On Peirce's Methodology of Logic and Philosophy

Sobre a metodologia da lógica e a filosofia de Peirce

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Abstract: In his paper "Explanation of Curiosity the First" (1908) Charles Peirce describes Euclid's procedure in proving theorems. Euclid first presents his theorem in general terms and then translates it into singular terms. Peirce pays attention to the fact that the generality of the statement is not lost by that move. The next step is construction, which is followed by demonstration. Finally the ergo-sentence repeats the original general proposition. Peirce lays much emphasis on the distinction between corollarial and theorematic reasoning in geometry. He takes an argument to be corollarial if no auxiliary construction is needed. For Peirce, construction is "the principal theoric step" of the demonstration. Peirce also stresses that it is the observation of diagrams that is essential to all reasoning and that even if no auxiliary constructions are made, there is always the step from a general to a singular statement in deductive reasoning; that means introducing a kind of diagram to reasoning.

This paper seeks to argue for two theses. One is that the way of thinking of reasoning which stresses the role of observation is essential to Peirce's discovery of the new logic. The other is that Peirce wants to consider both logical reasoning and philosophical thinking in general via the model of geometry. However, it is argued in this paper that it is the model of problematic analysis in geometry that is the most important for Peirce when he discovers the new structure of propositions. Moreover, this paper seeks to show that this very feature in Peirce's thought connects him with philosophers and logicians who emphasise the role of praxis instead of theoretic contemplation and that one who wants to reconstruct Peirce's metaphilosophical views must pay special attention to the model of problematic analysis.

Key-words: construction; demonstration; corollarial and theorematic reasoning; diagrams.

Resumo: Em seu artigo "Explanation of Curiosity the First" (1908), Charles Peirce descreve o procedimento de Euclides para demonstrar teoremas. Euclides primeiramente apresenta seu teorema em termos gerais, e então o traduz para termos singulares. Peirce presta atenção ao fato de que a generalidade do enunciado não é perdida por esse movimento. O próximo passo é a construção, que é seguida pela demonstração. Finalmente, a sentença-ergo repete a proposição geral original. Peirce dá bastante ênfase à distinção entre o raciocínio corolarial e o teoremático em geometria. Ele considera um argumento como corolarial se nenhuma construção auxiliar

se fizer necessária. Para Peirce, a construção é "o principal passo teórico" da demonstração. Peirce também enfatiza que a observação de diagramas é essencial para todo raciocínio e que, mesmo se não são feitas construções auxiliares, há sempre o passo de um enunciado geral para o particular no raciocínio dedutivo; o que significa introduzir um tipo de diagrama no raciocínio.

Este artigo procura defender duas teses. A primeira, que a maneira de pensar o raciocínio que enfatiza o papel da observação é essencial para a descoberta da nova lógica por Peirce. A outra, que Peirce deseja considerar tanto o raciocínio lógico quanto o pensamento filosófico em geral pelo modelo da geometria. Entretanto, defende-se neste artigo que o modelo de análise problemática em geometria é o mais importante para Peirce, quando ele descobre a nova estrutura das proposições. Este artigo procura mostrar, ainda, que é exatamente essa característica do pensamento de Peirce que o liga aos filósofos e lógicos que enfatizam o papel da práxis em vez da contemplação teórica, e que quem deseje reconstruir as visões metafilosóficas de Peirce deve prestar atenção especial ao modelo da análise problemática.

Palavras-chave: construção, demonstração, raciocínios corolarial e teoremático, diagrama.

1. Introduction

In his paper 'Explanation of Curiosity the First' (1908) Charles Peirce describes Euclid's procedure in proving theorems. Euclid first presents his theorem in general terms and then translates it into singular terms. Peirce pays attention to the fact that the generality of the statement is not lost by that move. The next step is construction, which is followed by demonstration. Finally the *ergo*-sentence repeats the original general proposition. Peirce lays much emphasis on the distinction between corollarial and theorematic reasoning in geometry. He takes an argument to be corollarial if no auxiliary construction is needed. For Peirce, construction is 'the principal theoric step' of the demonstration (CP 4.616). Peirce also stresses that it is the observation of diagrams that is essential to all reasoning and that even if no auxiliary constructions are made, there is always the step from a general to a singular statement in deductive reasoning; that means introducing a kind of diagram to reasoning.

