

Continuity and Discontinuity in Boundary Issues

Continuidade e Descontinuidade nas Questões de Fronteira

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Abstract: Notwithstanding the little importance given in the commentaries to the statute of boundary elements worked by Peirce, this theme remains present during more than a decade in Peirce's writings. Always returning to the same examples such as the colors to be attributed to the boundary elements of two adjacent surfaces, one red and the other blue, or the present moment in the flow of time, Peirce progressively advances the best theoretical explanation he can find to this so intriguing question. After a long inquiry involving the statute of the continuum, Peirce finally takes a very personal position defending the infinitesimal nature of the boundary element due to its affirmative potential mode of being. Assuming this thesis, Peirce can reread Aristotle, redistributing the roles to be conferred to matter, form and entelechy, as well as the true concept of continuity.

Keywords: Boundary. Border. Continuity. Aristotelicity.

Resumo: *Malgrado a pouca importância dada pelos comentários ao estatuto dos elementos fronteirços trabalhado por Peirce, esse tema permanece presente durante mais de uma década nos seus escritos. Retornando sempre aos mesmos exemplos tais como o das cores a serem atribuídas aos elementos fronteirços de duas superfícies adjacentes, uma vermelha e a outra azul, ou o do momento presente no fluxo do tempo, Peirce progressivamente faz avançar a melhor explicação teórica por ele encontrada para essa tão intrigante questão. Após uma longa investigação envolvendo o estatuto do continuum, Peirce finalmente toma uma posição muito pessoal defendendo a natureza infinitesimal do elemento fronteirço devido a seu modo potencial afirmativo de ser. Assumindo essa tese, Peirce pode reler Aristóteles, redistribuindo os papéis a serem conferidos à matéria, à forma e à entelêquia, assim como o verdadeiro conceito de continuidade.*

Palavras-chave: *Fronteira. Borda. Continuidade. Aristotelicidade.*

Introduction

Since “A Guess at the Riddle” at the earliest, a text written by Peirce between 1887 and 1888¹, the possibility to reiteratedly represent a boundary element, and under aspects

¹ The date given when the text was written is supported by Houser, Nathan & Christian Kloesel, EP1 p. viii. It is worth noticing, however that in the Collected Papers of Charles S. Peirce, the text is dated 1890.

not always coincident in that author's work, was tried by him. Without any pretension to exhaust the textual research, his works contain texts about the theme, dated until 1905, covering, thus, a period of at least fifteen years. If I am not mistaken, however, little attention has been given by Peircean scholars concerning this question. An exception to the rule is Antônio Machuco Rosa's thesis, whose title is *O conceito de continuidade em Charles S. Peirce*², which, in two instances in its explanation, approaches the concept of boundary or "border" according to Peirce. At first,³ when dealing with the gamma part of the Existential Graphs, Rosa mentions that, according to Peirce, this part of his logic enables one to cross the boundary from the domain of necessity to the domain of possibility through an identity line, traversing a cut, exactly because, in the case under consideration, it is not about a simple confrontation of the principle of identity, when one would assert and deny a certain predicate of the subject at the same time. The point in question is that one tries to modalize as impossible the attribution of a certain predicate to a subject. At a second moment, exactly under the subtitle *The Concept of Boundary*, he characterizes the problem of ascribing contrary predicates to elements pertaining to boundaries. After specifying the several solutions, which are apparently contradictory among themselves, and which are present in Peirce's text, he emphasizes, in the vagueness of a subject of attribution pertaining to a boundary, the reason for the indeterminateness immanent to this subject's predication of a certain predicate or its negative. Even the authors who have devoted themselves to the study of Existential Graphs or those who address mainly the vagueness that is intrinsic to signs, according to Peirce, do not seem to have paid special attention to this aspect, which, though briefly, is exposed and discussed by Machuco Rosa.

