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Implications of Synechism: Continuity and Second-Order Vagueness

Implicações do Sinequismo: Lógica Triádica e Vagueza de Segunda Ordem

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Abstract: My aim is to consider one of the most pressing problems for multi-valued logic and second order vagueness, in the light of Peirce's theory of Synechism. I will begin with a presentation of two sets of principles that we may argue form Peirce's work: the first one is directly taken from his results in the field of the Logic of Relations, while the second arises from a general consideration of the common root shared by the Phaneroscopic, the Semeiotic and the Pragmatic part of his system. In order to clarify the relation between Triadic Logic and the problem of Continuity, I will then turn to the famous example of the blot of ink, given by Pierce himself - in his logical notebook - dated 1909. Finally, I will discuss the general objection of second order vagueness for Triadic Logic, of which Peirce seems to have been unaware, enquiring if Synechism, the doctrine of Triadic Continuity, could be regarded as a possible theoretical solution for it.

Keywords: Synechism. Infinitesimals. Triadic logic. Second-order indeterminacy.

Resumo: *Meu objetivo é refletir sobre um dos problemas mais prementes para a lógica de múltiplos valores e a vagueza de segunda ordem, à luz da teoria peirceana do sinequismo. Começarei com uma apresentação de dois conjuntos de princípios que, podemos dizer, formam a obra de Peirce: o primeiro é diretamente retirado de seus resultados no campo da lógica das relações, ao passo que o segundo advém de uma reflexão geral da raiz comum compartilhada pelas partes de seu sistema, a faneroscopia, a semiótica e a pragmática. Para esclarecer a relação entre a lógica triádica e o problema da continuidade, farei uso do famoso exemplo do borrão de tinta, utilizado pelo próprio Peirce, em seu caderno de lógica em 1909. Finalmente, discutirei a objeção geral da vagueza de segunda ordem para a lógica triádica, da qual Peirce não parece ter se dado conta, indagando se o sinequismo, a doutrina da continuidade triádica, poderia ser considerada como uma possível solução teórica para a mesma.*

Palavras-chave: *Sinequismo. Infinitesimais. Lógica Triádica. Indeterminação de segunda ordem*

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The aim of this paper is to show that the objection of second-order vagueness is particular, not general; it exists at least one case of three-valued logic for which the dynamic of second-order vagueness could not be applied. This claim will be sustained taking into consideration the particular system of triadic logic developed by Charles Sanders Peirce and his principle of continuity: the synechism.

Second order-vagueness

The problem of second-order vagueness is one of the most relevant difficulties in founding a multi-valued logic. This objection seems to make a non-sense of every attempt to abandon the conceptual ground of reference offered by classical logic: the dichotomy true/false, the principles of excluded middle (PEM) and bivalence (PB). If you consider the following proposition: "When did Ulysses die?", you can approach this issue from two perspectives.

On the one hand, the first considers the passage between life and death to be a sharp passage: after a state of life follows immediately a state of death. This is the classical logic's view. On the other hand, the second perspective maintains that between the state of life and the state of death there is an intermediate state where Ulysses is neither clearly alive nor clearly death. This is the view of a hypothetical three-valued logic (truth, false, intermediate), which states that for the intermediate state the principles of excluded middle and bivalence should be considered violated or suspended. This perspective holds that "alive" is a vague predicate because it admits border-line cases.

Unfortunately, the borders of what is vague are vague themselves. This leads to a further problem: the same operation we used with two values could now be repeated also for three values. In fact, the question "When did Ulysses die" faces the same difficulties as the question "When did Ulysses *clearly* die? At some moment of the passage between life and death it is not clear if Ulysses is alive or not; at this stage, we should therefore assign an intermediate value between life and death because it is not clear if Ulysses is in the first or in the second state. But, if we admit that a sharp passage between life and death doesn't exist, then we are forced to maintain that such a sharp passage doesn't exist even between life, the intermediate state and death.

As a consequence, by placing an intermediate vague region we obtain two more vague boundaries: one between life and this region and the other between this region and death. We pass therefore from first-order vagueness to second-order vagueness. The regress, moreover, could be continued to infinity. Hence, we pass from a dynamic of second-order vagueness to a dynamic of higher-order vagueness that can be repeated *ad infinitum*. The classical objection to a multi-value logic is then: "if two values are not enough, three are not enough" (Williamson 1994: 111).

