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Calculating the volume of convex polyhedra using GeoGebra 3D

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ABSTRACT

This article presents the decomposition of the icosidodecahedron, an Archimedean convex polyhedron that is the dual of the rhombic triacontahedron, a Catalan polyhedron. The decomposition into regular triangular and pentagonal pyramids, performed in GeoGebra 3D, aims to establish a strategy for calculating the polyhedron's volume. In the polyhedron construction stage, which precedes the decomposition stage, we imported information, such as the coordinates of the vertices and faces, from the Visual Polyhedra Platform. In the calculation of the polyhedron's volume, we converted nested radicals into sums and differences of simple radicals. We made the decomposition available on a GeoGebra platform page accessible via an external link and illustrated the process with polyhedra of other classes. The decomposition strategy proved effective in calculating the volume, which agrees with data available in the literature.

Keywords: *icosidodecahedron; classes of polyhedra; nested radicals.*

Calculando o volume de poliedros convexos com o GeoGebra 3D

RESUMO

Apresentamos neste artigo a decomposição do icosidodecaedro, um poliedro convexo arquimediano, dual do triacontaedro rômbico, um poliedro de Catalan. A decomposição em pirâmides regulares triangulares e pentagonais, efetuada com o GeoGebra 3D, objetiva estabelecer uma estratégia para o cálculo do volume do poliedro. Na etapa de construção do poliedro, que antecede a etapa de decomposição, importamos informações, como coordenadas

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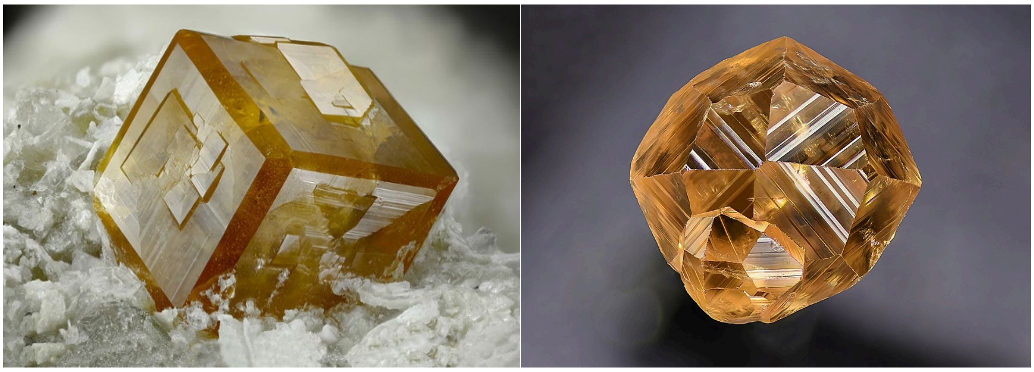
dos vértices e das faces, da Plataforma Visual Polyhedra. No cálculo do volume do poliedro, convertemos radicais duplos em somas e diferenças de radicais simples. Disponibilizamos a decomposição em uma página da plataforma GeoGebra, acessada por link externo, e ilustramos o processo com poliedros de outras classes. A estratégia de decomposição mostrou-se eficaz no cálculo do volume, cuja medida confere com os dados disponíveis na literatura.

Palavras-chave: *icosidodecaedro; classes de poliedros; radicais duplos.*

Introduction

Polyhedra appear naturally in the organization of matter, especially in molecular structures and crystals. Many molecules exhibit polyhedral geometry, such as methane, whose structure is a tetrahedron, or metallic complexes that form octahedra and other regular polyhedra. In crystals – Figure 1 – atoms or ions are organized into highly symmetrical three-dimensional networks, giving rise to external forms that reflect this internal organization, frequently associated with polyhedra, such as cubes, prisms, and dodecahedra.

Figure 1: Crystals: (a) spessartine; (b) grossular



(a) (b)

Source: (a) Mindat (2026a); (b) Mindat (2026b).

In the biological world, several viruses exhibit icosahedral capsids, a polyhedral shape that maximizes structural efficiency while minimizing the expenditure of genetic material. Radiolarians, microscopic marine organisms, produce mineral skeletons with impressive geometric shapes, often based on regular and stellated polyhedra, demonstrating how geometry arises spontaneously in natural processes.

Human constructions also draw inspiration from polyhedra to combine aesthetics, efficiency, and strength. Structures such as geodesic domes, buildings with faceted facades, and space bridges utilize polyhedral principles to distribute forces in a balanced way. Architects and engineers explore these forms for both visual beauty and material optimization, demonstrating that polyhedra connect mathematics, nature, and technology on different scales.

In this context, calculating the volume of a polyhedron is of great importance, as it allows us to quantify the space it occupies. This calculation is crucial for determining the amount of material required in construction, estimating storage capacities, analyzing crystal density, and understanding the physical properties of molecular and biological structures. Thus, the study of the volume of polyhedra is not just a mathematical exercise, but a fundamental tool for scientific, technological, and industrial applications.

Furthermore, calculating the volume of a convex polyhedron (Pugh, 1976), as defined in Definition 1 (Coxeter, 1973), is a relevant topic in the geometric training of mathematics teachers. However, this task can be complex. One possible strategy is to decompose the polyhedron into polyhedra whose volumes are known, such as prisms and pyramids.

Definition 1: A polyhedron is a finite, connected set of plane polygons, such that every side of each polygon belongs also to just one other polygon. The polyhedron is convex when the plane of each polygon leaves the others in the same half-space. The polyhedron is regular if its faces and vertex figures are regular (not necessarily convex) polygons.

