



PARACONSISTENT LOGICS FROM A PHILOSOPHICAL POINT OF VIEW

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Abstract: *This article begins with a general and abstract definition of logic and, particularly, of paraconsistent logics, to establish a common ground for the discussion. Briefly stating, these kinds of logics have the property of being non-explosive, that is, it is not possible to infer any conclusion from contradictories premises. Using these definitions, it is possible to analyze some of the philosophical aspects of paraconsistent logics, in particular, the relation between the notion of explosion and the law of non-contradiction, as well as the syntactic/semantic possibility and, above all, the metaphysical possibility of paraconsistent logics. I further analyze a stronger position towards paraconsistency, namely: the claim that there are true contradictions. This articles concludes with some possible critiques to paraconsistent logics – and their refutations as well –, and pose some open questions for further work.*

Keywords: *Logic. Paraconsistency. Explosion. Law of Non-Contradiction. Dialetheism*

LÓGICAS PARACONSISTENTES DE UM PONTO DE VISTA FILOSÓFICO

Resumo: Este artigo se inicia com a apresentação de uma noção geral e abstrata de lógica e, particularmente, de lógicas paraconsistentes, para estabelecer um pano de fundo para a discussão. De modo geral, estas lógicas são não-explosivas, isto é, não é possível inferir qualquer conclusão a partir de premissas contraditórias. Usando estas definições, é possível analisar alguns aspectos filosóficos das lógicas paraconsistentes; em particular, a relação entre a noção de explosão e a lei de não-contradição, assim como a possibilidade sintática, semântica e – acima disso – metafísica das lógicas paraconsistentes. Então, será discutida uma posição radical perante paraconsistência, a saber: a alegação de que há contradições verdadeiras. O artigo termina com algumas críticas possíveis às lógicas paraconsistentes – bem como suas refutações – e apresenta algumas questões abertas para pesquisas futuras.

Palavras-chave: Lógica. Paraconsistência. Explosão. Lei de não-contradição. Dialetheismo

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Introduction

The aim of this study is to discuss some philosophical aspects referred to the so called paraconsistent logics. Since they are non-classical logics one must first characterize precisely what is to be understood as logic throughout this paper. Well,

if one is to compare different logical systems they must be certain that they are in fact dealing with logical systems. One may obviously not identify logical system with the formulation of the classical logic – like many did and some still insist on doing – since any discussion on different logic would sum up in the following possibilities: either it is about a different formulation of the classical logic, but with no substantial changes¹, or it is not at all about a logical system. In being so, one must have a definition which comprehends the different logic and that, nevertheless, is rigorous.

We will start from an abstract idea of logic. *Logic* will be understood as any structure which has a set of sentences and a consequence relation. Formally, a *logic* L is a structure $L = \langle F_L, \Rightarrow_L \rangle$ such as:

- (i) F_L is a non-empty set, whose elements are called *formulas* of L ;
- (ii) \Rightarrow_L is a relation in $\mathcal{P}(F_L) \times F_L$ called *consequence relation* of L .

Intuitively the consequence relation points out, starting from a set of sentences, which sentences form the set of consequence from the original set. So, any formal system which fulfills the requirements above is a logic. Note that this is a very comprehensive version of logic, that is, we have no restrictions to the consequence operators. From this definition, objections such as those by Quine, who states that changing the logic is changing the subject, become empty. Logicians discussing different logical systems would be the equivalent to physicists discussing different conceptions of force.

Before we move on let's retake the classical definition of validity. An argument is *valid* if and only if there is no situation in which its premises are true and their conclusion is false. In the classical logic this definition has a particular consequence. Let us take the following example, named *Lewis' independent argument*:

It rains today	
It rains today or there is life in Mars	It doesn't rain today
So, there is life in Mars	

This argument uses two principles accepted by the classical logic: Addition ($\alpha \Rightarrow \alpha \vee \beta$) and Disjunctive Syllogism ($(\alpha \vee \beta, \neg\alpha) \Rightarrow \beta$). Even if the premises have no relevance for the conclusion – that is, they refer to totally different subjects – since there is no situation in which the premises are true and the conclusion is false, this is a valid argument, even if counter intuitive. Said in a different way, in the classical logic, starting from contradictory premises, one may deduce any conclusion. This property was named in Latin *ex contradictione quodlibet*, which means 'from a contradiction, everything follows'. In the contemporary logic it is called *explosion*, and the logic which has this property is considered explosive.

