Music and mathematics: a case study in the history of science

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Abstract

The Italian musical theorist and lutenist Vincenzo Galilei (1522?-1591), father of Galileo Galilei, held a life-long dispute with Gioseffo Zarlino (1517-1590), the famous theorist and *capellmeister* of San Marcos, in Venice. Daniel. P Walker and Claude V. Palisca - historians of science and music, respectively - thoroughly analyzed this dispute calling the attention to some quantitative aspects of music theory found in Galilei's work. These scholars understood the dispute as mainly opposing experimentalism (Galilei) to speculative mathematical reasoning (Zarlino). In the present article we suggest an alternative interpretation for this dispute based on newly found manuscripts by Galilei, according to which the dispute had more to do with the ideas Galilei had of science in general and the role played by arithmetic, geometry and sound in music.

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Keywords

History of science; Vincenzo Galilei; Mathematics and music; Renaissance science

Música e matemática: um estudo de caso na história da ciência

Resumo

Vincenzo Galilei (1522?-1591), pai de Galileo Galilei, foi um músico teórico italiano cujo trabalho teórico foi desenvolvido em grande parte pela disputa com o teórico renascentista Gioseffo Zarlino (1517-1590), *capellmeister* em Veneza. Os estudiosos da história da ciência e da música, respectivamente Daniel P. Walker e Claude V. Palisca estudaram esta disputa, focando os aspectos quantitativos da teoria musical de Galilei e interpretando a mesma no contexto do dualismo entre experimentalismo, personificado pelo trabalho de Galilei, e o racionalismo matemático de Zarlino. Neste artigo pretende-se prover uma nova interpretação da relação entre esses dois teóricos que se afasta do problema do experimentalismo e foca, principalmente, em novos manuscritos de Galilei, concluindo que o cerne da disputa estava nas diferentes definições de ciência utilizadas pelos autores e suas diferentes concepções sobre as participações da aritmética, da geometria e do som na música.

Palavras-chave

História da Ciência; Vincenzo Galilei; Matemática e música; Ciência renascentista

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Introduction

Music is seldom considered to be a subject relevant for studies in the history of science. The reason might be that in spite of being a field of knowledge, it did not always have the status of a science. Alternatively, research in music demands knowledge of a fully different system, which makes it considerably complex. These facts notwithstanding, many were the ones that in the 16th century performed their work within the realms of science and music, as for instance, two members of the Galilei family. One was Galileo, more famous in the history of science; the other was his father, Vincenzo, a music theorist, better known in the history of music.

According to the literature, Vincenzo Galilei was an independent practical musician who spent most of his life in a flourishing Florence, having no connection whatsoever to any court or institution.[1] The extant documentation on his life is scarce and consists in a small number of letters exchanged with his family and other scholars. However, having a life-long dispute with his master, Gioseffo Zarlino, the renaissance theorist and *capellmeister* in San Marcos, Venice, secured him a place in the history of music and the history of science alike.

According to the traditional view, the disagreement between Vincenzo Galilei and Zarlino concerned subjects such as the tuning systems and the definition of musical intervals. Zarlino argued based on textual sources seeking to validate his own system, while Galilei refused that approach on the grounds of experiments he had conducted with instruments.

However, as we discuss below, two traditions might be identified as concerns the historical studies on Vincenzo Galilei. One is essentially the one of historians of music, which considers Galilei a minor figure, to wit, a former disciple of Zarlino who had a part in the formulation of a musical concept known as 'monody'. The other tradition is mostly the one of historians of science and approaches Galilei mainly as an "experimentalist"[2]. As is known, the historiographical view adopted by any scholar makes he or she emphasizes some events at the expense or full neglect of others. For this reason the first part of the present article is devoted to a discussion of relevant historiographical considerations and the state of the art on scholarship in Vincenzo Galilei's conceptions on science and music. In the

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[1] Fabio Fano, ed., *La Camerata Fiorentina: Vincenzo Galilei (1520?-1591) la sua opera d'artista e di teorico come espressione di nuove idealità musicali* (Milano: Ricordi, 1934); Giulio Batelli, "Note per una Rilettura del "Dialogo della Musica antica et della moderna" di Vincenzio Galilei, *Annales della conferenza di Santa Maria a Monte*, 1992; Claude V. Palisca, "Girolamo Mei: Mentor of the Florentine Camerata," *Musical Quartely* 40, no. 1 (1954): 1-20; Palisca, *Seventeenth Century Science and the Arts*, (Princeton: Princeton University Press, 1961), 91-137; Palisca, *Girolamo Mei: Letters on Ancient and Modern Music to Vincenzo Galilei and Giovanni Bardi* (Stuttgart: Hänssler-Verlag, American Institute of Musicology, 1977); Palisca, "Galilei, Vincenzo," *The New Grove Dictionary of Music and Musicians* (Oxford: Oxford University Press, 1980), VII: 96-8; Palisca, *Humanism in Italian Renaissance Musical Thought* (New Haven [CT]: Yale University Press, 1985); Palisca, *The Documentary Studies and Translations: Florentine Camerata* (New Haven [CT]: Yale University Press, 1989); *Dialogue on Ancient and Modern Music by Vincenzo Galilei*, transl & comm. Claude V.Palisca (New Haven [CT]: Yale University Press, 2003).

[2] Claude V. Palisca, "Was Galileo's Father an Experimental Scientist?," in *Music and Science in the Age of Galileo*, ed. Victor Coelho (Netherlands: Springer, 1992), 143-52.

second part we present an alternative construction of the conceptual framework within which Galilei elaborated his ideas based on unpublished manuscripts - *Compendio* and *Critica* – as well as parts of published sources neglected by previous scholars precisely as a function of their historiographical approaches.

The basic assumption leading our study is that music and mathematics should be approached from the perspective of the 16th-century understanding of the subordinated sciences. While as a rule the literature stresses the mathematical classification of music, it tends to use an anachronistic terminology, probably for the sake of simplification. However, these anachronisms might result in deceptive conceptual frameworks. To avoid such risk we paid special attention to the terms used by Galilei, for while translations might be overall accurate and interpretations comprehensible - in that the mathematical or musical content of a passage remains understandable - any inaccurate use of words unavoidably leads to mistakes in ideas. Equally important is that the traditional historiography of mathematics sought to elaborate a chronological history of pure mathematical concepts, for which reason it commonly overlooked the relationship between the musical and the mathematical knowledge.[3]

To summarize, the aim of the present study was to identify concepts and definitions relative to mathematics and music theory in the works of Vincenzo Galilei by focusing on the enunciated propositions and the sources to which he had resource. For this purpose we analyzed original documents, instead of translations, not for the sake of philological accuracy, but because identification of an author's sources is the first methodological step in the understanding of the conceptual framework within which he worked.

Vincenzo Galilei and historiography

As was mentioned above, our review of the literature led us to identify at least two historiographical traditions among scholars who approached Vincenzo Galilei. One such tradition developed among historians of music, being supported by a homogeneous view formulated along the 19th century on positivistic grounds and through the assumption of an evolutionary model of history. This tradition is exemplary illustrated by the work of Lewis Lockwood, who followed Jacob Burckhardt,[4] as also many of the scholars who wrote about the Renaissance did. According to this view music was not a science belonging with the *quadrivium*, but with *trivium*, the Renaissance was the period when the links of music to the mathematical disciplines were severed to be music ultimately tied to the rhetorical sciences.[5] Hence, the musicologist and renaissance scholar Claude Palisca observed that "to link music

[3] For the opposite view, see Árpad Szabó, *The Beginnings of Greek Mathematics* (Dordrecht: D.Reidel, 1978).

