Thinking and Acting
The Logical Background of Peirce's Pragmatism[1]
Pensando e Agindo
O Arcabouço Lógico do Pragmatismo de Peirce

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Abstract: What is the logical background of pragmatism? The answer I want to suggest in this paper is that pragmatism is supported by some mathematical and logical ideas that provide a logical background for it. That is to say, they may be used to back up pragmatism's claim to give us a viable account of thought and knowledge acquisition that describes some of the crucial relations by which knowledge acquisition and action are guided. I will start by giving a short account of why some mathematical and logical ideas coming from the logic of relations, order theory in particular, might be helpful for pragmatism's view of knowledge and praxis. That is, I claim that they support, clarify and strengthen some of the claims made by the pragmatic maxim. In a second step I describe why these logical concepts and rules of reasoning acquire a normative meaning when they become part of pragmatism's semiotic, methodology and epistemology. In particular, the normative role of a semiotic concept of the identity of an object of different beliefs is discussed. In a concluding step some of the consequences of all of this for the pragmatistic account of thought for the theory of interpretations is developed.

Keywords: Pragmatism. Logic. Semiotic. Objects of thought.

Resumo: *Qual é o arcabouço lógico do pragmatismo? A resposta que desejo sugerir neste trabalho é que o pragmatismo lógico é sustentado por algumas idéias matemáticas e lógicas que fornecem um arcabouço para ele. Em outras palavras, elas podem ser utilizadas para apoiar a asserção do pragmatismo de nos fornecer uma explicação viável do pensamento e aquisição do conhecimento, que descreva algumas das relações cruciais pelas quais a aquisição do conhecimento e a ação são orientadas. Iniciarei dan-*

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do uma breve explicação porque algumas das idéias matemáticas e lógicas oriundas da lógica das relações – da teoria da ordem em particular – podem ser úteis para a visão do conhecimento e práxis do pragmatismo. Ou seja, afirmo que elas sustentam, esclarecem e fortalecem algumas das asserções da máxima pragmática. Em um segundo estágio, descrevo porque esses conceitos lógicos e regras de raciocínio adquirem um significado normativo quando se tornam parte da metodologia e epistemologia semiótica do pragmatismo. Em particular, discute-se o papel normativo de um conceito semiótico da identidade de um objeto de diferentes crenças. Em um estágio conclusivo, desenvolvem-se algumas das conseqüências de tudo isto para a explanação pragmatística do pensamento para a teoria das interpretações.

Palavras-chave: *Pragmatismo. Lógica. Semiótica. Objetos de pensamento.*

I. Introduction: Thought, Action and the Emptiness of the General

How is it that we are able to form thoughts in such a way that they – at least in some cases – guide our actions that successfully change and manipulate the world and ourselves? This is an important, interesting and amazingly complex question. But it has, for the purposes of this paper, one advantage: It is a way to formulate one of the key questions of the epistemological tradition in philosophy from a pragmatic point of view.

But, don't worry: I will not even try to answer this question. Rather, I will use this question as a sort of foil on which many aspects of Peirce's pragmatic account of thought, experience and action can be better understood. The reason for this is: the pragmatic account of thought gives priority to practice by insisting that the relation between thought and action develops even our theoretical thought. However, the results of Peirce's study of those logical properties which are required for the relation between theoretical thought and practice to hold has not been discussed in the literature on pragmatism, even though his logic of relations developed before or simultaneously with his pragmatism. Consequently, the thesis that this paper will defend is that some of the results in the logic of relations provides a substantial part of the logical background of pragmatism. In fact, I hold the even stronger claim that Peirce's work and results in the logic of relations did provide him with some of the crucial conceptual tools and modes of reasoning that played a role in developing his pragmatism.

The claim that pragmatism has such a backdrop in formal logic and mathematics that influenced its very shape and content, is consistent with Peirce's mature (and most of his early version) classification of the sciences: Pragmatism formulates a methodological thesis and is part of the third branch of logic, methodeutics. It presupposes and may use the results not only of the two other branches of logic. The first of this is Speculative Grammar (or Semiotics) which gives us a general analysis of the material form of signs, which Peirce calls "the physiology of forms" (MS 478, 1903)[2]. The second is Logical Critic, the theory of the validity and form of arguments that are truth-preserving. But Methodeutics and therefore pragmatism, may make also use of the results, principles and modes of reasoning of mathematics – and this is the place where the logic of relations belongs to.