Prior to Machuco Rosa's text, in an essay published in 1979,⁴ Carolyn Eisele makes a brief reference to Peirce's concern about the logical statute of the boundary elements. Although the aforementioned essay outlines Peirce's contributions to Mathematics and, as expected, is concerned with presenting Peirce's conception of the continuum, different from the Cantorian conception and, thus, highlighting the adoption by Peirce of the infinitesimals in the constitution of the real continuum, it refers in a very restrictive manner not to the merit, but to the opportunity for Peirce's reiterate concern about the statute of the boundary elements. The text says:

Peirce fusses with boundary problems throughout his topological studies. They bring to a third kind of existence in which an element on the boundary is not quite this nor that, but a bit related to both. This may account in part for his interest in the no metrical aspect of topology, for he had criticized Cantor for making his work depend upon "metrical considerations".⁵

² MACHUCO ROSA, A. *O conceito de continuidade*.

³ P. 106 -107.

⁴ EISELE, Carolyn. *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce*. Ed. by R.M. Martin. Paris; New York: The Hague, 1979.

⁵ P. 271.

Resorting to the verb *to fuss* to characterize the Peircean concern, what was possible to infer from the author's intention, who so diligently devoted her time and outstanding mathematical knowledge to study and publish Peirce's texts⁶ and texts about this author, is that the reiterate approach made by the author did not give any contribution to the advancement of knowledge on this subject.

However, she did not shun from showing subsequently that Peirce, as late as 1909, in a letter to William James,⁷ mentioned to be developing a three-value logic in which one of them contemplated, as the Peircean text allows us to understand, the specific case of boundary elements.

In fact, Peirce distinguishes in this letter the potentiality in three ways of being, assigning to each of them a specific logic statute: the "may-be" would consider the potential as a result of the insufficiency of knowledge; the "can be" would restrict the attribution of the subject's predicate to potentiality, due to the insufficiency of the grammatical Subject; and the "might be" to the insufficiency of the circumstances.

About potentiality, due to the insufficiency of knowledge to be able to predicate the subject in an accurate manner, when in English one categorizes the coupling under "may be", leaving an open field to "may-be" or "may-be not", provided that it is possible to predicate P from S, or non-P from S, Peirce will say what he reiteratedly said about the indefiniteness of predication to boundary elements, namely, and in such case, the principle of contradiction does not apply.⁸

We can conclude that, at such a late date, the logic statute of the boundary elements continues to be of great importance to Peirce, regardless of it being not justified, although the author provided different answers to characterize it at several stages. In fact, we could not be astonished if, to this three valued Logic, it would correspond to the Gamma part of Existential Graphs in its more advanced version.

Peirce's Texts

"A Guess at the Riddle" (1887-88), as mentioned, brings a double testimony: it raises the point of the boundary elements of identification, something which is directly or indirectly dealt with by Peirce's works until the last phase of his writings, as detected in the letter to William James, in December 1909, and still holds one's attention in search of a reasonable neurophysiological explanation for the perceptive phenomenon of such boundary elements.

In this text, as well as along the whole aforementioned article, Peirce approaches the principle of direct consciousness of Secondness, referred to as the *Polar Sense*. Certain joint discharges of nerve-cells would react, in a hypothetical manner, to the presence of a certain sense. It would result, thus, in our capacity to temporally discriminate

⁶ It is worth mentioning, as it is known, the edition of *New Elements of Mathematics* of Charles S. Peirce, in 4 volumes, where the third is divided into two tomes, in which Carolyn Eisele compiles an expressive collection of Peirce's mathematical and logical texts, collected from the manuscripts not published until then.

⁷ Letter dated December 25, 1909; published in EP2, p. 500-502.

⁸ EP2, p. 50.

the “before” and the “after” and, spatially, after a certain obscure perception, to distinguish among spatial boundary elements,⁹ like the red and blue colors that covered, respectively, adjacent surfaces. Therefore, both distinctions would occur without the intervention of a third element.