[4] Jacob Burckhardt, *Die Cultur der Renaissance in Italien: Ein Versuch* (Basel: Verlag, 1860).

[5] John Bergsagel, "A Musician among the 'Measurers," in *Learning, Language and Invention,* ed. W.D. Hackmann, & A.J. Turner, Aldershoot; Paris: Variorum; Société Intenationale de l'Astrolabe, 1994), 84-102, on 88. For one recent work still based on this historiographical tradition see, Daniel. K.L. Chua, "Vincenzo Galilei, Modernity and the Division of Nature," in *Music Theory and Natural Order*, ed. S. Clark, & A. Rehding, (Cambridge: Cambridge University Press, 2001), 17-29.

with verbal arts, with rhetoric and poetry, was as characteristic of the Renaissance as it was typical of the Middle Ages to ally music with the mathematical sciences"[6].

This reunion of music and text under an evolutionary view of history became the distinguishing feature of the historiography on Renaissance music. Within that context, the writings by the historian of Renaissance Edward Lowinsky supplied enough ammunition for many scholars affiliated with this tradition to describe the expressive-rhetorical goals subjected to the musical repertoire from the 14th to the 17th centuries.[7]

In the case of Galilei, his work was associated with vocal music. It was strongly believed that in *Dialogo della antica et della moderna musica* (1581) Galilei favored a monodic style of singing.[8] Moreover, this tradition adduced that Galilei's main concern was with the moral and ethical aspects of the modes, or scales, and their effects on men, which explained his preference for vocal music.

One further illustrative case is provided by Galilei's relationship to *Camerata Fiorentina,* a group of nobles, poets, musicians and scholars who gathered under the patronage of count Giovanni Bardi da Vernio (1534-1612), in Florence, to discuss the music and art of ancient Greeks. In time the *Camerata* came to be described in the music literature as the "melting pot" that gave rise to Baroque opera.[9] Galilei's participation as a performer is attested by a letter written by Pietro Bardi, Count Giovanni's son, dated to 1634.[10]

The connection of Galilei to the *Camerata* was addressed by the historiographical tradition we are discussing here with neglect of the mathematical and philosophical topics he discussed in his treatises. Thus, for instance, the scholars who studied Galilei's *Dialogo della antica et della moderna musica* (1581) merely sought to trace the history of monody within

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[6] Claude V. Palisca, *Humanism in Italian Renaissance*, 332. Palisca seemingly also advocated that the Renaissance started in Italy and that its chief source of inspiration was the revival of antiquity; Ibid., 22.

[7] Gustav Reese, *Music in the Renaissance* (New York: W.W. Norton, 1954); Howard M. Brown, *Music in the Renaissance* (Upper Saddle River [NJ]: Prentice-Hall Inc.,1976); Leo Schrade, "Renaissance: An Historical Conception of an Epoch," in *Kongress-Bericht der Internationale Gesellschaft für Musikwissenschaft*, Utrecht 1952, ed. A. Smijers (Amsterdam: G. Alsbach Verlag, 1953): 19-32; and Leo Schrade, *Monteverdi, Creator of Modern Music* (New York: Norton, 1950), part I, 17-74.

[8] Monody: a one-line musical texture. The musical definition of this term is not precise, for terms monody and monodic style do not denote any precise musical phenomenon. Their ambiguity has been often overlooked, although musicologists are aware of this problem, see Fano, *Camerata Fiorentina;* Claude V. Palisca, & Alfred Einstein, "Vincenzo Galilei and the Instructive Duo," *Music & Letters*, 18, no. 4 (1937): 360-8, on 360-1; Alfred Einstein, & Arthur Mendel, "Dante, on the Way to the Madrigal," *The Musical Quarterly*, 25, no. 2 (1939): 142-55; Palisca, "Galilei, Vincenzo"; Palisca, "Vincenzo Galilei's Counterpoint Treatise: A Code for the Seconda Prattica," *Journal of the American Musicological Society* 9, no. 2 (1956): 81-96; Palisca, "The Camerata Fiorentina: a Reppraisal," *Studi Musicali* I (1972): 206-36; and during the Renaissance Pietro Bardi's letter to Giovanni Battista Doni, in Angelo Solerti, *Le Origini del melodramma* (Torino: Fratelli Bocca Editore, 1903), 143-5, just to name a few.

[9] Alfred Einstein, "Galilei and the Instructive Duo," *Music and Letters*, 18 (1937): 360-8.

[10] Letter to Giovanni Battista Doni, from December 14th, 1634, in Solerti, 143-7.

the history of music.[11] That reading elicited some partial reaction, as scholarly examination did not support the traditional textbook claim that opera was the product of informal antiquarian debate at Bardi's *Camerata*: "One of the most monumental frauds ever recorded in the history of music […]"[12]. Nino Pirrotta points to various groups, in addition to the *Camerata,* that contributing from different perspectives to the reform of music in general, not of vocal music only. Victor Coelho, in turn, cited previously unknown sources to document the idea of monody that was held in 16th- and 17th-century Italy.[13]

The second historiographical tradition, which corresponds to historians of science, represents Vincenzo Galilei as an experimentalist amid a series of theorists that preceded the so-called scientific revolution that began in the 17th century. This approach is essentially represented by Daniel P. Walker and the above mentioned Palisca, who in the 60's and 70's called for the attention the first time to certain quantitative aspects of music theory found in Galilei's work.[14] Indeed, the writings of Palisca, the leading scholar on renaissance music, provide a good example on how the focus shifted from the *Camerata* - as the single group that could properly be considered as the center of research on song and drama -[15] to documenting the scientific discussions held at it,[16] while still placing music within the *trivium.*[17]

In regard to experimentalism, Palisca asserted that Vincenzo had conducted experiments, but Walker dismissed them, after having proved some errors in the results, more specifically in the ratio of the volumes of pipes.[18] Palisca replied to Walker in *The Florentine Camerata,*[19] and later on replicated Galilei's experiments, the results of which gave support to his contention that Galilei had designed an experiment to test a hypothesis.[20] Stilmann Drake's work corroborated Palisca's views, as "the manipulation of physical equipment set up to test a mathematical law had come much earlier than Galileo, and it came because of the conflict between numerology and physics in the field of music […]". And, "the fountainhead [of Renaissance music] was […] at least partly responsible for the

[11] Nino Pirrotta said that Galilei "cuts a poor figure as a theorist, and has nothing, or next to nothing, original to say", Nino Pirrota, "Temperaments and Tendencies in the Florentine Camerata," *Musical Quarterly* 40/2 (1954): 169-89, cited by Randall E. Goldberg, *Where Nature and Art Adjoin*, (Doctoral dissertation, Indiana University, 2011), 9.

[12] Ibid.

[13] Victor Coelho, "The Players of Florentine Monody in Context and in History, and a Newly Recognized Source for *Le nuove musiche,*" *Journal of Seventeenth Century Music 9* (2003): 48-67; Victor Coelho, ed., *Music and Science in the Age of Galileo* (Netherlands: Kluwer Academic Publishers, 1992).

