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On Aristotle's arguments for the Earth's smallness: argumentative gaps and context

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Abstract

In the last chapter of the second book of his *De Caelo*, Aristotle argues for the Earth's smallness. In this paper I will differentiate between three relevant ways in which this feature can be understood, and I will explain how Aristotle's arguments relate to them. I will argue that Aristotle's arguments, as they are presented in the relevant passages, are not conclusive, and finally I will provide a plausible context which might help understand Aristotle's text.

Keywords

Aristotle; Greek astronomy; *De Caelo*; Theory of the Earth

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Introduction

In the last chapter of the second book of his *De Caelo*, Aristotle tackles the problem of the size of the Earth. There, he argues that the Earth is small. However, he does not explicitly say what does he mean by small. In this paper I will differentiate between three relevant ways in which this feature can be understood, and I will explain how Aristotle's arguments relate to them. Then I will show how the argument, as it is presented in *De Caelo*, fails at providing a good argument for the conclusion it presents. Finally, I will explore a possible solution to this argumentative problem of the text.

Aristotle's arguments for the size of the Earth.

In book II, chapter 14 of Aristotle's *De Caelo*, we find various arguments to support four fundamental assertions about the Earth: 1) that it is in the center of the universe, 2) that it is at rest, 3) that it is spherical and 4) that it is small. That a work such as this dedicates a considerable space to discussions about the Earth should not come as a surprise: given that our observations of the celestial objects are made from the Earth, understanding these aspects about it, and how they affect our role as observers, is a preliminary step which is necessary in any serious astronomical text.

My discussion will be focused on the fourth position Aristotle wants to justify, that is, the assertion that the Earth is small. Aristotle's passage regarding this subject (297b30-298a20) can be divided in 4 sections: the first three are arguments, and the fourth is the conclusion. The first argument, which is the longest, taking 13 of the 25 lines of the entire discussion in Bekker's edition, is based on the fact that when an observer changes its latitude on the Earth, there is a corresponding variation in what stars he can see in the sky:

> "[…] it is of no great size, since a small change of position on our part southward or northward visibly alters the circle of the horizon, so that the stars above our heads change their position considerably, and we do not see the same stars as we move to the North or South. Certain stars are seen in Egypt and the neighborhood of Cyprus, which are invisible in more northerly lands, and stars which are continuously visible in the northern countries are observed to set in the others. This proves both that the earth is spherical and that its periphery is not large, for otherwise such a small change of position could not have had such an immediate effect."[1]

[1] Aristotle, *On the Heavens; with an English translation by W. K. C. Guthrie* (Cambridge: Harvard University Press, 1939), 253.

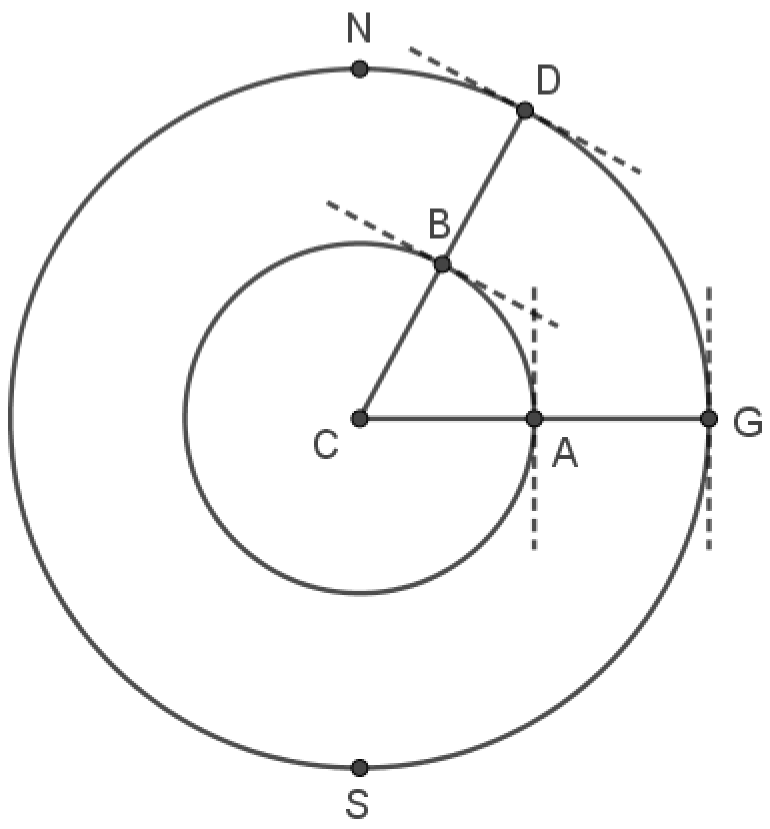

Figure 1. Comparison of two sizes for the Earth, and their relevance in the change of the horizon for two positions. Circle BA is an Earth centered on C, with a radius CB. Circle DG represents and Earth also centered on C, but with a radius CD twice as large as the previous one. Points N and S are the geographic north and south poles, respectively. The dashed lines represent the horizon for observers on A and B at the smaller Earth, and on G and D at the bigger one.