Dated from 1893, there are two significant texts about the boundary statute. Without seeking to establish an order of chronological origin among them, the first one,¹⁰ which is part of a longer manuscript entitled *Philosophy of Mind*, introduces, in the mind level, the concept of continuity when it says that “we naturally make all our distinctions too absolute. We are accustomed to speak of an external universe and an inner world of thought. But they are merely vicinities with no real boundary line between them”. It breaks, thus, the classic dichotomy that made it so difficult to establish the possible conditions of knowing what was not inside the “I think”. In fact, it starts introducing a phenomenological approach that, in the following century, would characterize all his philosophy and, one could say, launched the bases of the dialogic and semiotic knowledge statute, which became so well established in 1905, in the well-known passage “What Pragmatism is”, when he wrote

Two things here are all-important to assure oneself of and to remember. The first is that a person is not absolutely an individual. His thoughts are what he is “saying to himself”¹¹, that is, is saying to that other self that is just coming into life in the flow of time. When one reasons, it is that critical self that one is trying to persuade; and all thought whatsoever is a sign, and is mostly of the nature of language. The second thing to remember is that the man’s circle (however widely or narrowly this phrase may be understood) is a sort of loosely compacted person, in some respects, of a higher rank than the person of an individual organism.¹²

The elimination of a presumed rigidity of boundaries between the internal and the external world of mind, as will be more evident later on by the intervention of signs, preserves, at the same time, the distinction experienced between the impact of the external world and the almost exteriority of the imaginary experiences, which even produces concepts which are eminently eidetic in nature, like the mathematical idealities. To such considerations, the text dedicates the following paragraph:

Experience is double, as much as reality is. That is, there is an *outward* and an *inward experience*. Under this latter head ought particularly to be reckoned a mathematical experience, not usually so called, which has compelled the development of pure thought to take a determinate course.¹³

⁹ 1.386-390

¹⁰ CP 7.438-440.

¹¹ It is interesting to note that in CP 4.421, instead of “saying to himself”, the text says “saving to himself”. Although both versions make sense, the present paper adopted the Essential Peirce version, due to the fact that this edition is the last one, being elaborated in exceptionally better conditions than the first.

¹² EP2, p.338.

¹³ CP 7.440.

The second text,¹⁴ taken from *Elements of Logic*, when dealing with the correlation between the depth and breadth of the concepts, whose product is, according to Peirce, called Information, considers some cases in which that product either remains unchanged or has a zero value. The information would be null, in that case where the attribution subject (S) were situated in the border between two domains and, therefore, would remain indeterminate, comprising, we could say, separately, the P and non-P predicates. And the explanation for it would be the following:

If S is a particular term, it may have no breath, and then adds nothing to the breath of P. The latter case often occurs in metaphysics, and, on account of non-Pas well as P being predicated of S, gives rise to an appearance of contradiction where there really is none; for, as a contradiction consists in giving the contradictory terms some breath in common, it follows that the fact that, if the common subject of which they are predicated has no real breath, there is only a verbal and not a real contradiction. It is not really contradictory, for example, to say that a boundary is both within and without of what it bounds.

The next text to be considered, which comes before the discovery confessed by Peirce of potentiality as an affirmative mode of being, a discovery that would be decisive for the definite formulation of the concept of real continuum, is that text which is in the *Collected Papers*, under the title “The Immediate Neighborhood”, constitutes §7 of the *Logic of Quantity*. However, prior to 1896, it is dated from 1893 to 1895.

The continuum is preserved in order to transcend the Logic obstacle consisting of the boundary and its indefiniteness, and although not having yet found a fully satisfactory formulation, in this text it will find a significant progress.

The example that will be given of the presence of boundary elements and the conclusions that will follow hence show how close Peirce was to a solution for the establishment of the logical statute of the boundary elements, as well as to configure the theoretical obstacles to be surpassed to find a thoroughly persuasive answer. This would become possible through the reformulation of the concept of real continuum, which would place Peirce apart from the Cantorian conception of the transfinite series.

