On a Pragmatic Theory of Meaning and Knowledge

Sobre uma Teoria Pragmática da Significação e do Conhecimento

Risto Hilpinen
University of Miami, USA
hilpinen@miami.edu

Abstract: According to C. S. Peirce, there are two ways of explaining what a sign (an expression or a concept) means, namely, a *definition* and a *precept*. A precept tells the interpreters of a sign what the sign means by prescribing what they have to do in order to find or become acquainted with an object of the sign. A precept for a concept specifies how an interpreter can determine whether the concept is applicable to a given situation or object.

Peirce accepted the scholastic definition of truth, according to which a proposition is true if and only if its subject and predicate refer to the same thing, and applied this analysis to complex as well as singular propositions. However, this account does not tell how an interpreter can become acquainted with the objects of the predicate "true," that is, true propositions: it is not a good precept for the concept of truth. On the other hand, the so-called pragmatic conception of truth, truth as the limit or end of inquiry, can be regarded as a precept for truth, or as a general form of such a precept.

The requirement that concepts should have precepts attached to them is a version of Peirce's principle of pragmatism. (Concepts without precepts are meaningless.) The availability of precepts should make it possible for an interpreter (inquirer) to determine whether a concept is applicable to a given situation, or whether a given proposition is true. Thus the principle of pragmatism is closely related to the principle knowability, according to which any truth should be knowable. Some formulations of the principle of knowability lead to a paradox. The paper discusses several forms of the principle of knowability, and it is argued that the existence of precept for a proposition entails only a relatively weak form of the principle.

Keywords: Action. Inquiry. Knowledge. Meaning. Peirce. Pragmatism. Precept.

Resumo: *Segundo C. S. Peirce, há dois modos de explicar o que um signo (uma expressão ou um conceito) significa, a saber, uma* definição *e um* preceito*. Um preceito diz aos intérpretes de um signo o que o signo significa, prescrevendo o que eles devem fazer para encontrar ou inteirar-se de um objeto do signo. Um preceito para um conceito especifica como um intérprete pode determinar se o conceito é aplicável a uma dada situação ou a um dado objeto.*

Peirce aceitou a definição escolástica de verdade, segundo a qual uma pro-posição é verdadeira se, e somente se, seu sujeito e seu predicado se referirem à mesma coisa, e aplicou essa análise tanto a proposições complexas quanto a singulares. Entretanto, essa visão não informa como um intérprete pode inteirar-se dos objetos do predicado "verdadeiro", ou seja, proposições verda-deiras: não é um bom preceito para o conceito de verdade. De outro lado, a

assim chamada concepção pragmática de verdade, a verdade como o limite ou fim da investigação, pode ser vista como um preceito para a verdade, ou como uma forma geral de tal preceito.

A exigência de que conceitos tenham preceitos ligados a eles é uma versão do princípio do pragmatismo de Peirce (conceitos sem preceitos são vazios de sentido). A disponibilidade de preceitos deve tornar possível que um intérprete (investigador) determine se um conceito é aplicável a uma dada situação, ou seja, se certa proposição é verdadeira. Portanto, o princípio do pragmatismo está intimamente ligado ao princípio da cognoscibilidade, segundo o qual toda verdade pode ser conhecida. Algumas formulações do princípio da cognoscibilidade levam a um paradoxo. O artigo discute algumas formas não-paradoxais do princípio da cognoscibilidade, defendendo que a existência de um preceito para uma proposição exige apenas uma forma relativamente fraca do princípio.

Palavras-chave: *Ação. Investigação. Conhecimento. Significação. Peirce. Pragmatismo. Preceito.*

I

The concept of *precept* plays a central role in C. S. Peirce's pragmatic theory of meaning and knowledge. The word "precept" means a maxim or a rule of conduct, and a precept in Peirce's sense can be regarded as a prescription for interpreting a sign. In his manuscript *On some topics of logic* (1903), Peirce observes that there are two different ways of explaining what a sign (an expression or a concept) means, namely, a *definition* and a *precept*.[1] He uses "lithium" as an example:

> If you look into a textbook of chemistry for a definition if *lithium*, you may be told that it is that element whose atomic weight is 7 very nearly. But if the author has a more logical mind he will tell you that if you search among minerals that are vitreous, translucent, grey or white, very hard, brittle, and insoluble, for one which imparts a crimson tinge to an unluminous flame, this mineral being triturated with lime or witherite rats-bane, and then fused, can be partly dissolved in muriatic acid; and if this solution be evaporated, and the residue be extracted with sulphuric acid, and duly purified, it can be converted by ordinary methods into a chloride, which being obtained in the solid state, fused, and electrolyzed with half a dozen powerful cells, will yield a globule of a pinkish silvery metal that will float on gasolene; and the material of *that* is a specimen of lithium. (CP 2.330)

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[1] I shall follow the usual practice of citing from the *Collected papers of Charles Sanders Peirce* (PEIRCE 1931-35, 1958) by volume number and paragraph number, preceded by "CP". *The new elements of mathematics* (PEIRCE, 1976) will be abbreviated "NE", followed by volume number, and the chronological edition of Peirce's writings (PEIRCE, 1982-2000) by "WCSP", followed by volume number. References to the microfilm edition of Peirce's papers (Widener Library, Harvard University) will be indicated by "MS", followed by the manuscript number. The page numbers used in the references to the manuscripts are those used by Peirce (and contained in the microfilm edition of the manuscripts).

