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Spatial Thinking and Movements in Geometry: GeoGebra with a focus on building Animated Scenarios¹

Pensamento Espacial e Movimentos em Geometria: O GeoGebra com Enfoque na Construção de Cenários Animados

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ABSTRACT

Animated Scenarios are constructions created in GeoGebra with mathematical elements and movement, constituting an animated scene. Movement is fundamental to the construction of Animated Scenarios, it is also essential to spatial thinking and spatial abilities. This theoretical essay aims to discuss the potential of building Animated Scenarios in GeoGebra in/for the recognition of flat and spatial geometric objects, articulated with the movements associated with spatial thinking. For this, the House Animated Scenario is considered. Thus, we identified that movements performed during this construction allow rotating and modifying the shape of objects and that these movements are related to spatial abilities. Furthermore, the use of these movements in construction contributes to the recognition of important characteristics that distinguish flat and spatial objects.

Key-words: Spatial Abilities. Movements in GeoGebra. Plane and Spatial Geometry.

RESUMO

Os Cenários Animados são construções realizadas no GeoGebra com elementos matemáticos e movimento, constituindo uma cena animada. O movimento, fundamental para as construções de Cenários Animados, também se faz importante ao pensamento espacial e às habilidades espaciais. Este ensaio teórico tem como objetivo discutir o potencial da construção de Cenários Animados no GeoGebra no/para o reconhecimento de objetos geométricos planos e espaciais, articulado com os movimentos associados ao pensamento espacial. Para isso, é considerada a construção do Cenário Animado Casa. Foi identificado que movimentos executados no GeoGebra durante essa construção permitem rotacionar e modificar o formato de objetos, os quais estão relacionados a habilidades espaciais, e contribuem para o reconhecimento de características importantes que distinguem objetos geométricos planos e espaciais.

Palavras-chave: Habilidades Espaciais. Movimentos no GeoGebra. Geometria Plana e Espacial.

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Introdução

Spatial thinking is the basis of how we speak and think about space, time, people, and other elements (Tversky, 2019). In addition to being considered one of the main basis for cognition (Gutiérrez, 1996). In this sense, it is indicated that situations that stimulate spatial thinking are proposed in schools and by the families, as they play an important role in learning and doing day-to-day tasks (Tversky, 2019). In Mathematics, some notions of spatial thinking, such as position, location, and displacement appear in the study of Geometry. Psychologist Bárbara Tversky (2019) presents neuroscientific evidence that the movement associated with spatial thinking is the basis for the formation of other types of thinking, especially abstract thinking, which is related to mathematical and geometric thinking. Some of the movements associated with spatial thinking are mental movements, such as rotating, zooming, shifting, and changing the perspective of objects in the mind (Tversky, 2019).

In the software GeoGebra, it is possible to perform movements similar to these using tools that provide movement to mathematical objects built into the software. This feature of GeoGebra enables the construction of Animated Scenarios, which are a type of construction involving mathematical elements in a context/scene that in the end is animated, that is, it has movement (Bueno & Basniak, 2020). These constructions also provide an opportunity to discuss concepts and mathematical representations during their development. In this work the discussed Animated Scenario involves plane and spatial geometric objects. As a result, the Animated Scenarios can be used to work on Mathematics in classrooms. In addition to being a type of task that can stimulate spatial thinking from the movements associated with it.

Thus, by relating the theoretical framework built especially based on the work of Tversky (2019), with our subjectivity in the construction of an Animated Scenario, the aim of this work is to discuss the potential of building Animated Scenarios in GeoGebra in/for the recognition of flat and spatial geometric objects, articulated with movements associated with spatial thinking. For this, we developed this theoretical essay, permeated by the “permanent relationship between subject and object, a coming-to-be constituted by the interaction of subjectivity with the objectivity of those involved” (Meneghetti, 2011, p. 1, our translation).

We point out evidenced the originality of the research, as there are still no works discussing the Animated Scenarios built in GeoGebra on the focus of movements associated with spatial thinking. In the same way, studies by Tversky (2019) on thoughts and movements do not articulate explicitly mathematical educational practices involving the GeoGebra. We emphasize that this paper is part of Koftun's multipaper master's thesis (2023), in which other discussions about movements associated to spatial thinking can be found along with the constructions of the Animated Scenarios.

Thus, this study relates the mental movements associated with spatial thinking with an Animated Scenario, presenting possibilities for action/practice through this construction in GeoGebra.