In the text under consideration, the number series, albeit infinite, remains composed of discrete elements, imposing, just like the Cantorian series, an obstacle against the full resolution of the boundary element. Peirce intuitively believed, however, that the obstacle would be overcome, when he presumed the factual and discrete non-existence of this element.

The selected fragments are believed to show the state in which the treatment of the issue was at that very moment.

Except for a better judgment, the continuum seems to be characterized similarly to that proposed by Cantor. In fact, the author said:¹⁵

My definition of a continuum only prescribes that, after every innumerable series of points, there shall be a *next* following point, and does not forbid this to

¹⁴ CP 2.420.

¹⁵ CP 4.126.

follow at the interval of a mile. That, therefore, certainly permits cracks everywhere; for there is no ordinal place in the series where such a limit point is not inserted.

Given the inherent abstraction of the mathematical entities, he concludes that the indefinite approximation of the decimal numbers does not find any logical contradiction to its peculiar existence:

... there is no logical contradiction in supposing the existence all the numbers to which decimals can indefinitely approximate to exist, i.e., as all the objects of mathematics exist, as abstract hypotheses.

A step forward towards the conception of the real continuum, in which the parts are potential, although the text states not having carried out the adequate analysis of the continuum, it admits that the numbers are not part of the continuum and, without breaking it, they are only separated in the borders. He states, though, that “the incommensurable numbers taken by themselves do not constitute a continuum”.

An example then takes place where the boundary elements are logically characterized, stating the conclusion of their factual inexistence, to the exact extent that they belong to the continuum.¹⁶

A drop of ink has fallen upon the paper and I have walled it round. Now, every point of the area within the walls is either black or white; no point is both white and black. This is plain. The black is, however, all in one spot or blot; it is within bounds. There is a line of demarcation between the black and the white. Now I ask about the points of this line, are they black or white? Why one more than the other? Are they **(A)** both black and white or **(B)** neither black nor white? Why **A** more than **B**, or **B** more than **A**? It is certainly true, first, that all the points in this area are black or white; second, that no point is at the same time black and white; third, that the points of the boundary are not more white than black, or more black than white.

Thence it follows that:

The logical conclusion from these three propositions is that the points of the boundary do not exist. That is, they do not exist in such a sense as to have entirely determine characters attributed to them for such reasons as have operated to produce the above premises. This leads us to reflect that it is only as they are connected together into a continuous surface that the points are colored; taken singly, they have no color, and are neither black nor white, none of them.

In a letter to William James in 1897,¹⁷ Peirce will characterize the quality expressions by a certain vagueness, as they abstract a certain aspect from a complex group of aspects and, for the fact that it indicates the quality. Among other examples also mentioned in the text, there is that of attributing the red color to an object. The distinction of the concept of red as a sign of essence from its attribution to objects will show how, differently

¹⁶ § 127.

¹⁷ L 224, according to the numeration of Robin. Printed in NEM 3.2, p.799.

from the first sense, it will be a quality that varies infinitely according to each object, and, one could say, at each moment and in each situation of the same object. According to the text:

Everything is red or not-red; but were a thing capable of absolute determination it might be just on the border of redness, and then would neither red or not-red (or, if you please, both red and not red). Such an absolutely determinate object is a nothing. It therefore does not violate the dyadic laws of logic.

As for the qualities, we can conclude that they are potentialities and, as predicates, they abstractly represent a range of components and never determine an object in an absolute way. This margin of indetermination inherent to every quality assignable to a subject does not admit the condition so that the attribution can be real, it has to follow the principle of contradiction. This would only apply to logic abstractions.