Paraconsistent Logics

Paraconsistent logics are thus those which refute explosion, that is, $(\alpha, \neg\alpha \not\Rightarrow \beta)$. That means that from contradictory premises it is possible to make inferences

¹ Such as, for instance, start from axioms or different inference rules. Both formulations result in the same theorems and valid arguments.

without, however, deducing anything. This system is in fact characterized as logic – according to our initial definition – since paraconsistency is a property of consequence relation. Nevertheless, this definition is not at all without problems. Take for example the minimal logic. Even if it is paraconsistent, the inference $(\alpha, \neg\alpha \Rightarrow \neg\beta)$ is valid. That means that from contradictory premises it is possible to conclude the negation of any sentence, which seems to go against the paraconsistent motivation. Other formulations of paraconsistency were offered, but since all pose some kind of issue we will stick to the before mentioned definition, for its simplicity and elegance.

Note that when defining paraconsistency, the expression *paraconsistent logics* is being used in the plural. That means that there is more than one logical system which refutes the principle of explosion. In the present paper no preference will be given to any system in particular. The discussion will go over the questions related to paraconsistency in general. Also, no emphasis will be given to the formal aspect of such systems, i.e., it won't be discussed how each paraconsistent logic deals with contradictions and which properties result from it.

Let us move back to the definition of paraconsistent logic. At a first glance it may seem that rejecting the explosion would imply the rejection of the law of non-contradiction, which asserts that from two contradictory sentences one must be false². In despite of its possible different formulations – and not exactly equivalent ones – this law and the idea of explosion are fundamentally distinct. The formulation – and acceptance of the law of non-contradiction – dates back to Aristotle. The arguments that favor such law can be found at *Metaphysics*. However, the characterization and validity of such arguments are highly dubious. In being so, it is noteworthy that since then there is no significant work which properly justifies the law of non-contradiction.

Just as a matter of curiosity, I quote Horn, Laurence R, in what concerns to the defense of this law by Avicenna:

As for the obstinate, he must be plunged into fire, since fire and non-fire are identical. Let him be beaten, since suffering and not suffering are the same. Let him be deprived of food and drink, since eating and drinking are identical to abstaining³.

Even though, since then, the law of non-contradiction has been accepted as the fundamental law of logic. The acceptance of the explosion as a logical principle, on its turn, is a recent idea in the Logic. Aristotle's syllogism, for example, is non explosive, it is paraconsistent. We just have to see that the following argument is not valid, even with contradictory premises.

No planet is a red object

Some red object is a planet

So, all red objects are red objects⁴.

The first propositional logic was developed by the stoics. This is not explosive as well. Even if the Disjunctive Syllogism were accepted, the addition, in general,

² Formally, $\neg(\alpha \ \& \ \neg\alpha)$.

³ HORN, L. R. Contradiction. In: *The Stanford Encyclopedia of Philosophy (Winter 2010 Edition)*.

⁴ By the rules of syllogism, no affirmative conclusion follows from an argument with at least one negative premise.

would not be valid, since this was just accepted when the sentences involved were connected by the context.

In the Middle Age, Lewis' argument was developed and widely discussed by the logicians. It is supposed to have been invented by the French William de Soissons in the 12th century. It was defended by Scotus and Buridan, and rejected by the Cologne School in the end of the 15th century, which refuted the Disjunctive Syllogism. Anyway, in this period the acceptance of the principle of explosion was not established.

The explosion has just turned a considerably hegemonically accepted property by the end of the 19th century, with the modern logic's formal development, mainly headed by Boole, Frege and Russell.