Peirce's methodological interests are well known. In his 'Introductory Lecture on Logic' (1883) he makes an interesting remark on methodology. He writes:

> But modern logicians generally, particularly in Germany, do not regard Logic as an art but as a science. They do not conceive the logician as occupied in the study of methods of research, but only as describing what they call the normative laws of thought, or the essential maxims of all thinking. Now I have not a high respect for the Germans as logicians. I think them very unclear and obtuse. But I must admit that there is much to be said in favor of distinguishing Logic from Methodology. … Let us say then that Logic is not the art of method but the science which analyzes method. (W 4 pp. 509 – 510.)

As Peirce thus regards logic as science, it is no surprise that he is also interested in the methodological commitments and choices of the one who works in the science

of logic. What interests me here are the methods of discovery within that science. A logician can be said to discover axioms, theorems, rules of logical inference, or the structure of a logical language, hence, the vocabulary and the formation rules of that language. Here I am interested in Peirce's discovery of the structure of a logical language. I will argue that Peirce's methodology of logic ensues from his metaphilosophy, more precisely, from his views of the methods of philosophy.

This paper seeks to argue for two theses. One is that the way of thinking of reasoning which stresses the role of observation and construction is essential to Peirce's discovery of the new logic. The other is that Peirce wants to consider both logical reasoning and philosophical thinking in general via the methods of geometry. That has also been emphasized in Peirce scholarship. However, this paper is an attempt to focus on specific aspects of geometry. I will argue that it is the model of problematic analysis in geometry that guides Peirce when he discovers the new structure of propositions. Moreover, I will show that this very feature in Peirce's thought brings him close to Edmund Husserl, who emphasizes the role of praxis instead of theoretic contemplation. Special attention will be paid to the connections between geometry and Husserl's phenomenology, as these connections serve to illuminate Peirce's way of thinking.[1]

2. On Peirce's phenomenological method

The starting-point of Husserl's phenomenological project is the need to make a sharp distinction between psychological and epistemological questions. Phenomenological reductions mean an attempt to think of the realm of pure consciousness without presupposing any factual world. If that kind of attitude is within our reach, as Husserl assumes, we are able to make claims about consciousness and its objects without supposing anything about the natural world which is posited by the special sciences.

The same way of arguing can be found in Peirce's writings. Peirce is a phenomenologist at least in some sense of the word. He often uses the term 'phaneroscopy' instead of 'phenomenology'. In 1904 he writes:

Phaneroscopy is the description of the *phaneron*; and by the *phaneron* I mean the collective total of all that is in any way or in any sense present to the mind, quite regardless of whether it corresponds to any real thing or not. If you ask present *when*, and to *whose* mind, I reply that I leave these questions unanswered, never having entertained a doubt that those features of the phaneron that I have found in my mind are present at all times and to all minds. So far as I have developed this science of phaneroscopy, it is occupied with the formal elements of the phaneron. (CP 1.284.)

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1. The paper develops further the ideas discussed in HAAPARANTA (1993, 1994a, 1999). I have used Peirce's manuscripts 498, 499, 599, and 693a by permission of Houghton Library, Harvard University.

Like Husserl, Peirce parenthesizes the real world. He neither denies nor doubts its existence; the question concerning its existence does not arise in phaneroscopy. Peirce also excludes the empirical self. Moreover, like Husserl's phenomenology, phaneroscopy is a science which studies the structure of the world as it is given to us.