The decomposition of a convex polyhedron into pyramids can be done starting from the center of the sphere in which it is inscribed (when the inscription is possible): the bases of the pyramids are the faces of the convex polyhedron; the common vertex is the center of the sphere; the lateral edge of each pyramid is the radius of the sphere. This decomposition allows us to express the polyhedron's volume as the sum of the volumes of these pyramids.

We can use GeoGebra 3D (GeoGebra, 2026) to construct and decompose polyhedra (Nós & Silva, 2019a, 2019b; Silva, 2018; Silva & Nós, 2018, 2022). Through its three-dimensional visualization and dynamic manipulation capabilities, the software enables us to represent polyhedra, identify their centers, and construct segments that connect these points to the faces of the solid, highlighting the formation of pyramids with bases coinciding with the faces of the polyhedron. This approach facilitates the understanding of the metric relationships involved, such as volume. It enables the interactive exploration of various geometric configurations (Abar & Alencar, 2013; Mathias, Silva & Simas, 2024), making the decomposition process easier in both teaching and mathematical research.

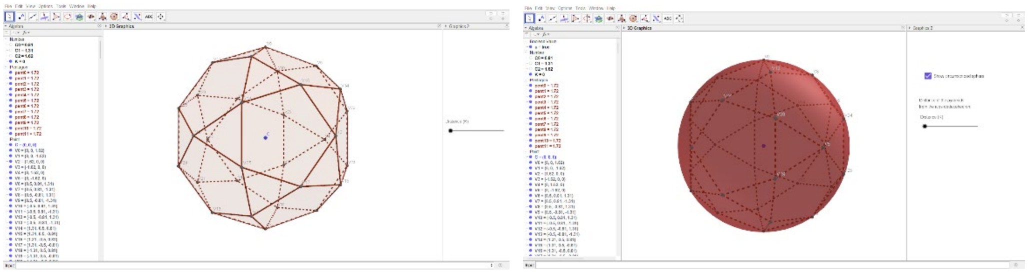
In this way, we present the decomposition of the icosidodecahedron, a convex Archimedean polyhedron. We used GeoGebra 3D to decompose the polyhedron into regular triangular and pentagonal pyramids. In the polyhedron construction phase, which precedes the decomposition stage, we imported the coordinates of the vertices and faces into GeoGebra (McCooey, 2015a, 2015b, 2015c). In calculating the volume as a function of the edge length, we converted nested radicals into sums and differences of simple radicals (Landau, 1994;

Rahal, 2017). We conclude by illustrating the decomposition of polyhedra from other classes, such as Platonic, Catalan, and Johnson solids (Catalan, 1865; Johnson, 1966; Cromwell, 2008).

1. Construction and decomposition of the icosidodecahedron

The icosidodecahedron is a semi-regular convex Archimedean polyhedron formed by 32 faces, consisting of 20 equilateral triangles and 12 regular pentagons, with 60 edges and 30 vertices. At each vertex, two triangles and two pentagons meet, giving it a highly symmetrical configuration. Figure 2 illustrates the icosidodecahedron and its inscription on a sphere.

Figure 2: Icosidodecahedron: (a) polyhedron; (b) inscription on a sphere

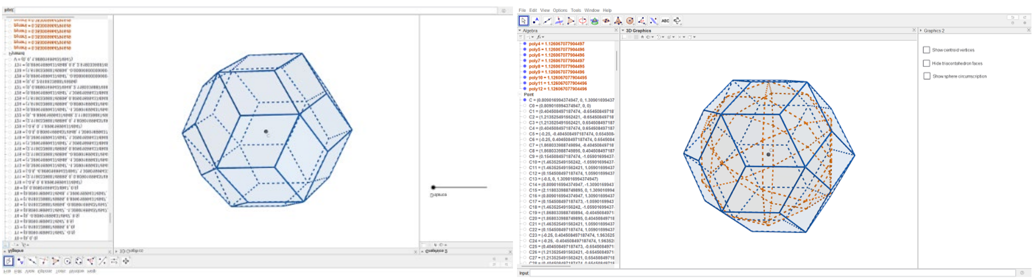


(a) (b)

Source: Authors.

The dual (Weisstein, 2026; Hart, 2025) of the icosidodecahedron is the rhombic triacontahedron, a convex Catalan polyhedron composed of 30 congruent rhombus-shaped faces, 60 edges, and 32 vertices - Figure 3. The duality relationship implies that each face of the triacontahedron corresponds to a vertex of the icosidodecahedron and vice versa, preserving the overall symmetry of the structure. Among its notable properties, the inscription and circumscription of spheres in both solids stand out, as do their significance in studies of geometry, crystallography, and the modeling of natural and artificial structures.

Figure 3: Rhombic triacontahedron: (a) polyhedron; (b) dual



(a) (b)

Source: Authors.

To calculate the volume of the icosidodecahedron – Theorem 1, we initially construct the polyhedron in GeoGebra 3D and then decompose it into regular triangular and pentagonal pyramids. The construction and decomposition process of the polyhedron is described in the following steps.

