GeoGebra as a Learning Mathematical Environment

GeoGebra como ambiente de aprendizagem matemática

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Abstract

GeoGebra, a software system for dynamic geometry and algebra in the plane, since its inception in 2001, has gone from a dynamic geometry software (DGS), to a powerful computational tool in several areas of mathematics. Powerful algebraic capabilities have joined GeoGebra, an efficient spreadsheet that can deal with many kinds of objects, an algebraic and symbolic calculation system and several graphical views that expand the possibility of multidimensional representations, namely, by using colouring domain techniques, expanded to representations in the Riemann sphere, making this DGS a powerful research tool in mathematics. On the other hand, GeoGebra can create applications easily and export to HTML, and the possibility to quickly integrating these applets in several web platforms provides this DGS with an excellent way to create strong collaborative environments to teach and learn mathematics. Recently was added to GeoGebra powerful capabilities that transform this software a real Learning Mathematical Environment, using the GeoGebraBooks and GeoGebraGroups, plain of collaborative functionality between students and teachers.

Keywords: *GeoGebra, Learning Mathematical Environment.*

Resumo

O GeoGebra, é um sistema de software para a geometria dinâmica e álgebra no plano, desde a sua criação em 2001, passou de um software de geometria dinâmica (DGS), a uma poderosa ferramenta computacional em várias áreas da matemática. Para além das capacidades algébricas do GeoGebra, foi adicionada uma folha de cálculo que comporta múltiplos tipos de objetos, um sistema de cálculo algébrico e simbólico e várias vistas gráficas que ampliam a possibilidade de representações multidimensionais, ou seja, usando técnicas de domínios coloridos, expandiu-se para representações na esfera de Riemann, tornando este DGS uma poderosa ferramenta de pesquisa matemática. Por outro lado, a facilidade com que se criam aplicações do GeoGebra, de fácil e exportação para HTML com a possibilidade de as integrar rapidamente em várias plataformas web, este software tornou possível criar ambientes de colaboração fortes para ensinar e aprender matemática. Recentemente foram adicionados vários recursos poderosos, que transformam o GeoGebra num verdadeiro ambiente de aprendizagem matemática, utilizando o GeoGebraBooks e GeoGebraGroups, permitindo varias funcionalidades de colaboração entre alunos e professores.

***Palavras-chave**: GeoGebra, Ambiente de Aprendizagem Matemática..*

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Introduction

GeoGebra, a software system for dynamic geometry and algebra in the plane (Hohenwarter, 2002), since its inception in 2001, has gone from a dynamic geometry software (DGS), to a powerful computational tool in several areas of mathematics. The software growth, based on an open-source philosophy, was possible due to its strong spread at a global level, alongside with the countless contributions of a strong community of researchers, teachers and students who develop and use it worldwide, transforming this DGS in a democratic tool (Jarvis, Hohenwarter, & Lavicza, 2011).

Powerful algebraic capabilities have joined Geogebra, an efficient spreadsheet that can deal with many kinds of objects, an algebraic and symbolic calculation system and several graphical views that expand the possibility of multidimensional representations, namely, by using colouring domain techniques (Breda, Dos Santos, & Trocado, 2013). expanded to representations in the Riemann sphere (Breda & Dos Santos, 2015), making this DGS a powerful research tool in mathematics. On the other hand, GeoGebra can create applications easily and export to HTML, and the possibility to quickly integrating these applets in several web platforms provides this DGS with an excellent way to create strong collaborative environments to teach and learn mathematics (Abar & Barbosa, 2010; Macias, 2011) . At the same time, the easy way to deal with javascript-based interfaces, and other web-based programming technologies makes it possible to join some prover methods to GeoGebra and establish a link to prover engines (Marić, Petrović & Janičić, 2012; Petrović & Janičić, 2012; Baeta & Quaresma, 2013; Botana, Hohenwarter, Janičić, Kovács, Petrović, Recio & Weitzhofer, 2015).

Finally, in the last five years some prototypes of collaborative environments have been developed. Initially implementations were based on behavioural learning, with little group interaction, characterised by proposals of simple tasks and closed answers; examples of these applications have been extensively developed on a Moodle platform by a group of researchers and implementers of Catalonia (Fernàndez, 2012). Later, rich environments interactions were created in the Virtual Math Teams projects in the United States, WGL, Portugal (Santos & Quaresma, 2013; Quaresma, Santos, & Bouallegue, 2013) and the GeoGebraLive project that will result in a collaborative functionality, characteristic of GeoGebra, which was presented in July 20015 in Linz, at the biannual conference of GeoGebra.

1. GeoGebra as a Learning Mathematical Environment

GeoGebra was created by Markus Hohenwarter in 2001/2002, and after that this software has been further developed. In the beginning this application was used for plane geometry but quickly the capabilities of the software expanded to other areas. Since 2002, the expansion of GeoGebra has become a phenomenon of popularity and developed a democratic access to a powerful mathematical tool to learn and create mathematical knowledge. We, alongside with others in the global GeoGebra community, consider that:

> “GeoGebra as a potentially defining moment in Kaput’s grand vision of democratic access; others maintain that the software still presupposes technological resources which are not always available in financially weak regions; and still others feel that the open-source status of the software is unsustainable in terms of development, growth, and quality control. However, we contend that these latter objections are likely unfounded, particularly in light of other longstanding and successful open-source initiatives (e.g., Linux), and the fact that several large software companies are now developing products that involve open-source technology in order to meet the challenges of sustainability and relevancy within a rapidly changing marketplace.” (Jarvis, Hohenwarter & Lavicza, 2011, pp. 232-233) [2].

For educational purposes the initial applications of GeoGebra were supported by tasks that invited students to visualize some geometric invariant or other mathematical properties, and the interaction of those students with the applications, with teacher and colleagues, stimulated them to acquire some geometrical reasoning and allowed the formulation of conjectures.

However, in the last decades some geometry automated theorem provers (GATP) have been developed, and the most successful use of algebraic methods, namely, Wu and Gröbner bases methods have been used in order to integrate this technology in GeoGebra (Marić, Petrović, & Janičić , 2012; Botana, Hohenwarter, Janičić, Kovács, Petrović, Recio, & Weitzhofer, 2015).

Learning environment with automatic feedback

The first applications with automatic feedback for the user appear in GeoGebra in the development of elementary geometry constructions. In fact, the possibility to create an algebraic output and check some properties of geometric objects was allowed, at the beginning, by applets with some interaction with the user; many of these applications of GeoGebra included questions and some answers. However, most of them were limited to some geometric issues and related to closed problems.