Refer to Figure 1. Aristotle is giving an argument that says that if an observer on a small Earth with radius CB moves from A to B, even if arc AB does not represent a great distance on the surface of the Earth, he will find that the change in the corresponding horizons is considerable: stars which were visible become invisible, and stars which set become circumpolar. If the direction of motion is from the north to the south, then the opposite happens. However, if the Earth had a radius CD, twice as big as before, then to experience the same change in the horizon an observer would have to move through an arc GD, which would represent a distance twice as large. Given that in fact one does not need to travel great distances in the south-north direction or vice versa to experience such changes in the horizon, that means that the circumference of the Earth is considerably small.

The second argument given by Aristotle says that, indeed, it is likely that the eastern and westernmost parts of the Earth known at the time in Greece, that is, India on one extreme, and the Strait of Gibraltar ("the Pillars of Heracles") on the other, are in fact two regions that are very close, and connected to each other. An indication of this connection is the fact that both regions were the habitats of elephants, an animal nowhere to be found except in those places. This argument would indicate that the Earth is small not only in the south-north direction, but also in the east-west one.

The last one is an *argumentum ad verecundiam*, where Aristotle simply states that "Mathematicians who try to calculate the circumference put it at 400,000 stades."[2]. He does not give any hint about how these mathematicians arrived at this value. However, in his commentary Simplicius[3] offers the method they supposedly used. It is a very simple one, and it bears some resemblance with the more famous one that Eratosthenes followed some 150 years after Aristotle, and which Cleomedes would later describe.[4]

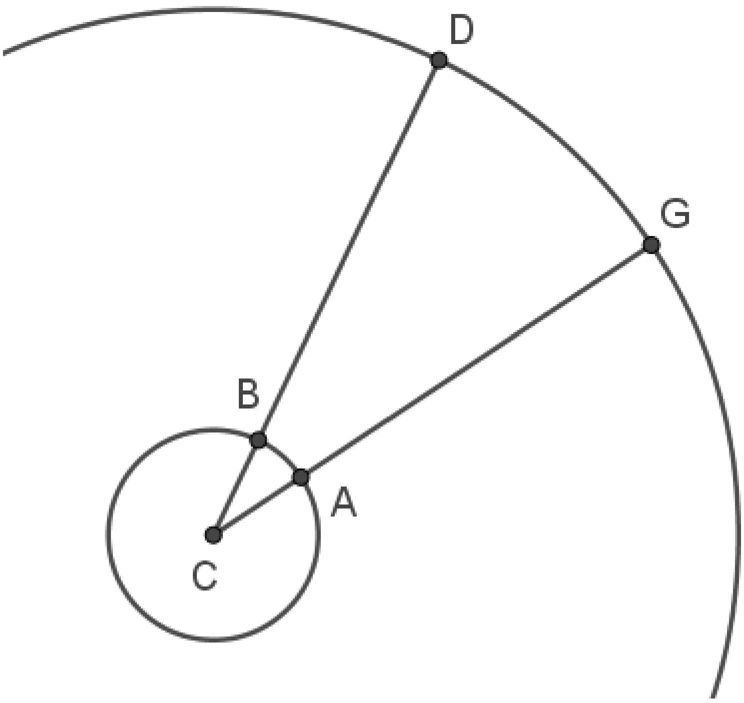

Figure 2. Circle BA is the Earth, centered on C. Circle GD is the celestial sphere, also centered on C. Locations A and B lie on the same meridian, therefore they will have stars G and D on their respective zeniths.