Boole formulated the classical treatment of negation, nowadays named *Boolean negation*. This accepts that $\neg\alpha$ is true only in situations in which α is false. It was also in this period that the classical notion of validity was established, which determines that an argument is *valid* if and only if there is no situation in which its premises are true and their conclusion is false. In addition to that, Frege and Russell defined the truth-functional treatment of logic connectives. None of these ideas are, *per se*, explosive ones. However, taken together they lead to explosion. That is, if the Boolean negation is truth-function, or α , or $\neg\alpha$ is valid in each situation. Given the classical definition of validity, it follows that there is no situation in which α and $\neg\alpha$ is true, and β is false. As a consequence, although counter-intuitive, $(\alpha \& \neg\alpha) \Rightarrow \beta$.

There is a curious anecdote about Russell in this respect⁵. When telling a colleague that from contradictory premises all can be deduced, he asked Russell to prove, from the fact that $2=3$, that Russell was the Pope. To that, Russell replied: 'Let's subtract 1 from the two sides of the equation, we have $1=2$. The Pope and I are two people. Since $1=2$, the Pope and I are one person. Therefore, I'm the Pope'.

To sum up, the principle of explosion is independent of the law of non-contradiction. It is possible to refute it, and however, keep the logic law. In addition to that, this law is not sacred either, and this way, there are logics which refute it.

From a formal point of view it is simple to construct a paraconsistent logic. One must, for example just take the classical logic without – however – suppose that true and false are exclusive, i.e., without rejecting the possibility that a single sentence is simultaneously true and false. Keeping the classical definition of validity, this paraconsistent system refutes Lewis' argument, but it keeps the law of non-contradiction as a logical truth. This system is called LP, *logic of paradox*, and was coined by Graham Priest⁶.

The discussion turns philosophically interesting when it is taken into account which the reasons are to use – or defend – a paraconsistent logic. And these motivations are present in the name of this logic itself.

The prefix 'para' in English has two meanings: 'quasi' (or 'similar to, modeled on') or 'beyond'. When the term 'paraconsistent' was coined by Miró Quesada [...] in 1976, he seems to have had the first

⁵ Cf. PRIEST, G. Paraconsistency and Dialetheism, Paraconsistency and Dialetheism. In: GABBAY, D., WOODS, J. (eds.). *Handbook of the History of Logic: The Many Valued and Nonmonotonic Turn in Logic*, Vol. 8. Oxford: Elsevier, 2007, p. 132.

⁶ PRIEST, G. The logic of paradox. In: *Journal of Philosophical Logic* 1979, 8, pp. 219-241.

meaning in mind. Many paraconsistent logicians, however, have taken it to mean the second⁷.

There are several occurrences in which the use of paraconsistent logic is desirable. One must just have a theory – or set of inconsistent information – and need – or wish – to make non trivial inferences from this set. It commonly takes place at data bases in a computer system.

We also have scientific theories which are notably inconsistent. Bohr's atomic theory predicts that there are electrons traveling around the nucleus of an atom without radiating energy. However, according to Maxwell's equations – which are explicitly incorporated in Bohr's theory – a traveling electron must radiate energy. It is clear that it is an inconsistent theory which, even though, does not let us infer that electrons are colorful marbles (which would be possible in case the logic to be used was the classical logic, and, thus, explosive). As a consequence this theory demands the use of a paraconsistent logic.

Similarly, the Newtonian mechanics is inconsistent with the observational data of the Mercury perihelion.

One can also find examples in the history of Mathematics. The first formulations of the Newtonian and Leibniz's calculus were inconsistent – and this was then recognized, like criticisms from Berkeley to these theories show. When calculating derivatives, the infinitesimals were sometimes considered zero and some other times non-zero⁸.

However, it is imperative to notice that in none of those cases the contradictions are interpreted as essential ones. In other words, theoretically it is possible to exclude them from the data base; just like reviewing or refusing inconsistent scientific theories. Nevertheless, it is not always clear how to exclude or remodel the inconsistent information. Besides, since the consistency is not decidable in complex theories, we cannot even be certain of having achieved the aimed goal. Thus, in such cases, the use of paraconsistent logic is highly desirable.