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[2] All numbers prefixed by "MS" refer to the manuscripts of C.S. Peirce, cf. bibliography.

However, I will not defend a reductive genetic interpretation of the development of pragmatism: I don't claim that the logic of relations is what is exclusively brought about or what in the first place motivated Peirce's pragmatism. It is rather likely that many other factors, e.g. the anti-ideological spirit of post-civil war Boston[3], Peirce's scientific education, and in particular his experience as an experimental scientist in chemistry, astronomy and geodesy paved the way to formulate his pragmatic methodology.[4] Nevertheless, even if all these other factors were instrumental in forging Peirce's 1878-pragmatism, it is no coincidence that the first version of the pragmatic methodological rule was formulated by Peirce in 1871, just one year after he had published his first comprehensive study of the logic of relations in DNLR, W 2, p. 359-429,[5] which stresses the equivalence of theoretical terms with regard to the sameness of their practical consequences: "A [...] rule for avoiding the deceits of language is this: Do things fulfil the same function practically? Then let them be signified by the same word. Do they not? Then let them be distinguished" (CP 8.33[6]).

Already in 1871 Peirce formulates a rule to improve thought by its relation to practice: Only because they have the same practical consequences will two thoughts have the same meaning. This implies that even the most general and formal concepts in logic and mathematics are to be interpreted in terms of some rather specific and mathematical practice in which they are involved. Therefore, formal or mathematical generality is never an epistemological value by and for itself. This is the reason why Peirce objected that the degree of generality for the solutions of logical problems, which Ernst Schröder tried to achieve in his *Algebra der Relative* (cf. CP 3.513f), was much too high. Therefore, Schröder's solutions were in danger of becoming empty and sterile and this is where he criticizes and attacks Schröders mathematical methodology directly:

> Professor Schröder attaches great importance to the generality of solutions. In my opinion, this is a mistake. It is not merely that he insists that solutions shall be *complete*, as for example when we require *every root* of a numerical equation, but further that they shall all be embraced under one algebraical expression. (CP 3.513)

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[3] See Louis Menand's colorful and insightful study of the spirit of the age which brought about pragmatism in The Metaphysical Club. A Story of Ideas in America. New York, 2001. Although Menand's understanding of Peirce's pragmatism and philosophy is one-sided and lacks philosophical depth, he conveys a good and reliable picture of what moved people at the time.

[4] I want to thank Prof. James Liszka, University of Alaska at Anchorage for pointing out to me that the thesis of this paper might be understood as a genetic claim about the invention of pragmatism. However, I am not interested in historical or genetic questions but in conceptual and logical issues which connect pragmatism with the rest of Peirce's thought.

[5] "DNLR" refers to "Description for a Notation for the Logic of Relatives, resulting from an Amplification of the Conceptions of Boole's Calculus of Logic". "W X", where X is a number between 1-6, refers to Writings of Charles S. Peirce. A Chronological Edition, cf. bibliography.

[6] CP 8.33 refers to the eighth volume and section 33 of the Collected Papers of Charles Sanders Peirce, cf. bibliography.

Of course, in pure mathematics some such general solutions are possible. However, they tend to be trivial and soon give way to another class of general solutions which are much more fruitful. Namely, those results in which "a general solution points the way to the particular solutions" (CP 3.514). And the reason for this crucial role of the particular mathematical solutions is an epistemic one. Even in mathematics, all understanding is in terms of a specific particular solution that gives us a "picture" – or as Peirce later will say – a diagrammatical representation: "... for it is only the particular solutions which picture to the mind the solution of a problem; and a form of words which fails to produce a definite picture in the mind is meaningless" (CP 3.514).