According to Simplicius, the ancient mathematicians found two stars separated by 1°, then located the two places on Earth where those stars culminated at the zenith, and measured the distance between those locations. Finally, they multiplied that distance by 360, and got the final circumference in degrees. Refer to Figure 2. If the two stars are points G and D, then $\angle GCD = 1°$. He says that the two locations A and B were found to be 500 stades apart. Therefore, the total circumference is $500 \text{ stades} \times 360 = 180{,}000 \text{ stades}$. Now, Simplicius wrote his commentary some 900 years after Aristotle, so his account is not, *prima facie*, too reliable. However, the observation of stars at culmination was a common practice in antiquity

[2] Aristotle, *On the Heavens*, 255.
[3] Simplicius, *On Aristotle On the Heavens (London: Bloomsbury*, 2005), 95-96.
[4] Morris R. Cohen & I. E. Drabkin, *A Source Book in Greek Science* (New York: McGraw-Hill Book Company, 1948), 151-153.

–for example, for the composition of *parapegmata*–: even Aristotle notes in his *Meteologica*[5] that α Coronae Borealis ("the Crown") culminated at the zenith for an observer in Athens. As I said, the method Simplicius is describing is very similar to the one of Erathostenes, from the 3rd century B.C. So it is not unlikely that Simplicius' account is an idealization based on a real procedure.

Having seen the three arguments, we can now ask what was Aristotle referring to when he said that they support the proposition that the Earth is small. Given that *small* is a relative term, it is not beside the point to ask with respect to what he thinks it is small. In this context, there are three relevant ways in which one can measure the Earth: a) with respect to some unit of measurement that is meaningful in our everyday experience (stade, mile, kilometer), b) with respect to the size of the celestial sphere, or to the size of the sphere of some celestial body, i.e., to the distance to the stars or planets. Finally, c) the Earth can be measured with respect to the size of the other celestial bodies, i.e., to the volumes or radii of the stars or planets. We shall call the first way the "geographic" way, the second the "cosmic" way, and the third one the "astronomical" way. It is clear that the second argument, the one that mentions the elephants, is only relevant in the geographic way: the argument does not mention any cosmic or astronomical body or distance. It is just proposing that the circumference of the Earth in the east-west direction is not much larger than the sum of the distances to the eastern and westernmost points on Earth that the Greeks were aware of, and the distance to which they had some approximate knowledge of. [6]

Even if they mention the stars, the first and third argument also point to this kind of geographic smallness: the first argument, although without giving any value, is simply saying that to a considerably small distance in the south-north direction, it corresponds a considerably big change in the part of the celestial sphere visible to the moving observer. The third argument, even if –in Simplicius' account– it uses the stars as reference, it is only to measure the angle that corresponds to a certain distance on the surface of the Earth. The comparison is only to a distance between geographical locations and through it, to the stade. When analyzed more carefully, it is even possible to conclude that the third argument is a more sophisticated version of the first one.

So it seems that all that interests Aristotle in this discussion is to prove that the Earth does not have a size that is incomparably bigger than the parts already known to the Greeks. This conclusion, though, is not at all the one that he gives in *De Caelo*. In fact, the fourth section of the passage we are analyzing says that

> "From these arguments we must conclude not only that the earth's mass is spherical but also that it is not large in comparison with the size of the other stars."[7]

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[5] Aristotle, *Meteorologica. Translated by H. D. P. Lee* (London: Heinemann, 1952), 181. See Per Collinder, "Dicaearchus and the 'Lysimachian' Measurement of the Earth," *Sudhoffs Archiv für Geschichte der Medizin und der Naturwissenschaften* 48, no 1 (1964): 63-78, on p. 71 for a discussion about the authenticity of the passage.

[6] As Aníbal Szapiro pointed out when he read a previous version of this paper, the *explanandum* Aristotle is addressing here, i.e., the existence of elephants at both extremes of the known world, could also be accounted for by assuming that the Earth is very big and that the known world is the only, small, section where elephants *do not* live on.

[7] Aristotle, *On the Heavens*, 255.

Aristotle's conclusion is truly unexpected, since he is jumping from a measurement in stades or some other unit, to a comparison to the size of "the other stars". That the expression πρὸς τὸ τῶν ἄλλων ἄστρων μέγεθος[8] must be understood as a reference to the size of the visible celestial bodies themselves, and not of the spheres that carry them –i.e., to our distance to them– is confirmed by Simplicius' commentary[9], which explains this much, and quotes Ptolemy's estimate for the ratio between the volumes of the Sun and the Earth's.[10]

In fact, Aristotle's third argument (Figure 2) in *De Caelo* assumes that the ratio between the radii of the Earth and the celestial sphere is that of a point to the radius of a sphere centered on it. It is clear that the argument only works if the angular distance between the two stars D and G, observed from points A or B on the surface of the Earth, is equal to that angular distance as seen from the center of the Earth C. That assumes that the distances CA or CB are negligible when compared to distances CD or CG.