1. Movement and Spatial Thinking

Movements follow the planet in its evolutionary strategies and are important for the survival of all species: animals move to hunt, plants move to grow and the Earth moves so that there is life. Thus, “constant motion in space is a given, the background for everything that has happened and that will happen [...] Action in space came long before language, as did thought based on action in space” (Tversky, 2019, p. 10).

Human beings perform movements in their daily lives all the time, even if unconsciously (Tversky, 2019). For example, if a person is asked to describe the way from home to the nearest supermarket, it is very likely that he will use gestures, which are a type of movement, to better situate the path taken. Or, imagine the movement whenever you cross the street when a car approaches, thinking whether if there is enough time to cross it or not. In this case, the movement occurs mentally as a way of judging an action to be performed in space.

In the classroom, movements are also used, especially gestures. In Math classes, for instance, the teacher can draw the representation of two straight lines, a circle, or any polygon in the air. You can also indicate with your hands the opening of an angle or locate the position of the x-axis and the y-axis in the Cartesian plane. Students, on the other hand, often perform movements while using their fingers to count, when gesturing symbols that are unfamiliar with the nomenclature, such as drawing a bar and two dots in the air to refer to the percentage symbol. Students can also move the notebook or book around, trying to see a representation from a different angle.

All these actions, carried out in the classroom to complement the teacher's practice or the students' reasoning, involve movement, however they are often carried out almost spontaneously, which makes their importance go unnoticed by those who carry them out and/or by whoever receives the information from the movements. In addition to gestures, other types of movement can be used in Math classes using software. GeoGebra allows you to freely move constructed mathematical objects, in addition to having tools that enable rotation and zoom movements, for instance.

To understand why movement is important in these and other situations, it is important to understand that there is a close relationship between movement and spatial thinking, and that movement helps us to think (Tversky, 2019). Considering that spatial thinking is a cognitive activity developed through our experience that allows us to understand the space around us, through its observation and when we move in it (Tversky 2019). In this sense, according to the author, in addition to thinking about space, we also use space and what exists in it to think.

The discovery of the relationship between movement and spatial thinking comes from experiments with rats, in which electrodes were implanted in neurons in regions of the brain called the hippocampus and entorhinal cortex, in which neuroscientists identified cells of place and later researchers identified grid cells respectively (Bortolossi, 2020; Tversky, 2019).

From the experiments, in which the rats were placed in a location to move around, it was identified that the place cells “triggered (emitted a more frequent electrical signal) when the rat was in certain places it already knew” (Bortolossi, 2020, p.102, our translation) in the same way as grid cells, but with different markings⁴. Therefore, they found that these cells serve as a spatial cognitive map, that is, they act as a GPS, and are the basis of spatial orientation and disorientation (Tversky, 2019). Tests carried out with humans involving virtual navigation through the environment reveal that the hippocampus is activated in these situations, that is, when the person is inserted and virtually navigates/explores an environment (Bortolossi, 2020). Therefore, the movement, in virtual environments, has influence on spatial thinking, as the region responsible for the location in space was activated through these simulations.

The results of a survey involving London taxi drivers who need to know the more than 25,000 streets and 20,000 landmarks of the city, highlight this aspect, as they point out that the posterior portion of the hippocampus of taxi drivers gets bigger and bigger as they spend more time in this profession, as their spatial notions are stimulated at all times (Tversky, 2019). According to Tversky (2019), in addition to representing places, place cells in the hippocampus represent other sets of characteristics such as *events, ideas and individuals*, regardless of the relationship they had. And grid cells in the entorhinal cortex also represent relationships between *spatial, temporal, or conceptual information*. The discovery of a function deviation of the studied brain regions was the highlight of this study.

In summary, the same neural structure of the brain regions that acquired the primary function of guiding us in terms of movement and space (movement activates the spatial GPS), due to evolution, began to manifest other functionalities (Tversky, 2019). Therefore, “the same brain mechanisms in humans that represent actual places in real spaces also represent ideas in conceptual spaces⁵. Spatial thinking enables abstract thinking” (Tversky, 2019, p. 72), that is, spatial thinking and the movements associated with this type of thinking are the basis for the formation of other thoughts, in particular spatial abstract thinking (Bortolossi, 2020).