Peirce characterizes this precept as follows:

> The peculiarity of this definition – or rather this precept that is more serviceable than a definition – is that it tells you what the word lithium denotes by prescribing what you are to *do* in order to gain perceptual acquaintance with the object of the word. (Ibid.)

The word "lithium" denotes samples of lithium, and a precept for lithium tells an inquirer what he ought to do in order to find (or become acquainted with) such samples. A precept of this kind is not unlike a cookbook recipe for making, for example, key lime pie. The precept (or recipe) enables its interpreter to become acquainted with an object of the expression "key lime pie" (what the expression denotes) in the most direct and intimate way: the interpreter can eat an object of the sign (a slice of pie) after having prepared it. A sign denotes its objects; thus the expression "key lime pie" denotes all key lime pies or slices of key lime pie, in other words, servings of key lime pie can be regarded as objects of the expression "key lime pie", just as samples of lithium can be regarded as objects of the sign "lithium". The main difference between these two examples is that a precept for key lime pie is a precept for making objects of the sign, whereas a precept for lithium enables an inquirer to extract samples from lithium from certain minerals and thus make them cognitively accessible: key lime pies, unlike instances of lithium, are essentially artifactual objects. In this respect a precept (or recipe) for key lime pie resembles a blueprint of a house or a score of a musical work. A precept for lithium may be called an *epistemic* precept, whereas the score of a musical work or the recipe for key lime pie may be regarded as a *constitutive* precept.

The view that the meaning of a sign can be articulated or spelled out by means of precepts can be regarded as an application of Peirce's *principle of pragmatism*. In his 1905 manuscript entitled *Pragmaticism* (MS 291) Peirce formulated the principle as follows:

(Prg1)   The intellectual purport of a concept consists in the truth of certain conditional propositions asserting that if the concept be applicable, and the utterer of the proposition or his fellow have a certain purpose in view, he would act in a certain way. (CP 5.528)

Peirce's reference to "the utterer [of a proposition] or his fellow" is based on his analysis of speech acts and communication situations as an interaction between two participants, an utterer and an interpreter of a sign (a proposition). In the examples considered above, the purpose of the utterer and the interpreter is to find specimens of lithium or make key lime pie in order to verify some proposition about lithium or key lime pie (for example, that key lime pie is tasty).

II

The actions prescribed by a precept connect a sign to its object or objects. Thus precepts make the semantic connections between signs and their objects accessible to the utterer and the interpreter and establish such connections. Peirce understood the relationship between a sign and its object or objects (the relation of reference) in accordance with the traditional, pre-Fregean theory of reference: the object of a singular term, for example,

a proper name, is the individual denoted by the name, whereas a general term "F" refers to each F; for example, every horse is an object of the term "horse". In this respect Peirce's concept of an object of a sign differs from Frege's concept of *Bedeutung* (reference): according to Frege, concept-words refer to (have as their *Bedeutung*) concepts, not to the objects falling under the concepts (cf. FREGE, 1891/1997b, p.149). Peirce also accepted the traditional view that any proposition can be analyzed as consisting of a subject (or subjects) and a predicate. However, his interpretation of this doctrine differs from the traditional view, according to which a proposition is a combination of two terms or concepts, a subject-term and a predicate-term. Peirce defines the subject of a proposition as follows: "A proposition is a symbol which separately *indicates* its object, and the representation in the proposition of that object is called the subject of the proposition" (PEIRCE, 1903/97, p. 176; cf. CP 5.139, NE 4, p. 242).

A proposition can indicate more than one object and can be regarded as having more than one subject. For example, the proposition

(1)   Alicia loves Bernardo

has Alicia and Bernardo as its objects, and the two proper names are the subjects of the proposition (or the two names together can be regarded as constituting the subject). More generally, Peirce means by a *logical subject* of a proposition "every part of a proposition which might be replaced by a proper name, and still leave the proposition a proposition" (CP 4.438). For example, in a quantified proposition

(2)   Some woman loves Bernardo,

the substitution of "Alicia" for "some woman" produces proposition (1); thus "some woman" and "Bernardo" can be regarded as the subjects of (2), and the predicate is "loves", or more accurately expressed, "__ loves __"; the horizontal lines in the latter expression represent the empty places or blanks attached to the predicate. Like Frege, Peirce characterized the predicate of a proposition as an incomplete or "unsaturated" expression, "a blank form of proposition produced by such erasures as can be filled, each with a proper name, to make a proposition again" (CP 3.420-421, 4.438; cf. FREGE, 1891/1997a, p. 133). (2) is equivalent to

(3)   Someone is a woman who loves Bernardo.

In (3), the substitution of "Alicia" for "someone" yields

(4)   Alicia is a woman who loves Bernardo.

According to this analysis, the subjects of (3) are "someone" and "Bernardo", and the predicate is "__ is a woman who loves __". Thus the subjects of a quantified sentence include the quantifiers or quantifier phrases that occur in the sentence. According to Peirce, a sentence can be divided into the subjects and the predicate in different ways (cf. PEIRCE, 1903/1997, pp. 180-81). In general, the function of the subject or subjects of a proposition is to indicate its object or objects (what the proposition is about), and the predicate, when attached to the subjects, conveys information (states something) about the objects (cf. CP 2.357-358; HILPINEN, 1992, pp. 472-78; 1998, pp. 155-59).