The first argument, though, seems to work even with a celestial sphere that is small in comparison to the Earth. Refer to Figure 3.

[8] August Immanuel Bekker , *Aristotelis Opera,* vol. II (Oxford: Oxford University Press, 1837), 287.
[9] Simplicius, *On Aristotle*, 96.
[10] Claudius Ptolemy, *Ptolemy´s Almagest. Translated by Gerald Toomer* (Princeton: Princeton University Press, 1984), 257. Simplicius says that if Ptolemy's calculation is correct, then an observer at the Sun would see the Earth having an apparent size 170 times smaller than the apparent size the Sun shows for an observer on the Earth. This is of course incorrect, because here Ptolemy is giving the relation between the volumes of the bodies, and not the radii. A few lines earlier, Ptolemy had given the ratio between radii as 5.5 to 1, which is the one that would also apply to apparent sizes from equal distances.

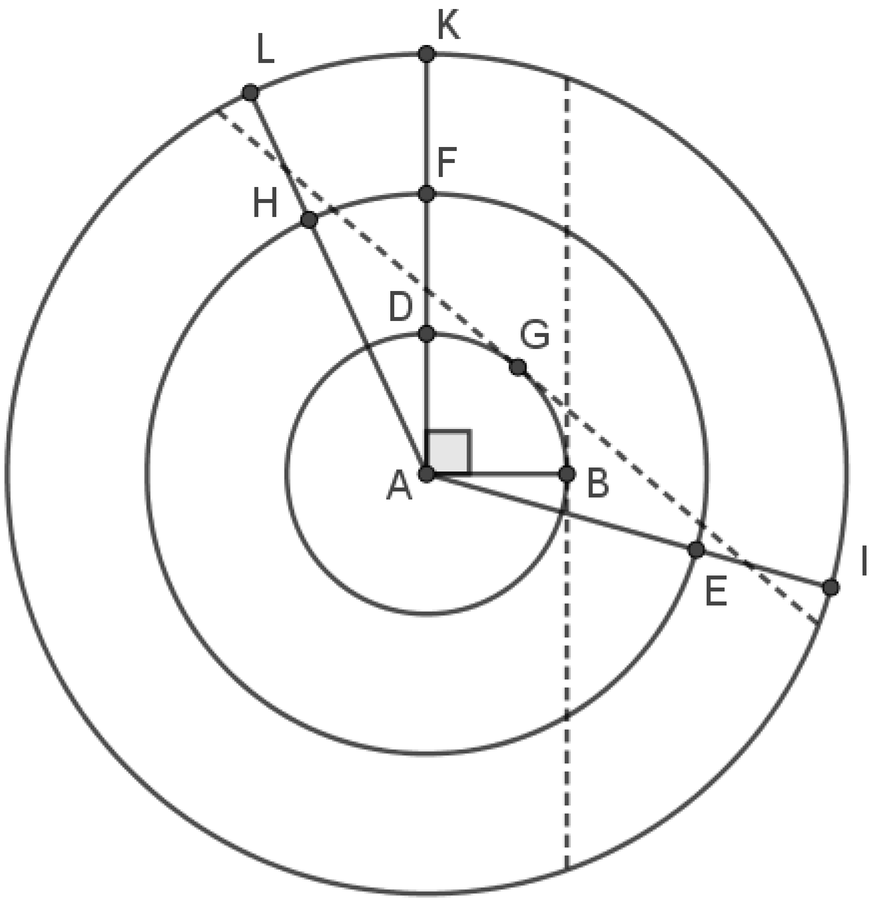

Figure 3. Point A is the center of the Earth BGD, and of the small celestial sphere EFH, and the big celestial sphere IKL. Points B and G are two positions on the Earth's surface with the same longitude. Both have their respective horizons marked with dashed lines. Points I, K, and L are three stars, and their corresponding locations on the small sphere are E, F and H respectively. Star K/F is Polaris on the celestial north pole, making D the geographic north pole. Finally, because ∠BAD is a right angle, B is a position on the equator.