Spatial thinking is mirrored in abstract thought, in social thought, in

⁴ The fire emitted by the grid cells is identified in agglomerations that together form a kind of triangular mesh as if representing a map of the place, while the fire emitted by the place cells is concentrated in a specific region (Bortolossi, 2020).

⁵ The conceptual space is like a structure in the mind that originates and organizes ideas from the relationship between any set of ideas (Tversky, 2019).

cognitive thought, in thought about what makes people tick, in thought about art and about science. Thinking is thinking, whatever the domain, and spatial thinking is core to our very existence. [...] we are far better and more experienced at spatial thinking than at abstract thinking. Abstract thought can be far more difficult in and of itself, but fortunately [...] spacial thinking can substitute for and scaffold abstract thought (Tversky, 2019, p. 59)

Therefore, it is possible to stimulate other types of thinking from spatial thinking and the movements associated with it, such as gestures that express actions based on ideas and other mental movements (Tversky, 2019). A contribution from the movements is identified in the project by Pawan Sinha (2009) which aims to teach congenitally blind Indians to be able to see after surgery. One of the main elements in this process is movement, because “the only thing the visual system needs to start analyzing the world is dynamic information” (Sinha, 2009, video [11:47-11:57]). In an experiment performed with these people, when a static image of a triangle is displayed on a computer screen, the Indian who observes it cannot identify the object, however when the triangle is set in motion, he identifies it (Sinha, 2009; Bortolossi, 2020).

Movement also plays an important role in Animated Scenarios built in GeoGebra, as we define and present an example in the following section.

2. Building Animated Scenarios in GeoGebra

The Animated Scenarios built in GeoGebra present movement as a fundamental component, as the final product of this construction is an animated scene involving mathematical elements (Bueno & Basniak, 2020). The *slider* tool is commonly used in order to animate the construction, but there are other possibilities in GeoGebra for assigning movement to the constructed scene. In addition, Animated Scenarios may or may not feature characters, everyday and/or imaginary situations, however they must have movement and a context/setting. Our main objective when working with Animated Scenarios has been to promote discussions about mathematical contents during construction, using the dynamicity made possible by GeoGebra.

The *House* Animated Scenario, discussed in this work, does not involve characters, but proposes the construction of a house, which is a representative element of our daily life. In the construction, we propose to discuss Geometry concepts related to plane and spatial geometric objects, such as polyhedrons and polygons, as well as other geometric objects such as points and segments. However, another characteristic of Animated Scenarios is that in their construction it is common to involve more than one area of mathematics, and thus, for some *details* of the construction, other contents were used, such as logic, algebra and integers and decimals.

We emphasize that the objective is not to restrict a content to be studied through the construction of the Animated Scenario, but to show how the mathematical contents

are related and can be worked together in the same situation, although the main objective may be related to one of the contents addressed.

The *House* scenario can be built using only the 3D visualization window of GeoGebra, but to meet the objective of this construction, which is to recognize and differentiate characteristics of flat and spatial geometric objects, it is essential to establish relations between the representations of the 3D window, the 2D window and also from the algebra window (Figure 1).

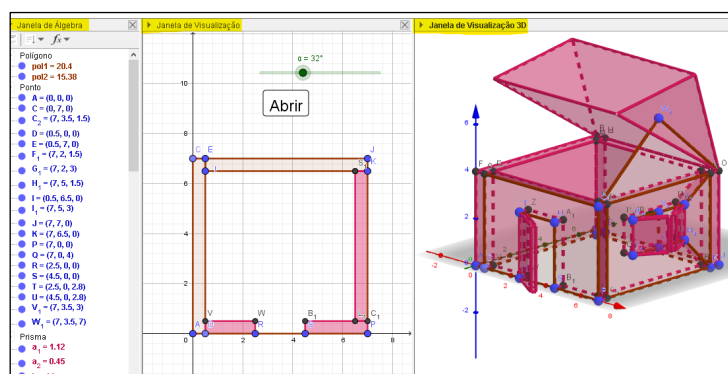


FIGURE 1: Construction of the *House* Animated Scenario in the different windows of GeoGebra
FONTE: Research data (2022)

The movement that characterizes the finished construction as an Animated Scenario is performed by some objects or characters in the scene and has some specificities: (i) it must be continuous until the objective of the scene is completed and, (ii) devoid of repeated action from the subject. In this way, after clicking on buttons or selecting the *animate* option, movement is initiated and should remain, without the need to repeatedly trigger the movements. In the *House* Animated Scenario⁶, the objects that perform movement are the door, the window, and the roof, the movement is started by clicking on the *open* button (Figure 2).