III

In Peirce's classification of signs, the subject of a proposition is either an index or an indexical symbol, and the predicate is an icon or an iconic symbol. A sign is an icon of its object insofar as it resembles the object (in some respect), and a sign is an index of an object by virtue of some existential relation to the object, for example, contiguity or some causal connection. It seems clear that any two objects resemble each other in some respect; thus the concept of iconicity becomes empty unless it is relativized some *ground* of representation, that is, some specific feature of the sign and its objects (cf. CP 1.551, 2.228). The concept of indexicality must be relativized in an analogous way. Iconic symbols are conventional signs which function like icons and whose meaning can be explained by means of iconic signs or (to use Peirce's terminology) "interpretants": iconic symbols are "apt to determine iconic interpretants" (NE 4, p. 243). In the same way, indexical symbols, for example, personal and demonstrative pronouns and proper names, are symbols (conventional signs) which "act very much like indices" (ibid.).

In a generalized sense, a sign can be regarded as an icon (iconic sign) if its applicability to an object depends on some qualitative features of the sign and the object, and the functioning of the sign as a sign is independent of the existence of its object: "An icon is a representamen [a sign] which fulfills the function of a representamen by virtue of a character which it possesses in itself, and would possess just the same though its object did not exist" (PEIRCE, 1903/1997, p. 170). On the other hand, since indexical signs refer to their objects by virtue of some existential (for example, causal) connection, a genuine index is "a representamen [a sign] which fulfills the function of a representamen by virtue of a character it could not have if the object did not exist" (PEIRCE, 1903/1997, p. 170). One significant difference between icons and indices is that since iconicity depends on shared characters, it is meaningful to say that an object *fits* or does not fit an iconic sign, but the concept of fit is applicable to an indexical sign only indirectly, on the basis of some icons attached to the index which help to identify its object(s).

A proposition gives information about the object indicated by its subject by representing its predicate as an icon (iconic symbol) of the object. Peirce gives the following analysis of propositional (and sentential) signs (NE, vol. 4, p. 242):

> It is remarkable that while neither a pure icon nor a pure index can assert anything, an index which forces something to be an icon, as a weather-cock does, or which forces us to regard it as an icon (of something), as the legend under a portrait does, does make an assertion, and forms a proposition.

IV

Peirce accepted the scholastic account of truth, according to which a proposition is true if and only if the subject and the predicate refer to the same thing (have the same object). For example, according to William Ockham, a singular proposition is true if and only its "subject and predicate supposit for the same thing" (WILLIAM OCKHAM, 1349/1980, pp. 8, 86-87; KAUFMANN, 1994, p. 177). Medieval logicians used the word

"supposition" for reference to an object; thus Ockham's definition is equivalent to saying that a proposition is true if and only if its subject and predicate are signs of the same object (or objects). Peirce defines the concept of truth in the same way:

(AT1)   A proposition has a subject (or a set of subjects) and a predicate. The subject is a sign; the predicate is a sign; and a proposition is a sign that the predicate is a sign of that of which the subject is a sign. If it be so, it is true. (CP 5.553)

It is clear that the conditional in the last sentence of (AT1) can be strengthened to an equivalence. Peirce also observes (CP 2.437): "Every assertion is an assertion that two different signs have the same object." Here the word "assertion" should be taken to mean what is asserted, that is, a proposition. An assertion is true if and only if the two signs, which make up the asserted proposition (the subject and the predicate), have the same object. This account of truth can also be formulated as follows:

(AT2)   A proposition is true if and only if its predicate is an iconic sign of the object of its subject, that is, if the object fits the predicate or the predicate conforms to the object,

or more briefly:

(AT2*)   A proposition is true if and only if its predicate and subject have (refer to) the same object.

(AT1) and (AT2) can be applied to a proposition with several subjects (a relational proposition) by regarding the ordered set of its subjects as *a* single "collective" subject (CP 3.467) and by taking the object of the proposition to be an ordered set (n-tuple) of individual objects. If the "conformity of a representamen to its object" (CP 5.553) is called (a form of) "correspondence", Peirce's neo-scholastic account of truth can be regarded as a version of the so-called correspondence theory.

V

Peirce applied the scholastic formula for truth to logically complex as well as singular propositions. This was made possible by his view that logical constants – propositional connectives, quantifiers, and modal expressions – are part of the subject of a proposition and function as precepts which tell how the utterer and the interpreter can find a singular object or singular objects which are objects of the predicate if the proposition is true. Here the precepts assume the role of semantic game rules for logical expressions. The function of the subject of a proposition is to indicate its object, but the subject need not be an index in the strict sense of the word:

> It does not follow that the subject of a proposition must literally be an index, although it indicates the subject of a representamen in a manner like the mode of representation of an index. It may be a precept by following which a singular could be found. Take for example the proposition
>
> Some woman is adored by every Catholic.