The dynamic of higher-order vagueness is indeed a serious challenge to every endeavour to build a calculus with three or more truth-values because it undermines its theoretical foundation. Why should we abandon classical logic, with his force and simplicity, if the other multi-valued alternatives face the same deficiency that they have previously ascribed to the former?

Triadicity and continuity

Peirce was one of the most important and original thinkers and logicians of the 19th century and was also the first to conceive a complete system of triadic logic. Complete, because the six operators and the matrixes defined for them are enough to describe a functional complete system for three values. As noted by Fish and Turquette, these six operators were rediscovered by later logicians, and the work of Peirce should be considered as a prelude of three-valued logic developed in 1920 by Jan Lukasiewicz and Emil Post.[1]

But Peirce's is different to all the others three-valued calculus due to its theoretical character. In fact, it represents the attempt to transfer in formal logical terms all the previous results that the American thinker had achieved in every other field of thought to which he had turn upon his attention. The peircean's system is a highly articulated system which, due to the application of a logically founded theoretical base, aims to produce a unitary theory ranging from mathematic to metaphysic. This theoretical base coincides in Peirce with the centrality assigned to the triadic-continuous

[1] Fisch e Turquette 1966: 79.

relation. The tricotomic logic that we may find in the Logical Notebook dating from 1909 represent an attempt to transfer this theoretical base in formal logic. While the centrality of the triadic-continuum relation was, as a matter of fact, well established in every other field of research (faneroscopy, semiotic, epistemology, cosmology, etc…), logic was still asymmetrically confined to the dichotomy true/false.

Moreover, the mathematical research, to which Peirce had hardly dedicated himself, had hinted the possibility of a tricotomic mathematic which suggested in turn the possibility of an equal logical perspective, modelled on three values.

In particular, the tricotomic calculus could be regarded as the consequence of Peirce's extreme logical realism which states that the possibilities (can be) and the generals (would be) should be considered as real as the existent actualities ("have beens, is's, will be's"). Classical logic proves to be universally true only when we analyse universes of discourse in which are present only actualities, singular-wholly determinate objects, while it is almost inadequate to treat "potentialities, real possibilities, universal or any other general" (Parker 1992: 72).

Peirce claims therefore that, in sharp contrast to an epistemic or semantic theory of vagueness, there are *real and objectively indeterminate* entities which can't be processed adequately by a system of classical logic.

In order to understand Peirce's particular position, we should now turn our attention to two sets of considerations. The former is derived from the theorems that Peirce had proved with the logic of relatives, while the second arise from his mathematical reflections centred on the concept of continuity. Thanks to these two set of principles it is possible to show the rationale which is behind the tricotomic calculus. The purpose of the present paper is to enquire if the theoretical difference that distinguishes Peirce's perspective translates itself into a practical difference through which the system can escape the objection of second-order vagueness.

We shall begin to examine Peirce's theorems that we may find in the logic of relatives, whose aim is to prove the generative centrality of the triadic relation:

1. all polyads can be generated out of triads;
2. a triadic relation can not be reduced to the sum of monadic and dyadic relations;
3. monadic and dyadic relations can be generated from triadic relations (CP 3.483)

These three points can be expressed by a single principle: the triad is the primitive relative. The triad is generative and at the same time irreducible: every relation can be generated from a triadic relation by intersection (relations with four or more terms) or by prescission (monadic or dyadic relations), but it is impossible to generate triads from the composition of monadic and dyadic relations.

Of course, the centrality of the triadic relation doesn't limit itself to the logical field, but has the merit to prove in a formal way what the preceding research had clearly already indicated. For example, at a faneroscopical level, this formulation decline itself into the three conceptions of First, Second and Third, later defined as the categories of Firstness, Secondness, Thirdness:

> *Three conceptions are perpetually turning up at every point in every theory of logic, and in the most rounded systems they occur in connection with one another. […] I call them the conceptions of First, Second, Third. First is the conception of being or existing independent of anything else. Second is the*

conception of being relative to, the conception of reaction with, something else. Third is the conception of mediation, whereby a first and a second are brought into relation (CP 6.32).