II. The Mathematico-Logical Background of Peirce's Pragmatism

The reason why general questions in mathematics and logic can only be solved in a satisfactory way if they are understood in terms of some specific, individual problem is an epistemic and semiotic one: For reformulating general formal questions in terms of particular cases is a cognitive activity, similar to a transformation joining symbolic and indexical signs that interprets some individual, particular case in terms of the concepts relevant to the question to be answered. And only such a "definite picture" of the solution will count as valid and legitimate. So this is the pragmatic methodological background from which Peirce attacked Schröder's thesis that the solution of a mathematical problem will have to consist in a single algebraic expression, and that such an expression is the most general formula that solves a mathematical or logical problem. In particular, Peirce denies that there ever is just one form of expression of a mathematical problem. He argues in the following way:

> To seek a formula for all logical problems is to ask what it is, in general terms, that men inquire. To answer that question [is to] ask what the essence of a question, in general, is. Now a question is a rational contrivance or device, and in order to understand any rational contrivance, experience shows that the best way is to begin by considering what circumstance of need prompted the contrivance, and then upon what general principle its action is designed to fill that need. (CP 3.515)

Even very general theoretical questions are controlled by some demand for a process of thought that is capable of fulfilling an interest or, as Peirce insists, some want or need. In this context he distinguishes two kinds of need:

> • First, there is a need for *generalization.* Generalization is required when we want to express in one proposition a new property common to a number of different situations or occasions that were experienced because our deliberate actions brought that property about. In this case we have experienced a new property instantiated in a large number of such occasions. Or as Peirce puts it, a new idea which is the consequence of a number of different antecedents has to be captured by a concept of what unifies the antecedents of its occurrence. In this case we reduce the "multitude of subjects" by a new hypothesis.

> • Second, there is a need for *theory*. This need arises when we encounter a new situation that possesses a large number of new predicates that can be explained *only* if they are described as the consequence of an hypothetical state of things.

In this case we need to introduce a theory to reduce the number of new predicates by explaining them as consequences of this theory. From the distinction between the demand for generalization and theory it follows that if all growth of knowledge falls into one of the categories, it is always the case that the introduction of a new knowledge-enlarging predicate will concern epistemic relations to a finite set of single cases.

This relation to single cases, in turn, requires transforming the solution into some indexical statements. They will perform an epistemic control function over the general theoretical version of the solution reached because only indexical statements express propositions which are true and false because they refer to individuals. Indexical statements are a way to check the empirical content of a theoretical claim or establish some other interpretative relation between an individual fact and a theoretical problem, claim or open question. In fact, Peirce argued that the empirical meaning of any sort of theoretical statement can be determined by looking for indexical, experiential confrontation with the individual objects the theory is about. He insisted: "This is a fact that every reader of philosophy should constantly bear in mind, translating every abstractly expressed proposition into its precise meaning in reference to an individual experience" (CP 3.2.315, 1903). This empirical translation thesis has been misunderstood as expressing a sort of verificationist theory of meaning. Quite to the contrary, I think that this passage rather points out that pragmatism highlights a universal methodological feature of all human thought and understanding: in the humanities, in philosophy as well as in the hard sciences the relation between theoretical thoughts and individual cases has a decisive and creative role to play.[7]

Peirce's response to Schröder's research project to look for the most general formulation of a problem, is the telling objection that "... special solutions are the only ones which directly mean anything or embody any knowledge; and general solutions are only useful when they happen to suggest what the special solutions will be" (CP 3.516). This is pragmatism applied to mathematical reasoning. As has been pointed out in the literature, Peirce opposed logicism and did not look for any logical foundation of mathematics. The discussion of Schröder's work makes it evident that Peirce thought that the pragmatic methodology is relevant for mathematics and formal logic. Nevertheless, I think that it is philosophy that has a lesson to learn from the problem mathematics confronts if it's theories become too general. For in Schröder's case it is the relation of interaction between theoretical thought and specific individual cases that is replaced by a sterile, methodological ideal of pure generality.

For a better understanding of Schröder, let us look at the converse of the relation between pragmatism and mathematics. One would ask: What are the modes of reasoning, concepts and principles that may be understood as coming from mathematics and being applied in pragmatism to the effect that they support the crucial role of the "special solutions"? Or, for short: What is the mathematical and logical background of pragmatism? Do mathematical and logical concepts and modes of reasoning from the logic of relation indeed provide a better understanding of the relation on which pragmatism relies on?