In sum, a paraconsistent logician may use this type of logic for practical reasons, lack of better options, or even the intrinsic interest of some scientific theories considered false, without considering true the inconsistent information. From a formal-semantic point of view, briefly discussed above it is not at all necessary to defend that a sentence may – really – be true and false. One just has to consider interpretations that take this bivaluation. These can be understood, in the last instance, as impossible situations. In such cases the motivation is related to the first sense of the word *paraconsistency*, that is, quasi consistent.

Dialetheism

Moreover, there are logicians that defend that there are true contradictions, which are called *dialetheia*. This positioning is known as *dialetheism*. Such terms were coined by Priest and Routley.

⁷ PRIEST, G.; TANAKA, K. Paraconsistent Logic. In: *The Stanford Encyclopedia of Philosophy (Summer 2009 Edition)*.

⁸ For a comprehensive analysis of inconsistent mathematical theories, cf. MORTENSEN, C. *Inconsistent Mathematics*. Dordrecht: Kluwer Academic Publishers, 1995.

The inspiration for the name was a passage in Wittgenstein's *Remarks on the Foundations of Mathematics*, where he describes the Liar sentence [...] as a Janus-headed figure facing both truth and falsity (1978, IV.59). Hence a dialetheia is a two (-way) truth⁹.

Let's see precisely what a *dialetheia* means. Asserting that there are true contradictions means essentially defending that there are sentences α , $\neg\alpha$, so that both are true (and false). From the conjunction, we have $(\alpha \ \& \ \neg\alpha)$ as true. If a sentence α had only one truth value, its negation would also have a single value, and, by the conjunction rule, the sentence $(\alpha \ \& \ \neg\alpha)$ could not be true.

It is imperative to realize that the dialetheism is different from trivialism. The first one states that some contradictions are true, while the trivialist defends that all contradictions are true, or, similarly, that everything is true. Great part of the criticism to dialetheism incurs in this lack of distinction.

Obviously, if a logician defends the dialetheism, he will make use of some kind of paraconsistent logic; the inverse, on the other hand, is not necessary. Thus, the dialetheism bases on the second meaning of paraconsistency, i.e., it will go beyond the consistency.

Again, things get interesting when we analyze the motivations for dialetheism. There are some situations which would tell the existence of dialetheias. Such occurrences may be divided – in general – in two categories: real and abstract.

The first case would deal with the existence of true contradictions in the real world. For instance, when sitting, there is an exact moment in which one is sitting and standing, that is, not sitting. Or yet, when entering a room, at some point one is inside and outside the room.

The second category, and on which we'll focus, is in what concerns the true but abstract contradictions. The most common cases are the self-reference paradoxes. Take M as the name of the following sentence: 'Sentence M is false'. Is M true or false? If it is true, then what it says is the case. But since it says that it is itself false, M is false. On the other hand, if M is false, so what it says is not the case. Then, M is true. That is, if M is true, it is false; if M is false, it is true. This paradox is known as the Liar Paradox, attributed to Eubulides in the 4th century BC. In other formulations it is possible to create a similar paradox without using self-referring sentences. Imagine a card with the following sentence: "The sentence on the other side of this card is false". When we turn the card, there is another sentence saying: "The sentence on the other side is true". Following the same previous reasoning one concludes that each sentence is true if and only if it is false. This is the 'post-card paradox'.

Let us move now to the last paradox. Some sets are elements of themselves, others aren't. So, the set of abstract ideas is element of itself, while the set of marbles isn't. So far, so good. Let us take now the set of the sets which are not elements of it. Call it R . Does R belong to this set or not? If R is an element of itself, it doesn't belong to this set, since its elements are not elements of itself. On the other hand, if R is not an element of itself, it fulfills the requirements to belong to R , i.e., not being an element of it. So, if R belongs to R , R does not belong to R , and if R doesn't

⁹ PRIEST, G.; BERTO, F. Dialetheism. In: *The Stanford Encyclopedia of Philosophy* (Summer 2010 Edition).

belong to R it belongs then to R . This paradox was formulated by Russell, and this way, it receives his name.