Among the three categories, it is in particular the third which mostly mark the difference between the philosophy of Peirce and the pragmatism of Dewey and James or, at a semiotic level, between the triadicity of the peircean sign relation and the dyadicity of the saussurian perspective. Thirdness is the category of mediation, representation, the function of connection between a First and a Second; but it is also the category of law, thought, habits: the principal agent of the progress of uniformity in the cosmos. Moreover, thirdness is the category of regularity, homogeneity and relation: it "[…] represents continuity almost to perfection" (CP 1.337).

This is a fundamental point, because it is due to the connection between the concept of thirdness and the concept of continuity that arise the theoretical ground of the tricotomic calculus conceived by Peirce.

Therefore, it is worth it to focus our attention on Peirce's theory of continuum, highlighting some of his features through the definition of continuity which Peirce reached in mathematics. In the early 1890's, Peirce's interest for the theory of mathematical continuity grew until it become central. His reflection in these years followed two coordinates: the first was cosmological, the second was mathematical. Both these lines of research led to the same result: the doctrine of triadic continuity, from now on defined as synechism (from the greek word *suneches*, "brought-together-by-surgery"), should be considered as "the leading conception of science" (CP 1.62), "the supreme guide in framing philosophical hypotheses" (CP 6.101).

Mathematical continuity

In mathematics, Peirce entered the contemporary debate which, thanks to Dedekind's and Cantor's advanced theories, defined continuity and infinity as one of the primary tasks of the process aimed at reconsider the foundations of mathematics. Thanks to the comparison with the cantorean scripts, Peirce reached a concept of continuity for many aspect innovative and very different from the one elaborated by the German mathematician. Cantor defines a continuous series as a "concatenated and perfect" series:

By a concatenated series, he means such a one that if any two points are given in it at any finite distance, however small, it is possible from the first point to the second through a succession of points of the series each at a distance, from the preceding one, less than the given distance. […] By a perfect series, he means one which contains every point such that there is no distance so small that this points has not an infinity of points of the series within that distance of it (CP 6.121).

In the cantorean model the continuity is represented by the biunique correspondence between a set of individual points and a set of real numbers. If we imagine a line, it is possible to make correspondence between every point and a real number, and vice versa. This method has the advantage of individualize with precision the spot whereon each point is placed.

This advantage however comes at a price. Where did the continuity of the line come from? The cantorean answer is substantially counterintuitive: the continuity of the line comes from amassing a transfinite series of individual points which take up every possible space on it. A-dimensional, individual and discrete entities therefore produce the dimensional continuum of the line with which they don't share any essential features.

Peirce suggests a very different concept of continuity. In a paper entitled *The Continuum*, Peirce defines continuity as a conjunction of two features: Kanticity and Aristotelicy. The Kanticity states that every continuum is composed of parts of the same kind, while the Aristotelicy states that in every continuum the parts have their limit in common. Thus the difference is that, while the cantorean model is composed of individual-discrete points, the peircean model doesn't possess any individual or ultimate parts: every part of a continuum is again a continuum. Every line is made of lines, every temporal interval is made of others temporal intervals. Any numerical system can never exhaust the continuum, since "the numbers are insufficient for exactitude" (NEM 3126-127). They always constitute a discrete collection independently of their multitude.

Thus, for Peirce the real numbers represent a model of pseudo-continuity opposite to the true-continuity wherein singular simple parts don't exist as such. In a true-continuum every sufficient small part loses its individual identity to become "welded" with the other parts:

> *You have then so crowed the field of possibility that the units of the aggregate lose their individual identity: it ceases to be a collection because it is now a continuum. […] A truly continuous line upon which there is room for any multitude of points whatsoever. Then the multitude or what correspond to multitude of possible points, exceeds all multitude. These points are pure possibilities. […] On continuous line there are not really any points at all (CP 3.388).*
>
> *The result is that we have altogether eliminated points. […] There are no points in such a line; there is no exact boundary between any parts. […] There is no flow in an instant. Hence, the present is not an instant. […] When the scale of number, rational and irrational, is applied to a line, the numbers are insufficient for exactitude; and it is intrinsically doubtful precisely where each number is placed. But the environs of each number is called a point. Thus, a point is the hazily outlined part of the line whereon is placed a single number. When we say is placed, we mean would be placed, could the placing of the numbers be made as precise as the nature of numbers permits (NEM 3126-127).*

At a closer look, the question is of a genetic kind. For Cantor the continuity of the line arises from the gathering of individual and discrete points while for Peirce the mathematical continuity has the same features of the triad showed before in the logic of relatives. Exactly like the triad, the peircean continuity is synthetical and generative; every effect of discreteness could only happen as a secondary effect of an original continuity. The continuum, moreover, is irreducible to the discrete; no collection of discrete individuals can ever generate a continuum, as before when it wasn't possible to generate triadic relations from dyadic and monadic ones. "Breaking grains of sand more and more will only make the sand more broken. It will not weld the grains into unbroken continuity" (Century Dictionary).

