decide whether the expanded complementary program halts? If the decider program's output is 'yes-halts', then the expanded program will run indefinitely. If the decider program's output is 'no-runs indefinitely', then the expanded complementary program will halt—an obvious contradiction. Thus, the decider program cannot decide the expanded complementary program without contradiction.

The halting problem in the convergence theory is similar to a point. Since the test of the truth of a claim is whether an inquiry will eventually converge over time in regard to that claim, then it would seem that the truth of a claim could not be decided ahead of that convergence. Even if there is a convergence at some relatively short period of time, it's possible that it's spurious, and the process goes on. To this extent, the convergence theory shares a similarity to the halting problem. However, the convergence theory itself is not analogous to a program that would

decide whether a particular claim is true or not prior to the point of convergence, it simply sets the criterion for what some such program *would* count as true. However, although it does not generate inconsistency, it creates another problem—circularity. The only way to prove the convergence theory of truth would appear to be to wait indefinitely to see if all inquiries into claims that are true end with convergence of the results of those inquiries, and the convergence of opinion concerning those results over time.

Perhaps a simpler way to see the problem with Peirce's convergence theory of truth is to conjure up Meno's paradox about inquiry: Why inquire? If inquirers do not already know whether a belief is true, how would they know it, even if they came across it in their inquiries by chance? (PLATO, *Meno* 80d-e).

As to the charge of circularity, as his classification of sciences show, Peirce argues for a hierarchy among the sciences—mathematics being the first philosophy, so to speak, followed by phenomenology, then the normative sciences of logic, ethics, and esthetics, in turn followed by metaphysics. This group makes up the formal sciences that serve as a propaedeutic to the empirical sciences (CP 1.180ff, 1901). The hierarchy is such that the sciences higher in the hierarchy provide guiding principles for the lower ones (CP 3.427, 1896). Stated plainly, the convergence theory of truth rests on the validity of induction, and the latter is proved mathematically by the remarkable Law of Large Numbers. Circularity is avoided since the proof of the theory does not depend on itself, but on a proven mathematical theorem.

As to a “program” that can decide prior to convergence at the end of all inquiry, Peirce argues for the scientific method, specifically induction as such a program, so to speak. Essentially, induction solves the halting problem by inferring the character of the whole, from a sample of its parts, thus avoiding the need to wait until some indefinite end to inquiry.

However, there is something of a price to this method of testing. Although induction is mathematically grounded, actual inductive tests are not mathematically certain, but approximate to the truth, to the point of inductive certainty. Although the scientific method does not provide a decision program that can absolutely decide in favor or not, it does so to a satisfactory degree for the practical certainty of “established truths.”

Peirce addresses this problem in his response to Josiah Royce's criticisms of his convergence theory of truth in 1885. Peirce first notes that there are qualifications to the claim that “The final opinion which would be sure to result from sufficient investigation may possibly, in reference to a given question, never be actually attained [...]” This, may be because of catastrophes such as a final extinction of intellectual life, “or for some other reason”—of which there are many.

However, Peirce emphasizes that “in that sense, this final judgment is not certain but only possible.” So that Peirce has not reversed his opinion about its possibility and, in fact, criticizes Royce when he says that “bare possibility is blank nothingness,” since the possibility of the theory is “but a hair's breadth from entire certainty.” Peirce then provides, interestingly, an inductive argument to support the indicative version of the convergence theory of truth.

Let us reason upon this matter by inductive logic [...].In the first place, then, upon innumerable questions, we have already

reached the final opinion. How do we know that? [...] throwing off as probably erroneous a thousandth or even a hundredth of all the beliefs established beyond present doubt, there must remain a vast multitude in which the final opinion has been reached. Every directory, guide-book, dictionary, history, and work of science is crammed with such facts [...]. The proportion of these which have in point of fact been conclusively settled very soon after the prediction has been surprisingly large. Our experience in this direction warrants us in saying with the highest degree of *empirical confidence* that questions that are either practical or could conceivably become so are susceptible of receiving final solutions provided the existence of the human race be indefinitely prolonged and the particular question excite sufficient interest [...] [emphasis added]. In that case, there is but an infinitesimal proportion of questions which do not get answered, although the multitude of unanswered questions is forever on the increase. It plainly is not fair to call a judgment which is certain to be made a 'barely possible' one [...]. "From this practical and economical point of view, it really makes no difference whether or not all questions are actually answered, by man or by God, so long as we are satisfied that investigation has a universal tendency toward the settlement of opinion[...]" (CP 8.43, c. 1885).

From a theoretical scientific point of view, strong inductive support of a hypothesis does not guarantee its absolute certainty. However,

As Practice apprehends it, the conclusion [...] is inductively supported. For a large sample has now been drawn from the entire collection of occasions in which the theory comes into comparison with fact, and an overwhelming proportion, in fact all the cases that have presented themselves, have been found to bear out the theory. And so, says Practice, I can safely presume that so it will be with the great bulk of the cases in which I shall go upon the theory; especially as they will closely resemble those which have been well tried. In other words there is now reason to believe in the theory, for belief is the willingness to risk a great deal upon a proposition [...]. We call them in science established truths [...] there are certain inferences which, scientifically considered, are undoubtedly hypotheses and yet which practically are perfectly certain [...]. These are established *truths* (CP 5.589, 1898).