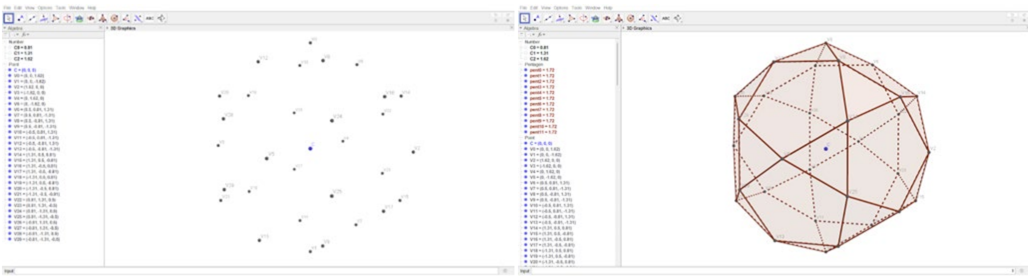
Theorem 1: The volume $\mathcal{V}$ of the icosidodecahedron with edge length a is

$$\mathcal{V} = \frac{17\sqrt{5} + 45}{6} a^3.$$

1.1 Vertices and faces coordinates

During the polyhedron construction phase, we imported the vertex and face coordinates from the *Visual Polyhedra Platform* (McCooey, 2015c) into GeoGebra. For the vertices, we have C0=(1+sqrt(5))/4, C1=(3+sqrt(5))/4, and C2=(1+sqrt(5))/2. With these values, the thirty vertices are defined: V0=(0.0, 0.0,C2); V1=(0.0,0.0,-C2); V2=(C2,0.0,0.0); V3=(-C2,0.0,0.0); V4=(0.0,C2,0.0); V5=(0.0,-C2,0.0); V6=(0.5,C0,C1); V7=(0.5,C0,-C1); V8=(0.5,-C0,C1); V9=(0.5,-C0,-C1); V10=(-0.5,C0,C1); V11=(-0.5,C0,-C1); V12=(-0.5,-C0,C1); V13=(-0.5,-C0,-C1); V14=(C1,0.5,C0); V15=(C1,0.5,-C0); V16=(C1,-0.5,C0); V17=(C1,-0.5,-C0); V18=(-C1,0.5,C0); V19=(-C1,0.5,-C0); V20=(-C1,-0.5,C0); V21=(-C1,-0.5,-C0); V22=(C0,C1,0.5); V23=(C0,C1,-0.5); V24=(C0,-C1,0.5); V25=(C0,-C1,-0.5); V26=(-C0,C1,0.5); V27=(-C0,C1,-0.5); V28=(-C0,-C1,0.5); V29=(-C0,-C1,-0.5). Figure 4(a) illustrates the determination of the vertices of the icosidodecahedron in GeoGebra 3D.

Figure 4: Icosidodecahedron coordinates: (a) vertices; (b) faces



(a) (b)

Source: Authors.

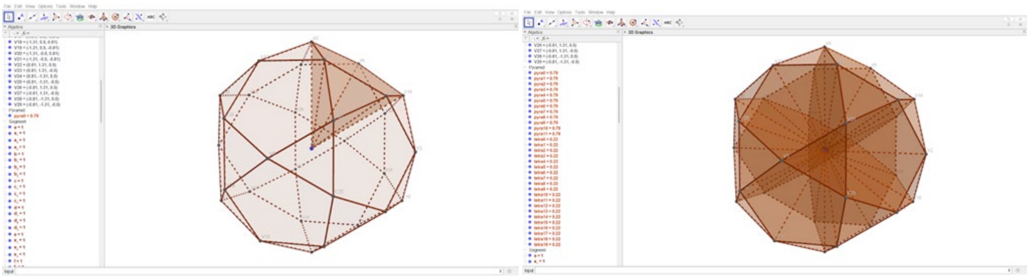
After configuring the vertices, we entered the coordinates (McCooey, 2015c) of the thirty-two faces: Polygon(V0,V8,V16,V14,V6); Polygon(V0,V10,V18,V20,V12); Polygon(V1,V7,V15,V17,V9); Polygon(V1,V13,V21,V19,V11); Polygon(V2,V15,V23,V22,V14); Polygon(V2,V16,V24,V25,V17); Polygon(V3,V18,V26,V27,V19);

Polygon(V3,V21,V29,V28,V20); Polygon(V4,V23,V7,V11,V27); Polygon(V4,V26,V10,V6,V22); Polygon (V5,V24,V8,V12,V28); Polygon(V5,V29,V13,V9,V25); Polygon (V0,V6,V10); Polygon(V0,V12,V8); Polygon(V1,V9,V13); Polygon(V1,V11,V7); Polygon(V2,V14,V16); Polygon(V2,V17,V15); Polygon(V3,V19,V21); Polygon(V3,V20,V18); Polygon(V4,V22,V23); Polygon(V4,V27,V26); Polygon(V5,V25,V24); Polygon(V5,V28,V29); Polygon(V6,V14,V22); Polygon(V7,V23,V15); Polygon(V8,V24,V16); Polygon(V9,V17,V25); Polygon(V10,V26,V18); Polygon(V11,V19,V27); Polygon(V12,V20,V28); Polygon(V13,V29,V21). Figure 4(b) illustrates the definition of the faces of the icosidodecahedron in GeoGebra 3D.

1.2 Decomposition strategy

We decompose the icosidodecahedron into pentagonal pyramids and triangular pyramids, with the faces of the solid serving as the bases and the geometric center of the polyhedron, which coincides with the center of the circumscribed sphere, as the common vertex. Initially, we determine the geometric center – Figure 5(a) – using the command C = Midpoint(V0, V1), where C is an arbitrarily assigned name and V0 and V1 are diametrically opposite points. After marking the center, we define the pyramids associated with the faces of the solid using the command Pyramid(FACE_NAME, C), where FACE_NAME corresponds to the name of the face, which can be defined arbitrarily or automatically by GeoGebra. We repeat this procedure for all 32 faces of the polyhedron, obtaining 12 pentagonal pyramids and 20 triangular pyramids – Figure 5(b).