What matters most both for Husserl and for Peirce is the question of the method of philosophy, and it is precisely that question to which phenomenology seeks to be an answer. The origins of Peirce's phenomenology can be found in his early writing 'On a New List of Categories' (1867). In deducing his basic categories, Peirce uses his method of precision, which is a special kind of abstraction. He states:

> The terms "precision" and "abstraction", which were formerly applied to every kind of separation, are now limited, not merely to mental separation, but to that which arises from attention to one element and neglect of the other. Exclusive attention consists in a definite conception or supposition of one part of an object, without any supposition of the other. (CP 1.549)

In the footnote, Peirce refers to the ways in which Duns Scotus and other scholastics used the word '*praecisio*'. In the text, Peirce continues:

> Abstraction or precision ought to be carefully distinguished from two other modes of mental separation, which may be termed discrimination and dissociation. Discrimination has to do merely with the senses of terms, and only draws a distinction in meaning. Dissociation is that separation which, in the absence of a constant association, is permitted by the law of the association of images. It is the consciousness of one thing, without the necessary simultaneous consciousness of the other. (ibid.)

Peirce's concept of precision, which supposes a greater separation than discrimination, but a less separation than dissociation, is a methodological concept. For example, we cannot separate colour from extension by precision, because we cannot suppose that in any possible universe color as a quality of an object exists without extension. However, we can separate colour from extension by an act of discrimination. Moreover, Peirce states that precision is not a reciprocal process; it often happens that while A cannot be prescinded from B, B can be prescinded from A (CP 1.549). For example, he states, impressions cannot be definitely attended to if an elementary conception which reduces them to unity is neglected; on the contrary, when such an explaining elementary conception is obtained, impressions can be neglected; hence, the explaining basic factor can be prescinded from impressions, while impressions cannot be prescinded from that factor.

Husserl's concept of phenomenological abstraction is close to Peirce's early method of precision. Husserl's transcendental phenomenology in the *Ideen I* amounts to studying, and making claims about, consciousness without making claims about the natural world or empirical consciousnesses. In Peircean terms, Husserl assumes that pure consciousness can be prescinded from empirical consciousnesses but empirical consciousnesses cannot be prescinded from pure consciousness, hence, certain formal structures of consciousness are exemplified in human minds. Asking how it is possible to prescind or to abstract pure consciousness from that which is empirical is asking how pure epistemology, that is, epistemology which is not intertwined with psychology, is possible.[2] For Husserl, this basic abstraction comes along with several other abstractions,

which lead to finding the essences of the considered phenomenon.

In his Carnegie Application (L 75, 1902) Peirce describes his own project in contrast to those of his philosophical predecessors and his contemporary psychologists. He states that his list of categories differs from those of Aristotle, Kant and Hegel in that, unlike those philosophers, he goes back to examining the phenomena to see what is observed there (NE 4, p. 19). He then describes his own conception of the mind and stresses that when a logician speaks about the operations of the mind, he means by mind something quite different from the object of study of the psychologist (NE 4, p. 20). In his 'Minute Logic' in 1902 – 1903, Peirce writes that he gives an analysis of what appears to us and that his study is not metaphysics but logic. He remarks that 'we do not ask what really is, but only what appears to everyone of us in every minute of our lives' (CP 2.84).[3]

In his manuscript 'Reason's Conscience: A Practical Treatise on the Theory of Discovery; Wherein Logic is conceived as Semeiotic' Peirce writes: "Phenomenology is that branch of philosophy which endeavors to describe in a general way the features of whatever may come before the mind in any way" (MS 693a p. 82.).

Moreover, he states:

> The work of discovery of the phenomenologist, and most difficult work it is, consists in disentangling or drawing out, from human thought, certain threads that [are] seen through it, and in showing what marks each has that distinguishes it from every other. (MS 693a p. 118.)

He concludes that the results of the phenomenologist's studies are extremely useful for a logician (MS 693a p. 120).

