Editing geometries: The geometry volumes in Klein's encyclopedia

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Abstract

In the present article I describe the structure of the volumes on geometry in the *Encyclopedia of Mathematical Sciences* designed by the German mathematician Felix Klein (1849-1925) and compare it to the one of the *Jahrbuch über die Fortschritte der Mathematik*. I will argue that some global features, like lack of opposition between synthetic and analytic geometry, were present in the very first volumes of Klein's encyclopedia, thus foreshadowing similar changes later made in the *Jahrbuch*. The regular participation of young mathematicians as coauthors allowed for the inclusion of innovative subjects in the *Encyclopedia*. In addition, changes were made in the structure of this work along its 30 years of continuous publication. Finally, I included some information as to the reception and obsolescence of Klein's publishing project.

Keywords

History of scientific publication; Geometry; Felix Klein's encyclopedia.

Editando geometrias: os volumes de geometria na enciclopédia de Klein

Resumo

Descrevemos a estrutura editorial dos volumes de geometria da enciclopédia de ciências matemáticas idealizada pelo matemático alemão Felix Klein. Esta estrutura é comparada com a organização editorial do *Jahrbuch über die Fortschritte der Mathematik*. Argumentamos que aspectos gerais do projeto editorial, tais como uma não oposição entre as geometrias sintéticas e analíticas, já estavam presentes desde a publicação dos seus primeiros volumes e anteciparam mudanças similares que ocorrerão no *Jahrbuch*. A presença regular de jovens matemáticos como autores da enciclopédia é associada à presença de novos temas. Ela também indica que alguns aspectos do projeto foram decididos ao longo dos trinta anos de sua edição. Alguns dados sobre a recepção e obsolescência do projeto são discutidos no decorrer do texto.

Palavras-chave

História da edição científica; Geometria; Enciclopédia de Felix Klein.

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Introduction

About 4,800 pages, 40 entries, written by 33 authors and distributed across six volumes devoted to geometry: the numbers speak by themselves.[1] When placed on a bookcase shelf side by side with the further 18 volumes on arithmetic, analysis, mechanics, physics, geophysics and astronomy, these books make a strong impression. However, on a closer look, the homogeneity and precision of this collection fades away as a function of several mistakes and irregularities.

The *Enzyklopädie der mathemathischen Wissenschaften mit Einschluss ihrer Anwendungen* (EMW – Encyclopedia of the mathematical sciences, including their applications, published by B.G. Teubner-Verlag from 1904 to 1935) was not compiled based on the alphabetical order of its entries, but on one resulting from by surveys commissioned to several editors by Felix Klein, the Göttingen don of mathematics, who intended to convey his vision on how the field should look like. The editors invited by Klein to organize the geometry volumes, Wilhelm Franz Meyer (1856-1934) from Königsberg, Prussia, and Hans Mohrmann (1881-1941) from Basel, Switzerland, seemingly did not set rigid standards. As a result, the length of the entries is rather variable, ranging from 30 to more than 400 pages. Then, the interval from the elaboration to the actual publication of entries could be quite long in some cases. For instance, the last line in Corrado Segre's (1863-1924) article on multidimensional spaces, published as a separate brochure in March 1921, informs that it had been delivered to the editors nine years earlier. For that reason, many of the entries conveyed outdated information. Another example is the one of Georg Scheffers (1866-1945), author of "Besondere transzendenten Kurven" (Particular transcendent curves), who by May 1903 was in Darmstadt, but the editors indicated he had moved to Charlottenburg, near Berlin, in September. As a result, a cloud of 'paratexts'[2] is indicative of the continuous editorial work performed to keep the sections as free from error as possible.