Figure 5: Icosidodecahedron decomposition: (a) geometric center; (b) pyramids

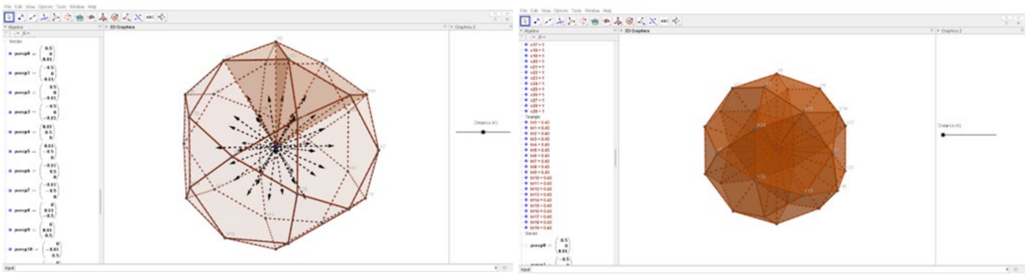


(a) (b)

Source: Authors.

Right after, in the window Graphics 2(Ctrl + Shift + 2) or in View > Graphics 2, we create a slider using the command K = Slider(0,5,0.5), where K is an arbitrary name. From this, we need to determine two types of vectors: a unit vector perpendicular to each face of the polyhedron and a vector obtained by multiplying this unit vector by the scalar K. To construct the unit vector perpendicular to a face, we use the command PerpendicularVector(FACE_NAME), where FACE_NAME represents the name of the corresponding face. We repeat this procedure for all 32 faces of the polyhedron – Figure 6(a).

Figure 6: Icosidodecahedron decomposition: (a) vectors; (b) translated pyramids



Source: Authors.

Next, we generate the vectors corresponding to the product of the scalar K by the perpendicular vectors using the command K*{PERPENDICULAR_VECTOR_NAME}, where PERPENDICULAR_VECTOR_NAME is the name assigned to the perpendicular vector. We repeat this process for all 32 vectors. Then, we hide all the vectors using the show button in the Algebra window. With the vectors defined, we translate the pyramids using the command Translate(PYRAMID_NAME, VECTOR_NAME), where PYRAMID_NAME is the name of the previously created pyramid, and VECTOR_NAME corresponds to the vector associated with the face that defines the pyramid's base. We repeated this procedure for the 12 pentagonal pyramids and the 20 triangular pyramids. Translated pyramids receive an apostrophe (‘) in their nomenclature. For better visualization, we hid the original pyramids (without ‘). By changing the value of slider K, we can move the pyramids, thus highlighting the decomposition of the solid – Figure 6(b). Finally, for a better visualization of the polyhedron without decomposition, we recommend using the “Condition to Show Object” option for the translated pyramids, with the condition K > 0, where K is the name of the created slider.

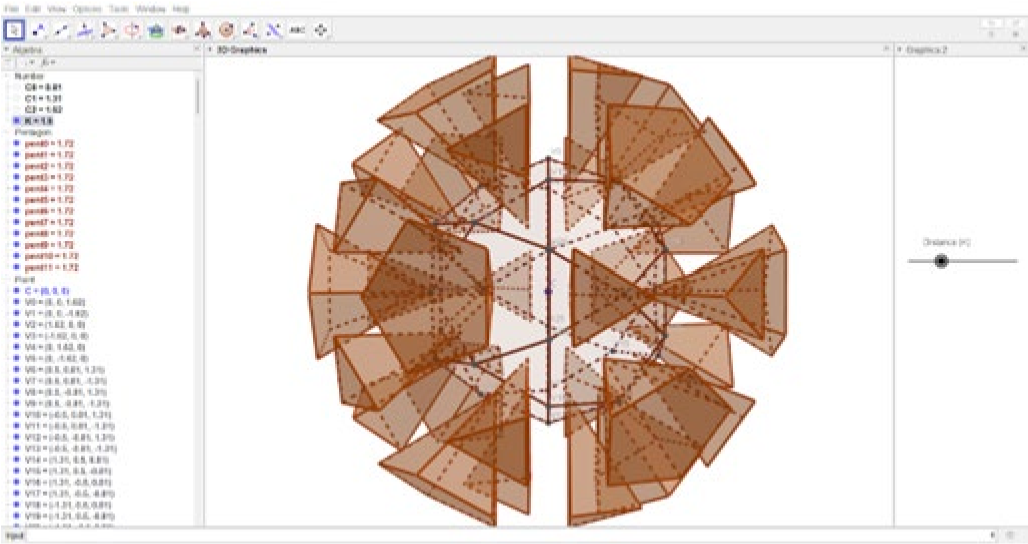
To finalize the construction and display the sphere circumscribed to the polyhedron, we use the command sph = Sphere(C,V0), where sph is an arbitrarily assigned name. This command creates a sphere with center at C and passing through point V0, resulting in the sphere circumscribed to the polyhedron – Figure 2(b). To improve visualization, we create a checkbox in the Graphics 2 window (Ctrl + Shift + 2) using the command check = Checkbox(), where check is an arbitrary name. Then, we select the sphere, either from the list of objects in the Algebra window or directly in the 3D Graphics window. In the “Advanced” options, in the "Condition to Show Object" field, we enter the name of the created checkbox. In this way, the sphere can be displayed or hidden more easily.

The decomposition of the icosidodecahedron into pyramids is illustrated in Figure 7. We have made the decomposition available on a page of the GeoGebra platform, accessible via the link

https://www.geogebra.org/classic/a7gaswy9.