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[7] For more on this topic see my paper "Searching for Traces. Towards a Peircean Account of an Indexical Methodology for the Humanities".

To do so these mathematical concepts must be able to be used to back up pragmatism's claim to give us a viable account of thought and knowledge acquisition that describes some of the crucial relations by which knowledge acquisition is guided and successfully executed.

In the scope of this paper I cannot discuss all the issues that a comprehensive account of pragmatism's logical backdrop would require.[8] I will start with a short account of why some mathematical and logical ideas are relevant for pragmatism. That is to say, why they support, explain and even strengthen some of the claims implicit in Peirce's pragmatic maxim. That is to say, I will show how some mathematical concepts and rules of reasoning acquire a practical role because they become part of pragmatism's methodology and epistemology. In particular this holds for the normative role of a semiotic conception of the interpretational identity of an object of different beliefs. In a concluding step I will trace out some of the consequences of the pragmatistic account of thought for the theory of interpretations in general.

III. Pragmatism, Relations and the Algebraic-Relational Notion of Identity

This is how in 1878 in "How to make our ideas clear" Peirce formulates what is nowadays called "pragmatic maxim" (PM). But keep in mind that at the time he had good reasons to call it a "rule for attaining [...] clearness of apprehension". So, at the moment of its birth, pragmatism was phrased in the imperative, as a methodological advice: "Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object" (CP 5.402).

The most important and most misunderstood part of this rule is the crucial concept of an object. What the pragmatic maxim proposes is a normative suggestion, a rule of thought and imagination. It entails the claim that this is the most efficient way in which we should go about, if we want to achieve clarification of our thoughts. Understood as a piece of methodological advice implying some strategy, this is what PM says:

> Replace your thought about an *object* by an account of the *object* phrased in terms of its practical effects by constructing - in your thought and imagination - a relation between your object of thought and your object's practical effects that allows you to connect both types of belief. Then you will be able to clarify your theoretical thought about this very object.

Note that, if we use the 1897 terminology of the objection against Schröder, PM requires us to construct both a generalization and a theory. Furthermore, it is a relation preserving the object that connects both generalizations, that is to say, the many occasions of introducing a new predicate. And which allows introducing a new theory, that is, a connection that relates a number of predicates to one object.

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[8] I have discussed the role of normative commitments in "Pragmatism and the Normativity of Assertion".

Let us start with the question of the relation of "the object of our thoughts" to the "conception of the object's practical effects". To put it bluntly, though not quite correctly, what is at stake here is that Peirce's pragmatism claims nothing more than the interpretation of the identity relation between the objects of theoretical thought and the objects of practical belief in terms of concepts of an ordering between these beliefs. Now, the formal properties of order relations Peirce described in the algebra of relations.

Already in his early epistemology papers in 1868 Peirce had claimed that there can be only one general form of the rules of valid reasoning that holds for all inferences, if this rule is to transfer semantic validity. He argues for this in the following way: Consider a premise *P* that truly represents a state of things and a conclusion *C* that truly represents a state of things. There is no common term in the formulation of *P* and *C* that would allow us to deduce *C* from *P*. Obviously, in order to show that *C* follows from *P* we need a second premise, *Q*. Now, whatever *Q* will say, it will always have the same semantical force. Peirce argues that *Q* will "virtually assert that every state of things such as is represented by *C* is the state of things represented in *P*" (W 2, p. 221). So all syllogistic inferences must have at least the property of transitivity that the so called principle of syllogism

$$[(P \Rightarrow Q) \;\&\; (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)]$$

expresses.

But transitivity alone will not suffice to adequately require that a relation will capture the semantic validity of a true belief applicable to the same individual object. What is needed is something more demanding: It is a relation we nowadays call an ordering relation. That is to say, a relation that is transitive, reflexive and anti-symmetric. The individual objects of our beliefs will not be preserved, if these beliefs are connected by any arbitrary transitive relation. We need some feature that characterizes relevant relations between beliefs that establish an epistemic connection between the objects of our experience.