3. Husserl's transcendental phenomenology and geometric analysis

The link between geometry and phenomenology which I want to make explicit in this paper is not recognized by the contemporary followers of Husserl.[4] I will argue that there is a connection between Husserl's phenomenology and the tradition of geometric analysis. Wilbur Knorr tells in his book titled *The Ancient Tradition of Geometric Problems* (1986) that in ancient geometry there were two ways of understanding the nature of geometry. There were the Platonists, the theoreticians, and those geometers who were close to geometric practice. For theoreticians, the main point of interest were the theorems, for the practical men they were the problems that mattered more. There are certain features in problematic analysis and synthesis, which are particularly relevant to my approach. Solving geometrical problems in Euclid's geometry had to do with making certain constructions, which were described in

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2. See HAAPARANTA (1999).
3. For Peirce's phenomenology, also see ROSENSOHN (1974).
4. I have tried to argue that there is such a connection and that the model of geometrical problem-solving helps us understand the nature of Husserl's phenomenology. See, for example, HAAPARANTA (1994a, 1994b, 1999).

the given problem. Analysis was the general method which the Greeks used for finding the solutions. In geometrical analysis, one takes that which is sought *as if* it were admitted and moves from it via its consequences to something that *is* admitted. Taking something as if it were already admitted normally means drawing a model-figure, which then becomes the object of analysis. The methods of analysis and synthesis were used both in proving theorems and in solving problems. However, Thomas Heath, who comments on the *Elements*, tells us that the ancient analysis had the greatest significance in relation to problems (HEATH, *Introduction*, p. 140). Knorr also stresses in his work on ancient geometry that the method of analysis was basically meant to offer heuristic power to the ancients in their search for solutions to geometrical problems (KNORR, 1986, p. 356). He calls special attention to the fact that the activity of investigating problems of construction was prominent in ancient geometry and that the questions of construction primarily concerned problems; what was primarily applicable to problems was then transferred to theorems (*ibid*., p. 360 and p. 368).

To put things more concretely, when we try to solve a geometrical problem, we may first draw a figure, which is meant to be a model of what the problem requires to be constructed. In order to find out what we have to do, we then analyze our figure. That is, we try to find out what we ought to do with the material given in the original problem, for example, segments of a line, in order to manage to construct the desired figure. When we have found out the conditions for the realization of the figure, we are able to construct it on the basis of the very information we have received from it by 'stepping backwards' from the imagined end-state of our constructing activities.

Let us consider one geometrical example. If we have to draw a triangle the sides of which are known, we have three segments of a line as our given data. In order to find out how to construct the desired figure, we take it as if we already had the figure. We then try to find out how its various parts are constructed starting from the given elements. The model-figure, say, the triangle *ABC*, which we try to analyze is the imagined end-state of our constructing activities. We ask questions like "Where are all the points, such as *B*, which are at the distance *a* from the point *A*?", "Where are all the points, such as *C*, which are at the distance *b* from the point *A*?", and "Where are all the points, such as *C*, which are at the distance *c* from the point *B*?". By answering these questions, we find out what we have to do in order to construct the figure. That is, even if we have the model-figure, we still have to find its parts in order to solve the geometrical problem. After having analyzed the model-figure, we may proceed with synthesis, that is, we may construct the required triangle.

The peculiar thing in geometric analysis is that even if we draw the model-figure in the beginning, that is, even if we seem to construct the figure, in the real sense of the word we have not constructed it. That is because we do not know how to construct it, which means that we do not have the intuition, the immediate knowledge, which is presupposed by constructive activity. The imagined end state, that is, the model-figure, is from which we as it were step backwards in analysis; in analysis we reveal the content and the form, which the subject gives to the figure in the act of drawing. After analysis it is easy to carry out the construction.

If we compare Husserl's phenomenology with geometry, we may think as follows: In phenomenological analysis there is the given, the experience, and we have it as we have the model-figure in geometry, that is, we do not know our experience in the

beginning; it is not intuitively given, as we do not know its various layers. Like geometric analysis, phenomenological analysis is stepping backwards, research into how experience is structured. The phenomenological description is the phase of construction, that is, of synthesis. The phenomenologist constructs in the peculiar sense that he or she writes down the structure of experience. One may ask what in phenomenology corresponds to the geometrical problem. At this point there is no detailed analogy; however, what the phenomenologist starts with is philosophical awakening or wonder, hence, a desire to reflect on the objects of experience and to reveal what lies hidden in them.