If the observer moves from B to G, he will find that the section of the celestial sphere has changed. Polaris is now in view, no matter which of the two sizes for the sphere is the correct one. However, if we assume the big sphere, we can see that more stars have come into the field of view than if we assume the small one: while point L is above the horizon, point H is not. Nevertheless, we can note that the opposite happens when we consider the stars that disappeared. Although point E has gone under the horizon in a model with a small sphere, its corresponding point I is still visible if the sphere is bigger. So, both possibilities can be accommodated with the Aristotelian text, which speaks of changes in the visibility of the stars. However, it is easy to see that, no matter the size of the sphere, the observer will have to move the same distance on the surface of the Earth, i. e. arc BD, to get Polaris to be at his zenith. If that distance is small in terms of some unit of measurement, then he can be sure that it represents one quarter of the Earth's circumference, and therefore that Earth is also small. The only difference he would find if he could experience the different sizes of the sphere is that the bigger it gets, the more uniformly the stars would change its altitude. If the sphere is infinitely big, then all the stars would change at the same rate given some change in latitude. As it gets smaller, he would notice that the variation in altitude is more pronounced the closer the star is to his zenith.

Now, this way of understanding the first argument only has the goal of showing that, strictly speaking, the argument needs not a small Earth in astronomical terms in order to work: only a geographically small one. Nevertheless, given the third argument, it is likely the case that its author also had this kind of cosmic smallness in mind, and it is in that context that he wanted his audience to understand it.

So Aristotle's passage is confusing: the three arguments only aim at proving the geographical smallness of the Earth. Moreover, –if Simplicius' commentary is correct– the third argument assumes a cosmic smallness, and it is likely that the author of the first argument also assumed that. However, the conclusion says that from these arguments we should arrive to the astronomical smallness.

The argumentative gap.

The text is thus one where there is a major argumentative gap, since it proves the geographic smallness, but then goes on to concluding the astronomical smallness. This gap needs an explanation.

If one assumes that the gap is simply an argumentative error, and also that such an error is not one which Aristotle would have fallen into, then it is reasonable to assume that the passage in question is only a later interpolation made by a less capable copyist or disciple. If it is an interpolation, then it is an ancient one, because in Themistius' commentary in the second half of the fourth century we already find it as part of the Aristotelian text.[11]

There is, however, another possibility which is in my opinion more interesting and fruitful: maybe Aristotle's text is not giving us the entire discussion, and there is a subtext which was well known to Aristotle's contemporaries, but which is not known to us. In order to imagine how this unwritten context might have looked like, we need to look again at the argumentative structure of the whole passage.

As we saw, the three arguments prove that the Earth is small in the geographic sense. This means that it is small when measured in some unit of measurement that is proportionate to the everyday human experience of distance, and thus much smaller than the Earth itself. But the conclusion states that the Earth as a body is smaller than the body of "the other stars", and by this he simply means the celestial bodies. In order for this to make sense, we must assume that he is comparing the size of the Earth and of those bodies *in the unit of measurement* he used to measure the Earth, or in some other unit convertible to it. Now, this could be achieved in two ways:

[11] Themistius, *Themistii in Libros Aristotelis de Caelo Paraphrasis Hebraice et Latine* (Berlin: Georgius Reimerus, 1902), 143-144.

a) we know the size of the Earth in those units, and we know the proportion between the sizes of the Earth and the celestial body. Therefore, we can *then* know how both sizes are compared in the selected unit of measurement.

This path is the one which, for example, Averroes follows in one of his commentaries to Aristotle's work. He briefly explains the procedure which Simplicius had referred to and we represented in Figure 2. He says that once we obtain the circumference of the Earth in that manner, we can calculate the size of the Sun and Moon from eclipses –presumably having in mind the method exemplified in the *Almagest*–.[12] From that, we can conclude that it "[…] is very small with respect to heaven, as it was said in the beginning."[13]. But this procedure has a major flaw in it, because if one wants to know the *astronomical* size of the Earth, i.e., the proportion between the size of the Earth and the celestial body, then there is no need to first calculate the size of the Earth in the chosen units of measurement. The eclipse diagram method which Averroes alludes to is in itself a calculation of the astronomical size of the Earth which uses as the unit of measurement the radius of the Earth! Any other path that calculates the proportion of the two bodies independently of how big is the Earth in terms of the everyday human experience would have the same problem. On the contrary, Aristotle's text suggests that we can know that the Earth is small in astronomical terms precisely because we first got to know its size in geographical terms, e.g., measured in stades. This leads us to the other path:

b) we independently know the size of the Earth *and* of the celestial body in the chosen unit of measurement, and this allows us to obtain the proportion between the sizes of the two bodies.

Although I am not aware of many methods that achieve this, there are certainly ways to do it.