This means that a well-disposed person with sufficient means [i.e., the utterer] could find an index whose object should be a woman such that allowing an ill-disposed person [the interpreter] to select an index whose object should be Catholic, that Catholic would adore that woman.

Thus the subject of a proposition if not an index is a precept prescribing the conditions under which an index is to be had. (PEIRCE, 1903/1997, p. 176)

Logical constants tell the utterer and the interpreter how they can find an object of the statement in which the constants are used. For example, an existential quantifier gives the utterer of a proposition the right to choose from the universe of discourse (domain of objects) an individual which can be regarded as an object of the proposition, whereas a universal quantifier transfers this choice to the interpreter of the proposition. In each case, the proposition can be regarded as being made about the individual selected by the utterer or the interpreter. In the same way, a disjunctive proposition gives the utterer the right to choose one of the disjuncts, and a conjunction sign signifies the interpreter's choice of a conjunct. In the latter case, the utterer's original statement is false if it is possible for the interpreter to choose a conjunct which can subsequently be shown to be false. In a semantical game of this kind, the utterer's objective is to defend the sentence and find an interpretation (a singular proposition) which is true, whereas the interpreter is the opponent of the proposition, and tries to find an interpretation which falsifies the proposition. A proposition is true if and only if the utterer has a winning strategy in the game associated with the proposition, that is, can reach a true atomic proposition as the outcome of the game regardless of the choices made by the interpreter. Peirce explained the opposition between the interests of the two players by the use of a sentence in an assertive speech act (cf. HILPINEN, 1995, pp. 273-75, 292-95). In an assertive speech act, the utterer of a proposition "assumes responsibility" for its truth and is assumed to suffer some untoward consequences if the sentence turns out to be false, and the hearer or the interpreter will suffer the negative effects of the acceptance of false proposition unless he detects its falsity. Thus the utterer is interested in finding an interpretation which makes the sentence true, and the falsifier must look for the interpretations which might falsify it. Peirce's semantic games are assertion games.

Considered as precepts, logical constants (quantifiers, propositional connectives, and modal expressions) differ from the epistemic precepts and the constitutive precepts discussed earlier. Logical constants are syncategorematic expressions which do not have objects of their own; the precepts for such expressions tell the utterer and the interpreter of a sentence how they should determine which object (or objects) the sentence is to be regarded as representing. Such precepts may be called *semantic* precepts: they express the meaning of logical constants. Semantic precepts provide a "pragmatic interpretation" for logical constants (cf. CP 2.328).

VI

The epistemic precepts associated with non-logical signs tell the utterer and the interpreter how they can find an object of the sign, but they need not express the signification of the sign or its meaning in the customary sense of the word. Proper names serve as a

good example of this. Peirce accepted the traditional view that proper names do not have any "connotation". However, Peirce notes that the object of a proper name can be recognized only by means of marks, that is, on the basis of some distinctive characteristics of the object in question, and the application of a proper name to an individual is guided by some such marks.

> The object of a proper name, say the name of an acquaintance of the interpreter, can only be recognized by him by means of marks; and when he hears the name mentioned, the image excited in his imagination will be composed of marks (so to say;) and any action he may take in consequence will be guided by those marks. Nay more: it may be granted that the name was conferred in the first instance and its use has been maintained ever since with the definite intention that the individual should be recognized in the manner thus described. (MS 283, "assorted pages" numbered 143-144 near the end of the MS; cf. HILPINEN, 1995, pp. 284-85)

The signs which signify the marks or characteristics of an object are iconic signs; any object having these characteristics is an object of such icons. But these icons are not essentially associated with the name: they are not part of its "meaning"; a proper name does not "signify" any iconic signs (predicates of its object):

> Yet it does not follow and could only very rarely be true that the name *signifies* certain defining marks, so as to be applicable to anything that should possess those marks, and to nothing else. For not to speak of the fact that the interpreter only uses the marks as aids in guessing at his acquaintance's identity, and may possibly be mistaken, however extraordinary they may be, there will be no one *definite* set of marks which the name signifies rather than another set of equally conclusive marks. (MS 283, "assorted pages" numbered 144-145 near the end of the MS)

Peirce's use of the expression "signification" is consistent with its customary use in traditional logic: it was used to mean both denotation and connotation (DE JONG, 1982, p. 13). Proper names have no connotation, and do not signify any attributes. However, to be capable of being used as signs, they need precepts. The marks on the basis of which the object of the name is recognized provide such precepts: they help the interpreter to find and identify the object of the name, and this is precisely the function of precepts.

The semantic precepts which determine the meaning of logical expressions must be distinguished from the epistemic precepts which enable an interpreter to be become acquainted with the objects of nonlogical expressions. For example, an interpreter knows, on the basis of the semantic precepts for the universal quantifier and the conditional, that to refute the proposition that all ravens are black, he has to find a raven which is not black. But to be able to determine the truth-value of the general proposition, he must be able to find (become acquainted with) ravens, and for this he needs a precept for the predicate "raven".

VII

The pragmatic maxim (or the principle of pragmatism) can be expressed by means of the concept of precept as follows:

(PP1)   The precepts associated with a sign (an expression, concept or proposition) constitute its pragmatic meaning.

(PP2)   Signs (concepts) without precepts are pragmatically defective (not fully meaningful).