In addition, another feature of Animated Scenarios is that, after finishing the construction, changes can be made in the visual aspect, so that it better represents the constructed object. For this, the actions performed are: changing the colors; inserting images or mathematical objects into the background and complementing the scene; and hiding points, segments, and other math elements that don't contribute to the Animated

⁶ Movements in the finished *House* scenario: <https://youtu.be/WsoHeXbp0ek>

Scenario appearance. From this, the explicit visual relation between the Animated Scenario and the mathematical objects ceases to be evident for those who observe it.

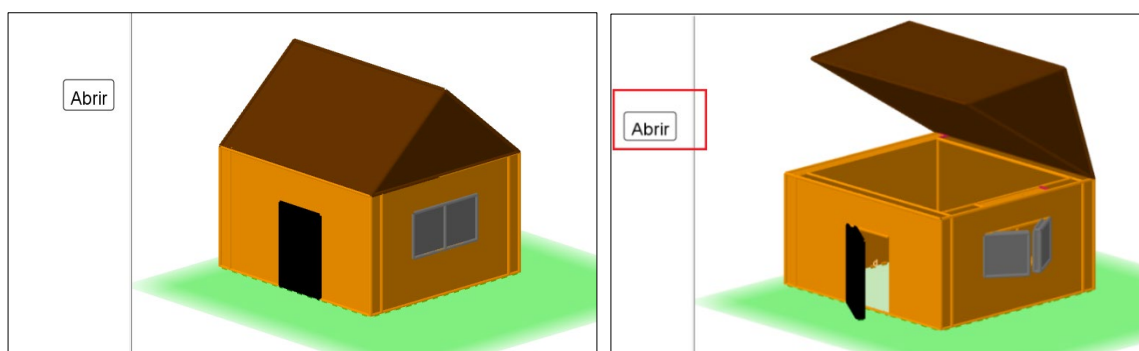


FIGURE 2: Finished *House* Animated Scenario
FONTE: Research data (2022)

Mathematical elements hidden from the viewport cannot be deleted, they are still displayed in the algebra window, otherwise the construction would also be deleted. The visualization of these objects (Figure 1) is not necessary at the end of the construction, since the discussions around the mathematical contents are carried out during the scenario construction process.

In the following sections, we discuss the flat and spatial geometric objects used in the construction of the *House* Animated Scenario, as well as establishing an articulation with the movements associated with spatial thinking based on the procedures we adopted during the construction of this Animated Scenario.

3. Movements and Spatial Thinking in the *House* Animated Scenario

The movement is an important characteristic for a scene built in GeoGebra to be considered as an Animated Scenario. These movements must appear in the final construction product, but they also appear during the stages of its construction.

It is important to emphasize that these are two different movements. When the scene is finished, the movement we observe is continuous and executed by some object that composes the scene from GeoGebra commands. On the other hand, there are also movements that are provided by the subject's action on the mathematical objects that make up the scenario, using GeoGebra tools that allow performing these movements. They can be used to observe the construction from different perspectives, analyzing whether the objects present the expected behavior. In this sense, the movements discussed in this section refer to this second case.

When using the 3D visualization window, the importance of the movements becomes more evident, because in this case, when the constructed object is three-dimensional, it is not possible to observe all its sides by visualizing it only in one

position, so it is necessary to move it to observe the features of the object that stand out as the viewing perspective changes.

GeoGebra has some tools (Figure 3) that allow to perform different types of movement, either from the visualization window or with the objects, allowing to visualize them in/from different perspectives.

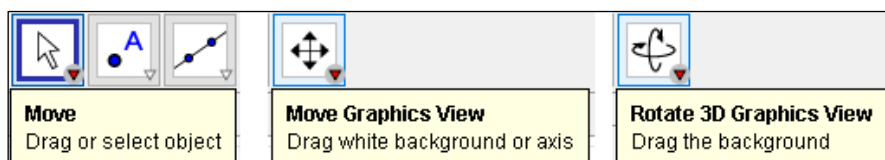


FIGURE 3: GeoGebra tools
FONTE: Research data (2022)

We identified that the movements that can be performed from these GeoGebra tools are associated with some spatial abilities (SA). According to Tversky (2019), SA are manifestations of spatial thinking associated with ways of thinking and acting on space. The word *ability* is often related to people who have *talent* or *aptitude* in some area, in this case with notions of space. According to Tversky (2019) this is not incorrect, as many people find it easier than others with location and arrangement in space. However, SA are not only innate, but can also be developed with practical activities that stimulate them, including school activities. We highlight below activities that stimulate the development of specific SA.