Peirce on judgements, signs and diagrams

In his manuscript 'On Existential Graphs as an Instrument of Logical Research' (1896; MS 498) Peirce states that the most important question which has troubled logicians for a decade has been the nature of propositions or judgements. He then presents the alternative views, one being that a proposition is built up of subject and predicate and the other being that a proposition is an act called assertion. Peirce's own view is formulated in the manuscript 'On the System of Existential Graphs Considered as an Instrument for the Investigation of Logic' (MS 499). There he asks what constitutes a judgement or a proposition and answers that ' the essence of the proposition does not lie in its being compound, but on the contrary upon its being asserted or at least conceived to be asserted'. Moreover, he points out that assertion does not add any new element to thought, as it is a deed. My interpretational model, which stresses the similarities between phenomenology and geometry, would require that Peirce takes those judgements which appear to us either as mental items or as sentences of natural language to be like model-figures, hence, the starting-points of a logician's process of analysis. It would also require that after revealing the structure of a model-figure, Peirce thinks to be able to draw the desired figure in his notation.

Can such a claim receive support from Peirce's texts? That Peirce proceeds this way cannot be given in any direct documentation. However, there is plenty of indirect evidence for the suggested interpretation which makes it at least plausible that Peirce would have thought of his discovery in the manner described above. First, starting from his earliest writings, Peirce seeks for the logical categories in what is given in experience. In his 'New List' of 1867 he uses the method of precision, and the starting-point of Peirce's study there is the manifold of substance. Even if Peirce later considered monadic, dyadic and triadic relations to be equally abstract, he still believed in the relevance of a step-by-step procedure. That claim can be supported by Peirce's statement in a manuscript dealing with existential graphs. There he remarks that one type of phaneroscopic analysis is precision and that whenever a higher valency is present, every lower valency is present (MS 499(s)). That point, which is written down by Peirce some time at the turn of the century, clearly testifies that he had not given up the analytic step-by-step procedure when revealing the categories.

The second fact which supports my interpretational model is the very fact that Peirce regards propositions or judgements as acts or potential acts, not as compositions of subjects and predicates. Peirce thus confesses the priority of judgements over the concepts which are its constituents. The third fact which is at least compatible with the

suggested model is that Peirce identifies his phaneroscopy with semiotics or the theory of signs and regards logic as formal semiotic or the formal doctrine of signs (CP 2.227; NE 4, p. 20). Hence, for Peirce, the study of mind and the study of language cannot be separated. However, what the relationship between logic and the analysis of mental and linguistic signs amounts to must be considered in more detail.

There are difficulties in such terms as 'judgement', 'assertion' and 'proposition'. So far, I have used the terms interchangeably, when I have discussed Peirce's logic. However, in some of his manuscripts, Peirce distinguishes a proposition from an assertion or affirmation.[5] He points out that one and the same proposition may be affirmed, denied, doubted etc., even if the normal use of a proposition is to affirm it (MS 517, NE 4, p. 248). Peirce defines a proposition as the sign of which the judgement is one replica and the linguistical expression another. For Peirce, a sign is something that exists in replicas, for example, in writing, in oral speech or in silent thought, and logic is precisely the study of the essential nature of signs. In his terminology, a judgement is the mental replica of a proposition plus its acceptance (*ibid.*).

After discovering his general algebra of logic, Peirce develops his theory of signs in a new way which shows the traces of the new logical discovery. In that theory propositions have icons and indices as their constituents. In his notes in 1895 Peirce characterizes icons by saying that they have no dynamical connections with the objects which they represent, but that the connections are merely based on resemblance (CP 2.299). An icon has a relation to the object which it signifies on the basis of similarity, that is, the sign and the object have common qualities or maybe a common structure (CP 2.276). Peirce states:

> Hence, every assertion must contain an icon or a set of icons, or else must contain signs whose meaning is only explicable by icons. The idea which the set of icons (or the equivalent of the set of icons) contained in an assertion signifies may be termed the predicate of the assertion. (CP 2.278.)