Established truths "[...] merely means propositions to which no competent man today demurs" (CP 1.635, 1903); "[...] established truths [...] are propositions into which the economy of endeavor prescribes that, for the time being, further inquiry shall cease. (CP 5.589, 1898).

In its indicative form, the convergence theory of truth—that inquiry, sufficiently pursued, *will* attain the truth—is an empirical claim and, like any hypothesis, predictive of certain results. The first consequence would be that its inductive testing

shows stable results, such that further inquiries do not change the results of earlier inquiries by much or if at all. That is, inquiries have reached an approximation to the truth. Another consequence would be that other inquiries have, independently reproduced these results. Related to this, it could also be the case that the same results have occurred in inquiries that are independent of one another, as in the case of multiple discoveries. It could also be the case that inquiries into different subjects are consistent with and support the results of the inquiry in question. The third consequence is a consensus or relative consensus among those competent inquirers as to the matter being investigated. The practical consequences of a true claim or hypothesis is that it leads to successful inquiries and consensus of opinion among inquirers. These consequences, should they be present, then serve as *indications* of the truth of any hypothesis.

## 6 The regulative version of the convergence theory of truth

If it is *possible* to attain the truth, and many inquiries have already resulted in “established truths,” then certainly inquirers can reasonably *hope* that their inquiries will converge to the truth. In this way, the proof of possibility can serve also as a *regulative* principle for inquiry. As Peirce says, “[...] it is more satisfactory to see these things set forth in a purely logical way and deduced mathematically, than to have them treated at their first presentation as regulative principles” (CP 4.81, 1893). Indeed, in his more optimistic moments, he believed that the very desire to inquire would be sufficient to ultimately be successful: “[...] there is but one thing needful for learning the truth, and that is a hearty and active desire to learn what is true” (CP 5.582, 1898). At times he is strongly fatalistic about the outcomes of inquiries. In 1878, in “How to Make Our Ideas Clear,” he writes: “the progress of investigation carries them by a force outside themselves [...] to a foreordained goal [...] like the operation of destiny. No modification of the point of view taken, no selection of other facts for study, no natural bent of mind even, can enable a man to escape the predestinate opinion” (CP 5.407, 1878). Indeed, Peirce thinks that on many questions “the final agreement is already reached, on all it will be reached if time enough is given” (CP 8.12, 1871).

But later, Peirce tempers this attitude toward the success of inquiry. Consider this striking passage Peirce made in 1893:

[...] we cannot be quite sure that the community ever will settle down to an unalterable conclusion upon any given question. Even if they do so for the most part, we have no reason to think the unanimity will be quite complete, nor can we rationally presume any overwhelming *consensus* of opinion will be reached upon every question. All that we are entitled to assume is in the form of a *hope* that such conclusion may be substantially reached concerning the particular questions with which our inquiries are busied (CP 6.610, 1893).

In *Regenerated Logic*, Peirce argues that when it is said that inquiry presupposes some truth to the matter of inquiry, “what can this possibly mean except it be that

there is one destined upshot of inquiry with reference to the question in hand—one result, which when reached will never be overthrown? Undoubtedly, we hope that this, *or something approximating to this*, is so, or we would not trouble ourselves to make the inquiry. But we do not necessarily have much confidence that it *is* so” (CP 3.432, 1896). Peirce makes it clear in 1893, in a response to criticisms by Paul Carus, that he never claimed that the community of inquirers *will* ultimately settle down to some opinion that is “inevitable”. “I confess I never anticipated that anybody would urge that.” He continues:

We cannot be quite sure that the community ever will settle down to an unalterable conclusions upon any given question. Even if they do so for the most part, we have no reason to think the unanimity will be quite complete, nor can we rationally presume any overwhelming *consensus* of opinion will be reached upon every question. All that we are entitled to assume is in the form of a *hope* that such conclusion may be substantially reached concerning the particular questions with which our inquirers are busied (CP 6.610, 1893).

The editors of *The Collected Papers* note Peirce, in the same year as these responses to Carus, changes the wording of key passages about the convergence theory of truth in *How to Make Our Ideas Clear*. Thus, instead of arguing that “the followers of science are “fully persuaded”, “that the processes of investigation, if only pushed far enough, will give one certain solution to each question to which they apply it,” Peirce changes this to “are animated by a cheerful hope.” That inquiry will reach “the predestinate opinion” is changed from the original, as a great “law” to one of a great “hope” (CP 5.407, 1878).

Christopher Hookway takes these passages to mean that the older Peirce became more and more skeptical about his earlier exuberance for reaching “final opinions” and, therefore, for the effectiveness of convergence as constitutive of truth. Hookway writes, “as Peirce’s philosophy developed after 1878, he soon came to give his account of truth a regulative status: we hope we will converge on the truth if we inquire long enough and well enough” (2004, p. 135). Cheryl Misak had come to the same conclusion earlier on, not so much on the basis of textual support, but on the claim that Peirce’s theory, if constitutive of truth, would suggest that unsuccessful inquiries would make something false, even if true (1991, p. 140). That is to say, it would seem to make the inquiry the determination of whether something is true or false, rather than the supposition that something can be true or false independent of any inquiry.