[14] Daniel P. Walker, *Studies in Musical Science in the Late Renaissance* (London; Leiden: The Warburg Institute; E.J. Brill, 1978),14-33; Claude V. Palisca, "Scientific Empiricism in Musical Thought," in *Studies in the History of Italian Music and Music Theory*, ed. C.V. Palisca (Oxford: Clarendon Press, 1994, rep. 2001), 200-35; and Palisca, "Was Galileo's Father?".

[15] Claude V. Palisca, "The Camerata Fiorentina: a Reppraisal," *Studi Musicali* I (1972): 206-36.

[16] Palisca, "Girolamo Mei", 13-24; *Girolamo Mei*; *Documentary*; *Dialogue on Ancient and Modern Music*.

[17] Claude V. Palisca, *Studies in the History of Italian Music and Music Theory* (Oxford: Clarendon Press,1994, rep. 2001).

[18] Walker, 14-33. It seems Walker did not consult Galilei's works, for if he had, he would have found the correct results relative to the ratio between the volumes of pipes.

[19] Palisca, *Florentine Camerata,* 159.

[20] Palisca, "Scientific Empiricism", 200-35; 202-3.

emergence not of experimental science alone, but of a whole new approach to theoretical science that we know as mathematical physics".[21] The experimental hypothesis was particularly supported by scholars interested in replicating in the present experiments designed and conducted in the past.[22]

Galilei's works and the relationship between mathematics and music

There are sixteenth works attributed to Vincenzo Galilei held at *Biblioteca Nazionale Centrale di Firenze* (Ms Galileiana, *Anteriori di Galileo*) probably written between the 1560's and 1590's. This set includes two manuscripts that were never published, *Compendio della musica theorica* (1570) and *Critica fatta a Gioseffo Zarlino* (1591?).[23]

Compendio is in facsimile. The initial pages bring comments made by Galilei to Zarlino's first work, *Istitutioni*, pointing out errors in it, and also discuss Vitruvius' *De Architectura* Book IV. The proper treatise begins in folio 2 and provides a systematic ordering of mathematical-musical concepts corresponding to some topics in Zarlino's *Istitutioni,* first published in 1558.[24] The conceptual content of *Compendio* is mathematical and musical in that musical elements are explained after the corresponding mathematical definitions. The text ends abruptly.

Critica, which is not dated, is probably Galilei's last work.[25] It contains a detailed criticism of Zarlino's last work, *Sopplimenti*, published in 1588,[26] being Galilei's second work explicitly aiming at refuting this book. The earlier one was *Discorso*, published in 1589, in which Vincenzo presented a philosophical argumentation on the natural and artificial elements of music. In *Critica*, Galilei addresses almost every chapter of *Sopplimenti*, provides calculations of tetrachords and intervals, explains the differences among Ptolemy, Didymus and Aristoxenus' tunings, and describes acoustical experiments.

Renaissance treatises on music often began with questions on the nature of music and definitions of its elements, and also this was the case of Zarlino's *Istitutioni* and Galilei's

[21] Stillman Drake, "Renaissance Music and Experimental Science," *Journal of the History of Ideas*, 31 (1970): 483-500, on 499-500; and "Vincenzo Galilei and Galileo," in *Galileo Studies: Personality, Tradition and Revolution,* ed. S. Drake (Ann Arbor: The University Michigan Press, 1970), 43-62.

[22] H.F. Cohen, *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580-1650* (Dordrecht: D. Reidel Publ. Com., 1984), 75-114.

[23] These treatises were seldom mentioned by previous scholars, but are extensively quoted in the present article to illustrate some of the novelties they bring into scholarship and to demonstrate the coherence in Galilei's thought.

[24] Gioseffo Zarlino, *Le istitutioni harmoniche* (Venice: Francesco de Franchesi Senese). The book, which had several printed editions, was revised by Zarlino and had two versions, one from 1558 and the other from 1573; Lucille Corwin, "Le Istitutioni Harmoniche of Gioseffo Zarlino, Part I. A Translation with Introduction" (PhD Dissertation. The University of New York, 2008).

[25] Vincezo Galilei, *Critica fatta di Vincentio Galilei intorno ai sopplimenti musicali di Gioseffo Zarlino,* Biblioteca Nazionale Centrale di Firenze, Anteriori di Galileo, vol. 5, 3r-58r.

[26] Gioseffo Zarlino, *Sopplimenti musicali,* (Venice: Francesco de Franchesi Senese, 1588).

Compendio.[27] At that time, the component elements of music were not defined within the scope of acoustics, as they are in the present time, thefore they could not be measured as frequencies. Musical elements such as intervals, scales or modes were considered to be mathematical entities, and for this reason the mathematical-arithmetical definitions described at the beginning of the treatises had a crucial role, for they provided the basis for the derivation and calculation of the musical elements.[28]

According to the textual tradition, musical intervals were multitudes in relation one to another and were expressible and measured only as ratios of integers. [29] Hence, to both Zarlino and Galilei the intervals allowed by theory were not just the ones expressible as ratios, but exclusively as multiple or superparticular ratios.[30]

Defining the science of music

According to Zarlino, that specification of the admissible types of ratios had grounds in the textual tradition. To be sure, he believed in a line of transmission of knowledge that starting in ancient Greece had reached his own times through Boethius' (ca. 480-524) works *De istitutione arithmetica* and *De instituione musica*.[31] Boethius believed that the means to acquire knowledge in music was the Pythagorean one, i.e., the number and thus insisted in that music had quantitative nature,[32] and that as a part of mathematics, music contributed to

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[27] Zarlino, *Istitutioni*, parte prima, and Galilei, *Compendio,*1570, fols. 3-19.

[28]Zarlino, the proemio to *Istitutioni*, ***p.2***: "[…] dividendo l'opera in quattro parti, nella prima si ragionerá delli numeri, delle proportioni & delle loro operationi, non lasciando cosa alcuna, quantunque minima, che al Musico s'appartenga".

[29] For instance, in *Republic* (Indianapolis: Hackett Pub. Co., 1992). Plato asserts, "[they are] measurable audible concords against one another" (531a1-2), "they search for numbers in those audible concords" (531a1-2), which shows that intervals were conceived of as one in relation to another. In *Theorica musice*, Book 2, chapter 1, Gaffurio observes "[...] we proposed that the discipline of music, which Pythagoras and Plato called Philosophy, is in the present purpose a science of numbers proportionate to sounds that measures the differences of low from high sounds through sense and reason"; see *Franchino Gaffurio: The Theory of Music*, transl. W.K. Kreyszig, ed. C. Palisca (New Heaven [CT]: Yale University Press, 1993), 49.

[30] Multiple ratios as xn/n and superparticular ratios as (n+1/n).

[31]Boethius wrote *De instituione musica* probably around 505 AD, which was intended to be read together with *De institutione arithmetica*; see *Fundamentals of Music*, trans., introd. & notes C.M. Bower (New Heaven [CT]: Yale University Press, 1989), xix. When liberal learning saw a rebirth in the Carolingian era, Boethius' treatises on arithmetic and music reappeared as authoritative works on these disciplines, rivaled only by Martianus Capela's *De nuptiis Philologiae et Mercurii*. When a tradition of independent musical treatises started in the 9th century, the one by Boethius became the single source for mathematical grounding of music theory in the West; Ibid, xx; see also Andrew Barker, *The Science of Harmonics in Classical Greece* (Cambridge: Cambridge University Press, 2007); James M. Barbour, "The Persistence of the Pythagorean Tuning System", *Scripta Mathematica* 1 (1933): 286-304; and *Boethian Number Theory: A Translation of the De Institutione Arithmetica*, ed. & transl. Michael Masi (Amsterdam: Editions Rodopi B.V, 1983).