In 2008 the first applications using relation tool (Relation[<Object>, <Object>], $a \overset{?}{=} b$) appeared. For example, in figure 1 we see a simple application of the relation tool, where the position between two straight lines and the position of a straight line and a conic section were evaluated.

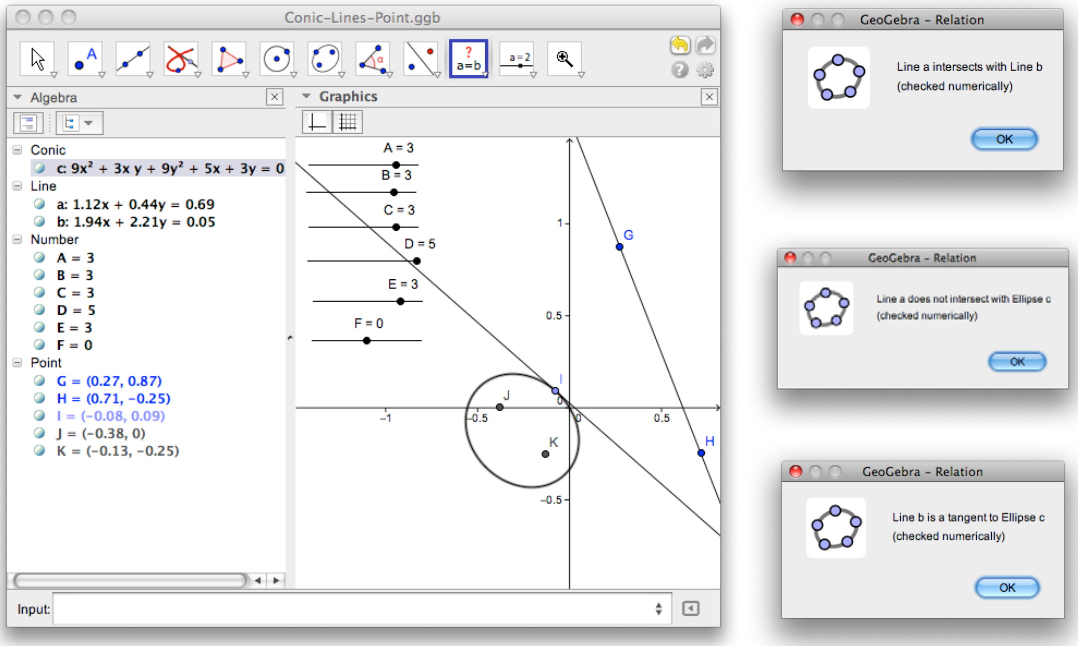


Figure 1 - Using a relation tool to check positions between straight lines and a conic section

Other example of these applications is a ruler and compass strategy for equilateral triangles. Indeed many of these applications uses the capabilities of GeoGebra to make some numerical calculations and answer questions such as: "Is straight line x parallel to line y?", "are points A, B and C collinear?", as you see in the example of figure 1, but in some cases the results are unexpected or inexact and may provide a wrong answer.

Another improvement occurred when it was possible to add dynamic colour to objects in GeoGebra, as Rafael Losada assigned a colour in a point with the objective to obtain the first representation of Mandelbrot set in GeoGebra. After, with the introduction of a spread sheet, a scanner technique for the plane, Rafael Losada, António Ribeiro and other members of the GeoGebra community improved the representations of some fractals set (see figure 2).

The dynamic colours were also used to develop several applications to study many connections between geometry and Algebra, which are important to study some algebraic curves. In spite of the possibility to represent an implicit curve with GeoGebra, for some of those studies the representation has some errors, in certain points, in other software.

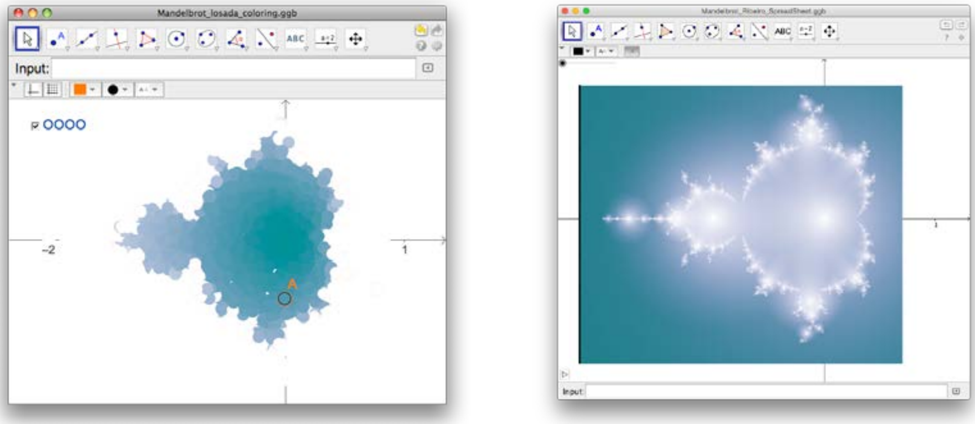
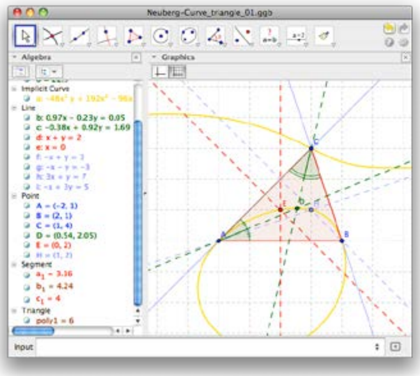


Figure 2 - Using dynamic colour and Spreadsheet in GeoGebra in order to obtain the Mandelbrot Set.

Now let us look at an illustration of how dynamic colour can help to detect some mistakes in the graphics outputs (Fig. 3). Let the points A, B and C and construct the Incenter, orthocenter and circumcenter of the triangle ABC. In GeoGebra we can directly obtain the Neuberg cubic, using the command Cubic [A, B, C, 1], and we can check, with the command relation, that Incenter, orthocenter and circumcenter belong to this curve.