Furthermore, we have to make sure that relations between beliefs cover semantically trustworthy areas of belief and that they are at least truth-preserving. In 1870, in his first comprehensive study of the Logic of Relatives (DNLR, W 2, p. 359-429), Peirce discovered a way to meet some of these demands for the case of theoretical and practical beliefs. In the algebra of relation the basic inferential relation is inclusion or what Peirce also calls "illation". Here he defines identity by the inferential relation and this basic inferential connection expressed by "if … then", constitutes an order relation for which transitivity, reflexivity and anti-symmetry hold, if they subsist in the same properly selected universe of discourse. If inferences are such order relations in a properly selected universe, we define identity from within the inferential relation in terms of inclusion.

Peirce provided a mathematical counterpart of such a definition in DNLR, where he says: "equality is the conjunction of being as small as and its converse. To say that x = y is to say that x -< y and y -< x." And he adds in a footnote: "Now all equality is inclusion in, but the converse is not true, hence inclusion in is a wider concept than equality, and therefore logically a simpler one" (W 2, 360 and fn. 1). If the converse does not hold, this means for the pragmatic relation between theoretical and practical beliefs that we cannot define the inferential relation by an equality relations between

objects only, though it is possible to infer equality relations from inferential relations in a properly selected universe of discourse.

This, then, is the logical background of pragmatism in the logic of relations: In the algebra of relations we can define the identity of an object in terms of the equality of an object in an order relation such as "inclusion in", if we assume an epistemic stable selection of a universe of discourse. Now, some inferential relations may exhibit some or all properties of such order relations. Viewed from the logic of relations, pragmatism is designed as a methodology that tells us how to structure the inferential relations between our thoughts and beliefs in such a way that they embody an order relation that leads our theoretical projects from theory to the practice of action and perception.

It might be objected that the condition of a "proper selection of a universe of discourse" in describing the logical properties of the relations required by pragmatism spoils the picture. The concept of a universe of discourse is often seen as committing a serious logical mistake: In its intuitive, vague idea of an unproblematic "suitable" selection, and in the Boolean tradition to which Peirce belongs, it occupied the place of a quantification theory. But Peirce developed quantified predicate logic in his own style. In doing this, he develops Boole's selection of universes of discourse into a sort of implicit quantification equivalent to second order quantified predicate logic.[9] Nevertheless, implicit quantification builds on Boole's idea of selecting universes of discourse. For example, in Peirce's graphical logic he introduces a two dimensional surface, the "sheet of assertion," as a sign for the semantical and logical relations between the objects of representations. Now, the sheet of assertion is something the interpreter and the author perceive: It is a sensibly accessible object they are both related to with their thoughts and their perceptions.[10]

Although we cannot discuss this implicit approach to quantified formal logic by means of a shared visual sign here in more detail, I think that this provides a paradigm for the way in which formal properties of relations can be transformed into normative conditions for a fruitful epistemic practice that relates thoughts and experience. In this way, in requiring a person to connect thought and practical experience of a surface identified as a sign, a universe of discourse is selected: It is the universe of signified and shared epistemic beliefs to which persons with normal cognitive abilities and knowledge have experiential access if they look at all the visual signs presented by the surface.

In particular, the epistemic threshold of theoretical thought in a person is constituted in such a way that relations between beliefs relevant for it satisfy the conditions

1. for keeping up the identity of an object of these beliefs effectively,

and

2. of connecting theoretical beliefs with a semantically relevant universe of beliefs, namely beliefs about action or perceptions.

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[9]   Cf. ZEMAN, J. Jay. "A System of Implicit Quantification".
[10] Peirce's revolutionary idea to solve the task of quantification by a sign in the form of a two-dimensional surface that embodies an equivalence of order I discuss in more detail in "The Pragmatic Logic of Ordered Representations".

But beliefs about the identity of objects of action and perception are what Peirce means by his conception of "practical bearings" in PM. And they are decisive for what we take for real, that is, what the state of things is like, independent of our thoughts about it.