Logical differences

For Cantor the points in a continuum are actual, determinate and existent, while for Peirce they are potential, indeterminate and real. The difference is not only terminological; rather, it is logical. We can't apply the classical principles of non contradiction and excluded middle to the points in a peircean true-continuum.

This is due to the fact these principles can be applied for Peirce only to fully determinate individuals "An *individuum* should be considered determined under every aspect, in such a way that the principle of excluded middle can always be applied to them" (NEM III: 763). But a determinate individual in a true continuum is only a real abstraction and not an existent thing; to exist as such the continuity must actually be interrupted. "Hence a point or indivisible place really does not exist unless there actually be something there to mark it, which, if there is, interrupts the continuity" (Century Dictionary).

If the proper characteristic of actuality (secondness) is determination, at a logical level its opposites are vagueness (firstness) and generality (thirdness), both being considered forms of indeterminacy. But what distinguishes vagueness from generality in Peirce's logical view? To answer this question we have to look at an article, written in 1905 and entitled *Issue of Pragmatism*, as well as follow the analysis that Robert Lane (1999) dedicated to this subject:

> *The general may be defined as that to which the principle of excluded middle does not apply. A triangle in general is not isosceles nor equilateral; nor is a triangle in general scalene. The vague might be defined as that to which the principle of contradiction does not apply. For it is false neither that an animal (in a vague sense) is male, nor that an animal is female.*

The PEM doesn't apply to the general, while the PNC doesn't apply to the vague. Lane ascribes to the logical Peirce of those years a distinction between determinate propositional terms and indeterminate propositional terms. The latter can be divided in two different classes: general terms and vague terms (which Lane calls *indefinites*). In order to express determinate terms in logic we usually refer to a letter, while to express indeterminate terms we refer respectively to the universal quantifier ("*All* the x") and to the existential quantifier ("*Some* x").

Lane maintains that the PEM used by Peirce differentiates itself from the PEM we use nowadays (of the form p V ¬p), and that it has to be understood as a principle that applies only to *singular subjects*. That is to say: for every singular term S and for every predicate P, the proposition "S is P or S is not-P" is true. This principle doesn't apply to the general case, because "It is not the case, with regard to every predicate "P" and every subject-term "S", that "S is P or S is not-P" is true", as we can notice in the case of the disjunction "every coloured person live in Africa or every coloured person don't live in Africa".

In the same way, the Peirce's PNC applies only to definite subjects; thus if S is a definite term, then "S is P" and "S is not-P" and neither of them are true. The PNC doesn't apply to vague terms because "it is not the case, with regard to every predicate "P" and every indefinite subject-term "S", that "S is P and S is not-P" is false"; sometimes these propositions are true, as in the case of "some men are bald and some men aren't bald".

The same argument can be repeated also for modal propositions; in fact, Peirce tends to associate generality and necessity, attributes of the third category, as well as vagueness and possibility, attributes of the first category. We should therefore carefully differentiate the cases where a principle doesn't apply, and those where it is applied and it is falsified:

> *[…] I do not say that the Principle of Contradiction is false of Indefinites. It could not be so without applying to them which is precisely what I deny of it. An argument against what I say, namely, that the Principle of Contradiction does not apply to "A man" because "A man is tall" and "A man is not tall", can only amount to saying that that man that is tall is not, while tall, not tall. That is true; and that is what I mean by refusing to say that the Principle of Contradiction is false of "A man" but when it is said of that man that is tall, then he is not not-tall, this is said of the existing man, which is not indefinite, but is, on the contrary, a certain man and no other (MS 641:24 2/3).*

Thus, in Peirce's view, the general propositions which express necessity are not "neither true nor false": for them the principle of excluded middle simply doesn't apply. The PEM and the PNC apply only to non-modal propositions concerning individual-definite terms. Only for them the PEM and the PNC should be considered as always applicable. Applicable but not absolutely true: for Peirce some cases do exist where they are applicable but *false*.