Then, EMW was not published as an integrated work. Articles, books, monographs… Some of the entries were sold as separate brochures, which at times could be more expensive than full books also published by B.G. Teubner. Thus, e.g., the 123-page translation of a monograph by Iwan Alexandroff, *Aufgaben aus der niederen Geometrie* (Exercises of elementary geometry), sold for 2.40 German marks, while the shorter, 96-page EMW entry by Otto Staude (1857-1928) sold for 2.80 German marks. Price differences like this were probably due to the market demand, as Alexandroff's book was intended for teachers, who usually represent a quite larger market compared to the researchers in mathematical sciences, i.e., the targeted readership of Klein's encyclopedia. Indeed, in 1942, the Norwegian-born American mathematician, Øystein Ore (1899-1968) remarked that, "The first encyclopedia was extremely expensive, and was for the most part supported by the

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[1]The full list of volumes, entries and authors is described in appendices 1 and 2.
[2]The concept of 'paratext' has been explored at length by Gérard Genette, *Paratextos Editoriais* (São Paulo: Ateliê Editorial, 2009).

subscribers of the libraries. Only few people could afford purchasing an entire encyclopedia. Even single volumes were not regularly sold as they were not generally affordable"[3]. It was thus a tough task to spread the Göttingen mathematical ideals across the world, especially in war times.

By the early 1930s, when the last volumes of the EMW appeared, the earliest entries were already 30 years old. Naturally, the scientific preferences and paradigms that had oriented its conception had changedalong that time. Klein died in 1925 and thus did not get to see the last entry on geometry, by the Italian mathematician Luigi Berzolari (1863-1949). That 432-page entry on algebraic transformations, the largest in the EMW, was edited in 1933 by Meyer, who died one year later. Ten years later, 22 out of the 33 authors of the geometry contributions to the EMW were also dead.[4] A French version of the whole enterprise, directed by Jules Molk (1857-1914), was discontinued at the beginning of World War I.

Nevertheless, the volumes outlived their authors and editors, as well as the reviews of their subjects, as documents for studies on the history of science.[5] Might they be taken as representatives of the status of the mathematical sciences at the beginning of the century? Might they provide a panorama of the geometry that was produced in Europe by that time?

Hierarchies for geometries

The six volumes of the EMW devoted to geometry were divided in four sections: A) pure geometrical theory; B) foundations of the applications of algebra and analysis to geometry; C) algebraic geometry; and D) differential geometry. A comparison of the former two to the latter two suggests that they represent a less cohesive group of entries. In sections A and B one finds multiple objects and techniques, such as projective geometry, topology, theory of polyhedrons, elementary geometry of triangles, philosophy of geometry and historical discussions on the synthetic and analytical methods in geometry. Under heading 'Algebraic Geometry', the editors included contributions that analyzed curves and surfaces on planes and in spaces, in addition to the enumerative method, birational transformations and multidimensional spaces. The last section also contains some studies on curves and surfaces from the perspective of calculus, one entry on differential invariants and another on Bernhard Riemann's (1826-1866) papers.

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[3]Øystein Ore, "Enzyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Book Review." *Bulletin of the American Mathematical Society* 48 (1942) Part 1: 653-8.

[4] Hélène Gispert, "German and French Editions of Klein-Molk Encyclopedia: Contrasted Images," in Umberto Bottazzini, & Amy Dahan-Dalmedico, ed., *Changing Images in Mathematics: From the French Revolution to the New Millenium* (London: Routledge, 2001), 113-35.

[5]Ore, "Enzyklopädie der mathematischen Wissenschaften".

While the outlines of each category had been delimited at the very inception of the EMW, many changes were introduced over the following three decades. B.G. Teubner's advertising in the *Mathematischen Annalen*, in 1904 (Fig. 1), points to the existence of three parts (*Teile* I, II and III), which indeed was the final plan of organization, with sections A and B subsumed in *Teil* I (T1), section C corresponding to *Teil* II (T2) and section D to *Teil* III (T3). However, I could establish that the various parts comprised several volumes or issues (*Hefte*), as follows: T1, two volumes, H1 and H2; T2, three volumes, H1, H2A and H2B; and T3, only one issue, to a total of six volumes.