In the case of the movements identified from the GeoGebra tools in figure 3, two SA are involved, and which are intrinsically related. There are the *mental rotation* and the *perspective* that is based on the movement and the visualization (mental or not), in addition to the SA of *spatial judgment* and *mental construction* that we discuss ahead in this work.

Mental rotation consists of imagining something in a different orientation (Tversky 2019). This ability is used when we assemble puzzles and play tetris, for example. Tversky (2019) observed from tests performed that when solving problems that require *mental rotation*, many people spontaneously rotated their hands as if they were rotating an object, and achieved faster and more accurate performance than people who did not. Gestures appear as a support for the development of this SA in a way that “physical rotation helps to internalize mental rotation” (Tversky, 2019, p. 90).

With the GeoGebra tools it is possible to reproduce on mathematical objects in the software some actions related to this SA, such as rotating and repositioning objects. These movements are important to recognize them in positions different from the usual ones and to identify hidden characteristics of the construction. In the *House* scenario, the position we see in Figure 2 does not allow identifying the back walls of the house, for which it is necessary to change the position of the building.

It is possible to modify the position of the objects present in the construction with the *move* and *rotate* GeoGebra tools from the rotation. In this manner we change our *perspective*⁷, which is another SA that is directly related to the *mental rotation*. This SA is important to recognize the position and the characteristics of objects in space. For this we can follow things around us, assuming the perspective of an observer that is external to the object, or when we imagine ourselves in the place of another person or object, for an internal perspective (Tversky, 2019). When we rotate an object in GeoGebra or mentally, we assume different *perspectives* for observation. Depending on the object, the frontal, lateral or rear position may have different characteristics and may be confused with the representation of another object⁸.

In Figure 4 (left), the *House* scenario has been rotated and assumed the rear/side position. In this case, we were unable to visualize the geometric solids that represent the window and the door, so that this position, in the *perspective* we assume, allows us to say that it is a cardboard box, for example, in which the parallelepiped represents the structure of the box and the triangular prism the raised box flaps.

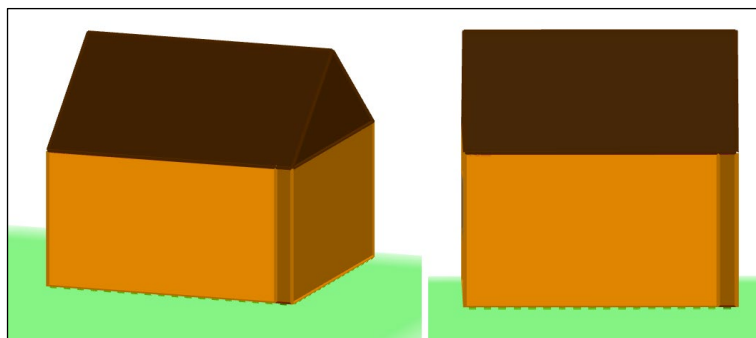


FIGURE 4: *House* Animated Scenario in back and side perspective (left) and rear perspective (right)
FONTE: Research data (2022)

In Figure 4 (right) we visualize only the rear position of the scenario. From the *perspective* assumed by us, it is not possible to understand that it is a three-dimensional object, and in the plan, the roof is formed by a triangular prism, it appears that the rectangular lateral face is straight, and not inclined. Again, we cannot say that the object is part of a house, as our *perspective* is of a rectangular figure. That's why it's important to rotate the object to visualize different representations that it can assume depending on the *perspective* of the observer. In these figures, the change of position occurs with the finished scenario, but these movements are also useful during the construction because you can rotate the object and modify the position when you only have a wall or the roof, for example, to check, from a different *perspective*, whether the construction represented what was expected or whether it was distorted. For the construction of the polyhedrons

⁷ Assumed, here, as the representation of objects as they appear to the eye, according to their position and sizes.

⁸ The *House* Animated Scenario finished at different perspectives <https://youtu.be/wKn6sxysOkY>

that represent walls, especially the polyhedra that have concavities to characterize as walls that house openings for the door and window, points were used to delimit the necessary format in each case (Figure 5).