Some authors divide these paradoxes in two categories: paradoxes of the set theory and semantic ones. Apart from what concerns their differences, we will focus on their similarities.

All these cases start from apparently evident principles, and by means of valid arguments, they reach contradictions. What is wrong with them? This question has been made throughout the history, and several solutions were proposed.

Among the most famous solutions is Tarski's theory of truth. It seems to exclude the possibility of self-reference paradoxes – in what concerns the truth predicate –, distinguishing object language from metalanguage, and forbidding the use of the predicate *true* in the object language. However, this solution implies a complex – and artificial – infinite language hierarchy, in which each language has the true predicate for the inferior members of this hierarchy. This way the Liar sentence cannot be formed in any language.

There is a series of possible criticism to this theory. Here, only three will be briefly mentioned. To start with, this theory seems to be subject to what is called *extended paradoxes*, i.e., similar reformulations of the Liar's paradox¹⁰. Secondly, the Liar sentence may be restructured in any natural language, such as the Portuguese, for example. Since Tarski's conception of truth is limited to the formal languages, what is to say about the paradox in semantically closed languages? Thirdly, a criticism that is more fundamental than the previous ones: What is the justification to differentiate object language and metalanguage, and impose a hierarchical restriction to the truth predicate? To a great extent, it is to avoid self-references paradoxes. And what is the justification for that? Well, self-reference paradoxes lead to contradictions. That is to say that, when arguing the impossibility of true contradictions, one starts exactly from this impossibility. As a consequence, it is a *petitio principii*, that is, what wants to be proved is presupposed.

Most of the proposed solutions for the paradoxes pose such flaws: they do not offer independent justifications for accepting or objecting the semantic principles involved, and mainly, they seem to be subject to paradoxes of some kind. According to the dialetheism, this situation shows that it is much simpler to just accept that the Liar sentence is true of false, that is, that it is a dialetheia.

Final remarks

To conclude, let us now analyze briefly some objections to the paraconsistency and the dialetheism, as well as their respective replies. From what has been said, it follows that a rejection to the paraconsistency implies in a rejection to the dialetheism. However, the dialetheism can be rejected, and even though, the idea of paraconsistency can be defended.

- i. Contradictions cannot be true, since they would imply everything.

In the context of the paraconsistent logic this argument makes no sense. Refuting the explosion is a syntactic/semantic possibility, and, above all, it is a metaphysical possibility. Its acceptance cannot be merely supposed.

¹⁰ Cf. PRIEST, G. Paraconsistency and Dialetheism, *ibidem*, p. 173.

- ii. Contradictions have no meaning. So, imposing interpretations in which contradictions are true makes no sense, nor does accepting that they are true.

This objection has a specific interest, since, in case it is taken to their last consequences, it wouldn't be a simple criticism to dialetheism, but to the classical logic in itself. There the contradictions have a meaning, they imply everything. In addition to that, if contradictions had no meaning, we would not be able to understand it, when somebody utter them, nor judge falsity – or truth – from what the person uttered.

- iii. The possibility of interpretations in which contradictions are true is excluded by the classical treatment of negation, which is correct.

That is an objection which points an essential aspect to paraconsistency, that is: the concept of negation. To start with, it is easy to show that the classic negation doesn't explain the behavior of the particle 'not' in the Portuguese language. Adding 'not' to the sentence doesn't necessarily mean negating it. Take as an example the sentence 'Every elephant is white'. Its negation is not 'Every elephant is not white', but 'Some elephant is not white'. In being so, what would the negation be? In the previous example, one may realize that the negation is a relation between contradictories. The core here is that there are several possible explanations to this relation. As we've seen, the classic treatment to the negation was established by Boole in the 19th century. There are other possible treatments, such as the intuitionist negation, for example. Similarly, there are different paraconsistent negation theories. In addition to that, what is the criterion – if there is one – to define the correct notion of negation? In sum, essential is to realize that the classic treatment to negation is one of the theories among other possible ones.