Fig. 1. Advertisement of the EMW in journal *Mathematischen Annalen*, 1904
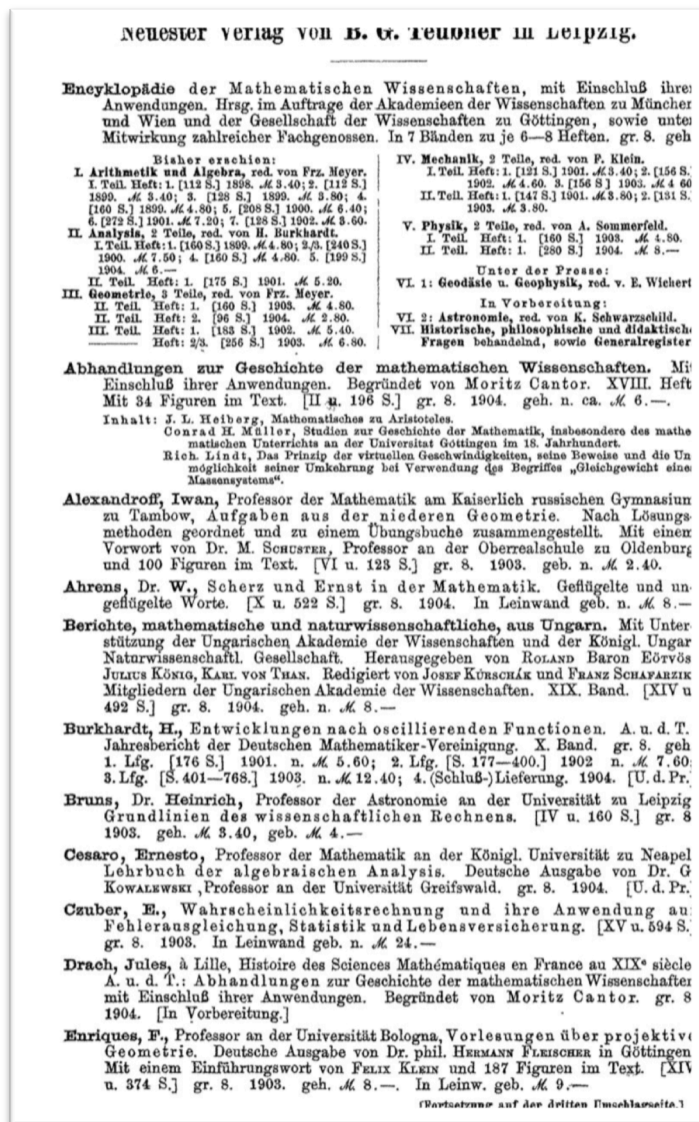
As it was mentioned above, each volume could be sold as separate brochures,which could be bound together later on, like the popular encyclopedias. Nine of the first pieces corresponding to T1 H1 were first sold as four separate brochures, which ought to be read in a particular sequence following Klein's grand design. In the very first brochure, for instance,

"Analysis situs", by Max Dehn (1878-1952) and Poul Heegard (1871-1948), was bound with Federigo Enriques' (1871-1946) principles of geometry and Hans C.F. von Mangoldt's (1854-1925) concepts of line and surface. Thus a philosophical air was added to that which, in 1907, was considered to be the ultimate foundation of geometry and that be later on came to be known as topology. To be sure, such relationship between analysis situs and philosophy also appears in the *Jahrbuch über die Fortschritte der Mathematik,* namely, the European journal of mathematical reviews, in which the entries dealing with the principles of geometry, explanations on continuity (i.e., analysis situs or topology), elementary geometry and descriptive geometry were always grouped together. Therefore, the EMW seemingly combined the *Jahrbuch* well-known system of hierarchies with publishing trade practices.

Such strategy, however, was not without some shortcomings. As an example, Segre's contribution on multidimensional spaces, the seventh in section C, could be alluded as "3, 2 H2A, 7", "3, 2, 7" or "3 C 7" (number '3' denotes the third main division of whole EMW, i.e., geometry). In fact, this true Babel-like reference system was maintained even when complete volumes were printed as such. In turn, readers who bought separate brochures sometimes bound them in random order or according to their different tastes or reading habits, as some preserved copies demonstrate.