> *Triadic logic is that logic which, though not rejecting entirely the principle of Excluded Middle, nevertheless recognizes that every proposition, S is P, is either true, or false, or else S has a lower mode of being such that it can neither be determinately P, nor determinately not-P, but is at the limit between P and not P (MS 339).*[2]

But which are those cases where a "lower mode of being at the limit" is required and for which the principle of non contradiction and excluded middle are false?

These are the cases where there is a continuity-breach. We have a continuity-breach every time that a topical-singular discontinuity (a relative discontinuity) marks a system, and in so doing it interrupts its homogeneity and continuity. If you recall the initial example, the decease divides the continuum of time into two distinguishable and determinate parts, the previous state of life and the successive state of death. However, there are three elements, not two to be taken into consideration. A boundary exists between the two determinate states.

One fundamental characteristic of Peirce's model is that these boundaries don't limit an intermediate region with any *specifiable* area (or length) between the two states they divide; rather they occupy an *infinitesimal* region that coincides with the limit of their relation.

The notion of infinitesimal plays a central role here. Peirce, following his father's path, holds, in contrast to the vast majority of the mathematicians of the day, that the concept of infinitesimal doesn't involve any kind of contradiction: "the illumination of the subject by a strict notation for the logic of relatives had show me clearly and evidently that the idea of an infinitesimal involves no contradiction" (CP 6.113).

An infinitesimal quantity is simply a positive quantity less than any specifiable quantity.[3] Where the cantorean model of continuity relies on the notion of "point", the

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[2] And, in a letter wrote only three days later to William James: "I have long felt that is a serious defect in existing logic that it takes no heed of the limit between two realms. I do not say that the Principle of Excluded middle is downright *false*; but I *do* say that in every field of thought whatsoever there is an indeterminate ground between *positive assertion* and *positive negation* which is just as Real as they (NEM 3:851).

[3] Parker 1992: 92.

peircean model of continuity uses the notion of infinitesimal instead. The most relevant difference is that the infinitesimals, against the individual points, are perfectly definite (both logically and mathematically) being at the same time *indeterminate*:

> *If we analyse the behaviour of a curve at x and at x+dx, x+dx is not strictly a point because two points necessary have to be at a certain length. Since infinitesimals are not quantities but variables instead, then x+dx is not a point but a variable pseudo-point. X and the pseudo-point x+dx are neither separate not identical, but they are welded together.* [4]

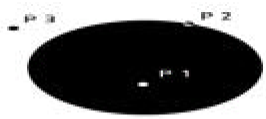
The infinitesimals express both an identity and a difference. In mathematics, at the limit point between a curve and tangent we have that the point in the curve in contact with the tangent and the point of the tangent in contact with the curve are no more distinguishable one from the other. Their distance is infinitesimal, not measurable, and so it follows that the two initial points should be considered as "welded together".

This is a central point for Peirce. Every continuity-breach provokes a relation of opposition between a First (the original continuum) and a Second (the relative discontinuity) through an infinitesimal limit placed between them. As before, between the curve and the tangent, at the limit we have an *objectively indeterminate singularity*. Every singular is an individual, every individual is definite. Therefore "indeterminate" refers here to an individual (*the* point, or *the* boundary, between the curve and the tangent) which possess "a lower mode of being" for which it is definite (one can apply the PNC) without being determined.

Triadic logic

Having presented Peirce's theory of mathematical continuity, and its logical differences from the cantorean model of continuity, we shall now see how this applies to the famous example of the blot of ink. Peirce used this example, in the *Local Notebook*, as an introduction to the problem of triadic logic:

> *Thus a blot is made in the sheet. Then every point of the sheet is unblackened or is blackened. But there are points on the boundary line, and those points are insusceptible of being unblackened or being blackened, since these predicates refer to the area about S and a line has no area about any point of it (MS 339).*



[4] Breger 1992: 79. This citation is taken from Paolucci (2005); in this paper, to which the present relation is profoundly indebted, the author focus his attention on the problem of the generative potency of the triads and on its application to the field of semiotic.