[32] Boethius, *Fundamentals*, 1.3, 2.20, and 4.1.

training the intellect.[33] As any mathematical study, music had no purpose of its own, but was instrumental to the larger science of philosophy. Although Boethius' treatise on music does not bear signs of direct Aristotelian influences, the medieval theorists on music believed his work was a product of Aristotelian doctrine, or of attempts at reconciling Plato and Aristotle at least.[34] In any case, the juxtaposition of arithmetic and music in Boethius' works was also propagated in Aristotelian treatises, *Posterior Analytics* and *Metaphysics* in particular; the former bore witness to the use of arithmetical demonstrations and theorems in harmonic science,[35] while the latter corroborated the biunivocal relationship between science and the first principles.[36]

The Pythagorean-Platonic tradition and Aristotle's notion of science were combined by Zarlino to demonstrate the high degree of certainty and the noble nature of music as a mathematical-speculative science.[37] Following the Philosopher, in *Istitutioni* (ch. 20) Zarlino subordinates music to arithmetic by classifying it as *mezzana* between mathematics and natural science,[38] and adds to the number the essential attribute of sonority as its diversifying quality, resulting in the *numero sonoro*.

In Zarlino's work the *numero sonoro* is the connection between *senario* – selected group of interval ratios - and sound. It is worth to observe that while term *numero sonoro* is not found in Boethius' works, it is frequent in musical treatises influenced by Aristotle's writings.[39] The notion of *numero sonoro* adopted by Zarlino resulted from his interpretation of a definition formulated earlier by Ludovico Fogliano (c.1475-1542) in *Musica theorica,* published in 1529.[40] Without giving proper credit to Fogliano, Zarlino observes that his own definition of *numero sonoro* suited better the purposes of music, as the previous one was incomplete and imperfect, because it could not be applied to the voice.[41]

In *Compendio*, Galilei defined the *numero sonoro* as the proper subject of music according to the four Aristotelian causes,[42] being sound the material cause and the number the formal cause. Galilei considered music to be a mathematical science, subaltern first to

[33] Boethius, *Arithmetica*, X.

[34] Leo Schrade, "Music in the Philosophy of Boethius," *The Musical Quarterly* 33, no. 2 (1947):188-200, on 189.

[35] Aristotle, *Posterior Analytics*, transl. Hugh Tredennick, & E.S. Forster Cambridge [MA]: Harvard University Press, 1960), 76b. 14-16.

[36] Ibid., I.30 46^a 17-22; most of the principles of each science are peculiar to it.

[37] Mostly in his first works, *Istitutioni* and *Dimostrationi*; Zarlino, *Istitutioni*, proemio, 2.

[38] However, here Zarlino opposes Aristotle, since he describes the science of music as having a more mathematical than natural nature.

[39] See Frank Hentschel, *Sinnlichkeit und Vernunft in der mittelalterlichen Musiktheorie: Strategien der Konsonazwertung und der gegenstand der musica Sonora um 1300* (Stuttgart: Franz Steiner, 2000).

[40] Lodovico Fogliano, "[Numerus sonorous]: appellatur: nihil aliud est: nisi numerus partium sonori corporis: utputa: chordate," in *Ludovici Foliani Mutinensis de musica theorica* (Venetiis: Io. Antonius et Fratres de Sabio, 1529), fol. 1r.

[41] Zarlino, *Istitutioni*, I: ch. 19, 29.

[42] Galilei, "Il suggetto della musica é Il numero sonoro", *Compendio*, fol. 8.

geometry and then to arithmetic.[43] Thus he differed from Zarlino, but tended to agree with Aristotle, who had defined it "as the most natural of the mathematical sciences"[44].

Aristotle's notion of the subalternation of the sciences seems to be at the very center of the debate between Zarlino and Galilei. Zarlino's final view on the subject asserted the priority of speculation over practice and of the mathematical over the natural classification of music. In regard to Aristotle's notions of science, Zarlino admitted that knowledge refers to causes and principles,[45] the first principles in particular.[46] To him the first principles of music were determined according to the arithmetic nature of music as described in his first work, *Istitutioni* and were: octave, as the primary interval that generates all others; [47] *senario*, as the complete set of consonant intervals;[48] and natural order, or right placement of intervals (*luoco*).[49] Since according to Aristotle the first principles belong with the primary science that provides the essential attributes of things, in *Dimostrationi* Zarlino assimilated definition to essential attributes and description to accidents, and thus definitions explained something by its essential attribute, while descriptions explained something through its accidents.[50]

Having expounded on the history and principles of his arithmetical theory in *Istitutioni*, Zarlino provides in *Dimonstrationi* the corresponding demonstrations,[51] for the fact that music "also derives other [principles] from arithmetic by means of its demonstrations, because [it is] through them [that] we have true knowledge of [music] science"[52]. Such demonstrations basically corresponded to the type of logical deduction described as necessary in Aristotle's texts, mainly in *Posterior Analytics*, 74b. 5-12, combined with the mathematical commentaries of Proclus and Campanus to Euclid's *Elements*.[53]

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[43] Galilei, *Compendio*, fol. 8.

[44] Aristotle, *Metaphysics*, transl. Hugh Tredennick (Cambridge [MA]: Harvard University Press, 1933), I: books 1-9.

[45] Ibid., 982a1-3.

[46]Aristotle, first part of A2,, 982a 4b-10. See Zarlino:"[..] però io primeramente ragionerò de I loro principij conciosia che allora diciamo di veramente conoscer le cose, quando li principij di esse conosciamo", *Istitutioni,* proemio, 2.

[47] Zarlino, *Istitutioni,* 1558, II, ch.48, 142; III, Ch.3, 149 ; *Dimostrationi*, 3-4, 6.

[48] Zarlino, *Istitutioni*, 1558, I, ch.15, 25-26.

[49] Ibid., I, 40, 52. Here we followed John E. Kelleher, "Zarlino's 'Dimostrationi Harmoniche' and demonstrative methodologies in the sixteenth century" (Doctoral dissertation, Columbia University, 1993).

[50] "La prima é detta Definittione: & è quell ache esplica la cosa per le cose essentiali, & la seconda è chiamata descrittione, & è quella che no dice la cosa per gli essentiali ma per li suoi accidenti"; Zarlino, *Dimostrationi*, rag. I, 9.

[51] Kelleher, "Zarlino's 'Dimostrationi".

[52] "[Musica] che havendo con l'Arithmetica per commune soggetto il Numero, aggiungendo a questo per sua differenza la sonorità, si fa ad essa Arithmetica subalternata, tenendo il numero sonoro per suo soggetto. […] ne piglia [principij] ancora de gli altri dall'Arithmetica, per li mezi delle sue demostrationi, percioche per essi havemo poi la vera cognitione della scienza"; Zarlino, *Istitutioni*, 1558, 30; Corwin, 322-3.

[53] Kelleher, "Zarlino's 'Dimostrationi".