Encyclopedia versus *Jahrbuch*: analytic versus synthetic geometry

The organization of EMW sections A and B becomes more clear when one compares the overall structure of Klein's encyclopedia to the grouping of the reviews on geometry in the *Jahrbuch.* Actually, the geometry reviews published from 1890 to 1915 in the *Jahrbuch* were distributed across two main sections, to wit, "Pure, elementary and synthetic geometry" and "Analytic geometry". EMW sections A and B were very similar to some of the secondary branches of the *Jahrbuch* first section, the headings of which were: principles of geometry, analysis situs, elementary geometry, descriptive geometry and new synthetic geometries. All the studies on both analytic and differential geometries were restricted to the second part of the *Jahrbuch*.

In 1915, 47% of the reviews in the first part of the *Jahrbuch* corresponded to papers on geometry, from which 23% were on plane geometry, trigonometry and stereometry. The subcategories of differential and algebraic geometry, included in EMW T2 and T3, were included as autonomous categories in the *Jahrbuch* only in 1915 and 1925, respectively, thus superseding the older system.

Such rearrangement hid the former opposition between analytic and synthetic geometries, which was akin to the ideas of Felix Klein. In his book, *Elementary Mathematics from an Advanced Standpoint*, he stated:

"In mathematics, however, as everywhere else, men are inclined to form parties, so that there arose schools of pure synthesists and schools of pure analysts, who placed chief emphasis upon absolute "purity of method," and who were thus more one-sided than the nature of the subject demanded. Thus analytic geometricians often lost themselves in blind calculations, devoid of any geometric representation. The synthesists, on the other hand, saw salvation in an artificial avoidance of all formulas, and thus they accomplished nothing more, finally, than to develop their own peculiar language formulas, different from ordinary formulas."[6]

In his contribution to the EMW, the Italian mathematician Gino Fano (1871-1952) acknowledged one geometry only. To him, both methods dealt with one and the same issue: "In fact, every synthetic reasoning may be 'analytically translated' […] and likewise, every calculation may be geometrically interpreted"[7]. Therefore, both editors and authors followed the ideas championed by Klein. Indeed, in 1933, the classification of reviews in the *Jahrbuch* was very similar to the one set in the EMW (Table 1).

Table 1. Classification and percent content of reviews in the *Jahrbuch*, 1933

Categories of reviews	Percent content
A) Generalities. Foundations of geometry	4%
B) Topology	16%
C) Elementary geometry	11%
D) Descriptive geometry	2%
E) Algebraic geometry	33%

Although as early as 1893, Ferdinand G. Frobenius (1849-1917) considered Klein's then projected EMW as an example of "overaged science"[8], the EMW probably anticipated some views that came to be held by the *Jahrbuch* editors, as well as certain changes amidst the European community of mathematicians. Nevertheless, such advancements might well have been due to circumstances other than an alleged (and indemonstrable) influence of Klein and his EMW.

All improvement and updating notwithstanding, the inclusion of Dehn and Heegard and their paper on analysis situs might represent a good example of the inconsistency of the

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[6]Felix Klein, *Elementary Mathematics from an Advanced Standpoint. Vol 2: Geometry* (New York: Dover, 2004), 55-6.
[7]Gino Fano, "Gegensatz von synthetischer und analytischer Geometrie in seiner historischen Entwicklung im XIX. Jahrhundert," *Enzyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, 3 T1 H1, 228.
[8]David Rowe, "Klein, Hilbert, and Göttingen Mathematical Tradition," *Osiris* 2nd series 5 (1989): 186-213, on 209.