A=(-2,1)
B=(2,1)
C=(1,4)
a=Cubic[A, B, C, 1]
Construct the circuncenter, point E, or use the command:
TriangleCentre[A,B,C,3]
Construct the incenter, point D, or use the command:
TriangleCentre[A,B,C,1]
Construct the orthocenter, point H, or use the command:
TriangleCentre[A,B,C,4]



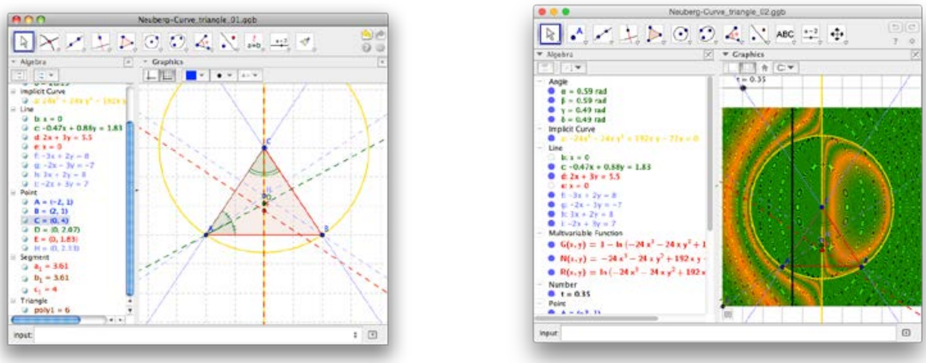


Figure 3 - Neuberg cubic of a triangle and Dynamic Colours.

Even though for some positions of points A, B and C the map of the Neuberg curve seems a circumference cut by a straight line, by using dynamic colours we can verify that it may be is a wrong idea in some proximal positions. In fact, the dynamic colours give to developers of GeoGebra a good way to have insights in several computational and mathematical problems (Fig. 3).

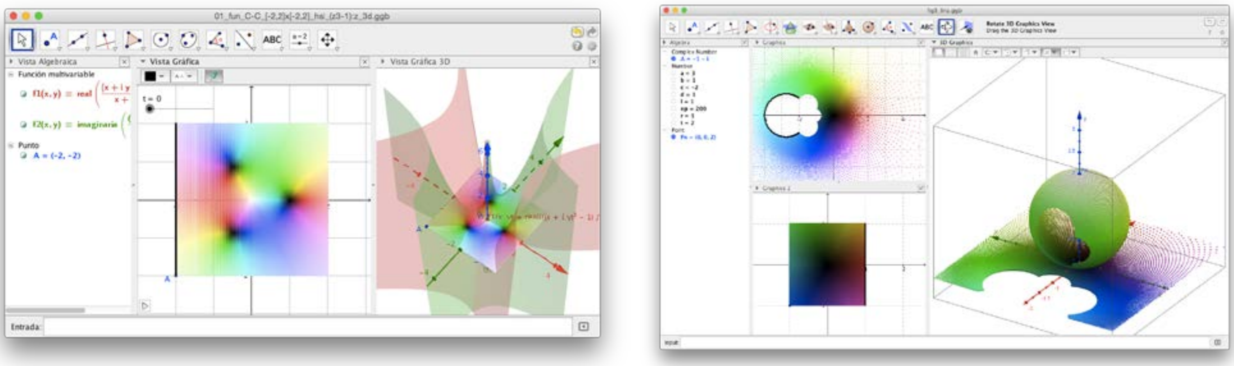


Figure 4 - Domain colouring techniques in GeoGebra in order to obtain maps of complex function with complex variable and in the Riemannian sphere.

More Recently, GeoGebra can reproduce the domain colouring techniques in 2d and 3d Windows (see fig. 4). These techniques in GeoGebra were used for the first time by Ana Breda and José Dos Santos to study the maps of function of $\mathbb{R}^2$ in $\mathbb{R}^2$ and maps of complex function with complex variable (Breda, A., Dos Santos, J. & Trocado, A., 2013); moreover, they were applied to represent maps in the Riemannian sphere and study the Möbius transformations in the Riemann sphere with GeoGebra (Breda, A. & Dos Santos, J., 2015). The colouring domain applied in GeoGebra provides a way to visualise graphs in four dimensions and extend the field of application of this software to research and educational purposes.

A new step occurred when GeoGebra included scripts, in javascript and python, and was integrated in the learning management system (LMS). The first integration of GeoGebra files in Moodle happened in 2009. The filter was created thanks to the work of Sara Arjona, Florian Sonner, and Christoph Reinisch with the collaboration of the Catalan Association of GeoGebra. The GeoGebra Moodle plugin allows the incorporation of GeoGebra activities in Moodle and saves its state (see fig. 5). It has been developed by the Departament of Education of Catalonia in collaboration with the Catalan Association of GeoGebra (ACG) and the GeoGebra development team.