In *The Logic of Continuity*,¹⁸ a text dated from 1898, Peirce outlines a peculiarly successful diagram so that the cosmic expression conditions of two of three categories later referred to as cenopythagorean can be genetically comprehended. Now, he recovers with a much cleaner diagrammatic form what he had outlined before, in an equally successful manner, but a logically constructive one, the genesis evolution and predestination of the cosmos, in “A Guess at the Riddle”.

Step by step, he presents to the reader the constructive phases of the multidimensional continuum, projected on the blackboard. What is presented bi-dimensionally must be understood as representing multiple dimensions, from a blackboard's total potentiality where nothing contrasts with it,¹⁹ and in which no discrete point has a place and emerges at the appearance level, to the projection of a globe representing the continuity, domain of the laws that will progressively define a certain cosmic configuration. In an intermediate phase, the potential continuum is broken by the always particular brutality of a chalk stroke, from which and by which a continuum third will be configured, at this time an effective general and determined one. The boundary element is also contemplated within this huge constructive experience, and this is what this research intends to focus on. It has a place exactly in the accurate rupture of the original indifference, through the merciless chalk stroke. Nothing would follow that indifference, if the mark was not derived from the continuum, keeping the continuum quality in its form and in the space it takes. In the second, they continue to oppose, collide and delimit one another. The mere potentiality takes an act form. The text will confirm that:

There is a certain continuity element in this line. Where did this continuity come from? It is nothing but the original continuity of the blackboard which makes everything upon it continuous. What I have really drawn there is an oval line. For this white chalk mark is not a *line*, it is a plane figure in Euclid's sense – a *surface*, and the only line there, is the line which forms the *limit* between the black surface and the white surface. Thus the discontinuity can only be produced upon that blackboard by the reaction between two continuous surfaces into which it is separated: the white surface and the black surface.

¹⁸ CP 6.203-209.

¹⁹ Along the text, such potentiality will be compared to the Aristotle's concept of matter.

The border is, therefore, the proper and irreducible way of being of a mutual negative. If it were not for the vagueness denounced by the previous text in the qualities assigned to its subject, the border would simply be *zero*.²⁰

In *The Law of Mind* (1900-1901), an important step forward will be given by Peirce towards the understanding of the boundary elements in light of the real continuum theory, from the Aristotelian standpoint.²¹ This property of the continuum will be termed by Peirce, *Aristotelicity*.

The approach bestowed on the continuum-related boundary elements will presume the following conceptual elucidations provided in the text: first, what Peirce means by Aristotelianism and, second, the concept of the infinitesimal in a series of numbers, a notion that will characterize the statute of the boundary elements.

By *Aristotelicity*, Peirce will understand the property of a continuum that “contains the end point belonging to every endless series of points which it contains,” thence giving origin to the corollary that “every continuum contains its limits.”²² The continuity series would cease, therefore, after the presence of its limits.

And the concept of infinitesimal will be circumstantially presented in three stages:

First, Peirce will present every real number as a limit which an infinite series of numbers tends to where, between whichever two real numbers, it is possible to have an innumerable series of numbers or points tending toward them.

Second, he will define infinitesimals as the ordinal number that will take the infinitesimal place of an incommensurable series of numbers.

Thence it follows that “continuity supposes infinitesimal quantities” in it, which can be done by applying the addition and multiplication functions, as long as the series is not required to be denumerable²³.

In the infinitesimal continuous series, the series size will not be changed, even if the series is finite, because it would be correct to assert that \mathbf{A} , as a finite series, and \mathbf{i} , an infinitesimal, $\mathbf{A} + \mathbf{i} = \mathbf{A}$.²⁴

The infinitesimal would be, therefore, an authentic potentiality and may enclose an authentically continuous series, endowed with *Aristotelicity*.

Next, Peirce approaches the boundary elements issue so as to better establish their statute.²⁵ As an example, he will use both the case of the colors covering adjacent surfaces and the statute of the present, along the time flow.

²⁰ Cf. NEM 3: 2, p. 799-800.