EMW editorial project. Dehn and Heegard's entry, published in 1907, has been deemed as one of the earliest systematic elaborations of analysis situs,[9] but this belief is questionable in the light of earlier work published by Frigyes Riesz (1880-1956), Arthur M. Schönflies (1853-1928) and Heinrich F.F. Tietze (1880-1964).[10]

It is worth to keep in mind that in the beginning of the 1900s, Dehn and Heegard, both students of Klein and David Hilbert's (1862-1843), were making their first steps in this field of research, and by 1907 their work had received just a few reviews in the *Jarhbuch*. This to say, although Dehn and Heegard were inexperienced scholars and theirwork had little visibility, they were nonetheless commissioned to write an entryfor the EMW. By comparison, other contributors to Klein's encyclopedia were much more widely acknowledged and had published more papers on analysis situs, like Schönflies and Ernest Steinitz (1871-1928). In 1930, Tietze published together with Leopold Vietoris (1891-2002), a young researcher, the last paper on this subject, "Relationships between the various branches of topology", included in the very first volume of the geometry division of the EMW (T1 H1).

Incidentally, it is worth to observe that some authors started their careers after 1907, i.e., when the first geometry entries of the EMW began to appear in bookstores. That is the case of Vietoris, Hermann Rothe (1882-1923), Ludwig Berwald (1883-1942), Paul Alfred Lotze (1882-1964) and Roland Weitzenböck (1885-1955). Indeed, the EMW was authored by two generations of mathematicians. An analysis of the experience of some of the authors, as measured by the number of years elapsed from the date they earned their PhDs and the date their contributions to the EMW were submitted, as well as of their notoriety, as indicated by the number of times their papers were reviewed in the *Jahrbuch*, tells of the heterogeneous characteristics of the collaborators recruited by the editors appointed by Klein (Fig. 2).

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[9] Charles S. Fisher, "Max Dehn," in *Dictionary of Scientific Biography*, ed. Charles C. Gillispie (New York: Charles Scribner's Sons, 1970-90), vol. 4, 9.

[10] See, for example: Heinrich Tietze, "Zur Analysis situs mehr dimensionaler Mannigfaltigkeiten," *Sitzungsberichte der Wiener Akademie* 115 (1906): 841-6; Arthur Schönflies, "Über einen grundlegen den Satz der Analysis situs," *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, Mathematisch-Physikalische Klasse (1902): 185-92; Frigyes Riesz, "Über einen Satz der Analysis situs," *Mathematischen Annalen* 59 (1904): 409-15.