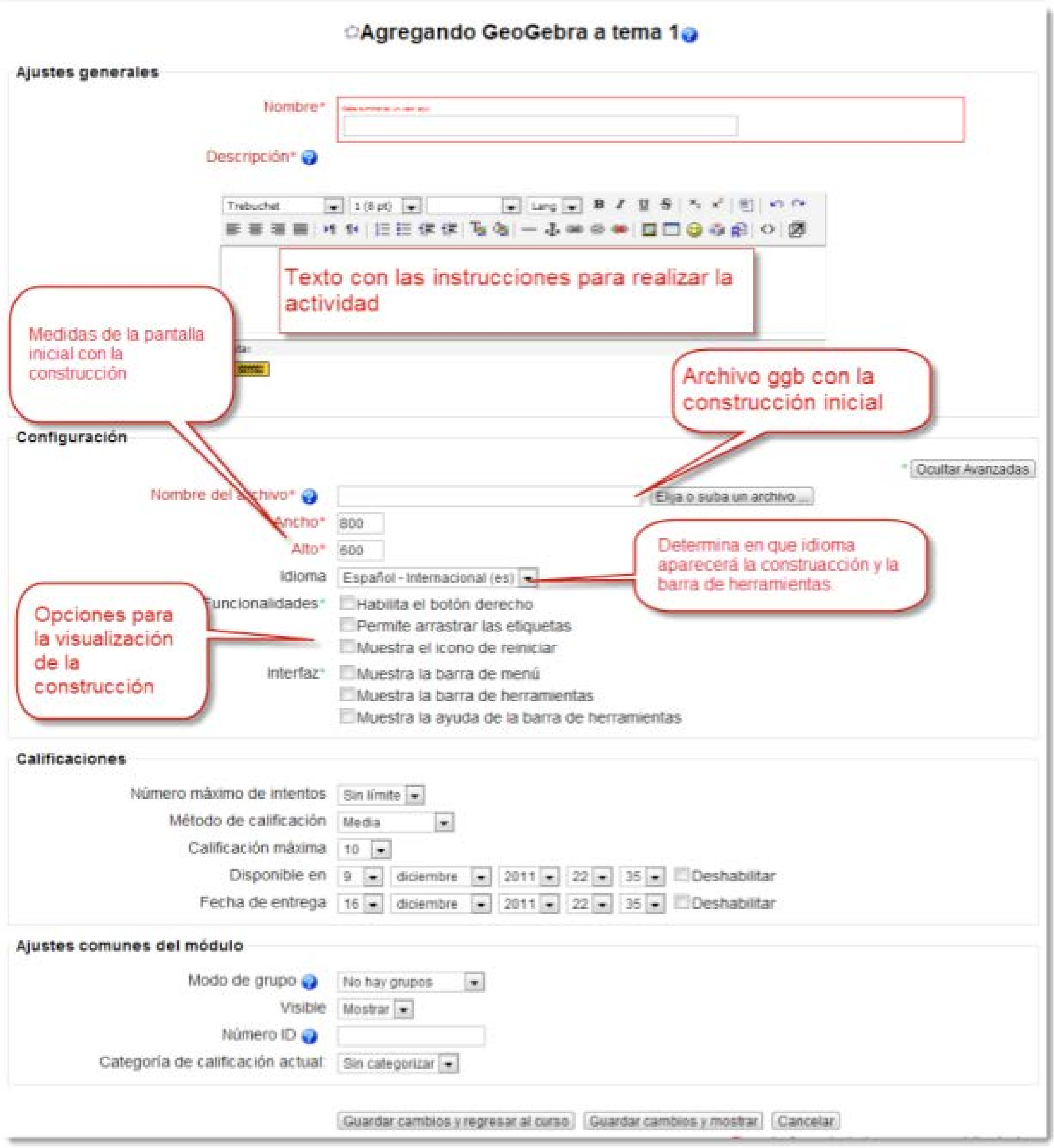


Figure 5 - Options in the Moodle plugging.

The first version was created in November of 2011. More of the applications built in moodle consist in ggb files that the student can edit; besides, the teacher can see the student's version in an asynchronic way. With the use of javascript and python programming it was possible to create some auto-valuable tasks with the Moodle package; the first of these applications was created by Joseph Lluis Cañadilla, and an example can be observed in figure 6.

The application in Moodle asks the user to build a straight line perpendicular to a.

The initial value of p is 1, but the answer is p=-1/b.

The grade attributes 10 points if the user builds a line with slope as less as 0,001, assuming that we work only with numbers with one decimal position.

Using the JavaScript Global, we can make a javascript function to evaluate the object properties built by the user.

```
A=(1,3)
B=(4,1)
a=line[A,B]
b=Slope[ a ]
p=1
grade:= if [abs(b + 1 / p) < 0.001, 10, 0]
```

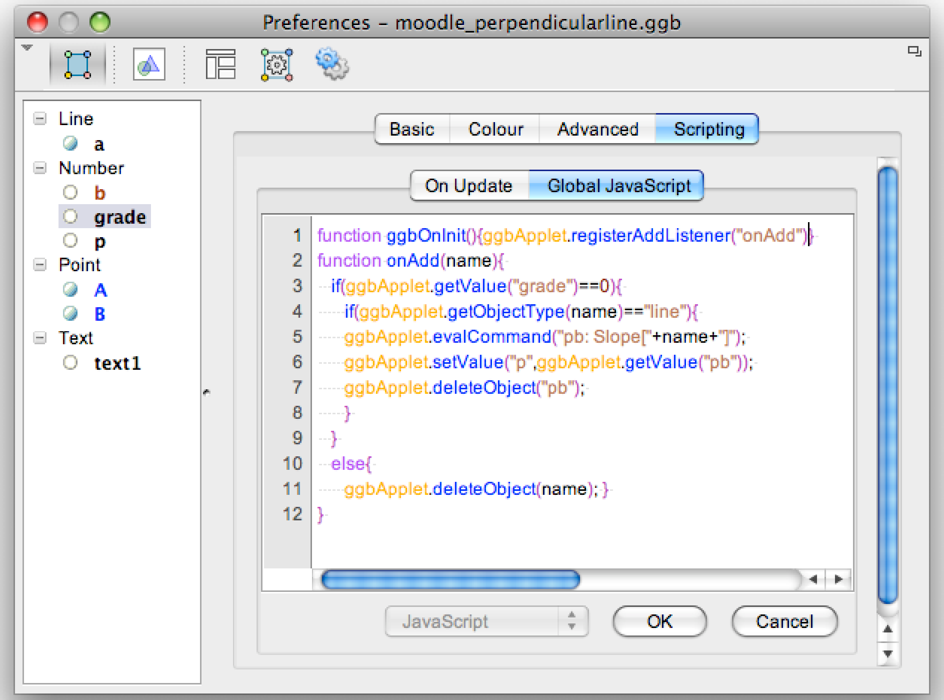


figure 6 - Script in GeoGebra that uses JavaScript in order to produce an auto evaluable task for Moodle.

At this point, most applications are based in a tutorial environment and tasks with closed problems. In fact, the challenge will be provided by making GeoGebra a true prover of capacities, which can help users who are looking for solutions to open problems. In version 5.0 GeoGebra there are several methods to decide if a statement is true in general or not, and some of these methods are based on Gröbner bases (Marić, Petrović, & Janičić, 2012); however much work is still needed. There are some positive experiences linking GeoGebra to other prover engines. One of the prover engines related to Geogebra is the Open Geo Prover (OGP), which is being developed by Ivan Petrović and Predrag Janičić (2012). Recently, Pedro Quaresma and Nuno Baeta (2013) applied the area method (geometric theorem proving method) in OGP and tested it in Geogebra. In fact, Geogebra can be linked to external provers to try to demonstrate a result using a determined heuristic.

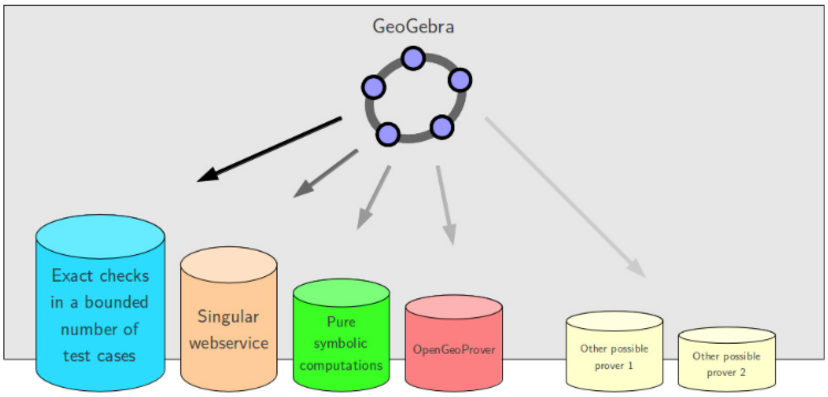


Figure 7 - Provers that GeoGebra can use. (Botana, et al, 2015)

The Singular program, placed on a server, supplies this limitation. To transfer the order of calculation GeoGebra Singular Web Service (singular WS), a web service that can accept Singular commands can be used through the web address (like a PHP instruction), thus allowing to calculate with them. Singular GeoGebra automatically calls when using one of these commands (or other equations as loci).