Peirce also distinguishes between different types of iconic signs. In 1902 he writes that if iconic signs have the same simple quality as their objects, they are images, if their parts have analogous relations to those of their object, they are diagrams, and if they represent the representative character of a sign by representing a parallelism in something else, they are metaphors (CP 2.277).[6] Indices have a factual, for example, a causal connection with the objects which they signify. Indices are thus physically connected with their objects and the interpreting mind merely notices the connection (CP 2.299). Icons and indices do not assert anything (CP 2.291). Therefore, a symbol is needed to mediate the icon and the index, hence, it gives a rule for connecting those two types of elements. Peirce states:

> A Symbol is a Representamen whose Representative Character consists precisely in its being a rule that will determine its Interpretant. All words, sentences, books and other conventional signs are symbols. (CP 2.292.)

Symbols belong to the category of thirdness, which is most immediately given in

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5. Cf. HILPINEN (1982), pp. 182 - 183.
6. Cf. TILES (1988), p. 173.

our experience, hence, which is the starting point of the analysis of experience. The category of thirdness is not represented separately in the new logical vocabulary, but it is united in all signs of the vocabulary. According to Peirce, the connection between the symbol and its object is established by the symbol-using mind; hence, the interpretative activity of the mind is necessary for creating the connection (CP 2.299). That amounts to saying that there are no symbols without the mind, that in the last analysis, symbols are creations of the mind.

Even if the symbolic element is mixed with every expression, it would not be true to claim that icons and indices have a secondary role in Peirce's philosophy. On the contrary, it seems that Peirce's view of mathematics, logic and philosophy presupposes that it is precisely the icon that has a central role in the practice of those disciplines. That point has also been stressed by some scholars, for example, Roberts (1980), Dougherty (1980) and Zeman (1989). A great number of documents are available if we want to show that, starting from 1882, Peirce developed his logic relying on the idea that the form of the argument is iconic.

Even more evidence of Peirce's general line of thought can be detected, as Peirce lays special stress on the role of observing figures in mathematical and logical reasoning. Joswick argues that as the nature of mathematics, according to Peirce, involves the construction and observation of a diagram, likewise the interpretation of any sign, according to Peirce, involves the construction and observation of an icon (Joswick, 1988, p. 107). This is a much stronger thesis than the one supported by Dougherty and Zeman. Joswick continues that Peirce did not deny the value of formalization but that he took it to be essential for mathematical practice, like for all scientific practice, to make experiments and observations and that a logician's practice also ought to follow this model (*ibid*., pp. 108-109).

Indeed, in about 1893 Peirce states quite clearly that 'the whole of inference consists in *observation*, namely in the observation of icons' (CP 7.557). Moreover, in 1898 Peirce states that a construction used by the mathematician is 'formed according to a precept furnished by the hypothesis'. After making the construction, the mathematician scrutinizes his or her diagram and comes to find new relations among its parts. Those discoveries are made by mental experimentation. Peirce concludes that

> the necessary reasoning of mathematics is performed by means of observation and experiment, and its necessary character is due simply to the circumstance that the subject of this observation and experiment is a diagram of our own creation, the conditions of whose being we know all about. (CP 3.560.)

In 1896, Peirce points out that 'logic rests upon observations of real facts about mental products' (NE 4, p. 267).

That Peirce wants to extend this way of thinking to philosophy in general can be easily seen from his remarks. For example, in his Carnegie Application he remarks that when a student has once read the first book of Euclid's *Elements*, he will have an idea of how philosophy must be read (NE 4, p. 72). Moreover, he points out that if we make our thought diagrammatical and mathematical and if we then experiment upon our diagram, 'the danger of error in philosophy can be reduced to a minimum' (CP 6.204). Peirce even writes that icons have to be used in all thinking (NE 4, p. xxi).