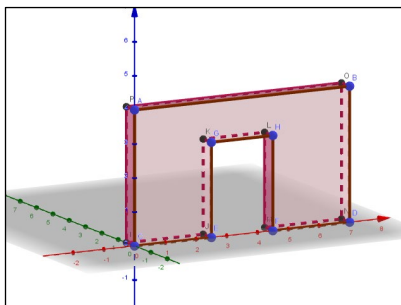


FIGURE 5: The house wall polyhedron with door opening
FONTE: Research data (2022)

For this, we use *spatial judgment* to locate the most convenient position for the points, evaluating the distance between the points that would be adequate to later insert a polyhedron in that space that represents the door, based on the size of the wall polyhedron, since the measurements must follow a proportion so that the visual aspect of the scenario is not compromised.

Spatial judgment is an SA that allows us to make comparisons on notions of distance, size and position, distinguishing what is near, far, small, large, right and left. It is an SA linked to sight, but which adds movement in imagination (Tversky, 2019). When we want to move furniture around or park a car, we use this SA. In Mathematics, we use *spatial judgment* to indicate and move coordinates in the plane and in Cartesian space, in addition to the study of scale and proportion that works directly with comparisons, as in the *House Animated Scenario*.

Before actually building these representations in GeoGebra, we already have an idea of what we want to represent, as we carry out a *mental construction* of the objects.

Since everyone knows the usual shape of a house, we don't need to look at a house to know the elements that compose it and thus build it in GeoGebra. Gutiérrez (1996) calls this element *external representation*, which is any type of representation we find in verbal, written, image, drawing, etc. format that directly influences the creation or transformation of *mental images*, that is, representations of objects, concepts or properties that are constructed in our mind. This happens because spatial thinking involves thinking and reasoning visually, with shapes and arrangements in space (Tversky, 2019). It is according to this previously mentally constructed image that we know what modifications are necessary for the GeoGebra construction, based on geometric objects, to have the characteristics of a house.

This *Mental construction* is also an SA, and involves actions such as imagining things rotating, changing size, shape, adding and removing parts of objects, and other possibilities, which are used to create infinite objects in the mind (Tversky, 2019). GeoGebra allows reproducing these representations of objects built in the mind in objects built in the software, stimulating spatial thinking.

In the construction of the *House* scenario, these (re)constructions are fundamental to obtaining the final representation in GeoGebra. For example, it was necessary to reevaluate the format of some geometric solids from those already created, removing parts to form the window and door openings. The creation of new shapes for these geometric objects is influenced by the accumulation of external representations that serve as a starting point for this improvement (Gutiérrez, 1996).

On the other hand, it is not only from visual inputs that we can build an object in the mind, it is possible to do it from a spoken description, without any visual input (Tversky, 2019), which makes it possible to build something abstract and /or visualize something that cannot be touched, such as geometric objects. On the other hand, the product of *mental construction* is associated with figurative representations, for example, when someone is asked to think of a chair, it is very likely that the person mentally constructs the image of a chair and not the word chair.

In addition to building static elements, the mind can also build animated images. Tversky (2019) points out that there are people who are specialists in this type of SA, while others may have great difficulties. A common example, also from mathematics and/or physics classes, is the systems of gears or pulleys in action, which for most people it is difficult to answer in which direction each pulley turns when it is necessary to mentally animate this system from a static representation.

Considering the construction stages, we identified that the four SA discussed in this work are closely related in this process. Starting with the *mental construction* of the house that we want to represent in GeoGebra, we need to do or redo the *mental construction* at each stage, depending on the representations obtained in the software that are analyzed by *rotating* the object, which makes it possible to observe it from different *perspectives*. Furthermore, the changes that occur on the object must conform to our *spatial judgment*.

The development of SA not only can but should be encouraged, according to the National Academy of Sciences committee, because SA can benefit school learning, performance in some professions, as well as day-to-day activities (Tversky, 2019). In schools, stimulation can occur from tasks that propose “understanding and creating maps, graphs [...] and visual explanations of not just of science and math but also of literature, history, social sciences” (Tversky, 2019, p. 102). Parents and teachers can complement this stimulus with spatial games, such as puzzles, Lego, board games and guessing games, in addition to drawing attention to comparisons, similarities,

differences, symmetries and analogies around them (Tversky, 2019). Another possibility presented here is to stimulate spatial thinking with the use of GeoGebra, as the software allows replicating actions and movements that are related to SA of *mental rotation*, *perspective*, *spatial judgment* and *mental construction*, manifested mentally.