- iv. Another important objection refers to the notion of rationality, and asserts that the rationality is closed under entailment.

Let us picture the following situation: somebody writes a non-fiction book about some topic. Each information α is established in the most empirical way possible. This way the author believes in all information in his book, that is, he rationally believes in $(\alpha_0 \ \& \ \alpha_1 \ \& \ \alpha_2 \ \& \ \dots \ \& \ \alpha_n)$. Regardless of that, the author is also aware that all books written pose some kind of incorrect information, i.e., false. So, he also believes that $\neg(\alpha_0 \ \& \ \alpha_1 \ \& \ \alpha_2 \ \& \ \dots \ \& \ \alpha_n)$. Logically, that would imply a contradiction, but the author doesn't believe in $(\alpha_0 \ \& \ \alpha_1 \ \& \ \alpha_2 \ \& \ \dots \ \& \ \alpha_n) \ \& \ \neg(\alpha_0 \ \& \ \alpha_1 \ \& \ \alpha_2 \ \& \ \dots \ \& \ \alpha_n)$. This story is known as the *preface paradox*. Although this paradox shows that the rational belief is not closed under entailment, it could seem that it would indicate that one may not rationally believe in contradictions, no matter if they are true or not. But this is also a false objection. Consistency is, in fact, a criterion to evaluate a rational belief, but it is not the only one. Let's take, for example, the field of science. Accepting a theory is not limited to the consistency criterion. Other aspects, such as application, proximity to empirical data, lack of *ad hoc* hypotheses or even elegance are also taken into account when one accepts – or refuses – a scientific theory. So, a theory can be inconsistent and anyhow be adopted by the scientific community, like seen previously.

Among all the previous objections, none of them offer a satisfying justification to reject the dialetheism. Apparently all incur in *petitio principii*. However, would that be reason enough to accept the existence of *dialetheias*? Moreover, there are other possible criticisms to the dialetheism.

Let's see an alternative formulation to the Liar's paradox. Take M as the sentence: 'Sentence M is not-true'. Following the same reasoning discussed above, we would have that M is true and not-true. Asserting that true and not-true are exclusive is something totally different from asserting that truth and falsity are not. But both are consequences of dialetheism. Would that be a desirable consequence?

The dialetheism seems to be subject to other forms of paradox. Consider the sentence P : 'Sentence P is not assertable'. We will once again that P is assertable and not assertable. How can we handle this problem, or then, how to defend that this is not a problem?

A possible way is to accept, like Priest did, that the dialetheism is a dialetheia in itself, that is, a true contradiction. That leads us to a capital question: Even if the dialetheism is false – and only false – how can it be proved? Showing that the dialetheism lead to contradictions can obviously not do the work, since a dialetheist does not only agree with that, but it accepts that it is essential. So, how to proceed then?

Such questions are still unanswered. In despite of the final verdict of dialetheism, its development and mainly the overall development of paraconsistent logic has allowed a new and deep debate about the central notions of logic. In addition to that, establishing the logical and metaphysical possibility of non-explosive theories have allowed as well the study of paraconsistent mathematical theories, such as the *naïve relevant set theory*, which accepts the *principle of comprehension* – which leads to the paradox of Russell.

Finally, the main philosophical contribution of paraconsistent logics is to question one of the deepest beliefs of philosophy, namely: consistency is a supposition of any knowledge form, that is, truth supposed consistency. In being so,

[...] the philosophical result [of the paraconsistent logic] may be the overthrow of another Aristotelian doctrine: that the truth is exhausted by the domain of the consistent¹¹.

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¹¹ PRIEST, G. Paraconsistent Logic. In: GABBAY, D., GUENTHNER, F. (eds.), *Handbook of philosophical logic*, vol. 6. Dordrecht: Kluwer Academic Publishers, 2002, pp.387-8.

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