Dimostrationi was designed as a dialogue between musicians and non-musicians, in which the former explain to the latter the practical problems posed by the application of mathematical precepts to musical sound.[54] Zarlino insisted in that anyone wanting to demonstrate matters related with music in a correct manner had to know the principles used and demonstrated by others to lead us to the purpose of the thing we are looking at.[55]

Although Galilei was well aware of Zarlino's statements, and probably took them into consideration up to a certain point, his works reflect a different understanding of the science of music.[56] By the time of the *Compendio,* Galilei had already published *Il Fronimo* and written *Libro d'Intavolatura di Liuto.*[57] In the latter he had performed a systematic exploration of the possibilities for tempering the lute, as well as for homophonic writing. [58] In the second part of *Intavolatura* there are *passamezzos* and *romanescas* composed in all twelve semitones of the octave scale and ascending in chromatic order from the sound of the open sixth course.[59]

The type of equal temperament used by Galilei did not fit with neither Zarlino's nor Boethius' theories.[60] That procedure had been, indeed, dismissed by Zarlino based on the claim that he was unable to locate anyone who had ever thought or explained such temperament;[61] therefore, its use must have been "introduced by chance"[62] by musicians, rather than in a learned manner.[63]

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[54] In the prologue to *Dimostrationi*, Zarlino apologizes for having to replace the explicit demonstrations of the complete axiomatic proofs in *Istitutioni* by practical demonstrations. In the first two discussions of *Dimostrationi* he presents the mathematical and rational side of harmonics, while the remainder of the chapters is devoted to its perceptible and physical aspects.

[55] "Onde bisogna sapere che a voler dimostrare perfettamente le cose della musica, bisogna à quei principij co'l mezo dei quali altri hanno dimostrato, aggiungervi quelle cose, che ne conducono al fine della cosa, che noi cerchiamo"; Zarlino, *Dimostrationi*, Rag. I, 6.

[56] "[...] atteso che lui [Zarlino] vuole le cose a modo suo et senz'alcuna ragione o cognitione dal vero, et io le voglio come realmente sono secondo la natura et veritá loro; però non me maraviglia se noi non conveghiamo insieme[…]'; Galilei, *Critica* , fol. 66.

[57] *Intavolatura de lauto di Vincenzo Galileo Fiorentino madrigal e ricercare, Libro primo* Roma: M. Valerio Dorico, 1563; and *Fronimo Dialogo di Vincentio Galilei Fiorentino nel quale si contengono le vere et necessarie regole del intavolare la musica nel liuto* (Venetia: Girolamo Scotto, 1568).

[58] Homophony is the term used to describe a composition with many voices in which one is autonomous and plays a distinct melody and the other accompany it; in polyphonic compositions many voices are simultaneously composed, all of them equally important and with equal possibilities to become the main one.

[59] Kuis Gasser, "Vincenzo Galilei's Manuscript: Libro d'intavolatura di liuto (1584): An Introductory Study" (Doctoral sissertation, Stanford University, 1991), 14, who used the second edition. Other lute composers from mid-16th century were aware that lute tuning approximated equal temperament and that Giacomo Gorzanis (Munich, Bayerische Staatbibliothek, Mus. Ms. 1511a) had been one of the first to intabulate music in all 24 postures.

[60] "Therefore, it must be argued and proved once again, albeit briefly, that no superparticular relation can be divided into an integral half by any known number"; Boethius, *Fundamentals*, Bk.3, I; Bower, 88.

[61] "Et tale temperamento, gli moderni chiamano participatione, della quale fin hora non so, che da alcun'altro sia stato ragionato, o mostrato cosa alcuna"; Zarlino, *Istitutioni*,1558, II: ch.41, 125.

The theoretical problem posed by the equal temperament depended on the assumption of the equal division of the tone in two parts, which was not allowed by Pythagorean-Platonic theory, because for being an epimoric or superparticular ratio, the tone could not be divided in two equal parts by definition.[64]

In *Il Fronimo*, Galilei had already calculated the ratio 18/17 for the semitone and showed that such ratio was appropriate for fretting the lute. The explanation he gave to account for the division of the tone in two equal semitones on the lute was logical and correct, as the frets were placed straight across the six strings, and the order of diatonic and chromatic semitones was the same in all the strings.[65] It is worth to observe that also other authors reported on equal temperament for fretted instruments. For instance, Francisco Salinas (1513-1590) described the tempering of the viol in *De musica libri septem*, published in 1577.[66] The division of a tone in two equal semitones is identifiable in the writings of the Greek writer Aristoxenus of Tarento (*Elementa harmonica* 2.46, 56-7). According to Palisca, Vincenzo Galilei copied the Latin translation of that book performed by Antonio Gogava, as well as Carlo Valgulio's proem to his translation of Plutarch's *De Musica*, which was known to include quotations from Aristoxenus.[67] As concerns Zarlino, although he had commissioned Gogava to perform the translation of Aristoxenus' book, it seems he was not interested in it, as his dismissal of the equal temperament indicates, but only paid considerable attention to Aristoxenus upon approaching the formation of tetrachords in later works. Nevertheless, both Zarlino's disciples, Vincenzo Galilei and Giovanni Maria Artusi (c.1540-1613), not only stood by Aristoxenus' division, but also reviewed the ancient objections of Ptolemy to it, which had also been mentioned by Boethius and Gaffurius.[68]

The equal division of the tone posed a theoretical and a practical problem to Zarlino. Some authors believed that Aristoxenus had divided the string in equal parts to get equal

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[62] "Tuttavia credo veramente, che tal temperamento, o participation sia stata introdutta a caso & non studiosamente"; Ibid., II: cap.41, 125.

[63] Ibid., ch. 42.

[64] Although Archytas had demonstrated a mean that allowed dividing an interval in two equal subintervals, he agreed on that epimoric ratios have no mean proportional. That Pythagoreans were aware they were unable to find the geometric means of those intervals through arithmetical procedures is seen in *Sectio canonis*, prop. 3 of Euclid's *Elements*, as well as in the works of all authors on music from Boethius to the 16th century.

[65] J. Murray Barbour, *Tuning and Temperament. A Historical Survey* (New York: Dover, 2004), 8.

[66] However, Salinas and Galilei disagreed on the calculation of the semitone, as well as in the explanations of the placement of the temperament in instruments. Although *Il Fronimo* makes broader contributions to music theory, Carol MacClintock, who performed an English translation of that book, and Philippe Canguilhem, who compared the two editions of *Il Fronimo*, restricted their analyses to the forms of intabulation and the rules of polyphonic composition and did not address the problem of the division of the string in a tempered instrument as the lute.

[67] Antonio Gogava, *Aristoxeni…harmonicorum elementorum libri iii…Cl.Ptolemaei harmonicorum…libri iii. Aristoteli de objecto auditus*, (Venitiis: V. Valgrisio, 1562) and *Proemium in musicam Plutarchi ad Titum Pyrrhinum,* (Brescia: G.A. de Gandino ditto de Caeguli, 1507); repr. and trans. in Palisca, *Florentine Camerata*, 21-44.

[68] Giovanni M. Artusi, *L' Artusi, ouero delle imperfettioni della moderna musica ragionamenti dui* (Venitiis: Giacomo Vincenti, 1600), fol. 25r; fols. 31v-35r.

intervals.[69] However, Vincenzo Galilei asserted that Aristoxenus was aware he was dividing sound in equal parts rather than "the quantity of line, string or space […] and thus [was] not [acting] as a simple mathematician around a continuous quantity"[70].

Galilei did not merely defended Aristoxenus' ideas, but held him an exemplary case that showed how modern authors like Zarlino, despite their recourse to the textual tradition, had fully misunderstood the structural elements of music.[71] Galilei's major criticism to Zarlino was directed to his misappropriation of the textual tradition and incorrect separation between the mathematical and physical properties of music.