If a blot of ink is walled round upon a sheet we get are two determinate regions. We have points completely black (P1) and points completely white (P3). But what Peirce found interesting was, of course, the line between the black and the white part. Which colour do these points have? Peirce's answer is that the points on the line are *at the limit* between white and black, neither fully black nor fully white and at the same time both white and black. They occupy a region of infinitesimal width, smaller then any assignable area, at the limit between black and white.

The sheet is considered as an original continuous surface, a firstness. On it a secondness is marked, the black blot of ink. The blot is a relatively discontinuous with regard to the white surface; it in a relation of reaction between a first and a second. However, it is in itself essentially continuous as well as the other surface with which it is in relation.

Exactly as before in mathematics, where the Aristotelian feature asserted that two parts in a continuum have their limit in common, also here the border between white and black has to be considered in common; it is the contemporary overlap of the two states, the articulation of their difference. This means that the points on the border-line are both white and black, but neither fully white nor fully black.

Obviously, this image is only an icon of a logical problem; it is sufficient to substitute "true" with "white" and "black" with "false". Peirce's tricotomic logic, beside the two classical truth-values "true" and "false", introduces a third value "L" to express the state of things at the limit-point between truth and falsity. As the classical tricotomic systems introduce a third value to include border-line (vague) cases, so the peircean system contemplates the "L" value to include particular cases of indeterminacy at the limit.

This value is assigned at these states of things because, in Peirce's view, the two assertions concerning the state of thing at an infinitesimal limit point, "the border is white" and "the border is black" are neither true nor false. These assertions are non modal-propositions concerning singular and definite states of things (P2). Hence, for what we have seen, the PEM as well as the PNC are applicable, both of which, given the incomplete determination of P2, are here falsified. It is true that the points on the boundary are black and white (violating the PNC) and it is false to say that "the points on the border are white or black" (violating the PEM).[5] Said in truth-values terms, the third value "L" is an obvious case of violation of the principle of bivalence for which a proposition concerning a state of things (of the form "it is the case that…") is either true or false.

Peirce's tricotomic calculus isn't projected to eliminate *tout-court* the three fundamental principles of logic; rather his aim is to extend classical logic so as to include objectively-indeterminate entities as the kind we met when we consider the states of things at limit in a truly-continuum system. The principle of bivalence is generally valid for assertions concerning the state of things; with the relevant exception of assertion concerning the state of things at the limit in continuous system.

In a system where there are only discrete terms, or when we want to use a perfectly defined system for our practical purpose, the third value "L" is simply not

[5] Here, I must distinguish my perspective from Lane's one. Lane holds that the PNC is true (thus applicable) for state of thing at the limit in continuous system; however, from my point of views, both the theoretical ground and the textual evidence (NEM III: 747 e CP 6.126) strongly suggest that we can apply the principle of non contradiction here, but it will results falsified.

assigned and classical logic should be considered completely valid. The continuum-valued logic of Lucasiewicz, for example, in which the truth-values are in a biunique correspondence with the real numbers between 0 and 1, could certainly be retained useful for some practical purposes. What Peirce would have objected to is that, by anchoring the truth values to a numerical system, we could represent only pseudo-continua and not true continuity just like that of a temporal series.

For Peirce, time represents one of the best models of continuity that we can directly find in experience. It is not a mere coincidence that many examples of Leibniz and Peirce focus on a rolling billiard ball that comes to rest, or on passages between life/death and sleep/wake. As the spatial continuum is not composed of ultimate-indivisible parts, also the temporal continuum is not composed of absolutely-individual instants. Every instant has to be considered in its "state of flow"; we could single out as many breach-points as we desire by marking them and every one of them will posses the same features of "L": it will be a state of things at the limit neither completely different nor identical to the instants that precede and follow. For this reason, when we consider passages at the limit in a temporal system, applying a pseudo-continuum model is almost inadequate for the phenomenic field we are describing.

Peirce' triadic logic and the dynamic of second order-vagueness

Having discussed the rationale of Peirce's tricotomic calculus, and its necessary connection with the theme of continuity, we shall now see if the objection of second-order vagueness leaves this perspective as meaningless as it does with the tricotomic calculus of Körner or Halldén.[6]

First of all, we have to note that, actually, following the peircean coordinates, here we don't have a problem of second-order vagueness but a problem of second-order indeterminacy. The points on the border violate both the PEM and the PNC; hence, they are in a state (the one at the limit) where they are both vague and general: in a word, indeterminate.