On this page, the distance from the pyramids' vertices to the center of the icosidodecahedron can be adjusted using a slider (Distance(K)).

Figure 7: Icosidodecahedron's decomposition into triangular and pentagonal pyramids



Source: Authors.

2. Icosidodecahedron's volume

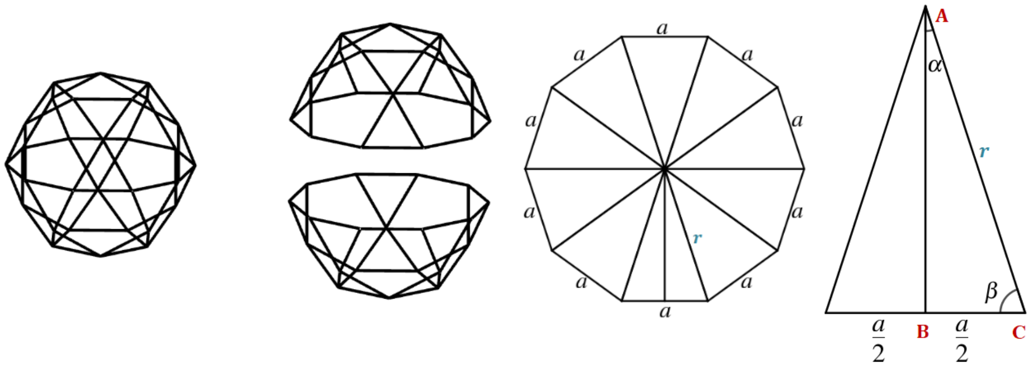
We decompose the icosidodecahedron into 12 pentagonal pyramids and 20 triangular pyramids. Thus, to prove Theorem 1, we consider the volume $\mathcal{V}$ of the icosidodecahedron equal to

$$\mathcal{V} = 12\mathcal{V}_p + 20\mathcal{V}_t, \tag{1}$$

where $\mathcal{V}_p$ and $\mathcal{V}_t$ represent, respectively, the volume of the pentagonal pyramid and the volume of the triangular pyramid.

The bases of the pentagonal and triangular pyramids are, respectively, regular pentagons and equilateral triangles of side a, where a is the measure of the edge of the icosidodecahedron; the lateral edge of these pyramids is the radius r of the sphere circumscribed to the icosidodecahedron, which can be decomposed into two pentagonal rotundas (Silva & Nós, 2018), the rotunda being a Johnson polyhedron (J6) – Figure 8(a). The base of the pentagonal rotunda is a regular decagon inscribed in the great circle of the sphere that circumscribes the icosidodecahedron.

Figure 8: Icosidodecahedron: (a) pentagonal rotunda; (b) regular decagon; (c) isosceles triangle



(a) (b) (c)

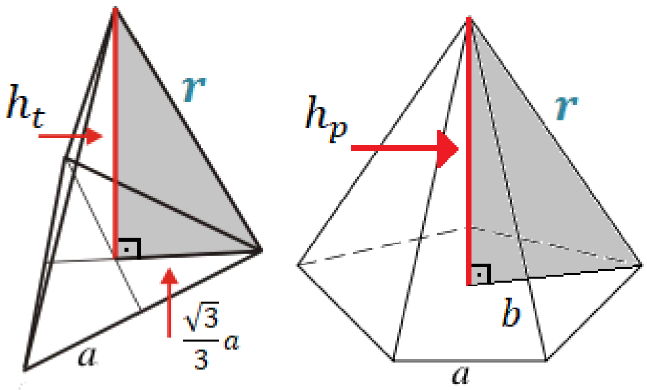
Source: Authors.

Decomposing the regular decagon into ten isosceles triangles – Figure 8(b), we find that angle α is equal to one-twentieth of the central angle. Therefore, $\alpha = 18°$. Since $\sin 18° = \frac{\sqrt{5}-1}{4}$ (Nós, Saito & Santos, 2017a), we have in the right triangle ABC – Figure 8(c) – that:

$$\sin 18° = \frac{\frac{a}{2}}{r} \Rightarrow r = \frac{\frac{a}{2}}{\frac{\sqrt{5}-1}{4}} \Rightarrow r = \frac{2a}{\sqrt{5}-1} \Rightarrow r = \frac{\sqrt{5}+1}{2}a. \qquad (2)$$

By measuring the lateral edge (2), we can calculate the heights h_t and h_p of the triangular and pentagonal pyramids, respectively, as illustrated in Figure 9.

Figure 9: Pyramid: (a) triangular; (b) pentagonal



(a) (b)

Source: Authors.

Using (2), we obtain by applying the Pythagorean theorem (Silva, Nós & Sano, 2023) to the right triangle in Figure 9(a):

$$r^2 = {h_t}^{\mathbf{2}} + \left(\frac{\sqrt{3}}{3}a\right)^{\mathbf{2}} \Rightarrow {h_t}^{\mathbf{2}} = \left(\frac{\sqrt{5}+1}{2}a\right)^{\mathbf{2}} - \frac{a^{\mathbf{2}}}{3} \Rightarrow {h_t}^{\mathbf{2}} = \frac{\sqrt{5}+3}{2}a^2 - \frac{a^{\mathbf{2}}}{3}$$

$$\Rightarrow {h_t}^{\mathbf{2}} = \frac{18\sqrt{5}+42}{36}a^2 \Rightarrow h_t = \frac{\sqrt{42+18\sqrt{5}}}{6}a. \quad (3)$$