However, both Hookway and Misak seem to think that Peirce settles for just the regulative sense of the convergence theory of truth. But as argued here, the regulative version is based on the mathematical certainty of the possibility of convergence towards the truth, and the induction that many inquiries have resulted already in established truths. As noted, this is emphasized by Peirce in his Carnegie application in 1902, long after Hookway argues he changes from thinking convergence as constitutive of truth to regulative only (L75, 1902, p. 268-270 Draft D). Thus, there is strong evidence to suggest that Peirce did not give up on convergence

as constitutive of truth. He makes it clear that inquiry may, historically speaking and in actuality, end prematurely at a point that is insufficient to resolve all salient inquiries and, *in this respect*, inquirers *can only hope* that it continues to the point of success. After all, human beings may destroy the ecology that sustains human life, or, they may initiate a nuclear holocaust that wipes out most, if not all of the human population and other life forms with them. But, otherwise, textual evidence suggests that Peirce still expresses confidence in his convergence theory of truth after 1878.

As Peirce defines a regulative principle—interpreting Kant—it is nothing more than an “intellectual hope” (CP 1.405, c. 1890). If it is not possible to attain truth through inquiry, then inquiry is delusional. Peirce argues that the “faith of the logician,” is that inquiries in the long run, will move “toward certain predestinate conclusions which are the same for all men,” and it is something “upon which all maxims of reasoning repose” (CP 3.161, 1880). In this way, inquirers can act *as if* their inquiries will resolve the question of the truth of their claims, even if they do not actually do so.

## 7 Conclusion

The textual evidence suggests that Peirce remained consistently confident and certain about the mathematical basis of his convergence theory of truth, and continued to claim that inquiry, sufficiently and competently pursued, *would* result in true beliefs, even if it didn't *actually* occur—although it has already in many cases resulted in established truth. For these reasons, inquirers can rationally hope that their future inquiries will converge to the truth. The convergence theory of truth is, therefore, not simply a regulative ideal but a mathematical certainty that often can be actualized.

Can it be said that inquiry *would be* successful in the long run? Yes, in the *mathematical* sense of certainty. Can it be said that inquiry *will be* successful in the long run? Possibly for some, but not likely for all inquiries. There will be some subset of propositions that are inductively certain, and some that will be considered practically certain and established truths. For Peirce, “The final opinion which would be sure to result from sufficient investigation may possibly, in reference to a given question, never be actually attained, owing to a final extinction of intellectual life or for some other reason. In that sense, this final judgment is not certain but only possible” (CP 8.43, c. 1885). As a result “the only attainment of truth by science is an eventual predestination, a predestination *aliquando denique*. Sooner or later, it will attain the truth, nothing more” (CP 7.78, c. 1900).

Despite some of the more dramatic expressions of the convergence theory of truth, Peirce's theory of truth is a quite reasonable and not surprising one. To say that truth is whatever is finally agreed to by all those who inquire into the content of that agreement, on the basis of results of good methods of inquiry, sounds much like what goes on in actual scientific practice. The way that Peirce sees it in *How to Make Our Ideas Clear* is something like the following. To take Peirce's example, consider the assertion that ‘diamonds are hard’. That assertion is true if, first, its meaning is clarified in accord with the pragmatic maxim, namely, in terms of its practical, observable consequences. Second, inductive sampling of those consequences would be sufficient to a high confidence interval, in accord with frequentist theory that it would be difficult for any competent and reasonable

inquirer to doubt, resulting, third, in a consensus of opinion about those results.

There are different types of hardness, but suppose the type the experimenter wants to examine is the *scratch hardness* of the diamond. Its clearest meaning is to predict what hard diamonds would do under certain specified conditions of scratching or being scratched by other objects. If the diamond is harder than an object like glass, then it should scratch glass rather than being scratched, that is, one should be able to observe a permanent fracture in the glass from the friction of the diamond being rubbed against it by means of a sclerometer. The clarification of the meaning of scratch hardness, also specifies for the experimenter, the means by which a hypothesis, an assertion about diamonds, can be formulated for testing, such that the results of the test, if they occurred as predicted, would under continued sampling show a frequency way above chance. The experiment could use the Mohs scale to see whether the diamond or the glass is harder, based on what scratches what. The experimenter might also see if the diamond scratches other substances thought to be hard, for example, quartz, or corundum, to get a sense of the relative scratch hardness of diamonds. After repeated trials with a variety of substances, the experimenters will and have concluded that the hypothesis that the diamond is hard, should not be rejected, particularly since all inductive evidence shows that it has harder scratch resistance than any other known substance. As Peirce would say, this is an “established truth,” “propositions which the economy of endeavor prescribes that, for the time being, further inquiry shall cease” (CP 5.589, 1898).

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