Fig. 2. Number of years after PhD thesis (in blue) and number of reviews in the *Jahrbuch*(in orange) in the year of submission of entries to EMW
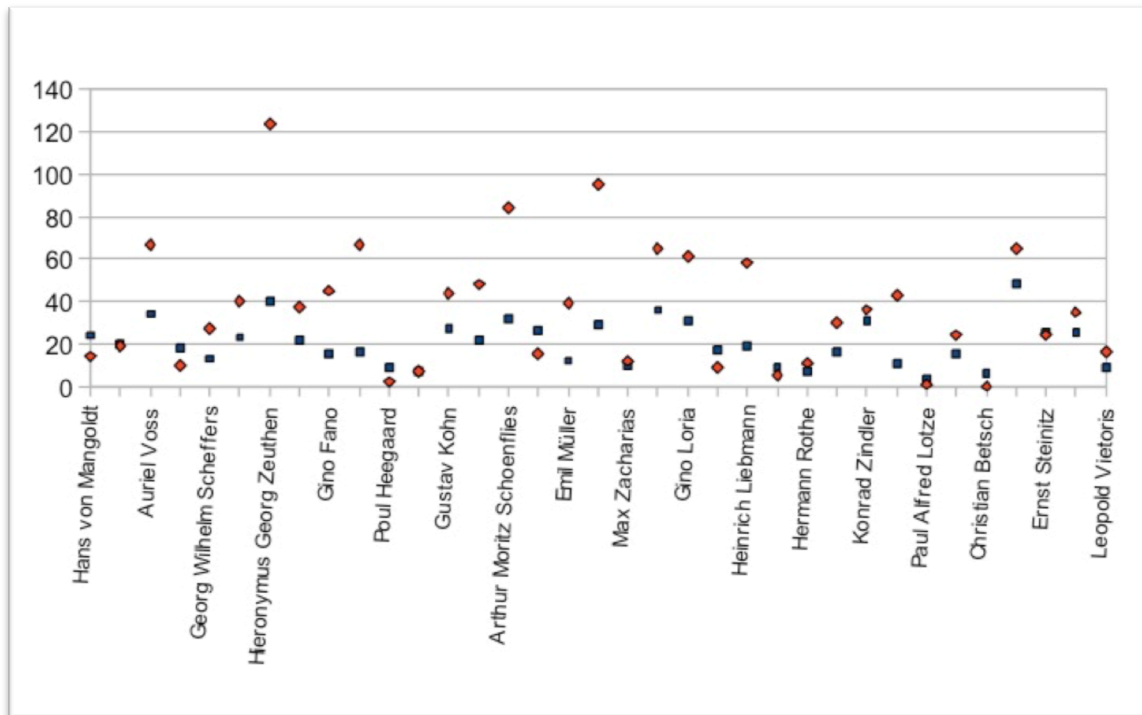
Back to the comparison to the *Jahrbuch*, the EMW included an entry on descriptive geometry, a theme in decline among practicing mathematicians, as indicated by the number of reviews compared to analysis situs. Its presence is probably due to Klein's views on the topic as having a significant connection with engineering.[11] This was not the case of geometrical optics, which was a sub-topic of differential geometry in the *Jahrbuch*. The entry by Albert Wangerin (1844- 1933) on this subject ended up with papers on electrostatics in the EMW physics division.

Postscript: may obsolescence be avoided?

Going back to Frobenius, who considered the EMW design outdated even before it actually started, one might ask: Would the cooperation of renowned scholars with young mathematicians and the addition of up-to-date papers on new topics be good remedial measures to avoid EMW falling into obsolescence? In the end, they were not. Footnote references to the EMW in papers published in a handful of European scientific journals

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[11]Gert Schubring. "Pure and Applied Mathematics in Divergent Institutional Settings in Germany: The Role and Impact of Félix Klein"; Renate Töbies, "On the Contribution of Mathematical Societies to Promoting Applications of Mathematics in Germany"; both in *The History of Modern Mathematics II: Institutions and Applications*, ed. David Rowe, & John McCleary (Boston: Academic Press, 1989), 177-220 and 223-248, respectively.

between 1905 and 1935 might help us assess the former's impact and the 'half-life' of the editorial enterprise.[12]

An analysis of the frequency of references to the encyclopedia shows that its entries were cited in about 4% of the papers published in 1905. The climax of its popularity was attained in 1920, when almost 7% of the papers made reference to the EMW. In 1935, the frequency of such references was below 3%.

These data confirm Ore's judgment, who as early as 1942 considered the EMW an historical object, and they also help us understand why Teubner, the publisher, decided to start a fully novel edition in 1939 with new editors, texts and authors.[13]

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[12]The journals selected for analysis as a function of their undisputed representativeness are: *Acta Mathematica, Journal für die reine und angewandte Mathematik, Mathematischen Annalen, Rendiconti del Circolo Matematico di Palermo, Jahresbericht DMV, Proceedings of the London Mathematical Society* and *Annalen der Physik*. The total number of papers published by all of them together from 1905 to 1935 was 2,479.

[13]Ore, "Enzyklopädie der mathematischen Wissenschaften".