Commands	Boolean Expression
Relation[<Object>, <Object>]	A==B or AreEqual[<Object>, <Object>]
Prove[<Boolean Expression>]	a // b or AreParallel[<Line>, <Line>]
ProveDetails[<Boolean Expression>]	a ⊥ b or ArePerpendicular[<Line>, <Line>]
Eliminate[<List of Polynomials>, <List of Variables>]	AreCollinear[<Point>, <Point>, <Point>]
GroebnerLex[<List of Polynomials>] GroebnerLex[<List of Polynomials>, <List of Variables>]	AreConcurrent[<Line>, <Line>, <Line>]
GroebnerLexDeg[<List of Polynomials>] GroebnerLexDeg[<List of Polynomials>, <List of Variables>]	AreConcyclic[<Point>, <Point>, <Point>, <Point>]
GroebnerDegRevLex[<List of Polynomials>] GroebnerDegRevLex[<List of Polynomials>, <List of Variables>]	

Figure 8 - Commands and boolean expression for automatic demonstration in GeoGebra

The automatic demonstration commands internally GeoGebra 5, as presented in figure 8. These commands are the result of mathematical understanding of two Spanish mathematicians, Tomas Recio (Cantabria University) and Francisco Botana (University of Vigo), who along with Simon Weitzhofer and Zoltan Kovacs (both from the University of Linz), developed GeoGebra. They are the architects of the procedures used internally in GeoGebra to execute these commands.

Let us see an example of how we can use the commands above. Suppose that we can demonstrate that:

the height of a triangle falls on the midpoint of its base if and only if the triangle is isosceles. (1)

In the application, as shown in figure 9, we use the relation command to compare points B and C. The response of figure 9 is based on an algebraic method (method of Wu), which shows that, relatively speaking of detailed exceptions, the geometric relationship is true. However, this method does not question the need for the added extra conditions, ie, the method of Wu guarantees that if those conditions are met (in our case, P and R are not equal) the result is true, but it is not sure that it is true if and only if these additional conditions are met.

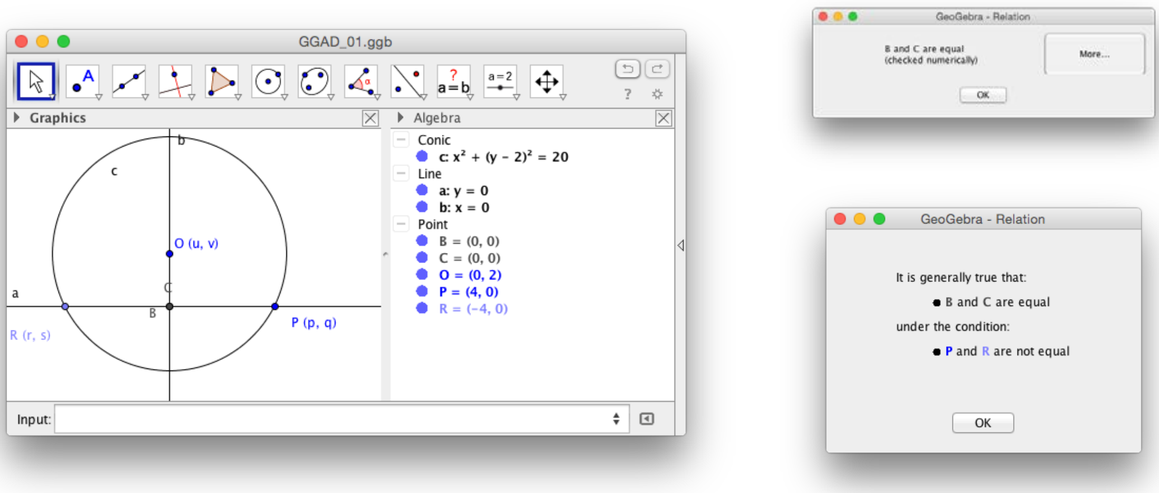


Figure 9 - Using Relation between B and C, where B=Intersect [Line[P, R], Perpendicular Line[O, a]] and C=Midpoint[P,R], to check "the height of a triangle falls on the midpoint of its base if and only if the triangle is isosceles".

Now we can see how we can use the Gröbner Method to prove (1) in GeoGebra. $\overrightarrow{PR}$ is the vector (r-p, s-q). Therefore, the equation of the line that is perpendicular to PR by the O point is (r-p) (x-u) + (s-q) (y-v) =0. If the midpoint (x, y) is perpendicular, it is to fulfil this equation. So, from the view of symbolic computation, we write the condition that height b passes through the point (x, y), ignoring the second term of the equation (as it will always be zero): L1={(r-p) (x-u) + (s-q) (y-v)}.