Without precepts, the utterer and the interpreter do not know what to do with a sign; preceptless signs are meaningless.

The definition or analysis of the concept of truth discussed above (that is, [AT2] or [AT2*]) is not a good precept for the concept of truth. A precept for the concept of truth should be a directive which tells how an inquirer can become acquainted with the objects of the predicate "true". According to Peirce, the objects of "true" are true sentences or propositions; thus a precept for truth should give instructions for finding true propositions, or for determining whether a given proposition p is true. The process of determining whether a proposition is true is called an *inquiry*. According to Peirce, the determination of the truth-value of a proposition p consists in reaching a settlement of opinion about the question whether p: the goal of inquiry is the settlement of opinion: "That the settlement of opinion is the sole end of inquiry is a very important proposition" (CP 5.375).

The nature of an inquiry into the truth (or falsity) of a sentence p depends of course on the content of p, but a general schema for a precept for the concept of truth can be formulated as follows:

(PT1)   A sentence p is true if and only if an inquiry into the question whether p is true would eventually lead to a settled (stable) opinion that p.

Truth is independent of the vagaries of individual opinions; here is one of Peirce's formulations of (PT1):

(PT2)   Truth's independence of individual opinions is due (so far as there is any "truth") to its being the predestined result to which sufficient inquiry would ultimately lead. (CP 5.494)

(PT2) suggests the following reformulation of (PT1):

(PT1*)   A sentence p is true if and only if a sufficient inquiry into the question whether p is true would lead to a settled (stable) opinion that p.

(PT1*) is a version of the so-called pragmatic theory truth; thus I want to suggest that the pragmatic theory can be regarded as general schema for a precept for truth, and is by no means incompatible with the scholastic conception or the "correspondence theory" of truth. On the contrary, Peirce's scholastic definition of truth indicates what kind of inquiry is required for different propositions or hypotheses. To determine whether a given proposition is true, it is sufficient to find the object or the objects of a proposition, and then determine whether these objects are objects of the predicate. If both the subject and the predicate are furnished with appropriate precepts, as required by the

principle of pragmatism, then following the precepts should make it possible for the utterer and the interpreter to find out whether the proposition is true or false, and reach a stable opinion about its truth-value. In the case of complex sentences, the required precepts include the semantic precepts for logical constants.

VIII

A stable (or "ultimate") opinion is an opinion which no further inquiry would shake. According to Peirce, such beliefs constitute an inquirer's knowledge. Peirce distinguished *knowing* that *p* from *having sufficient reason to be entirely confident* that *p* as follows:

> The only difference that there seems to be room for between these two is that what one *knows*, one will always have *reason to be confident of*, while what one now has ample reason to be entirely confident of, one may conceivably in the future, in the consequence of new light, find reason to doubt and ultimately to deny. (CP 4.253)

Given this account of the concept of knowledge, principle (PT1*) seems to entail a *pragmatic principle of knowability*:

(PTK1)   If *p* is true, then a sufficient inquiry into the question whether *p* would ultimately make it known that *p*.

If there is a condition *C* such that if *C* holds, a sufficient inquiry into the question whether *p* would make it known that *p*, then *p* can become known under *C*. This can be expressed simply by saying that under *C*, it is possible to know that *p* (in the sense that an inquirer can learn that *p*). Thus**,** principle (PT1*) seems to entail the principle of the knowability of (any) truth,

(TMK1)   If *p* is true, it is possible to know that *p*,

which some logicians and epistemologists have expressed by the modal formula

(TMK2) $p \supset \mathbf{MK}p$,

where "$\mathbf{M}$" is an appropriate possibility operator, and "$\mathbf{K}p$" means that *p* is known or that the inquirer knows that *p* (cf. FITCH, 1963, p. 138, WILLIAMSON, 2000, p. 217).

However, this formulation of the principle of knowability entails the proposition that there is no ignorance, that is, there is no proposition *q* such that $\neg \mathbf{K}q$ (FITCH, 1963, p. 138). The sentence

(5) $(q \,\&\, \neg \mathbf{K}q) \supset \mathbf{MK}(q \,\&\, \neg \mathbf{K}q)$

is a substitution instance of (TMK2). If *q* is a proposition not known to be true, in other words, if the antecedent of (5) is true, then, according to (5), the consequent is also true:

(6) $\mathbf{MK}(q \,\&\, \neg \mathbf{K}q)$.

According to the conjunctive distributivity of the K-operator, (6) entails

(7) $\mathbf{M}(\mathbf{K}q \,\&\, \mathbf{K}\neg\mathbf{K}q)$,

and since only truths can be known, (7) entails

(8) $\mathbf{M}(\mathbf{K}q \& \neg\mathbf{K}q)$,

which is logically false. Thus (TMK2) is inconsistent with the assumption that something is not known. The argument seems to show that all propositions are knowable only if there is no ignorance. This result has been called Fitch's paradox or the paradox of knowability (EDGINGTON, 1985; WILLIAMSON, 1987; LINDSTRÖM, 1997, p. 184). It may be misleading or an exaggeration to call the result a "paradox": it is obvious that one cannot know that q and simultaneously know that one does not know that q, in other words,

(9) $\neg(\mathbf{K}q \& \mathbf{K}\neg\mathbf{K}q)$

is necessarily true. Fitch's result shows that the modal formula (TMK2) is not an adequate representation of the pragmatic principle of knowability.