Appendix 1: *Entries in EMW geometry volumes*

Sections A and B: Pure geometrical theory and foundations of the application of algebra and analysis to geometry				
Volume	**Author**	**Title**	**Received**	**Published**
3 T1 H1	F. Enriques	Principles of Geometry	Apr 1907	Jun 26 1907
	H. von Mangoldt	The Concepts 'Line and Surface'	Sept 1906	
	M. Dehn & P. Heegard	Analysis Situs	Jan 1907	
	G. Fano	The Opposition between Synthetic and Analytic Geometry in its Historical Development in the 19th Century	May 1907	Aug 10 1907
	G. Fano	Continuous Geometrical Groups. The Group Theory as a Geometrical Principle of Classification.	Jul 1907	
	A. Schönflies	Projective Geometry	Jan 1909	May 28 1909
	E. Steinitz	Configurations of Projective Geometry	Apr 1910	Nov 22 1910
	E. Papperitz	Descriptive Geometry	Jul 1909	
	E. Müller	The Several Coordinate Systems	Jul 1910	
3 T1 H2	J. Sommer	Elementary Geometry from the New Analysis Standpoint	Jan 1914	Jun 8 1914
	M. Zacharias	Elementary and Non-Euclidean Geometry in the Synthetic Approach	Dec 1913	
	W.F. Meyer	Elementary and Non-Euclidean geometry in the Synthetic approach. With Appointments of Meyer	Dec 1913	Jan 3 1921
	G. Berkhan & W. F. Meyer	New Geometry of Triangles	Oct 1914	
	H. Rothe	Systems of Geometric Analysis I	Oct 1916	
	A. Lotze & C. Betsch	Systems of Geometric Analysis II	Mar 1923	Jul 10 1924
	E. Steinitz	Polyhedron and Classifications in the Three Dimensional Space	Aug 1929	May 15 1922
	H. Tietze & L. Vietoris	Relationships between the Various Branches of Topology	Oct 1929	Dec 10 1930

Section C: Algebraic Geometry				
Volume	**Author**	**Title**	**Received**	**Published**
3 T2 H1	F. Dingeldey	Conics and Systems of Conics	Jan 1903	Mar 10 1903
	O. Staude	Systems of Surfaces of 2nd Order and Its Curves	Mar 1904	Jun 1 1904
	G. Zeuthen	Enumerative Method	Dec 1905	
	L. Berzolari	General Theory of Superior Plane Algebraic Curves	Jun 1906	Aug 16 1906
	G. Kohn	Plane Curves of Third and Four Order	May 1908	Apr 23 1909

	G. Loria	Special Algebraic Place Curves of Order Greater than Four	Sept 1914	Apr 22 1915
	G. Castelnuovo & F. Enriques	Basic Properties of Algebraic Surfaces	Jan 1908	Sept 21 1915
	G. Castelnuovo & F. Enriques	The Algebraic Surfaces from the Point of View of Birational Transformations	Dec 1914	
3 T2 H2A	C. Segré	Multidimensional Spaces	Dec 1912	Jan 3 1921
	K. Zindler	Algebraic Line Geometry	Apr 1921	Dec 1 1922
	K. Rohn & L. Berzolari	Algebraic Space Curves and Developable Surfaces	Jul 1926	Jan 25 1928
	W.F. Meyer	Special Algebraic Surfaces. A) Surfaces of Order Three	Sept 1928	Nov 20 1928
3 T2 H2B	W.F. Meyer	Special Algebraic Surfaces. A) Surfaces of Order Four and Superior	Aug 1930	May 27 1931
	L. Berzolari	Algebraic Transformations and Correspondences	Dec 1932	Aug 17 1933

Section D: Differential Geometry

3 T3	H. von Mangoldt	Applications of the Differential and Integral Calculus to Curves and Surfaces	May 1902	Oct 14 1902
	R. von Lilienthal	Curves Inscribed in Surfaces	Aug 1902	
	G. Scheffers	Special Transcendental Curves	Jun 1903	
	R. von Lilienthal	Special Surfaces	Aug 1903	Sept 20 1909
	A. Voss	Applications and Isometries between Two Surfaces	Aug 1903	
	H. Liebmann	Contact Transformations	Oct 1914	May 14 1915
	H. Liebmann	Geometric Theory of Differential Equations	Oct 1914	
	E. Salkowski	Triply Orthogonal Systems of Surfaces	Apr 1920	Feb 8 1921
	R. Weitzenböck	New Works on the Theory of Algebraic Invariants	Mar 1921	Nov 1 1922
	L. Berwald	Differential Invariants in Geometry. Riemann Manifolds and Its Generalization	Oct 1923	Oct 15 1927