In (3), we have a nested radical that can be rewritten as the sum of simple radicals (Landau, 1994; Nós, Saito & Santos, 2017a, 2017b; Rahal, 2017):

$$\sqrt{42+18\sqrt{5}} = \sqrt{42+\sqrt{1620}} = 3\sqrt{3}+\sqrt{15}. \quad (4)$$

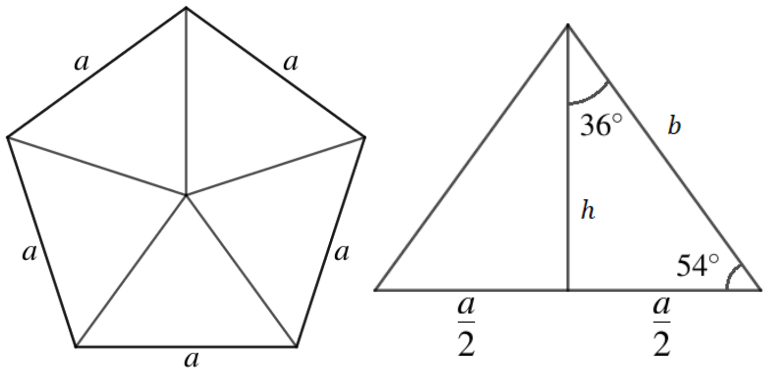
Substituting (4) into (3), we have the height h_t of the triangular pyramid expressed as a function of simple radicals. Thus, the volume $\mathcal{V}_t$ of the triangular pyramid, where $\mathcal{A}$ expresses the measure of the area, is equal to:

$$\mathcal{V}_t = \frac{1}{3}\mathcal{A}(equilateral\ triangle)h_t \Rightarrow \mathcal{V}_t = \frac{1}{3}\left(\frac{\sqrt{3}}{4}a^2\right)\left(\frac{3\sqrt{3}+\sqrt{15}}{6}a\right) \Rightarrow$$

$$\mathcal{V}_t = \frac{\sqrt{5}+3}{24}a^3. \quad (5)$$

To calculate the volume of the pentagonal pyramid, we first need to calculate the measure b of the congruent sides of the isosceles triangles that make up the regular pentagon – Figure 10(a).

Figure 10: Decomposition of a regular pentagon into isosceles triangles



(a) (b)

Source: Authors.

Knowing that $\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}$ and that $\tan 36° = \sqrt{5-2\sqrt{5}}$ (Silva, 2018), we have in the right triangle of Figure 10(b) that:

$$\sin 36° = \frac{\frac{a}{2}}{b} \Rightarrow b = \frac{\frac{a}{2}}{\frac{\sqrt{10-2\sqrt{5}}}{4}} \Rightarrow b = \frac{2a}{\sqrt{10-2\sqrt{5}}} \Rightarrow b = \frac{\sqrt{50+10\sqrt{5}}}{10}a; \quad (6)$$

$$\tan 36° = \frac{\frac{a}{2}}{h} \Rightarrow h = \frac{\frac{a}{2}}{\sqrt{5-2\sqrt{5}}} \Rightarrow h = \frac{\sqrt{5+2\sqrt{5}}}{2\sqrt{5}}a \Rightarrow h = \frac{\sqrt{25+10\sqrt{5}}}{10}a. \quad (7)$$

Applying the Pythagorean theorem to the right triangle in Figure 9(b), we obtain, using (2) and (6):

$$r^2 = h_p{}^2 + b^2 \Rightarrow h_p{}^2 = \left(\frac{\sqrt{5}+1}{2}a\right)^2 - \left(\frac{\sqrt{50+10\sqrt{5}}}{10}a\right)^2 \Rightarrow$$

$$h_p{}^2 = \frac{\sqrt{5}+3}{2}a^2 - \frac{50+10\sqrt{5}}{100}a^2 = \frac{25+10\sqrt{5}}{25} \Rightarrow h_p = \frac{\sqrt{25+10\sqrt{5}}}{5}a. \quad (8)$$

Thus, using (7) and (8) to determine the volume $\mathcal{V}_p$ of the pentagonal pyramid, we have that:

$$\mathcal{V}_p = \frac{1}{3}\mathcal{A}(regular\ pentagon)h_p \Rightarrow \mathcal{V}_p = \frac{1}{3}\left(\frac{5}{2}ah\right)\left(\frac{\sqrt{25+10\sqrt{5}}}{5}a\right) \Rightarrow$$

$$\mathcal{V}_p = \frac{1}{3}\left(\frac{5}{2}a\frac{\sqrt{25+10\sqrt{5}}}{10}a\right)\left(\frac{\sqrt{25+10\sqrt{5}}}{5}a\right) \Rightarrow \mathcal{V}_p = \frac{2\sqrt{5}+5}{12}a^3. \quad (9)$$

Substituting (5) and (9) into (1), we conclude that the volume $\mathcal{V}$ of the icosidodecahedron is equal to:

$$\mathcal{V} = 12\mathcal{V}_p + 20\mathcal{V}_t \Rightarrow \mathcal{V} = 12\left(\frac{2\sqrt{5}+5}{12}a^3\right) + 20\left(\frac{\sqrt{5}+3}{24}a^3\right) \Rightarrow$$

$$\mathcal{V} = \frac{17\sqrt{5}+45}{6}a^3. \quad (10)$$

Measure (10) confirms the thesis of Theorem 1 and agrees with the results available in the literature (McCooey, 2015b).