Therefore, here it would be more appropriate to speak of second-order indeterminacy rather then second-order vagueness. We are changing only the terms and the conceptual references, not the substance of the objection; the dynamics of the passage from a first level of vagueness/indeterminacy to a second level remain the same indeed. The objection of second-order indeterminacy can be as decisive for the peircean view as the objection of second-order vagueness is decisive for the other triadic perspectives.

At first sight, it seems that this objection applies also to the peircean view. We have two determinate regions and an indeterminate value in between. So it seems valid to state that in so doing we place two more boundaries; in the former example between "white" and "L" and between "L" and "black". Indeed, in the theoretical ground of Peirce's calculus, this kind of objection *can't be applied because it is simply beside the point*. If we consider that:

> *Let the clean blackboard be a sort of diagram of the original vague potentiality, or at any rate of some early stage of its determination, this is something more than a figure of speech; for after all continuity is generality.[…] This blackboard is a continuum of possible points. […]There are no points on this*

[6] These are the two perspectives for which the objection of second-order vagueness could be regarded as decisive, Williamson 1994: 111-113.

blackboard. There are no dimensions in that continuum. I draw a chalk line on the board. This discontinuity is one of those brute acts by which alone the original vagueness could have made a step towards definiteness. There is a certain element of continuity in this line. Where did this continuity come from? It is nothing but the original continuity of the blackboard which makes everything upon it continuous. […] Thus the discontinuity can only be produced upon that blackboard by the reaction between two continuous surfaces into which it is separated, the white surface and the black surface. […] But the boundary between the black and white is neither black, nor white, nor neither, nor both. It is the pairedness of the two. It is for the white the active Secondness of the black; for the black the active Secondness of the white (CP 6.203).

In this example, which precedes the ink-blot example by a few years, we can see two important points. Firstly, all the surfaces (the blackboard, the white line, the boundary) are continuous. Secondly, we have a boundary where the parts are in "*active* relation".

But what do we mean by "active relation"? Simply, that the boundary, as we have described it, emerges *only because it mediates and permits the relation* between the white and the black determinate and actual surface. This means that, if we consider a further division between only one of the two determinate values and the indeterminate one, without the second determinate region in an active relation with the former, the boundary as such disappears. It exists only in the infinitesimal width at the limit between white and black. In itself, the boundary doesn't exist, it isn't actual. In the same way the third category works as the mediation between a first and a second, thereby permitting their relation, also the boundary functions as the third relative placed in between the two extreme limits. But the third relative comes into being only because of the mediation between a first and a second; in itself, without a first and a second, the third isn't even real. By removing one extreme, we remove the boundary too.

If we imagine an ideal separation of the two surfaces, the point (or, better, the infinitesimal region) on the boundary will be divided in two parts: one goes with the black surface, the other with the white surface, and the indeterminate "point" P2 disappears. Disappears as such, but in reality it will split into two other points, one determinately black, and the other determinately white. Moreover, if we rejoin the two surfaces and we put them together again, the boundary reappears with the same indeterminate features as before (NEM 4:342-3).

This view is the exact opposite of the cantorean model, which has to observe Dedekind's theorem of the cut. This theorem states that, due to the fact that we consider the point as an individual, discrete, determinate and indivisible entity, if we cut a segment AB in a point t then the point t must join *only one* of he two halves of the segment.

The difference is, again, that in a peircean continuum only potential places upon which we *can* mark points exist. These places, before the actual mark, are only potential and real, while in the cantorean continuum the points are always conceived as actual and existent.

As we have seen before, for Peirce using a synechistic theory and the concept of infinitesimal quantity does not involve any contradiction at all. But what's more, according to Peirce, it is the cantorean model of continuity that leads straight to paradoxes like, for example, Zeno's:

> *All the arguments of Zeno depend upon that a continuum has ultimate parts. But a continuum is precisely that, every part of which has parts, in the same sense. Hence, he makes out his contradictions only by making a self-contradictory supposition. In ordinary and mathematical language, we allow ourselves to speak of such parts -points- and whenever we are led into contradiction thereby, we have simply to express ourselves more accurately to resolve the difficulty (CP 5.335).*

Paradoxes like "Achilles and the tortoise" are created because we suppose that a continuum has ultimate parts like a (transfinite) series of actually existent points instead of an original continuum of potentialities. This potential continuum can always be interrupted by marking on it as many point as required for our practical purpose but, in that process, we always have to remember that these "points" are only abstractions which shouldn't be substituted for the real topological features of the continuum.