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[12] Ptolemy, *Ptolemy´s Almagest*, 251-257. See Olaf Pedersen, *A Survey of the Almagest: with annotation and new commentary by Alexander Jones* (New York: Springer, 2010), 207-213 for a detailed explanation of Ptolemy's procedure, and Christián Carman, "Rounding numbers: Ptolemy's calculation of the Earth-Sun distance," *Archives for the History of Exact Sciences* 63, no 2 (2009): 205-242 for a study of Ptolemy's handling of the observational data for this case.

[13] Averroes, *Averrois Cordubensis commentum magnum super libro De celo et mundo Aristotelis*, vol. II, ed. Rüdiger Arnzen (Louvain: Peeters, 2003), 480. The Islamic philosopher argues for the cosmic smallness, instead of the astronomical one, which is the one Aristotle is referring to. Also, Averroes seems to be aware that the Aristotelian argument had some flaw: "But perhaps someone inquires and declares which smallness he was referring to: because small and big are said with respect [to something], and if he meant that it is small with respect to the sphere, this is not known in that way [i. e., Aristotle's first argument] but in others […]" (p. 479).

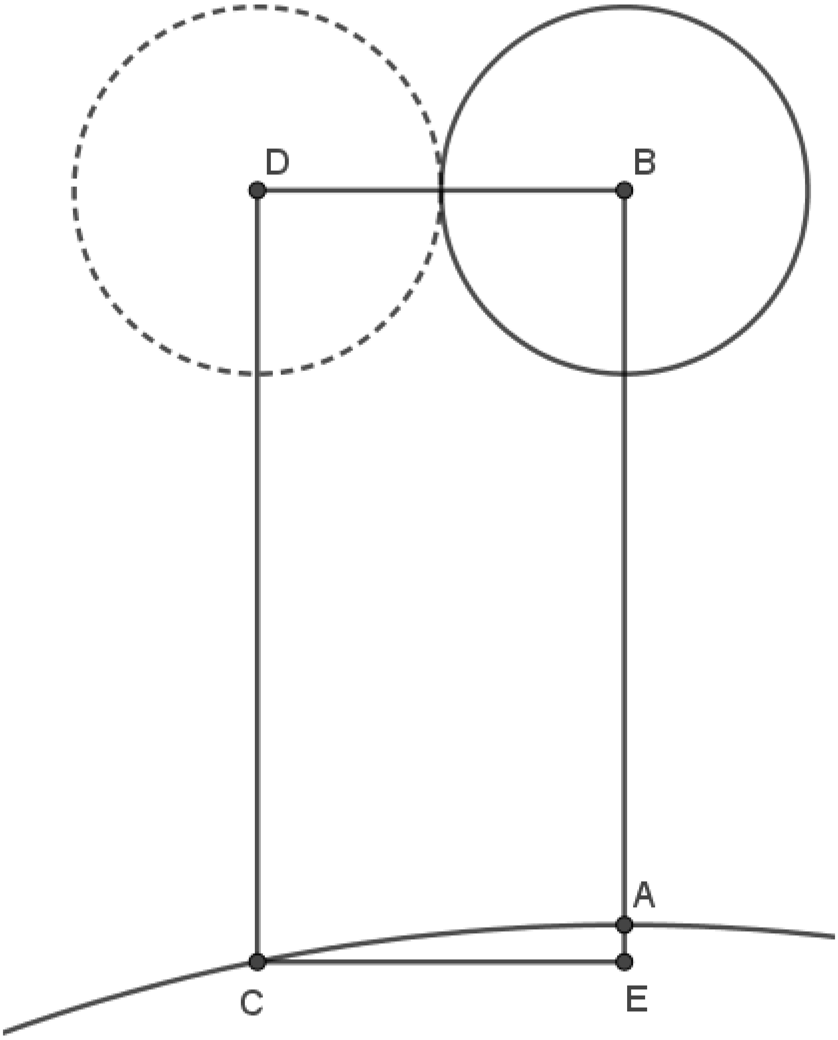

Figure 4. Calculation of the size of the Moon. Points A and C are locations on the surface of the Earth. Point B is the center of the Moon, and point D is the projection of the center of the Sun at the lunar distance. Point E is the intersection of BA and a line perpendicular to it determined by C.