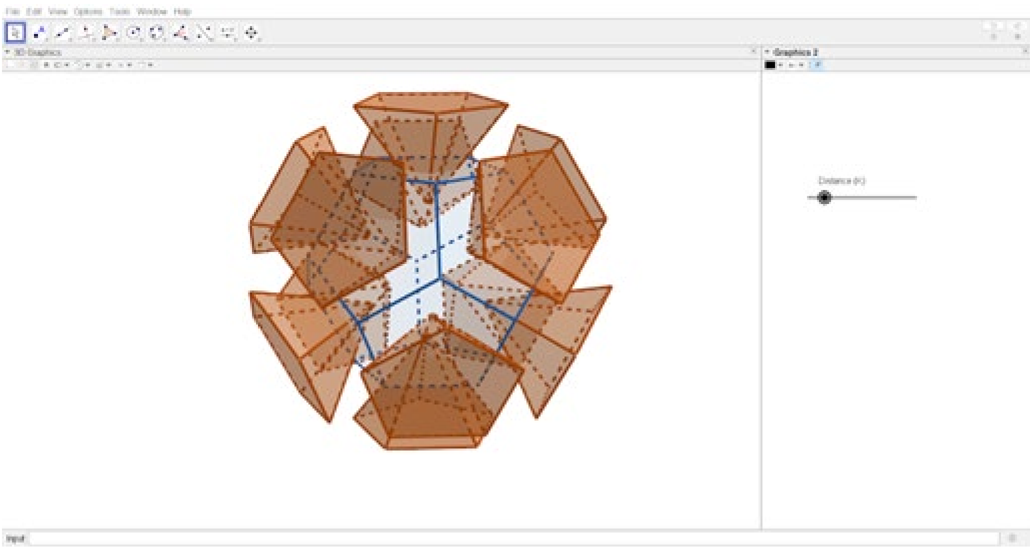
3. Other decompositions

In this section, we present the decomposition of polyhedra into three classes: Platonic, Catalan, and Johnson. These decompositions can be performed in GeoGebra by following the procedure previously described for the icosidodecahedron, except for the decomposition of the triakis octahedron.

3.1 Regular dodecahedron

The regular dodecahedron is a Platonic solid, the dual of the regular icosahedron, another Platonic solid. The regular dodecahedron has 12 pentagonal faces, 30 edges, and 20 vertices. We can decompose it into 12 pentagonal pyramids, each starting from the geometric center. The decomposition, illustrated in Figure 11, is available on a GeoGebra page accessible via the link https://www.geogebra.org/classic/dafz2vqn. The volume of the regular dodecahedron is calculated in Silva (2018).

Figure 11: Regular dodecahedron decomposition

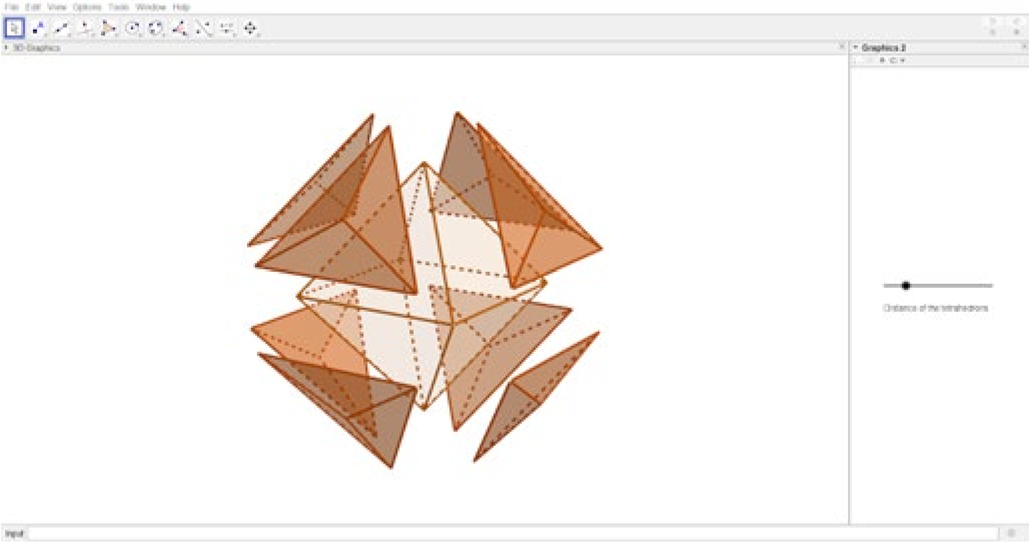


Source: Authors.

3.2 Triakis octahedron

The triakis octahedron is a Catalan polyhedron, the dual of the truncated cube, an Archimedean solid. This polyhedron has 24 isosceles triangular faces, 36 edges, and 14 vertices. We can decompose it into a regular octahedron (a Platonic solid) and eight triangular pyramids. This decomposition, illustrated in Figure 12, was created on a GeoGebra page and can be accessed via the link https://www.geogebra.org/classic/exjhujax. The calculation of the volume of the triakis octahedron is presented in Silva & Nós (2022).