Peirce's logic of logical discovery and geometric analysis

Peirce's distinction between corollarial and theorematic reasoning, which was mentioned in the beginning, deserves special attention. There has been a discussion in the literature concerning the true import of Peirce's view of theorematic reasoning.[7] In what follows, I will list the points in Peirce's view on which the different scholars seem to agree and which can be supported by Peirce's own statements on the matter. In his writings after 1885, for example, in the middle of the 1890's, Peirce clarifies the distinction. He thinks that in corollarial deduction it is only necessary to imagine any case in which the premises are true in order to realize that the conclusion is true; instead, in theorematic deduction it is necessary to "experiment in the imagination upon the image of the premiss in order from the result of such experiment to make corollarial deductions to the truth of the conclusion" (NE 4, p. 38). When Peirce describes the conclusions, he states that a corollary, that is, the conclusion of corollarial deduction, is a proposition which is deduced directly from propositions already established "without the use of any other construction than one necessarily suggested in apprehending the enunciation of the proposition" (NE 4, p. 288). In Peirce's terminology, a theorem, that is, the conclusion of theorematic deduction, is a proposition which can be deduced from previously established propositions only by imagining something more than is supposed in the conditions (NE 4, p. 289).

In the beginning of this paper I already referred to Peirce's writing 'Explanation of Curiosity the First', published in *The Monist* in 1908, where Peirce describes Euclid's procedure in proving theorems. He lays much emphasis on his distinction between corollarial and theorematic reasoning. In his Carnegie Application he even remarks that it was his first real discovery about mathematical procedure (NE 4, p. 49). Peirce seems to connect his insight with his logical discoveries, as he remarks that Kant was unaware of theorematic reasoning, because he had not studied the logic of relatives (NE 4, p. 59). The central role of the logic of relatives also comes up in Peirce's writings in 1911. Peirce remarks that deductive logic can only be understood by means of the logic of relatives, as that logic rescues us from a number of errors of thought. In Peirce's view, one such error is that demonstrative reasoning is something completely different from observation (CP 3.641). He then continues:

> But the intricate forms of inference of relative logic call for such studied scrutiny of the representations of the facts, which representations are of an iconic kind, in that they represent relations in the fact by analogous relations in the representation that we cannot fail to remark that it is by observation of diagrams that the reasoning proceeds in such cases. (ibid.)

Even if the interpreters disagree on the contemporary relevance of Peirce's distinction between corollarial and theorematic reasoning, they are ready to agree on two things. One is that, for Peirce, mathematics and logic are observational sciences, that is, they experiment upon diagrams. The other is that Peirce does make a distinction between corollarial and theorematic reasoning and that in those types

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7.See HINTIKKA (1980), KETNER (1985) and JOSWICK (1988).

of reasoning constructions have two roles. One of the roles is to serve as instances of general terms. The other is to serve as auxiliary constructions. The former task is performed by icons, that is, constructions, both in corollarial and in theorematic reasoning, while the latter role is played by them only in theorematic reasoning. However, as Peirce writes to William James in 1909, it is essential that "every Deduction involves the observation of a Diagram" (NE 3, p. 869). Peirce also writes in his Carnegie Application that all knowledge without exception comes from observation (NE 4, pp. 47-48). If for Peirce, all knowledge comes from observation in the described manner, we may quite confidently proceed with the hypothesis that he takes that to be valid also for the one who tries to discover the structure of a logical language.

So far we are allowed to conclude that Peirce wants to consider logical reasoning and philosophical thinking in general via the model offered by corollarial and theorematic reasoning, hence, by keeping in mind how the proofs of geometrical theorems are found and presented. However, my special interest does not lie in theorems. Instead, I wish to show that Peirce relies on the model of problematic analysis, when he discovers the new structure of propositions. That Peirce is especially interested in constructions is easily seen from his remarks. For example, in his "Minute Logic" (1902) he states as follows: "Thinking in general terms is not enough. It is necessary that something should be DONE" (CP 4.233.).