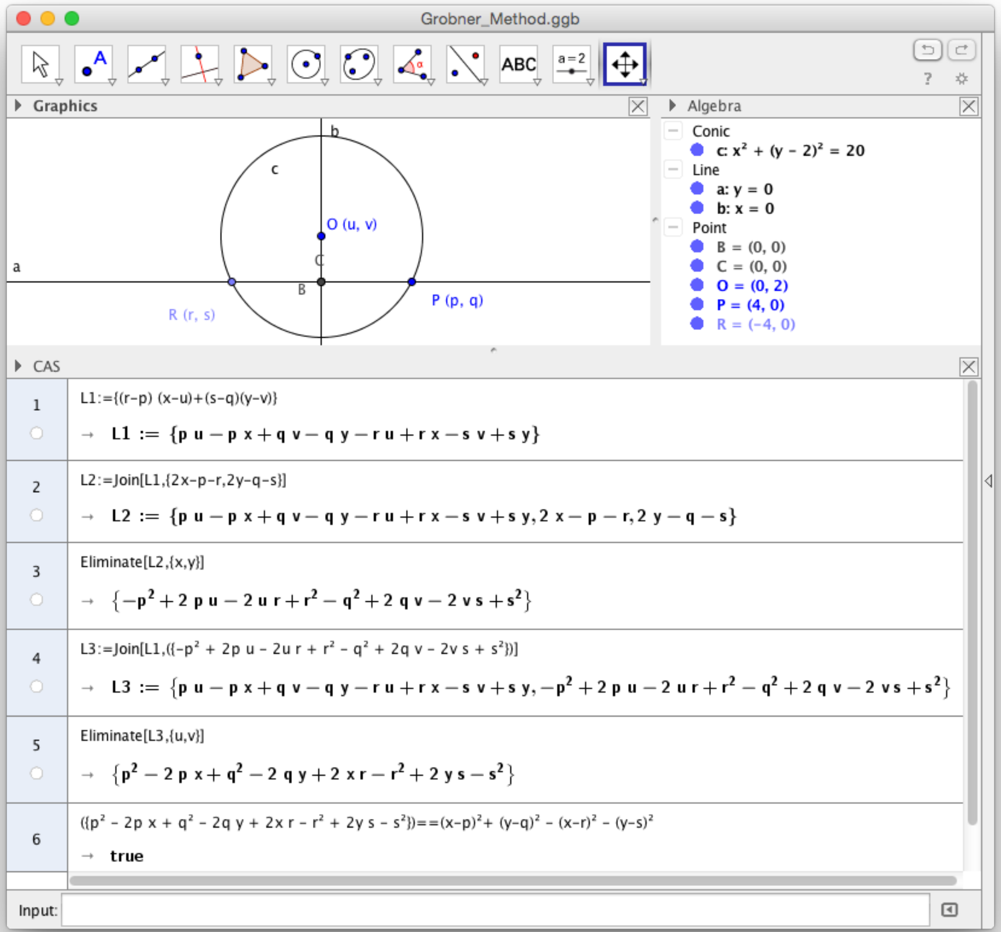


Figure 10 - Using Gröbner Method to check “the height of a triangle falls on the midpoint of its base if and only if the triangle is isosceles”.

Point C (x, y) also to be the midpoint of P and R. Therefore, x =(p + r)/2, y=(q+s) / 2. Add these conditions to the above list: L2:=Join[L1,{2x-p-r,2y-q-s}].

Now, we need to eliminate x and y in L2 and obtain L3:=Join [L2,Eliminate[x,y]].

Finally we obtain:

$$-p^2 + 2p\, u - 2u\, r + r^2 - q^2 + 2q\, v - 2v\, s + s;$$

But this polynomial is the same as

$$(p-u)^2 + (q-v)^2 -((r-u)^2 + (s-v)^2).$$

We can check in the CAS view writing:

$$-p^2 + 2p\, u - 2u\, r + r^2 - q^2 + 2q\, v - 2v\, s + s = = (p-u)^2 + (q-v)^2 -((r-u)^2 + (s-v)^2);$$

and evaluate obtaining True.

The coordinate of P, R an O satisfied the equation:

$$(p-u)^2+ (q-v)^2 -((r-u)^2 + (s-v)^2)=0$$

iden est,

$$(p-u)^2+ (q-v)^2 =(r-u)^2 + (s-v)^2.$$

And then we have demonstrated that P and R are equidistant of O. QED

The automatic prove in GeoGebra is a long way to discover. Many mathematicians, developers and GeoGebra lovers all over the world work together to improve this software, with a democratic issue in their agenda, and trying to engage more people in a deep understanding of mathematics. But how can GeoGebra include such collaboration in its core?

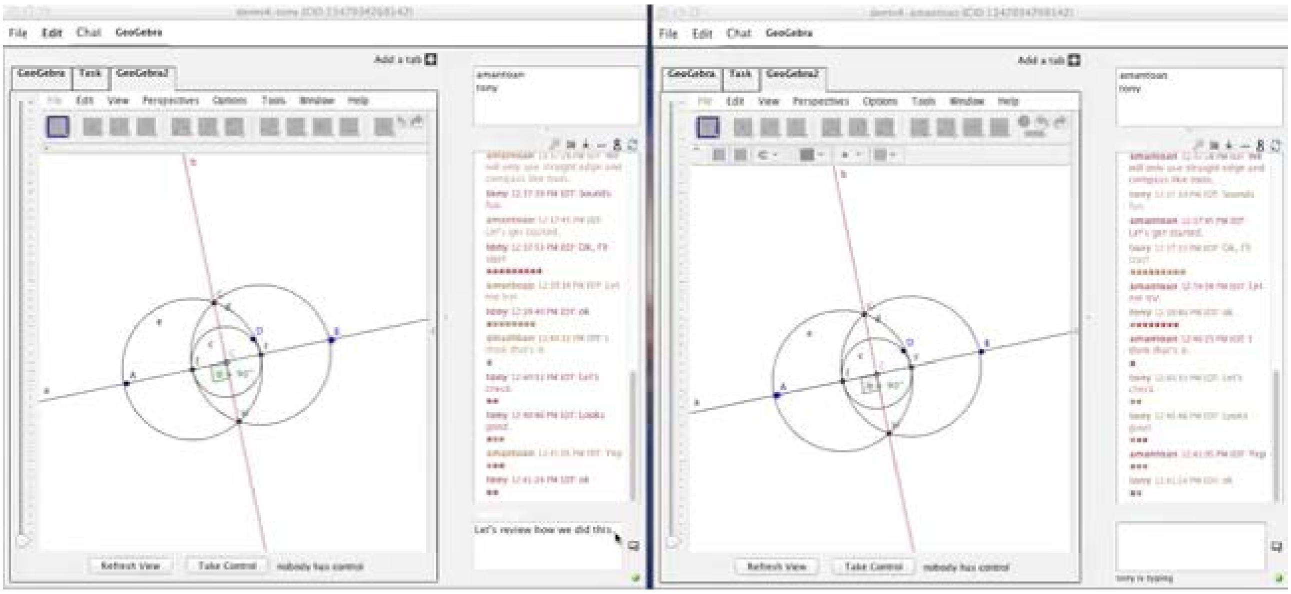
Learning environment with colaboration

In the last pages we have tried to demonstrate that GeoGebra is a learning environment to teach and produce mathematics. We have also shown that GeoGebra can be integrated in a LMS, but the community of GeoGebra around the world maintains several Wiki´s in many languages, has created a large contacts network, produces and shares materials in GeoGebra tube. There are two central questions: Will a live interaction between user in real time and in the web be possible? Can GeoGebra become an autonomous Learning Managing System? In our opinion the answer is YES! Let us see the new features that are ready to be included in GeoGebra’s core.