In *Dialogo* (1581), a work written as a response to Zarlino's *Istitutioni* and *Dimostrationi*, Galilei demonstrated the wide variety of abstract intervals that could occur in the syntonic diatonic system advocated by Zarlino. Galilei set out to compile a list of intervals by comparing the ratios provided by theorists, such as Zarlino, Ptolemy and Didymus. The initial part of *Dialogo* is devoted to demonstrate that combinations of syntonic diatonic intervals, as defined by their accepted respective ratios, created dissonances. For example, while Zarlino divided the perfect fifth (3:2) into the consonant major third (5:4) and minor third (6:5), Galilei claimed that a fifth was also composed of a fourth plus a tone and therefore, a fifth constructed from a perfect fourth (4:3) and the minor tone (10:9) was a possible interval in the syntonic system. However, the fourth plus a minor tone (4:3 + 10:9), as given by Zarlino, produced a dissonant ratio 40:27,[72] while Galilei succeeded in proving that many more dissonant intervals could still be possible.[73]

In the history of music, the most important method to render sounds visible involved the use of an instrument called *kanōn* or monochord, which allowed musical relationships become quantifiable. It consisted of a measuring rod, known since ancient times and well

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69 Ptolemy, Pythagoreans, Valgulio, in Claude V. Palisca, "Aristoxenus Redeemed in the Renaissance," in *Studies in the History of Italian Music and Music Theory*, ed. C.V. Palisca (Oxford: Clarendon Press, 1994), 189-99, on 191

70 "Sapeva Aristosseno, d'havere a distribuire in parti uguali la qualità del suono, & non la quantità della linea, corda & spatio […] & non come semplice matematico intorno la continua quantità"; Galilei, *Dialogo*, 53.

71 According to Galilei, Zarlino believed he used Ptolemy's *syntono* and other authors, like Gaffurio, Glareano, Fabro and Valgulio, believed they used the ancient Diatonic *ditoneo*; see Galilei, *Dialogo*, 2; 4; 9.

72 Kelleher, 134.

73 Galileo, *Dialogo*: lemma, major and minor semiton, on 7-8; major and minor tone, on 9; interval of the minor third on 9-10; the major third on 12; the fourth on 13, tritono on 14; the diminished fifth or *semidiapente* on 15-6; the fifth on 17-8; the minor sixth on19; the major sixth on 20-1; the minor septima pn 21-2, the major septima on 23-4; the octave on 24. In many passages of other works Galilei explains that the modern authors confounded the physical sounds with their musical definition in the attempt to make them conform to ancient ratios. Some examples: "Questo nostro semiditono é l'istesso di quello degli antichi? Non è l'istesso in modo alcuno; impeorche questo nostro é consonante [...] & vien prodotto nel genere superparticolare dalla proportione sesquiquinta; & quello come affermano i musici tutti dissonanti, contenuto nel genere superpartiente tra questi numeri 32.27"; *Dialogo*, 10; and "Di manera che tra le due corde più acute contenevvano un triemitono dissonante dell'antico Diatono, et non una sesquiquinta consonante del sintono come lui scrive pigliando inoltre Il detto triemitono per l'istesso dalla sesquiquinta"; Galilei, *Critica*, fol. 33.

illustrated in history through a Pythagorean legend.[74] According to this legend about the invention of consonances, different types of instruments, including *auloi*, panpipes and canons, subjected to different degrees of tension through attached weights and vessels filled with a liquid to various levels yielded the same proportional ratios. Contrariwise, experiments conducted with vessels, coins, strings and pipes of different materials and sizes, as well as with his lute, allowed Vincenzo Galilei demonstrate that the Pythagorean set of ratios could not be verified in all types of instruments.[75]

Ever since his *Istitutioni* Zarlino was convinced that the *senario* would work in any type of instrument: "[…] and these parts [senario's parts] are ordered in such a way that if one were *to take six strings of any instrument* derived according to the numbers shown and *strike them together* not only would one be unable to hear any discrepancy in the intervals that would arise from the aforementioned tones [strings], but such a harmony would be produced from them that the listener would take great pleasure [from them] and the contrary would occur if this order were altered in any way"[76]. In *Dimostrationi* he explained that by placing the larger terms and the larger ratio before the smaller terms and the smaller ratio, the visual ordering of the natural places of the intervals was preserved, as "[…] the musician proceeds by making and obtaining results of his causes from the whole and the part of the sonorous body, *be it a string or anything else often used* that is divisible into infinity […]"[77].

In the definition of *corpo sonoro* given in *Dimostrationi*, Zarlino explains that since the interval was a distance between high and low sounds which bear some proportion, the best any musician wanting to know what such distance can do is to measure the *corpi* out of which sound is made.[78] Zarlino believed that the division of a sonorous body retained both

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[74] Iamblicus, Nicomachus and Theon of Smyrna told the legend of the invention of consonances by Pythagoras in which the monochord appears. See Iamblichus, *On the Pythagorean Life*, transl. Gillian Clark (Liverpool: Liverpool University Press, 1989), M. du Sautoy, *The Music of the Primes: Searching to Solve the Greatest Mystery in Mathematics*, (United Kingdom: Fourth State and Harper Collins, 2003), 77.

[75] Galilei established that the weight required to produce the tension that corresponded to a given pitch was the inverse square of the length of string. He also proved that two strings of dissimilar materials stretched on a lute and tuned to a unison did not yield unisons when stopped at the frets on the instrument's fingerboard. Thus he evidenced the relevance of the material that composed the vibrating bodies in the production of sound; Galilei, "Discorso particolare intorno alla diversità delle forme del diapason," ed & trans C.V. Palisca, in *Florentine Camerata*, 188-91, fols. 44-54; and "Discorso particolare intorno all'unissono," in *Florentine Camerata*, 198-207, fols. 55-61.

[76] Italics are ours. "[..] che quando si pigliassero sei chorde in qual si voglia istrumento, tirate sotto la ragione de I mostrati numeri, & si percuotessero insieme; ne i suoni, che nascerebbeno dale predette chorde, non solo non si udirebbe alcuna discrepanza; ma da essi ne uscirebbe una tal harmonia, che l'udito ne pigliarebbe sommo piacere, & il contrario averebbe quando tal ordine in parte alcuna fusse mutato"; Zarlino, *Istitutione* I: ch.15, 26; English translation in Kelleher, 287.

[77] "[...] il musico va facendo e cavando le sue ragioni dal tutto e delle parti fatte del corpo sonoro: sia poi corda, o qual si voglia altra cosa, che torni al proposito: il qual corpo è divisibile in infinito"; Zarlino, *Dimostrationi*, 55; English translation in Kelleher, 102; italics are ours.