However, even if the role of constructions is stressed by Peirce, he seems to be interested in constructions merely as tools, hence, in order to prove something. Thus, the suggestion that Peirce tried to reach the structure of propositions by following the model of problematic analysis may still sound unbelievable. As I mentioned above, we must show that for Peirce propositions are picture-like and that discovering the structure of those pictures is like analyzing model-figures in geometry. But can we find any evidence for *those* claims? Joswick seems to suggest a somewhat similar line of thought, when he says that understanding a proposition according to Peirce is analogous to the mathematician's use of a diagram (Joswick, 1988, p. 116). He refers to Peirce's statement in about 1902 that when one reads a sentence, a picture begins to be painted in the imagination (Joswick, 1988, p. 117; MS 599). In his manuscript 'Reason's Rules' in about 1902 Peirce quotes Jesus' command in the Bible that his disciples ought to go to a village and fetch an ass there. Peirce claims that as this was said by Jesus, it created a picture in the disciples' imagination and that picture of an ass accompanied by a young colt is an icon (MS 599, p. 7). When Jesus uses the expression 'this village over against us', he uses an index, according to Peirce (*ibid.*, p. 8). Peirce also claims that those languages in which the predicate comes first in a sentence are more readable than our own languages. He suggests that this is precisely because a picture begins to be painted in the imagination almost as soon as the utterance begins and that we then put details in the picture when the utterance proceeds. He also points out that when we listen to a German or Latin sentence, we first get the material for building up the idea and it is not until we have heard the whole sentence that we can consider our material and find out what we have got (MS 599, p. 33). In the manuscript on which Joswick relies Peirce describes propositions as the significations of signs which represent that some icon is applicable to that which is indicated by an index (MS 599 p. 11, Robin, 1967, p. 74).

Joswick also states that Peirce was not interested in the psychological mechanisms of constructing diagrams but that he was concerned with the formal

representation of thought (Joswick, 1988, p. 118). He quotes Peirce's statement in *Prolegomena to an Apology for Pragmaticism* (1906) that the chief need for the icon is to "show the Forms of the synthesis of the elements of thought" (CP 4.544). He then concludes that, for Peirce, the role of the icon is to reveal the skeletal structure of the proposition (Joswick, 1988, p. 118). In his *Grand Logic* (about 1893), Peirce also makes the following interesting remark: "However, when I just think of the bird calling, I do not think the idea of connection so distinctly. Nevertheless, I *do* think it, and think of the call and the visual bird as belonging to it" (CP 7.426.).

Joswick cites this and continues:

> Something exhibits the relation of bird to call. These logical icons of connection may not be clearly apprehended, but some icon is necessary in order to understand the relation of thought expressed by the proposition. (Joswick, 1988, p. 119.)

That Peirce stresses the iconic nature of propositions is also supported by a passage which he wrote about 1895 (CP 2.279). He first tells us that every diagram is an icon, even if there is no sensuous resemblance between it and its object, but only an analogy between the relations of the parts of each. Peirce is especially interested in icons in which the likeness is aided by conventional rules. He mentions that an algebraic formula is an icon which is rendered such by the rules of commutation, association and distribution of the symbols. He notes that it may seem arbitrary to call an algebraic expression an icon but that it may be better to regard it as a compound conventional sign. However, Peirce tries to persuade his reader that this is not the case. He writes:

> For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction. Thus, by means of two photographs a map can be drawn, etc. Given a conventional or other general sign of an object, to deduce any other truth than that which it explicitly signifies, it is necessary, in all cases, to replace that sign by an icon. This capacity of revealing unexpected truth is precisely that wherein the utility of algebraical formulae consists, so that the iconic character is the prevailing one. (CP 2.279.)

It is noteworthy that in his paper *On the Algebra of Logic* (1885) Peirce labels the formulas of his general algebra of logic as icons (CP 3.363). Our problem is how Peirce comes up with those kinds of formulas. We saw above that in the beginning of the 1880's Peirce already takes propositions to be prior to concepts. We also noted above that Peirce assumes propositions to have a number of replicas. That view suggests that the different replicas have iconic relations with each other. Without the new formula language, propositions appear to the mind as mental replicas or as the sentences of natural language. As these replicas are nothing but diagrams, in which the likeness is aided by conventional rules, and as they are the objects of Peirce's phenomenological studies, it is not a far-fetched thesis that they serve in the same role as the model-figures of problematic analysis. By means of those replicas Peirce seeks for the new formula language. After finding their structure, Peirce is able to *do* something, that is, to draw figures in his new logical notation.

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