What does this belief-dependent extension of the relation of identity do for a better interpretation and understanding of pragmatism? In what follows I will reject some of the standard beliefs about pragmatism. According to some standard views, pragmatism gives us a *theory* of meaning and truth that explains meaning in terms of practical consequences and truth in terms of their verification. It will suffice to concentrate on two points of disagreement with these standard views, the first negative and the second positive. The most important is perhaps the first one:

> (i) Peirce's pragmatism formulates *no* explicit theory of meaning. It does not commit itself to any general claim of what the nature of e.g. linguistic meaning consists. In particular, it does not claim that the meaning of a theoretical sentence has to be reduced to the verification in terms of its practical consequences, perceptions and actions. If the logical properties of order relations constitute the pragmatic clarification of meaning, it is not possible to identify meaning with the practical consequence themselves.

And

> (ii) what Peirce's pragmatism proposes is a methodological strategy for establishing a relevant and selectively ordered relation between theoretical and practical beliefs about specific actions and perceptions that are supposed to clarify the theoretical beliefs.

The standard views about pragmatism are correct only to this extent: Peirce's pragmati-sm stresses the *theoretical* meaning and the relational position of *prac-tical* consequences as end points of explicative processes. What Peirce was interested in when he proposed the PM was a methodological rule that advises us to create a connection between different types of beliefs that we are well advised to rely on if we want to enlarge and clarify our thought and knowledge about the issue in question. The relevant beliefs so connected are theoretical, fairly general and therefore risky beliefs on the one hand, and practical, more concrete and reliable beliefs about perceptions and actions on the other. To clarify a theoretical belief, you are advised to interpret it in terms of its possible consequences or applications. But if finding or construing reliable connections between practical and theoretical beliefs can be turned in inferentially governed cognitive process, we can say what it means to clarify our "high-risk" theoretical thought in terms of the more concrete beliefs about applications or about some practical matters in a fruitful, explanatory manner.

IV. Logic as a Normative Science: Pragmatism and the Normative Status of Identity

Pragmatism, so far, has turned out to be a methodological thesis based on the order-theoretic definition of identity in terms of rich, and relevant inferential relations. However, the whole approach hinges on the possibility of identifying the cognitive relations between theoretical and practical beliefs with a sufficiently strong inferential relation that satisfies, at least sometimes, these order-theoretic properties.

To justify this interpretation of pragmatism two lines of development suggest themselves: First, to describe a logic of identity of objects for situations when that there

is a relation between thought or beliefs, where one belief interprets the other and show that this interpretation relation has the order-theoretic properties required. This might be called a logic of interpretative identity. The second line of development, already hinted at above, is to understand logic, including methodology, as a normative discipline describing our cognitive practice. Now, among other things, a normative logic will understand both the identity of an object of our beliefs and the beliefs themselves as assuming, under suitable conditions, a normative status. The task of these normative requirements is to make sure that we handle our beliefs in such a way as to make sure that the internal conditions of the relations between beliefs guarantees that the order-theoretic properties of the inferential relations are satisfied

Peirce did pursue both lines of development, but I can only give you a sketch of some of his basic ideas. First, let us take a look at the logic of interpretative identity. The identity relation between different objects of a number of beliefs interrelated by an antisymmetric interpretation relation allows for a complex notion of identity. Assume we have two theoretical beliefs P and Q and some individuals denoted a, b, c. Now P that implies that

$$a = b.$$

Our second belief, let us say about the application of P to some practical circumstances, implies that

$$b = c.$$

In this setting we have two valid identity statements. If P is an interpretation of Q and because they both imply a identity relation with b, it follows by transitivity that

$$a = c$$

also holds. The identity relation between beliefs allows further inferences about the individual objects a, b, c and about any other object we succeed in identifying them with. So in order to develop an adequate account of the role of the identity of the object in interpreting a series of beliefs we need at least a conjunction of two identity statements such as

$$a = b \ \& \ b = c.$$

It is this conjunction that allows us to draw further inferences from interrelated beliefs with regard to the identity of a, b, and c. For Peirce, the conjunction of two informative identity statements introduces a new aspect of the identity relations involved. This conjunctive identity describes new epistemic situations where two identity statements are represented as connected. Therefore, Peirce introduces a new name for this sort of complex identity. He calls this new complex concept of identity "teridentity" or "triadic identity": "It is identity and identity, but this "and" is a distinct concept, and is precisely that of teridentity" (CP 4.516).