[78] "Ma perche ogni intervalli musicali ha distanza, che si trova tra il suono grave & acuto: la quale senza dubio cade sotto alcuna proportione: però volendo i musici havere la ragione de tale distanza:

the nature of continuous quantity – because it is a body, i.e., a real object - and the nature of discrete quantity - because upon dividing it, each division should follow the ratios contained in the natural order of numbers.[79] Zarlino, who advanced a detailed system of division of the monochord in *Dimonstrationi*, discussion 4, was probably aware that the Greeks had performed manual division of the string. One of the categories in that technique comprised diatonic divisions based on superparticular proportions that could be applied to the monochord with a compass. Vincenzo Galilei, nonetheless, correctly argued that Zarlino was mistaken in his proposal to apply the same proportions directly to real instruments. Galilei explained that Zarlino's mistake derived from his belief in that a same method of division could be equally applied to both monochord and real instrument, because both exhibited characteristics proper to continuous quantity.[80]

In the attempt to legitimate his musical practice on theoretical grounds Zarlino resorted to Euclid's *Elements*, and thus defined term proportion as "the relation of one quantity to another similar in kind". However, he added a third type to Euclid's quantities, which he named harmonic, resulting in discrete quantity (numerical, whereby one term is enumerated by another), continuous quantity (one term is measured by another) and harmonic quantity, a combination of both.[81]

According to Galilei, these novelties and corruptions of terms in truth only did was to create more serious problems. In regard to proportions, the meaning of the parts was crucial, bacause the status of ratios and proportions changes when they are represented arithmetically or geometrically, or are to be taken as actual measurement units applicable to real bodies. Zarlino, in turn, treated numerical (arithmetical and discrete) ratios as if they were subjected to the attributes of magnitude (geometrical and continuous).[82]

Vincenzo detected and denounced Zarlino's analogical thought. Zarlino had first tried to transform geometrical into aritmethical elements to then treat geometrical entities as if they were aritmethical by virtue of simple analogy.[83] In *Dialogo,* Galilei warned Zarlino that, "adding four *unissonos* [together] will never result in [the interval of] a fourth", for intervals were not to be treated as *arithmos*, as Zarlino did sometimes.[84] Zarlino was not alone in this regard, as another 16th- century author, Nicola Vicentino (1511- c.1576), had also fell prey to the same mistake. In *L'antica musica ridotta alla moderna prattica,* aiming to prove that the unison was not a perfect consonance, Vicentino posited an analogy between the concepts of unity and unison. According to him, just as in mathematics the unity was not a number, neither in music the unison was a consonance, but the origin of consonance. The musicians ought to realize that just as nature had arranged the natural numbers in the arithmetic progression 1 2 3 4 5 6 7 8 9, also harmony was born out of numbers. They should

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non hanno ritrovato meglior mezzo, quanto la misura de i nominate corpi dalli quali nascono i suoni"; Zarlino, *Dimostrationi*, Rag.I, def. iii, 22.

[79]Zarlino, *Istitutioni*, I: 40, 52.

[80] Galilei, *Il Fronimo*, 1584; *Dialogo*, 9-25, 30.

[81]Zarlino, *Istitutioni*, ch.40, 51.

[82] Zarlino, *Dimostrationi,* Rag. I, 36; Galilei, *Discorso*, 1589, 57-8.

[83] See Fumikazu Saito, & Carla Bromberg, "Measuring the Invisible: A Process among Arithmetic, Geometry and Music," *Circumscribere* 16 (2015):17-37.

[84] Zarlino, *Istitutione*, I: ch.3.

also consider the reinforcement of one number by another, because each one exceeds the following in one unit only:

> "[…] when unity wishes to form the binary number, it counts itself twice to generate two […] the same thing happens with the unison […] when it wishes to generate the first dissonance it multiplies itself twice, […] the interval called the second in practice. And then, when it wishes to create the consonance called the third it adds itself three times and puts together three steps."[85]

Since Zarlino wished to attribute the epistemological priority to his arithmetical generation of intervals and the manner to organize them, every single definition and demonstration unrelated with arithmetic posed a serious problem to him, including his very approach to the real instrument. To make the confusion even worse, Zarlino used term sounding body *(corpo sonoro)* in a too restricted sense. While it was commonly used to designate anything from which sounds emanates and could be approached from a mathematical perspective, Zarlino applied it to the string alone. One further mistake he made was to think that the string division was analogous to the division of a string in a musical instrument. Galilei warned Zarlino that neither sounding bodies nor strings were the same as the musical instruments. Galilei made drawings of ancient instruments and devoted a part of *Dialogo* and several short *discorsi* to explain the various characteristics of instruments. Instruments were complex structures. They belonged to the natural world, and thus, physical investigation would show that several variables, especially the ones involved in the constitution of instruments, needed to be taken into account when dividing or tuning instruments. The strings of a musical instrument, except for the monochord,[86] had to be distributed according to their function and shape (including arm frets and curves). Galilei thus showed, in *Dialogo*, that the consonant intervals advocated by Zarlino within his syntonic system could sound dissonant in real instruments.[87]

One further point of disagreement concerned the direct relationship of proportions to consonances. The definitions of consonance and dissonance given by Galilei bear no direct connection with nor depend on ratios. The definitions of consonance and dissonance in *Compendio* match the ones given by Boethius in *De musica*, according to which consonance is a mixture of low and high sound that is pleasing to the ear, while dissonance is a mixture of

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[85] "Et cosi come l'unità multiplicata in se tre volte fa nascere il numero ternário, cosi anco l'unisono tolto a se tre altri unisoni ascendenti, overo discendenti, crea un grado, overo un salto della consonanza detta terza"; Vicentino, *L'antica musica ridotta alla moderna pratica*, book II, 28-9; English translationby Maria R. Maniates, *Nicola Vicentino - Ancient Music Adapted to Modern Practice* (New Haven [CT]: Yale University Press, 2011), 88-9.

[86] As is known, there were monochords with various strings; see: Cecil Adkins, "The Theory and Practice of the Monochord" (Doctoral dissertation, State University of Iowa, 1964).

[87] Galilei used the fifth 16:15 as example; Galilei, *Dialogo*, 1581, 17. Here he also criticizes Zarlino's propo.II, 30, *Dimostrationi*, which states that the fifth consists of two sesquioctaval and one sesquinonal tones and one large semitone in ratio 16:15. Galilei then describes the opposite case in syntonic tuning between a fifth wherein two of the tones are small sesquinonal tones, and thus when one adds the fourth 4;3 to the small tone 10:9, an impurely tuned fifth results 40:27.

low and high sound that offends the ear, while no explicit mention of ratios was made.[88] In *Il Fronimo,* quoting Boethius' *De musica*,[89] Galilei tells again the legend about Pythagoras' invention of consonances to Zarlino.[90] Vincenzo agrees on that Pythagoras had looked for the reasons behind the diversity of sounds and as a consequence found their numbers. Nevertheless, for Vincenzo Pythagoras had not actually found the consonances, because their cause was not in the numbers, but in sound.[91] Zarlino's explanation of consonances as ratios seemed to have no precedents: "*Consonance is a ratio of numbers* contained between two sounds or voices, the one low and the other high, which comes smoothly to our hearing"[92].

Final remarks

In the present paper we sought to bring new light on how Galilei's beliefs and methods shaped his musical science. Music science had two parts, one mathematical and one physical. To Zarlino, the mathematical nature of music was defined by numbers that determined the consonant and dissonant intervals. To Galilei, the physical nature of music had in numbers a simple description of intervals, i.e., a quantitative process that did not evaluate the interval as a function of its consonant or dissonant character.