Appendix 2: *Authors of geometry entries*

Author	Birth/death	City	Country	First paper	PhD thesis	EMW entry (first)
Reinhold von Lilienthal	1857-1935	Münster	Germany (Prussia)	1882	1882	1902
Auriel Voss	1845-1931	Munich	Germany (Bavaria)	1872	1869	1903
Friedrich Dingeldey	1859-1939	Darmstadt	Germany (Hesse)	1885	1885	1903
Georg W. Scheffers	1866-1945	Charlottenburg	Germany (Prussia)	1889	1890	1903
Hans von Mangoldt	1854-1925	Danzig	Germany (Prussia)	1889	1890	1903
Otto Staude	1857-1928	Rostock	Germany (Prussia)	1881	1881	1904
Hyeronimus G. Zeuthen	1839-1920	Copenhagen	Denmark	1868	1865	1905
Luigi Berzolari	1863-1949	Pavia	Italy	1885	1884	1906
Max Dehn	1878-1952	Frankfurt	Germany (Hesse)	1900	1900	1907
Gino Fano	1871-1952	Turin	Italy	1892	1892	1907
Federigo Enriques	1871-1946	Rome	Italy	1890	1891	1907
Poul Heegard	1871-1948	Christiania (Oslo)	Norway	1898	1898	1907
Gustav Kohn	1859-1921	Vienna	Austria-Hungary	1879	1881	1908
Guido Castelnuovo	1865-1952	Rome	Italy (Papal States)	1886	1886	1908
Arthur M. Schönflies	1853-1928	Frankfurt	Germany (Hesse)	1885	1877	1909
Erwin Papperitz	1857-1938	Freiberg	Germany (Saxony)	1885	1883	1909
Emil Müller	1861-1927	Vienna	Austria-Hungary	1910	1898	1910
Corrado Segre	1863-1924	Turin	Italy	1883	1883	1912
Max Zacharias	1873-1962	Berlin	Germany (Prussia)	1903	1903	1913
Gustav W. Berhan	1882-1914	Hamburg	Germany	1905	1905	1914
Julius Sommer	1871-1943	Danzig	Germany (Prussia)	1897	1897	1914
Heinrich Liebmann	1874-1939	Heidelberg	Germany (Baden-Württenberg)	1895	1895	1914
Wilhelm F. Meyer	1856-1934	Königsberg	Germany (Prussia)	1874	1878	1914
Gino Loria	1862-1954	Genoa	Italy	1884	1883	1914
Hermann Rothe	1882-1923	Vienna	Austria-Hungary	1908	1909	1916
Erich Salkowski	1881-1943	Hannover	Germany (Lower Saxony)	1904	1904	1920
Konrad Zindler	1866-1934	Innsbruck	Austria-Hungary	1988	1890	1921
Roland Weitzenböck	1885-1955	Amsterdam	Netherlands	1908	1910	1921
Ludwig Berwald	1883-1942	Prague	Austria-Hungary	1909	1908	1923
Paul A. Lotze	1882-1964	Stuttgart	Germany (Baden-	1922	1920	1923

Christian Betsch	1888-1934	Stuttgart	Württenberg) Germany (Baden-Württenberg)	1924	1917	1923
Karl Rohn	1855-1920	Leipzig	Germany (Saxony)	1878	1878	1926
Leopold Vietoris	1891-2002	Vienna	Austria-Hungary	1916	1920	1929
Heinrich F.F. Tietze	1880-1964	Munich	Germany (Bavaria)	1905	1904	1929
Ernst Steinitz	1871-1928	Kiel	Germany (Schleswig-Holstein)	1894	1904	1929