The crucial point for our discussion is that teridentity is the identity of an object that we understand only if we succeed in interpreting and connecting thereby successfully at least two different beliefs. But this is the very structure of a relation between beliefs, which the pragmatic maxim asks us to create. Namely, a relation between the very object of our theoretical thought and our practical beliefs. Or, to put it differently,

pragmatism recommends a connection between generalization and theory that expands our understanding of some belief we started from. To see this more clearly let us take a look at an example Peirce suggests.

> I see a man on Monday. On Tuesday I see a man, and I exclaim, "Why, that is the very man I saw on Monday." We may say, with sufficient accuracy, that I directly experienced the identity. On Wednesday I see a man and I say, "That is the same man I saw on Tuesday, and consequently is the same I saw on Monday." There is a recognition of triadic identity; but it is only brought about as a conclusion from two premisses, which is itself a triadic relation. If I see two men at once, I cannot by any such direct experience identify both of them with a man I saw before. I can only identify them if I regard them, not as the very same, but as two different manifestations of the same man. But the idea of manifestation is the idea of a sign. Now a sign is something, A, which denotes some fact or object, B, to some interpretant thought, C. (CP 1.346)

Peirce uses this example to argue that triadic identity is the relevant notion of identity, if we want to understand informative meaning relations between at least two different beliefs that can be true independently of one another but involve the same object. Triadic identity depends on an inferential relation or a sign-relation between our beliefs and our ability to establish a relation of sign-interpretation that connects two identities into a third one.

This, by the way, also allows for an implicit characterization of what Peirce may have meant by the expression that a real object is "independent of our thought": Its identity can be understood in terms of a triadic identity relation connecting theoretical beliefs with some beliefs about practical effects.

These are, to my mind, important results. For example, we can see now that it holds for Peirce's pragmatism that the pragmatic maxim can be seen as the result of creating an inferential-ordertheoretic methodology based on special requirements for the identity relation between explanatory beliefs. Namely, for those beliefs for which a general theoretical thesis needs clarification. And from this order-theoretic view of the logic and identity of the objects of explication and explanation it follows that a characterization of any theoretical thought by an empirical content can only be arrived at by finding an identity relation in a relevant interpretational context of knowledge about practical effects relevant for action. That is to say, we will look for a possible triadic identity relation in concrete cases involving objects which might, in an informative way, be said to be the same as the objects we had thought about before.

V. Pragmatic Interpretations: A Semiotic Account of the Objects of Thought

Let us take a last and short look into the second line of development. How does the claim that logic is a normative science influence the concept of an object of belief? As we saw above, Peirce understands the PM as a normative methodological principle for discovering or constructing identity relations between the objects of theoretical and practical beliefs. Late in his career Peirce realized that he also generalized this normative feature into a feature that applies to all logic. So logic became a normative discipline. And, of course, the construction of identity relations between the objects of our relevant beliefs became a normative requirement. Or, as he phrased this once, it became the

"ethical" aspect of logic. This is what he said in 1908:

> We think in signs; and indeed meditation takes the form of a dialogue in which one makes constant appeal to his self of a subsequent moment for ratification of his meaning in respect to his thought = signs really representing the objects they profess to represent. Logic therefore is almost a branch of ethics, being the theory of the control of signs in respect to their relation to their objects. (PEIRCE in a letter to P.E.B. Jourdain, 5.12. 1908, NEM,[11] v. III, p. 886)

What does it mean to describe logic – which in this context is tantamount to semiotic – as "the theory of the control of signs in respect to their relation to their objects"?[12] This passage can best be understood if we read it as the contention that in every sequence of signs a principle of semiotic identity (PSI) is at work. This principle may be stated in the following form:

(PSI)   For any sequence of signs, if some sign S_2 acts as an interpretant of another sign S_1, then there is one object b for which they both stand if and only if it holds that S_1 allows us to identify b in one aspect a, so that $a = b$ is true and S_2 allows us to identify b with another aspect c, such that $b = c$ is true and it holds that these identity relations in which b occurs are a product of the sign relation.