IX

It is easy to see why (TMK2) is not a satisfactory formalization of (TMK1). In (TMK2), the concept of possibility is construed as a propositional operator, and the consequent of (TMK2) should, strictly speaking, be read:

(10)   It is possible *that* it is known that p.

This is not synonymous with the consequent of (TMK1),

(11)   It is possible *to* know that p,

which can also be expressed as

(12) p *can* be known.

The difference between (10) and (11) is parallel to the difference between the concepts of ought-to-be and ought-to-do in deontic logic (cf. HORTY, 2001, pp. 34-95). The modality in the consequent of (TMK1) is what G. H. von Wright has called a *dynamic* modality, that is, possibility in the sense of *ability* (VON WRIGHT, 1951, p. 54; cf. KENNY, 1976, p. 209). As a formulation of a pragmatic principle of knowability, (TMK1) should be regarded as equivalent to (PTK1) which states that if p is true, an inquirer *can come to know* that p by means of an inquiry. (The expression "inquirer" can be taken to refer here to an individual inquirer or to an unlimited community of inquirers; cf. CP 5.311.) If q is a true proposition which is not known to be true, the pragmatic principle of knowability entails that an inquirer should be able to *come to know* that q and come to know that he *did not* know that q, in other words, an inquirer should be able to learn that he was ignorant about the truth of q. Here the time of an inquirer's ignorance must obviously be different from the time of his knowledge of his ignorance. Coming to know something involves a change in an inquirer's epistemic state; thus a correct formulation of the pragmatic principle of knowability involves reference to at least two epistemic states, the inquirer's original state of ignorance and a state which results from the inquiry.

According to the standard semantics of modal (epistemic) logic, formula (6),

$$\mathbf{MK}(q \,\&\, \neg\mathbf{K}q),$$

may be read:

It is possible that the following is known: q and it is not known that q,

which is evidently inconsistent. If the two occurrences of the K-operator are relativized to different points of time (or different epistemic situations), the inconsistency is avoided. Such a reformulation of (6) can be written as

(13) $\mathbf{MK}(q \,\&\, \neg\mathbf{K}(q)t_1)t_2,$

where "$\mathbf{K}(q)t_j$" means that it is known at time j that q, and q is a temporally definite (complete) proposition. (13) states:

It is possible to know at t_2 that q and that it was not known at t_1 that q,

which is not inconsistent. If the inquirer's knowledge at t_2 is the result of an inquiry into the truth of q undertaken at time t_1, t_1 is temporally prior to t_2. Here is as slightly revised version of an example given by Lindström (1997, p. 193): Suppose that I had on November 1, 2003, an even number of books in my bookcase, but I did not know it then. I counted the books on November 2, 2003, and found out that the number of books was 148. If I knew that the number of books had not changed, I knew on November 3, 2003, that the following proposition is true:

There was an even number of books in my bookcase on November 1, and I did not know it on November 1, 2003.

Counting the books was an action (that is, an inquiry) which changed the situation in which I did not know the number of my books (the situation on November 1) into a situation in which knew that I my bookcase contains an even number of books. The truth of any epistemic proposition (i.e., that an inquirer knows or does not know that some proposition p is true) is relative to an epistemic situation. Lindström solves Fitch's problem by relativizing the concept of truth, especially the truth of modal (including epistemic) propositions, to situations (LINDSTRÖM, 1997, pp. 193-197). A satisfactory formal representation of the pragmatic principle of knowability requires a semantics which can represent *changes* in the world and in the inquirer's epistemic state.

Statements about the knowability of various propositions are also relative to points of time and the inquirer's epistemic states. The statement that an inquirer can at a certain point of time t come to know that p should in the present context be regarded as a statement about the inquirer's epistemic possibilities at t: it means that it is possible for the inquirer to transform his epistemic state at t into a state in which he knows that p. The sentence form "at t, i can know that p", where p is a temporally definite proposition, can be paraphrased as "at t, it is possible for i to come to know (find out) that p": t refers here to the starting-point of an inquiry. If p is an epistemic proposition, it may refer to t or to some other point of time. It is clear that an inquirer's epistemic possibilities change over time. In Lindström's example, I might not be able to find out on January 1, 2004, that I had an even number books on November 1, 2003, if I had not counted them earlier; I may have bought more books or given some of them to my friends.

X

If epistemic possibilities are understood in the way suggested above, it is possible to formulate different versions of the principle of knowability. A weak concept of epistemic possibility,

(MMK1)   At t, it is possible to come to know that p,

where p is a temporally definite proposition, can be defined by means of the following might-conditional:

(MMK2)   If an inquirer were to undertake at t an inquiry into the question whether p, he might come to know that p, that is, reach a state in which he knows that p.

Such a weak possibility can hardly be called an epistemic ability (cf. KENNY, 1976, p. 214). Peirce's principle of knowability involves a strong possibility, expressed by a would-conditional:

(MWK)   If an inquirer were to undertake at t an inquiry into the question whether p, he would come to know that p, that is, reach an epistemic state in which he knows that p.