First, we have to present three collaboration environments that were created and tested. These projects point to GeoGebra as an integrated and really collaborative tool for groups.

a) The Virtual Math Teams - VMT

Supported by the National Science Foundation, VMT is a research project aimed at supporting mathematical discourse and collaborative learning of mathematics with computer networking and support. Some applications involve the use of GeoGebra in order to create a collaborative learning environment between students (Stahl, Rosé, O’Hara, & Powell, 2010) . Students can chat with their colleagues about the problem and interact with the GeoGebra application, as we can see in figure 11.



Picture 11 - Example of a session in a Virtual Math Teams project.

VMT is a project designed to investigate small group collaborative learning in the Math Forum's popular "Problem of the Week" service. Small groups of students are invited to collaborate online solving math problems that require reflection and discussion. The project aims to investigate questions of group process and social roles in math problem solving in order to aid the design of the service, through the following research activities: observing students in math classrooms in Philadelphia public schools working collaboratively on math problems and analyzing their collaborative learning process so as to create Virtual Math Teams. The project produced some mechanisms, using GeoGebra, to bring together compatible online teams that develop math problems structured for discussion, exploration and solution by small groups of students (Stahl, 2012) .

b) Webb Geometry Lab – WGL

The Webb Geometry Lab is a project developed by the researchers Vanda Santos (CISUC - Coimbra University) and Pedro Quaresma (Coimbra University). The project’s goal is to build an adaptive and collaborative Web-environment, integrating dynamic geometry systems (DGS), particularly GeoGebra, and geometry automated theorem provers (GATP). The collaborative module is built in such a way that allows teachers to manage and assess collaborative working sessions and students to join these collaborative sessions and solve the tasks in collaborative working groups (Santos & Quaresma, 2013).

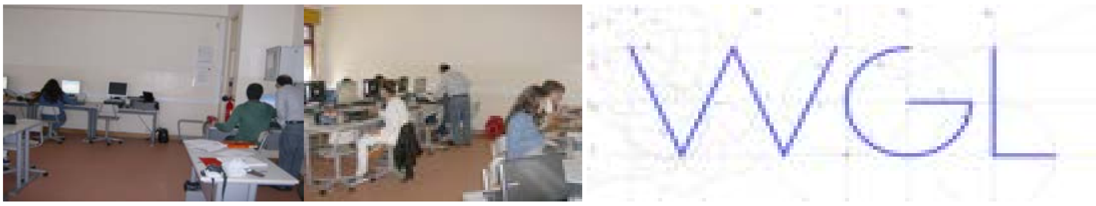


Figure 12 - Images of a session of WGL.

One of the authors of this paper had the opportunity to participate with students in some sessions using WGL. Even though these working sessions occurred at the beginning of the project, WGL proved to be a robust environment with clear evidence that students learn and improve their geometry knowledge. It should be noted that the interaction between the students was carried out without direct contact between them, except for the comments and buildings generated on the platform (figure 12). The teacher worked only as a facilitator in the process, and the researcher Vanda Santos was always present. In relation to student learning the teacher found that there was an improvement in the outcomes of students with lower performance participating in the experiment. These results were confirmed by Van Hiele's test. The investigator Vanda Santos reported a change from level 3 to level 4 in the comprehensive experience.

c) GeoGebra Live

GeoGebra Live was an application of GeoGebra in the Web. This application lets you share GeoGebra worksheets with other people for live collaborations in two ways: two people at the same time; one teacher and multiple students. It should be possible to have both open sessions (anybody can join in) and closed sessions (invitation code sent by e-mail or IM). An example of the first application can be seen in figure 13. The live app is not under active development now, but the development of this experiment has created a new way to learn with GeoGebra, a collaborative feature that will be part of the concept of the software.

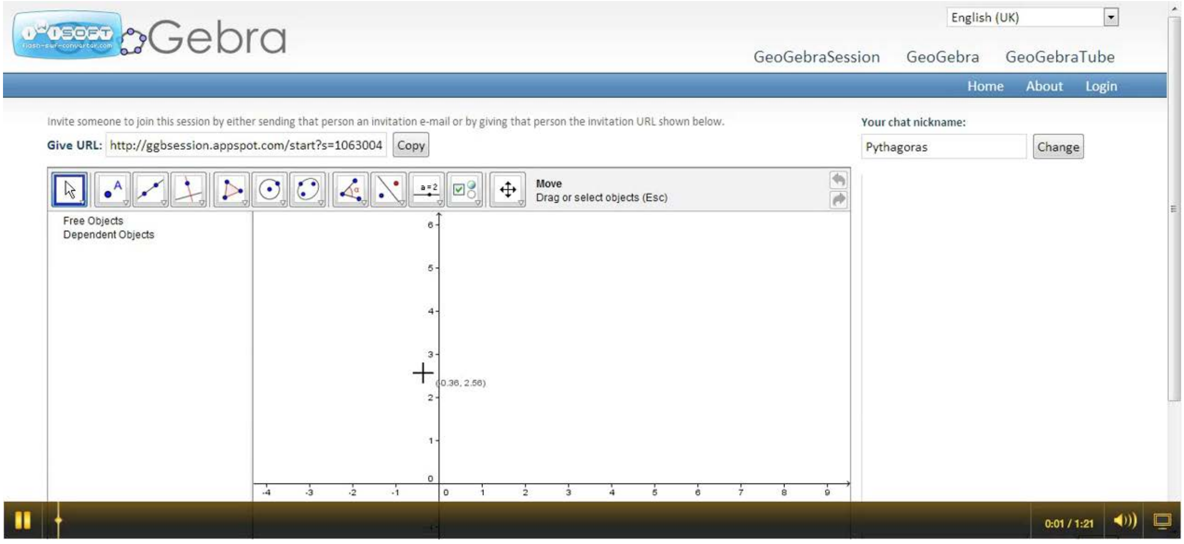


Figure 13 - Video that show the way to interact with GeoGebra Live (https://youtu.be/JXY_qcXLBXw)

d) GeoGebra Groups

Recently the Geogebra community has developed a virtual platform that allows teachers and students to cooperate in some tasks and interact as in a virtual classroom. With GeoGebra Groups teachers and students can share materials like applets from GeoGebra Tube, pdf files, videos, images and books from Geogebra Books.