The BNCC (Brasil, 2018) highlights that spatial thinking should be encouraged in schools and can enrich classes with sensory and visual experiences, especially in subjects such as Geography, Mathematics, and Art. The Mathematics curriculum in BNCC, for the early years of Elementary School⁹, presents objects of knowledge related to the development of spatial thinking in the field of Geometry (Table 1).

Thematic Unit	Objects of Knowledge
Geometry	Location and movement of people and objects in space, according to reference points, and indication of changes in direction and sense of direction
	Outline of simple roadmaps and plans
	Spatial geometric figures (cube, rectangular block, pyramid, cone, cylinder, and sphere): recognition and characteristics
	Flat geometric figures (circle, square, rectangle, and triangle): recognition and characteristics

Table 1. Geometry Curriculum BNCC early grades of Elementary School (Brasil, 2018, p. 282)

The development of these topics involves some of the SA already discussed, and they present contributions to studies in the area of Geometry itself in later school years. The location and movement of people and objects, described in BNCC, is associated with the SA of *perspective* and *spatial judgment* and is related to the location of Cartesian coordinates in the plane and in space. The change of direction, which carries traces of spatial judgment, can contribute to the study of vectors.

In the next section, we address in more detail the discussion of mathematical elements that can occur from the movements associated with SA. In this case, for the *House Animated Scenario*.

4. Building the Animated Scenario *House*: Mathematical Discussions

When we conceived the construction of the *House* scenario, the first step was to list the geometric objects that we considered necessary to use in the construction, in a way that they presented similarities with the objects we wanted to build. Lima and Almeida (2015) point out that students take to school an intuitive knowledge of the space

⁹ In Brazil, the early years of elementary school are a stage of education for children between the ages of 6 and 10.

in which they live in, and the teaching of Geometry must recover everyday experiences and shape for school knowledge.

We chose the construction of a house, as it is known from everyday life, so whoever builds it will be able to indicate the main characteristics that compose it. Thus, it is possible to make an association between the house built in GeoGebra and the material houses, observing their shape and characteristics to decide which geometric objects to use in the construction.

Differences between flat and spatial objects are evidenced when building the walls of the house from the insertion and comparison between them in GeoGebra, which leads to discussions about concepts and mathematical representations from the recognition of the characteristics of these objects. When we create a point in the 2D window and another in the 3D window, the difference in the structure of a point in the plane and in space is evident. Even if the point created in the 3D window presents only two non-zero coordinates, contained in the xy plane, it is represented algebraically considering three coordinates (x, y, z) showing that that environment is spatial.

When representing a polygon and a polyhedron in the GeoGebra 3D viewer, it is possible to compare these representations by imposing the *rotation* movement on these objects, identifying that they have different characteristics (Figure 6). The height of the polyhedron is an element that differentiates it from the polygon, which in turn represents the faces of the polyhedron. While the representation of a polygon involves vertices, edges, internal and external angles, and diagonals, the representation of a polyhedron relates all these elements in each of its faces.

Thus, the correct representation of the house walls has depth and therefore, they are polyhedrons and not polygons.

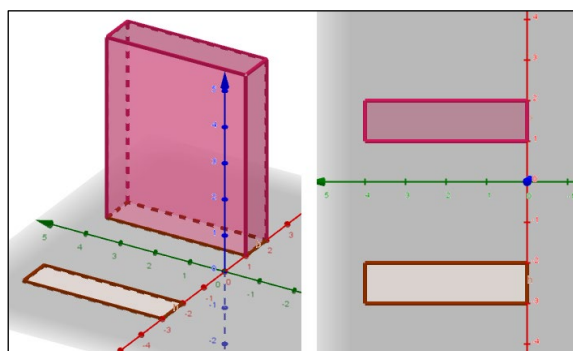


FIGURE 6: Polygon and polyhedron in 3D window, in different perspectives
FONTTE: Research data (2022)

However, without *rotating* the representations in the 3D window, depending on the assumed *perspective*, it is not possible to differentiate the flat object from the spatial one, whose faces have the same shape, if they are in the same position (Figure 6), since it is the *rotation* of the object that allows changing the *perspective* and identifying

elements that are hidden. Thus, the movements made possible by GeoGebra contribute to the recognition and differentiation of constructed objects, whether flat or spatial.