(For the distinction between might-conditionals and would-conditionals, see LEWIS, 1973, pp. 21-24.) It is clear that (MWK) is too strong: an inquirer might for various reasons fail to complete his work. Thus the pragmatic principle of knowability (PTK1) is expressed by means of the concept of *sufficient* inquiry. According to the analysis of epistemic ability proposed above, it is possible to define different versions of the principle of knowability on the basis of Peirce's principle (PTK1), for example:

(TSK1)   For any true proposition p, there is a time t such that a sufficient inquiry into the question whether p would change the inquirer's epistemic state at t into a state in which he knows that p.

(TSK2)   For any true proposition p and time t, a sufficient inquiry into the question whether p would change the inquirer's epistemic state at t into a state in which he knows that p.

Here the proposition p is assumed to be a temporally definite proposition, and if it is an epistemic proposition, it has to refer to a particular knowledge situation. (TSK1) is weaker and therefore more plausible than (TSK2). However, the expression "sufficient inquiry" used in (PTK1) and (TSK1)-(TSK2) may make the principles appear trivial: a "sufficient inquiry" may be understood as an inquiry which is sufficient for determining the truth-value of a given proposition p. More interesting versions of the principle of knowability can be obtained by relativizing the principle to the epistemic resources and procedures (methods of inquiry) available to an inquirer. Such procedures and methods are an inquirer's *epistemic precepts*. For example, the design of an experiment is an epistemic precept for acquiring experimental information. The following variants of (TSK1)-(TSK2) are not subject to the objection of triviality:

(TAK1)   For any true proposition p, there is a time t and an epistemic precept $A(p)$ such that acting in accordance with $A(p)$ would transform the inquirer's epistemic state at t into a state in which the he knows that p.

(TAK2)   For any true proposition p and time t, there is an epistemic precept $A(p)$ such that acting in accordance with $A(p)$ would change the inquirer's epistemic state at t into a state in which he knows that p.

(TAK3)   For any true proposition p, there is an epistemic precept $A(p)$ such that for any time t, acting in accordance with $A(p)$ would change the inquirer's epistemic state at t into a state in which he knows that p.

Above "$A(p)$" means an epistemic precept for the proposition p, that is, a precept for determining whether it is true that p. Such a precept can be quite complex, and the inquirer (or the community of inquirers) may construct it step by step in the course of the inquiry. Principle (TAK1) is weaker and more plausible than (TAK2) or (TAK3). According to (TAK3), any true proposition should have a precept which is applicable to any knowledge situation (a "universal precept"), and (TAK2) states that any knowledge situation in which p is not known to be true can be transformed by means of a suitable precept into one in which the inquirer knows that p. (TAK2) and (TAK3) are extreme and somewhat implausible forms of the principle of knowability. Using the language of Krister Segerberg's logic of action, (TAK1) can be expressed in the simple form

(TAK1*)   For any true proposition p, there is a time t and an epistemic precept $A(p)$ such that   t ⊨ $[A(p)]\mathbf{K}p$,

where "t ⊨ $[A]q$" means that every performance of the action A initiated at t terminates in the state q; "t" refers here to the starting point of the inquiry. (Cf. SEGERBERG, 1989, pp. 337-38; 1990, p. 206). This representation involves some idealization, because even good precepts need not invariably lead to the desired epistemic state: they must be reliable, but need not be infallible.

A precept for a proposition can be regarded as a conditional directive which says that an inquirer who wishes to determine the truth-value of the proposition should perform an action of a certain kind; thus the content of a precept is always a generic action or an action type. A given generic action need not be performable on all occasions, but only in situations satisfying suitable conditions. In general, the performability of an action means performability on some occasion (or on suitable occasions). According to (TAK1), a proposition is pragmatically meaningful if there is a precept which an inquirer can apply in suitable circumstances to determine whether the proposition is true.

It is possible to derive from any factual proposition q an action description by attaching to it a prefix "bring it about that" or "see to it that". For example, "to bring it about that $\mathbf{K}p \lor \mathbf{K}\neg p$" (to bring it about that one knows whether p) is a description of an epistemic action. Action descriptions derived in this way from the results of actions cannot be regarded as precepts. To be serviceable, precepts should characterize the required actions directly and spell out the way in which an inquirer can reach a desired epistemic state. Moreover, epistemic precepts should represent action types which are reliably performable by an inquirer or which an inquirer can choose to perform in a given knowledge situation. Krister Segerberg has called such action types *routines*: "To be able to do something is to have a routine available. To deliberate is to search for a routine" (SEGERBERG, 1985, p. 188). The epistemic routines available to an inquirer can be regarded as tools which the inquirer can use for changing his epistemic state. As was observed earlier, epistemic precepts can be quite complex; thus such precepts may consists of sequences of different routines. Cognitive progress consists partly in the development of new epistemic precepts for various propositions, including the invention of new experimental methods and instruments of knowledge acquisition.

XI

If epistemic propositions are relativized to points of time in the way suggested above, the pragmatic principle of knowability, even in its weakest form (TAK1), is subject to another kind of difficulty. An inquiry consists of actions, and actions change the world. Thus, if the truth-value of p has not been investigated by means of any precept $A(p)$, there is some true proposition r such that if an inquirer had acted in accordance with $A(p)$ at some time t, r would have been false. An inquiry $A(p)$ undertaken at t_i, briefly expressed "$A(p)t_i$", may be then called a *falsifier* of r. If $A(p)t_1$ would make the truth of p known to the inquirer at some later point to time, say t_2, it is clear that $A(p)t_1$ would falsify the epistemic proposition $\neg\mathbf{K}(p)t_2$ and consequently also the proposition

(14) p & $\neg\mathbf{K}(p)t_2$.