Refer to Figure 4. If an observer is located on A during a total solar eclipse with the center of the Moon on B, then the center of the Sun will also be on line AB. If at the same time another observer is at C, and C is the closest location to A where the eclipse is not even partial, then he will see the center of the Sun in the direction of D. If the Sun is assumed to be sufficiently far away, then lines AB and CD can be considered to be parallel. Also, both lines BD and EC are, by construction, perpendicular to AB. So DBEC is a parallelogram, and line CE is equal to DB. But because the lunar and solar apparent radii are equal, then line DB is equal to a lunar diameter. Therefore, line CE is equal to a lunar diameter. If we assume arc CA to be equal to line CE, that is if we assume that the effect of the Earth's sphericity is negligible for the distance between the observers, then the distance between the two observers is equal to the lunar radius.

This calculation may be the very one behind Martianus Capella's rendering of an argument for the calculation of the size of the Moon *De Nuptiis Philologiae et Mercurii*, at the beginning of the fifth century A.D.[14] He mentions Meroe and the mouth of the Borysthenes river as the two observing locations.[15] The fact that the exposition of the argument is so confused in Capella's work is a sign that he is simply conveying an argument that was already being proposed and was not fully understood by him. How old was this method, it is hard to say. But Capella's source for his chapter on astronomy is likely to be found in Varro.[16] Varro belongs to the end of the second century B.C., and was only compiling and popularizing previous material. So, it is not unlikely that the argument we just saw could be traced back to the first half of the fourth century, when the application of geometrical arguments to astronomical problems was taking off among Greek philosophers.

Whether it is this one, or a similar method, it does not seem farfetched that the mathematicians of Aristotle's time were carrying out some kind of procedure to calculate the size of the Moon in stades, without the need to use the Earth's radius, for example, as a unit of measurement, but instead directly using the stade or some other unit.

Now, the results of such a method would only yield a value for the Moon, which is a celestial body that is smaller than the Earth. However, Aristotle himself tells us that at the time the distance to the Sun was already being calculated. In his *Meteorologica* he says:

> "[…] astronomical researches have now shown that the size of the sun is greater than that of the earth and that the stars are far farther away than the sun from the earth, just as the sun is farther than the moon from the earth […]"[17]

It is possible, for example, that he had in mind the simple method which Aristarchus made famous some decades later in his *Treatise on the size and distances of the Sun and Moon*[18].

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[14] Christián Carman, "Martianus Capella's calculation of the size of the moon," *Archives for the History of Exact Sciences* 71, no. 2 (2017): 193-210.

[15] This is in fact a very close approximation, since there are about 3,300 km. between the two locations, and the lunar penumbra has a radius that varies between 3,400 and 3,650 km.

[16] William Stahl, *The quadrivium of Martianus Capella. Latin traditions in the mathematical sciences, 50 B.C.-A.D. 1250*, vol. I (New York: Columbia University Press, 1971), 50-53.

[17] Aristotle, *Meteorologica*, 61.

[18] Thomas Heath, *Aristarchus of Samos; the ancient Copernicus. A history of Greek astronomy to Aristarchus together with Aristarchus' Treatise on the sizes and distances of the Sun and Moon,* (Oxford: Oxford University Press, 1913). When I say that the method is a simple one, I refer to the core argument. Aristarchus' presentation, however, is at times extremly complicated. This is in part due to the pre-trigonometrical methods he had at his disposal, but also due to Aristarchus' over scrupulous attitude in dealing with the observational data he had.

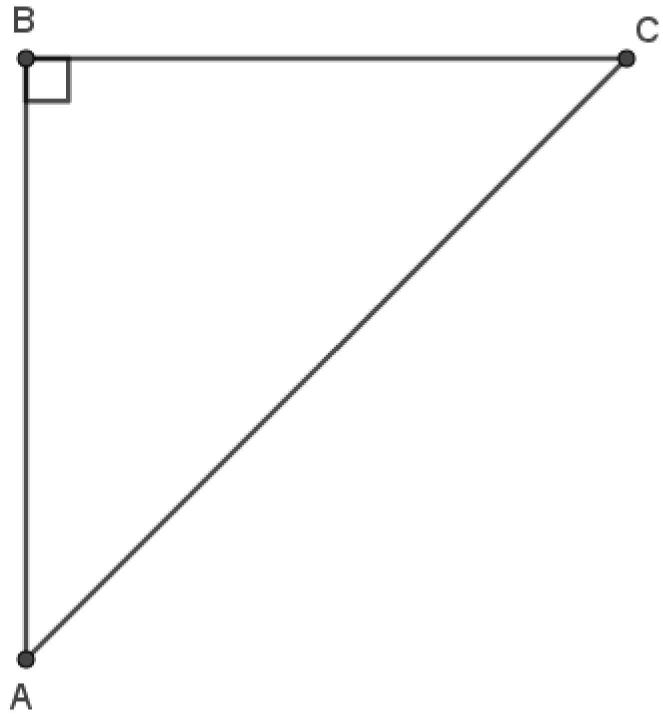

Figure 5. Basic diagram in Aristarchus' method. The observer is at A, point B is the center of the Moon, and point C the center of the Sun.