Figure 12: Triakis octahedron decomposition

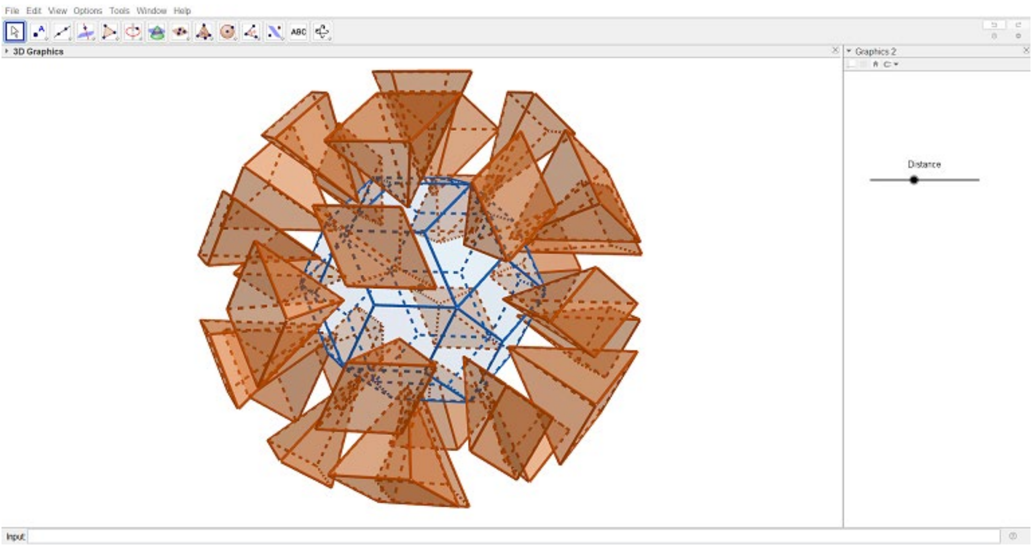


Source: Authors.

3.3 Rhombic triacontahedron

The rhombic triacontahedron is a Catalan polyhedron, the dual of the icosidodecahedron, which is an Archimedean polyhedron. The rhombic triacontahedron has 60 edges, 32 vertices, and 30 faces, all of which are rhombus-shaped. We decompose the polyhedron into 30 pyramids with rhombus bases, starting from its geometric center. The decomposition, illustrated in Figure 13, is available on a GeoGebra page, accessed via the link https://www.geogebra.org/classic/yygj4mxp. The volume of the rhombic triacontahedron is given in McCooey (2015d).

Figure 13: Rhombic triacontahedron decomposition

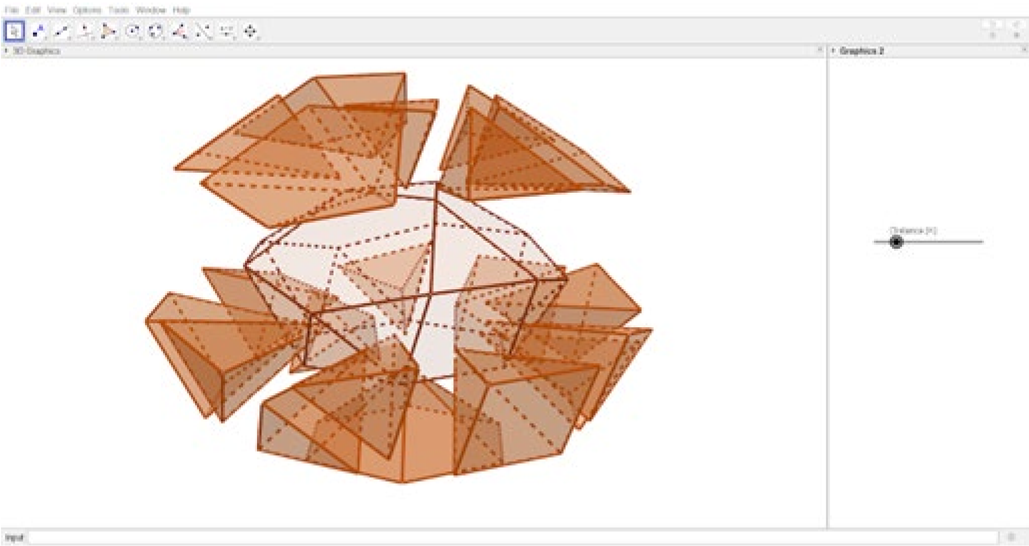


Source: Authors.

3.4 Triangular hebesphenorotunda (J92)

The triangular hebesphenorotunda is a Johnson polyhedron (J92) with 36 edges, 18 vertices, and 20 faces: 13 equilateral triangles, 3 squares, 3 regular pentagons, and 1 regular hexagon. The decomposition of this polyhedron into 20 pyramids, illustrated in Figure 14, was performed from the geometric center and is available on a GeoGebra page accessible via the link https://www.geogebra.org/classic/jwvw8nf5. The volume of the triangular hebesphenorotunda is given in McCooey (2015e).

Figure 14: Triangular hebesphenorotunda (J92) decomposition



Source: Authors.

Concluding remarks

We used GeoGebra 3D to decompose the icosidodecahedron into pyramids and then calculate its volume. We made the decomposition available on a GeoGebra page via an external link and used GeoGebra to construct two- and three-dimensional figures.

In the polyhedron construction stage, we imported the coordinates of the vertices and faces of the polyhedra selected for this article from the *Visual Polyhedra Platform* (McCooey, 2015a, 2015b, 2015c) into GeoGebra 3D. Since these coordinates are generally irrational, we emphasize the importance of defining the maximum number of significant digits (15) in GeoGebra (floating-point arithmetic). Using a few significant digits, such as two decimal digits, results in the construction of polyhedra with distortions (non-convex) due to the propagation of truncation and rounding errors.

We highlight that, despite the limited range of polyhedra in GeoGebra 3D's library, it is an excellent tool for constructing and decomposing polyhedra. Decomposition has proven to be an effective strategy for calculating the volume of polyhedra, a task that is often complex.

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We used the Grammarly tool (https://www.grammarly.com/) in the development of this article to assist with linguistic revision and the reformulation of text sections, ensuring greater clarity and argumentative cohesion. The tool was used across all sections of the text, under the authors' critical supervision and complete review, and the authors assume full responsibility for the content presented.

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