This platform can act like a virtual classroom where the teacher can post tasks or questions (multiple choice or open questions) and give students some feedback on their resolution. The platform GeoGebra Groups can be boosted by GeoGebra Books (fig. 14) that, as the name says, are virtual books formed by static and interactive content as GeoGebra Applets and Videos. All of these together in the same platform can mean a much different interaction from the traditional classroom.

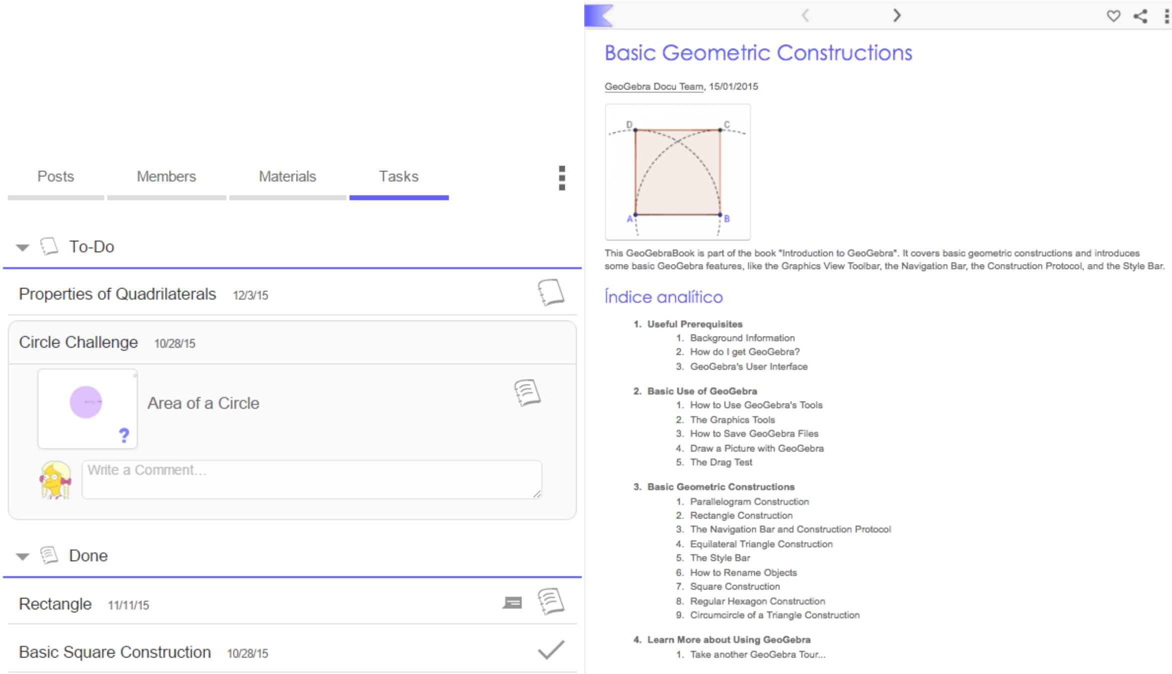


Figure 14 - Output of GeoGebra Groups on the left and GeoGebra Books, on the right.

e) GeoGebra - Exam Mode

This is a GeoGebra version that allows students to use this tool during exams on paper (like a graphic calculator). During the exam the GeoGebra window is showed maximized with a blue bar on top displaying a clock (fig. 15). The student will neither be allowed to exit that window nor save or open any file in the computer or GeoGebra Tube.

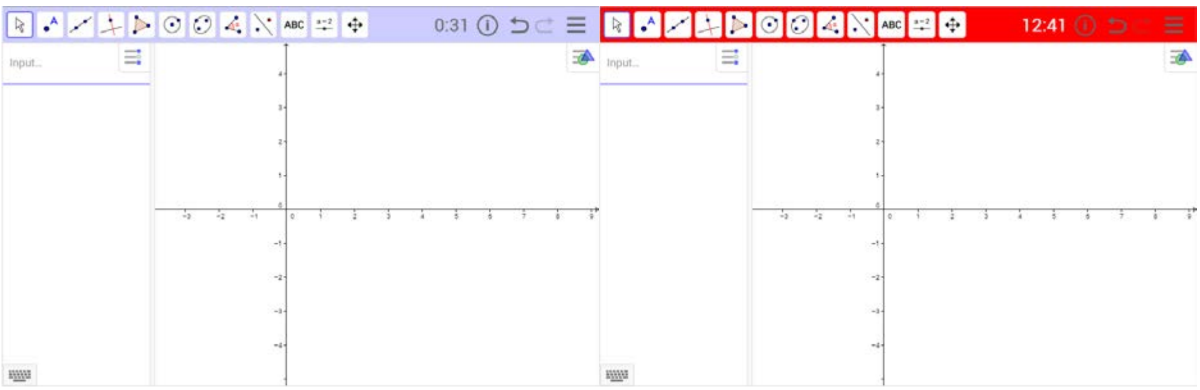


Figure 15- Output of GeoGebra Exam Mode.

If the Exam Mode is left, a visual alert that is easily detectable and recorded in the Exam Log is showed in the GeoGebra window. The log where all events that occurred during the exam were registered is showed at the end, as you can see in figure 16.

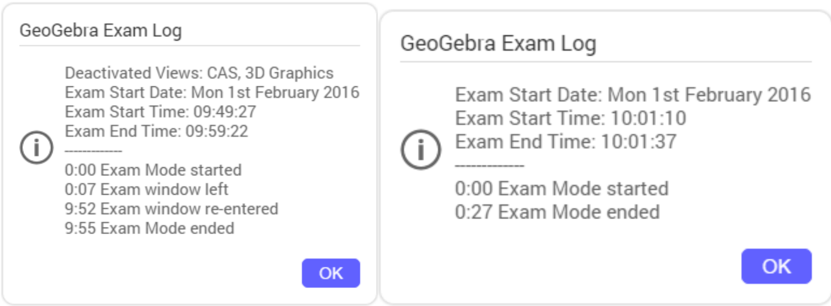


Figure 16 - Output of events during GeoGebra Exam Mode.

Conclusions

With the collection of presented facts and ideas, we intend to demonstrate how GeoGebra can integrate and develop learning and mathematical knowledge on its users. We also attempt to show how the software can play an important role in mathematics disclosure, besides making clear that GeoGebra has great importance in the mathematical knowledge democratisation process. We also present the beginning of the incorporation of automatic demonstration in GeoGebra, creating sophisticated environments and collaborative learning communities. Thus, it seems reasonable to conclude that GeoGebra is soon to be assumed as an autonomous learning management system, initially in mathematics but likely to extend to other subjects of exact sciences.

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