With the construction finished, movements that simulate *opening and closing* are assigned to the solids that represent the door, the window, and the roof from the *Rotate* command of GeoGebra, which rotates an object according to an angle in relation to an axis of rotation. In this case, the axis of rotation chosen was one of the edges of each polyhedron, with the angle varying according to the parameters established in the *slider* tool¹⁰. This movement helps to identify that geometric objects do not have a *correct* position to be represented. For example, when the roof is closed, the triangular prism is in one position, on the other hand, when the roof opens, the prism begins to be tilted and rotated, assuming different positions. However, the object does not undergo deformations and its properties are maintained, so it remains a triangular prism, which can be observed in different positions¹¹. This comparison can be made with geometric solids and also with the flat shapes of their faces.

In textbooks, geometric objects are usually presented in prototypical positions, considering a standard perspective and observation angle (Machado *et al.*, 2019). For instance, a triangle is usually represented as acute, equilateral, or isosceles and with one side parallel to the edges of the sheet. In the scenario, it is possible to identify other representations for the triangle belonging to the prism from the movement. In this wise, the movement helps in the differentiation between properties of a specific representation and properties of the geometric object, which are usually confused and/or understood as the same, since the static representation carries particular characteristics that do not belong to the definition of the object (Machado *et al.*, 2019).

With this joint approach between flat and spatial objects, associated with movement in GeoGebra, it is possible to identify that plane and space do not deal with objects of the same nature. For example, a rectangular prism and a rectangle are not the same object, although they are often confused, even though the rectangle is part of the prism because it is one of its faces. Therefore, in order to avoid such kind of confusion, the teaching of Spatial Geometry must be associated with the teaching of Plane Geometry, as it happens in the construction of the *House Animated Scenario*, which involves plane and spatial geometric objects concomitantly and provides discussions about their characteristics.

Final Considerations

In the construction of the *House Animated Scenario*, geometric objects such as points, segments, polygons, and polyhedra are involved. When we use them to build the

¹⁰ Using the *Rotate* command in GeoGebra <https://youtu.be/13FTIjC19KU>

¹¹ Roof triangle in different positions <https://youtu.be/h2lu3eveeE8>

House, it is possible to carry out discussions about their properties, representations, nomenclature, and the nature of these objects.

It is necessary to rotate, reposition, change the perspective, enlarge and reduce the built geometric objects, to adjust them to the shape of the house. These movements that are associated with SA, in addition to helping in the construction process, also provide opportunities for mathematical discussions, which were identified taking into account the construction process discussed in this work.

To start building, it is necessary to analyze the characteristics of geometric objects, looking for those that are most similar to the particularities of the house and, for this, it is recommended to compare objects that have characteristics in common. When building a rectangle and a prism with a rectangular base, the *rotation* movements and the change of *perspective* on them show that the rectangle has no height, unlike the prism, which can be considered a representation for the walls.

The shape of the objects must be rethought when the need arises to build walls with an opening, either for the door or the window. This shows, for example, that polygons are not restricted to the most common shapes, such as triangle, square, and rectangle. To build polygons with unusual shapes, the *rotation* movement of the GeoGebra environment helps in *spatial judgment*, so that the vertices are inserted in specific places that will depend on the *mental construction* that one has about the polygon that one intends to build.

In addition to movement, comparing objects in different GeoGebra windows also provide mathematical discussions. When a prism, for example, is compared in the 2D and 3D windows, it is evident that the flat environment does not support the representation of three-dimensional objects. If we add the algebra window to these comparisons, in addition to the figurative representation, it is possible to consider the algebraic representation of these objects, which also presents differences for plane and space. A point belonging to the plane will have two Cartesian coordinates, whereas a point created in the spatial environment, which has a height value, will have three Cartesian coordinates. From these comparisons, the recognition of important characteristics that belong to flat and spatial geometric objects and that differ from them can occur by movement, especially for space objects, in which it is not possible to observe all its parts by viewing only one position.

Thus, we identified the construction of the *House Animated Scenario* in GeoGebra as a type of educational practice to implement discussions about flat and spatial geometric objects in an articulated way. And the construction of this Animated Scenario can also be configured as a type of task that stimulates the development of the SA mentioned, as it involves movements made in mathematical objects with GeoGebra tools, associated with such SA.

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