Conclusion

The general dynamic of second-order vagueness/indeterminacy presents at least one case in which it can't be applied, this case being precisely the system of triadic logic developed by Peirce in order to process passages at the limit in truly-continuous systems in a formal way.

This particular feature of the peircean view is a consequence of the conjunction between his specific theory of triadic continuity, the hypothesis of synechism, and his necessary reference to the concept of infinitesimal. In Peirce's view, the boundary between two coloured surfaces, or the third truth-value "L", is real only in so far as it mediates two determinate terms created by a relative discontinuity that marks the original continuum. In itself, it is only potential, neither actual nor existent. That means that it disappears as soon as we consider only a dyadic relation between it and only one of the two determinate terms, blocking therefore the dynamic of an infinite regress.

In conclusion, Peirce's original conceptual ground enables him to define a complete system of triadic logic to which the dynamic of second-order vagueness simply can't be applied.

References

Collected Papers of Charles Sanders Peirce. (1931-5). Hartshorne C., Weiss P eds. Vol. 1-6; (1958). Burks eds., Vol. 7-8. Cambridge: Harvard University Press.

Writings of Charles S. Peirce. A Chronological Edition. (1982-1996). By the "Peirce Edition Project". Bloomington: Indiana University Press.

The New Elements of Mathematics, by Charles S. Peirce. (1976). 4 voll. By C. Eisele. The Hague: Mouton Publishers.

The Essential Peirce: Selected Philosophical Writings. (1992) by Houser, N., Kloesel, C. Bloomington and Indianapolis: Indiana University Press.

Studies in Logic, by Members of the John Hopkins University (1883). (1983) by C. S. Peirce, introduction by Fisch. Amsterdam-Philadelphia: M. John Benjamins Publisher.

Breger, H. (1992). *The growth of mathematical knowledge.* Boston: Kluwer Academic Publishers.

Cantor, G. (1897). Beträge zur Begrundung der transfiniten Mengenlhere. *Mathematische Annalen*,49, pp. 52-94.

Dedekind, R. (1969). *Was sind und was sollen die Zahlen?.* Braunschweig: Vieweg & Sohn.

De Morgan, A. (1859). On the Syllogism IV, and the Logic of Relations. *Cambridge Philosophical Transaction*, 10, pp. 331-58.

Fabbrichesi Leo, R. (1993). *Introduzione a Peirce.* Bari: Laterza.

Fabbrichesi Leo, R., Leoni, F. (2005). Continuità e variazione.Milano: Mimesis.

Lane, R. (1999). Peirce's Triadic Logic Revisited. *Transactions of the Charles S. Peirce Society*, 35, pp. 284-311.

Paolucci, C. (2005). Piegature della continuità: semiotica interpretativa e semiotica generativa. *Versus*, 97.

Parker, K. (1998). *The continuity of Peirce thought*. Nashville: Vanderbilt University Press.

Salanskis, J.H., Hourya, F. (1992). *Le labyrinthe du continu: colloque de Cerisy*. Paris: Springer-Verlag.

Turquette, A. (1969). Peirce’s Complete System of Triadic Logic. *Transaction of the Charles S. Peirce Society*, 5, pp. 199-210.

Turquette, A. (1976). Minimal Axioms for Peirce's Triadic Logic. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 22, pp. 169-176.

Turquette, A. (1978). Alternative Axioms for Peirce's Triadic Logic. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 24, pp. 443-444.

Turquette, A. (1981). Quantification for Peirce's Preferred System of Triadic Logic. *Studia Logica*, 40, pp. 373-382.

Williamson, T. (1994). *Vagueness*. Cornwall: Routledge.

Zalamea, F. (2001). *El continuo peirceano*. Bogotà: Universidad Nacional De Colombia.

Zeman, J. J. (1986). Peirce’s Philosophy of Logic. *Transaction of the Charles S. Peirce Society*, 22, pp. 1-22.