If the inquirer has an epistemic precept for p, he also has a precept for (14) if he can use $A(p)$ (or some other precept for p) at t_2 or at some later time. However, since an inquiry, especially an experimental inquiry, changes the inquirer's epistemic state by changing the world, any inquiry (epistemic action) is a potential falsifier of many nonepistemic propositions, that is, there are true nonepistemic propositions r such that $A(p)t_1$ would falsify r. Assume now that p is a true proposition such that as a matter of fact, the truth of p is not known and will never be known. If an inquirer had investigated the question whether p by means of a precept of $A(p)$, he would have found out that p is true, but in that case any proposition r falsified by $A(p)$ would have been false. It is clear that any epistemic action $A(p)$ is a falsifier of many propositions; thus, no matter how an inquirer might have determined the truth-value of p, there are true conjunctive propositions p & r whose truth-value cannot be determined by the same precept, because the investigation of one of the conjuncts would falsify the other. (Note that the conjuncts p and r can refer to different points of time.) There may be good epistemic precepts for p and r, but no way of determining the truth-value of their conjunction. In other words, there may be true propositions p and r, times t and u, and epistemic precepts $A(p)$ and $A(r)$ such that

    $t \vDash [A(p)]\mathbf{K}p$

and

    $u \vDash [A(r)]\mathbf{K}r$,

but no time v and epistemic precept $A(p$ & $r)$ such that

    $v \vDash [A(p$ & $r)]\mathbf{K}(p$ & $r)$.

This result should not be surprising, because as a modality, the concept of knowability is analogous to the concept of possibility rather than necessity: p and r can be knowable, but their conjunction need not be knowable. The result can be avoided only if we assume that whenever a precept for a true proposition p falsifies r, there is another epistemic precept for p & r. The application of such a precept would falsify some true proposition s; thus we should assume that the truth-value of p can be determined by means of yet another precept which does not falsify r or s, and so on. The pragmatic principle of knowability seems to require not only an unlimited community of inquirers, but also an unlimited supply of epistemic precepts and other cognitive resources.

The preceding argument suggests that the pragmatic principle of knowability cannot hold in its unrestricted form, regardless of whether it is formulated as (TAK1) or as (TAK2). However, the argument does not affect Peirce's principle of pragmatism, as formulated in (PP1) and in (PP2). Any conjunctive proposition $p_1 \& p_2$ has a simple semantic precept: as was observed above (in section V), the semantic game rule for a conjunction gives the interpreter the right to choose one of the conjuncts, and a conjunction is true (the utterer has a winning strategy in the semantic game) if the interpreter is unable to choose a false conjunct. To show that a conjunction is true, the utterer must be able to verify each of the conjuncts. To become acquainted with the object or objects of a conjunctive sentence, it is necessary to have precepts for both conjuncts. In the example considered above, both p and r as well as their conjunction can have good precepts, even though the conjunction is not knowable. The principle of pragmatism does not entail the unrestricted principle of knowability. However, the pragmatic principle of knowability may hold in the following restricted or *distributive* form:

(TMK.dist)   For any true proposition p, there are propositions p_1, p_2, ..., p_n such that p is equivalent to $p_1 \& p_2 \& ... \& p_n$ and each p_i (i = 1, 2,..., n) is knowable.

The principle of distributive knowability can be interpreted in different ways. The following interpretation of (TMK.dist) corresponds to (TAK1):

(TAK.dist.1)   For any true proposition p, there are true propositions p_1, p_2, ..., p_n, times t_1, t_2, ..., t_n, and precepts $A(p_1)$, $A(p_2)$,..., $A(p_n)$, such that p is equivalent to $p_1 \& p_2 \& ... \& p_n$, and acting in accordance with $A(p_i)$ at t_i would change the inquirer's epistemic state into a state in which the inquirer knows that p_i (i = 1,2,..., n).

(TAK1) is a special case of (TAK.dist.1) in which n = 1. The propositions p_i in (TMK.dist) and (TAK.dist.1) need not be singular propositions; they may also be general propositions, that is, general propositions with epistemic precepts. For example, the design of a controlled experiment can be regarded as an epistemic precept for a general proposition (cf. HINTIKKA, 1988/1999, pp. 156-57). I observed in section VII that Peirce's scholastic definition of truth shows what kind of inquiry is necessary for determining whether a given proposition is true: an inquirer needs precepts for the predicate and the subject of the proposition. Having a precept for the predicate and the subject (or subjects) is enough to make the proposition pragmatically meaningful. An inquirer should have epistemic precepts for all singular propositions; the meaningfulness of complex propositions is based the semantic precepts which determine the meanings of propositional connectives, quantifiers, and modal expressions. According to the pragmatic theory of meaning, meaningful singular (or atomic) statements should be knowable at least in the weak sense expressed by (TAK1), but the principle of knowability need not hold for all complex propositions.[2]

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