The epistemological dispute between Zarlino and Galilei was based on their different conception of definitions and demonstrations. To Zarlino, the relationship of definition to demonstration was hierarchical, for definitions proceeded from and were assumed in demonstrations, while demonstrations illustrated the non-essential attributes of the object being defined.[93] Zarlino defined music by its formal cause and dealt with the contradictions in music by creating categories and classifications. The presentation of the first principles - which did not derive from sensory data, but from arithmetics - became the basis for

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[88] "Consonanza non è altro che mistura di suono grave et acuto, che proviene alle nostre orechie soave et uniformemente" and "Dissonanza è medesimamente mistura di suono grave et acuto, la quale aspremente previene alle nostre orechie"; Galilei, *Compendio*, fol. 18v. Then:"Consonantia est acuti sono gravisque mixture suaviter uniformiterque auribus accidens" and "Dissonantia vero est duorum sonorum sibimet permixtorum ad aurem veniens aspera atque iniucunda percussion"; Boethius, *De istitutione musica*, liber I, fol. 195. VIII, see also Nichomacus, *Enchiridion* 12, in Carl von Jan, *Musici Scriptores Graeci* (Leipzig: Teubner, 1895) 262.1-2, 5-6.

[89] Boethius, *De musica*, book I, fol. 196-8. X.

[90] Theon of Smyrna, 57.1-10, 59.4-61-17; Nichomacus of Gerasa, *Harmonics*, 248.13-18 *apud* Andrew Barker, *The Science of Harmonics*, 408.

[91] "Dopo che Pitagora hebbe investigate la cagione da quello nascesse la diversità de suoni, & ch'egl'hebbe ritrovato da quail numeri fussero contenuti & non le consonanze o la musica come dicono alcuni semplice […] di maniera che per essere contenuta quella consonanza da quelli & non da questi numeri, non credete gia che fusse la cagione che questa piu di quella gli dilettasse ma che realmente nel suono fusse questa facultà"; Galilei, *Il Fronimo*, 1568, 95.

[92] Italics are ours."Consonanza è ragione dei numeri, contenuta da due suoni, o voci, l'uno grave e l'altro acuto: la quale soavemente viene al nuestro udito"; *Dimostratione*, 10. See also: Corwin, *The Istitutione Harmoniche of Gioseffo Zarlino*, 55. Some authors from ancient Greece, like Dydimus, had introduced divisions of the monochord in new genera - diatonic and chromatic- and placed the tetrachords under new ratios, such as 5:4 and 6:5, but none of them was thought to form consonances; Ptolemy, *Harmonics*, book II, ch.13, 68.15-25, in Barker, *Greek Musical Writings*, 343. These are the same ratios Zarlino included in his senario as being consonances.

[93] Kelleher, 106.

Zarlino's derivation of the musical intervals, which included the calculation of ratios and tempering.

Vincenzo Galilei suggested a new point of departure to define the scope of demonstrations. He studied the role geometry and arithmetic played in music, and by identifying Zarlino's difficulties to measure intervals in instruments he made room for an appropriate separation between the quantitative and the qualitative attributes of sound.

Unlike previous interpretations of Galilei's work, the present study shows that he worked within a mathematical framework and methodology. As was mentioned above, *Critica* was not the only work in which Galilei described his mathematical demonstrations, and even challenged Zarlino "or any other famous mathematician to demonstrate their conclusions better than I have done"[94]. The reason was that according to Galilei, his demonstrations were not practical, as Zarlino had argued, but numerical.[95]

As is known, arithmetic and geometry were two different fields of knowledge at that time and should be treated as such by modern scholars approaching 16th-century music. Included in two different conceptual frameworks, these two sciences had different methods for measuring, different methodologies and they were approached by different types of mathematicians. However, while Zarlino was, indeed, considered a mathematician, as is shown in Bernardino Baldi (1553-1617), *Vite dei matematici*,[96] Galilei never was so, even though he used the methods usually applied by the mathematicians known as 'practitioners'.

The role of experimentation with instruments was a part of the dispute between Galilei and Zarlino, being the two parts of music, namely, the theoretical (abstract) and the practical (experimental) present in their work. In spite of his attempts to dismiss the experimental side of Galilei's work based on the axiomatic structure he had elaborated, Zarlino himself asked him to report an experiment. In *Critica* Galilei described an experimental procedure Zarlino wanted a trombonist to perform. Zarlino, who could not understand the experiment, asked Galilei to explain it to him, but according to the latter, Zarlino was unable to make any profit from the many explanations he had attempted.[97] Still in *Critica*, Galilei calls attention to some "good gentlemen who were of good taste in the things of music", who wanted him to explain the differences between the Pythagorean and modern intervals. On that occasion Galilei had the intervals be played in different instruments, one fretted in equal temperament and the other not. The result was that theoretical intervals perfect in their abstract form differed in size when actually played.[98]

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[94] "Ci vediamo mai messer Gioseffo che Archimede o altro matematico famoso di mostri le sue conclusioni con piu ragioni di quelle che faccio io al presente"; Galilei, *Critica*, fols. 27r; 31v; 33v.

[95] Ibid., fols. 24r-26r.

[96] Bernardino Baldi, *Cronica de'matematici overo Epitome dell´istoria dele vite loro Opera Di Monsignor Bernardino Baldi da Urbino abate di Guastalla* (Urbino: per Angelo Ant. Ponticelli, 1707).

[97] Galilei, *Critica*, fol. 53r.

[98] "Ritrovandomi in compagnia di diversi gentil huomini di buon gusto delle cose di musica, *si vollono alla mia presenza chiarire, se nella distribuitione de Pitagora erano realmente dissonanti gl'intervalli da noi detti consonanze imperfette*; et accostadici ad'uno strumento di tasti d'uno trali altri le consonanza in

As is known, reconciling the physical world with the principles of mathematics was a main concern of 16th-century science. In regard to the notions of science and demonstration, Galilei proved that the axiomatic procedure based on a particular choice of first principles selected by Zarlino yielded nothing but definitions of rational-abstract objects. Ultimately Vincenzo demonstrated incongruity between mathematics and the physical reality. In this sense, Zarlino remained a traditional author, as he showed little interest in freeing his *modus operandis* from the traditional textual-philosophical and axiomatic-mathematical patterns. Galilei, in turn, succeeded in dismissing the authority of the textual tradition over scientific endeavors.

Scholars who approached Galilei devoted much effort and time to present him as a 'monodist' or as an 'experimentalist'. In the former case, the focus on monody led scholars to disregard the statements Galilei made about the mathematical nature of music as a science. In the latter, although discussions on experimentalism and on the relationship between theoretical explanation and experimental observation are legitimate, scholars seem to have been unaware of the mathematical content of Galilei's works, and ultimately did not consider the classification of mathematics in the 1500s, according to which arithmetic and geometry were two separate fields of knowledge and acoustics, or frequency, could have no theoretical role whatsoever.

Galilei successfully identified the roles arithmetic and geometry played in music, thus broadening its scope and even making it surpass its traditional boundaries within the field of mathematics. While Galilei did take into account the physical aspects of music, he did not go as far as to develop a theory of sound. He did perceive the changes within instruments, but working within a mathematical framework that ultimately he proved to be inefficient to describe the musical phenomenon.

eccellenza gli cominciai a fare temperare in esso le quinte di maniera con una mia regola; [...] et erano in conclusione tali, da poterci chiarire della nostra difficultà temperato che lui l'ebbe comincio a volere sonare, et udi in fatto che la cosa passava per Il verso [...] Egli havevo della qual cosa abastanza sodisfatti, gli feci vedere daquello cio fusse cagionato et di quanto si trovavano fuore dell'ordinaria forma loro"; Galilei, *Critica*, fol. 36v.