Refer to Figure 5. The argument assumes that the Moon is closer to the Earth than the Sun, a fact which is easily derivable from solar eclipses. The observation of the position of the Sun and Moon is to be done when the phase of the Moon is either first or last quarter, thus assuring that $\angle CBA = 90°$, and so ABC is a right-angled triangle. By observing the positions of the Sun and Moon, we get $\angle BAC$, and we can solve the triangle. In that way, we can get the proportion between the lunar and solar distances. It is well known that the method is so sensitive to the input value of $\angle BAC$, which in turn depends on the determination of the correct lunar phase, that it is very difficult to obtain accurate results. However, Aristarchus obtained that the solar distance was between 18 to 20 times greater than that of the Moon.

Given that the solar and lunar apparent sizes are equal, with these two methods we can obtain the solar diameter in the chosen unit, with a simple multiplication.

The entire path we have followed here does not make use of the Earth's radius to obtain the solar radius –and therefore its circumference and volume– in, let's say, stadia. So we have achieved our goal of showing how to get that value using methods of which we have some evidence that are likely to have been in circulation in Aristotle's times. Once we also get the value of the Earth's circumference –either an approximate one, like in the first two of Aristotle's arguments, or a more accurate value, like in the third argument– we can compare the sizes of both. It seems that there must have also been some kind of argument regarding the distance to the fixed stars, as the fragment from the *Meteorologica* implies. If this is the case, then an estimate of their angular sizes, combined with an assumed great distance to them, would have furnished the missing element to complete a similar calculation, and compare the Earth to them.

Conclusion.

The Aristotelian passage is a perplexing one. The disconnection between the arguments and the conclusion is such that it is tempting to assume that Aristotle himself, or what is more likely, some of the later interpreters, misunderstood the subject and introduced a conclusion which could not be supported by the previous arguments. In that sense, the analysis from the first section could support the idea that 298a18-20 –i.e., the conclusion– is a regrettable interpolation. However, I leave that judgement to a more philologically qualified scholar.

So, assuming that the whole discussion has come down to us as Aristotle wanted to, I have tried to show how the disconnection which I point out could be saved, by elucidating a possible background which the text does not make explicit. The path I propose here combines the likely argument for the lunar radius behind Martianus Capella's explanation, with the basic idea behind Aristarchus' dichotomy method for the proportion between the lunar and solar distances. It is just a theoretical path. But the methods they use where all available in Aristotle's times, and it is not unlikely that the arguments Aristotle elliptically refers to in *De caelo* and *Meteologica* are related to them.

Even if the proposed path is not the one Aristotle had in mind when he concluded the astronomical smallness of the Earth, my proposal illuminates what kind of arguments should complete the argumentative gap I point out at the beginning of the paper. Regarding this, my proposal does what –to my knowledge– ancient and medieval commentators failed to do in their interpretation of this passage. If they deal with it at all[19], they usually choose to furnish their commentaries with the Ptolemaic arguments from *Almagest* I, 6, which only aim at proving the cosmic smallness of the Earth.[20] Even if they argue for the astronomical smallness, they refer to the eclipse method which Ptolemy explains in *Almagest* V, 15, which uses the radius of the Earth as the unit of measurement. In general, there is in them a lack of understanding of the confusing nature of the Aristotelian passage, which they do little to clarify.

[19] Themistius' commentary, which I referenced in note 11, merely paraphrases the text, and makes very few additional remarks. Buridan (XIV cent.) does not even mention the problem (Cf. Johannes Buridanus, *Quaetiones Super Libri Quattuor De Caelo et Mundo* (Cambridge, Mass.: The Medieval Academy of America, 1942), 233-235.)

[20] In his commentary, Simplicius draws heavily on these arguments. So does Averroes, S. Albert the Great (cf. Albert the Great, *Beati Alberti Magni Opera Omnia,* vol. IV (Paris: Ludovicus Vives Bibliopola, 1890), 233-235), and S. Thomas Aquinas (cf. Thomas Aquinas, "In libros Aristotelis De caelo et mundo expositio." (1963). *isidore.com.* Accessed November 24, 2020. https://isidore.co/aquinas/DeCoelo.htm.).