If (PSI) holds for some signs, Peirce's triadic sign-relation or semeiosis (e.g. "A represents B for C") consists in those cases in the fact that a sign and its interpretant are related to one another because they represent the same object. That is to say, (PSI) can be seen as describing what it is for a sign and its interpretant to stand for one and the same object because of the sign's interpretation: the sign in itself and its interpretant constitute a complete sign, if and only if, they relate to the same object although with regard to different aspects of it.

Of course, the identity created by a sequence of signs is teridentity or triadic identity. To create and to continue a sequence of interpretations that reveals the teridentity of its objects is to pick out an object *as identical within this* process. The ontological assumption here is that it is not the single interpretations, e.g. single assertions, which allow us to identify the individuals correctly. Rather, it is the *logical form of a sequence of interpretations with identical objects that allows us to represent independent objects more and more comprehensively.* Much of Peirce's argument for his version of pragmatism is based on this idea of using the order-theoretic characterisation of the identity of objects as a normative requirement for the growth of knowledge and its approximation of an independent reality.

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[11] "NEM I-IV" refers to The New Elements of Mathematics by Charles S. PEIRCE. Ed. Carolyn Eisele. The Hague; Paris, 1976. Four volumes in five.

[12] The other part of this passage restates an old thesis of Peirce – adapted from Plato – that thinking is a sort of dialogue with one's future self.

VI. The Mathematical Background of Pragmatic Order: Order Relations and the Comparability of Meaning

I pointed out above that pragmatism is not a theory of meaning as the standard interpretations claim. In fact, Peirce pointed this out time and again. For example, in 1905 (CP 5.8) he said about pragmatism that "it does not undertake to say in what the meanings of all [representations] consist, but merely to lay down a method of *determining* the meaning of intellectual concepts, that is those upon which reasonings turn." And indeed this is what we saw above, namely that pragmatism is an account of a method how to construct meaning from within the network of inferential connections. That is to say, how to make sure that a consequence of beliefs gives us a specific and semantically relevant result for evaluating this theoretical belief. As in Peirce's argument for the semantic priority of specific mathematical solutions, determining meaning is always a question of how to combine and establish a balance between generalization – the common property of a number of independent cases – and theory, of introducing a theoretical predicate that allows to develop a space for a number of independently established specific predicates. Consequently, pragmatism becomes a method for comparing beliefs and placing them into a structure of interpretative relations that exhibits their content, or meaning.

What is the specific mathematical theory that supports a reading of pragmatism that understanding the identity of objects of beliefs is a special case of an inclusion relation, e.g. the relation between the compared contents of a sequence of thoughts, perceptions and actions? The modern mathematical background for this is, I propose, what is nowadays called "lattice theory" and what Peirce developed in his algebra of relations. One way to characterize lattice theory is to describe it as a mathematical theory which allows you to combine "ordering" and "connecting" as two ways of describing structured sets. Seen from an ordertheoretic perspective, a lattice is a set ordered e.g. by inclusion, such that for every two members of this set there is a lowest limit, an infimum, and a highest limit, a supremum. Now, suppose you apply this to beliefs. Then two beliefs that can be ordered in a lattice are comparable to one another, e.g. with respect to their practical results. Lattice theory introduces a generalization into the way in which we may describe structures. In this way it becomes possible to describe even conceptual structures in terms of lattice theory. And this is one of the reasons why in mathematics one can use lattice theory to describe structured sets coming from completely different areas, e.g. topology, algebra, logic or analysis.

Indeed, the common ground between pragmatism as a methodological theory and lattice theory is so strong that only two years after proposing the pragmatic maxim, Peirce in 1880 developed what we today would call the concept of a lattice in the paper *On the Algebra of Logic*. He did so (W 4, p. 18-88) by taking the set of all propositions for which disjunction and conjunction holds and by describing this set by associativity, commutativity and idempotency. Such a characterization is equivalent to an algebraic definition of an abstract lattice. But this only made explicit what ten years before he had described in DNLR, namely the properties of ordered relations that constitute identity and difference for objects of belief. This is, I propose, the most general and important logical background for Peirce's variety of pragmatism by insisting on the fruitfulness of interpreting mathematical and logical forms to allow for "special solutions".

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