²¹ CP 6.123-142.

²² § 123.

²³ Such would be the case of series of the integers, or whole numbers, which has zero as its origin. In CP 4.188, it can be read: “... the entire collection of whole numbers forms a denumerable collection. For zero is a whole number, which is not greater by one than any number, there is a number greater by one than any given whole number, and there is no number or numbers which could be struck out of the collection and still leave it true that zero belonged to the collection and that there was a number of the collection greater by one than each number of the collection.”

²⁴ § 124-125.

²⁵ § 126.

When attributing to the boundary elements the infinitesimal characteristics of innumerable series, Peirce, as he had previously shown that they were not worth the uniqueness imposed by the principle of contradiction, will state that, considering two adjacent surfaces where one is red and the other is blue, none of the parts being simultaneously red and blue, the color should extend beyond the boundary line in order to exist indeed, and the color is the color of the line's immediate vicinity and, therefore, considering the entirety, half red and half blue. With regard to time, he said:

In like manner, we find it necessary to hold that consciousness essentially occupies time; and what is present to the mind at any ordinary instant is what is present during a moment in which that instant occurs. Thus, the present is half past and half to come. Again, the color of the parts of a surface at any finite distance from a point has nothing to do with the color just at that point; and, in the parallel, the feeling of any finite interval from the present has nothing to do with the present feeling, except vicariously.

He presents, thus, another example, which can be dealt with in a similar manner. A particle's speed at a certain instant would be "its mean velocity during an infinitesimal instant in which time is contained".

The same allegation, which supports a continuum of qualities, continues in Chapter 4 of the text, whose title is the same as the previous one,²⁶ when the qualities occur within a certain vagueness amplitude, and, therefore, remaining as potentialities, they would not be subject to the principle of contradiction.

Here, the text focuses on every learning experience's temporal character, which consequently occurs along the continuous time flow. Even accepting every kind of learning as an inferential process that would follow the reasoning model, given this intrinsic indetermination, it would not be proper to ask for a certain step in reasoning as if this step were discrete and factual in its nature.

Thus, the text will say:

... every reasoning involves another reasoning, which in its turn involves another and so on *ad infinitum*. Every reasoning connects something that has just been learned with knowledge already acquired so that we thereby learn what has been unknown.

The time continuum, where learning takes place, would make, at any time, the present merge with the past and future, keeping the statute of the infinitesimals:

It is thus that the present is so welded to what is just past as to render what is just coming about inevitable. The consciousness of the present, as the boundary between past and future, involves both. The reasoning is a new experience which involves something old and something hitherto unknown.

²⁶ In CP 7.536, the text is mentioned as having an unknown date. However, it is likely to make a unit with the above mentioned The Law of Mind essay, in such a way that we could presume that it was written in 1900-01.

Learning is an ongoing process. A third, linked through the present, past and future or, in terms of consciousness, assimilating the self that comes to the present, the still undetermined and, however determinable, future non-self.

During the same years (1900-1901), Peirce, while exposing Euler's graphic system, expresses the characteristics of a boundary point inserted there.²⁷ The text goes:

If an oval already exists cutting the space in which the dot is to be placed, the latter should be put on the line of that oval, to show that it is doubtful on which side it belongs; or, if an oval should be drawn through the space where a dot is, it should be drawn through the dot; and it should further be remembered that if two dots lie on the boundaries of one compartment, there is nothing to prevent their being identical. The negative relation appears here as entirely outside of.

As the ovals represent predicate class domains, if there is not a point that represents a particular subject, all the predicates are attributed to the universal subject class, and the existential quantification of the attribution subject will correspond to the point insertion. Then, the text will say that as there is another domain inside the domain where the subject must be inserted, attributing or not this new predicate also to it becomes something that can not be decided. The subject will be, therefore, considered as a boundary element, to which the principle of contradiction shall not apply, and must be placed on the trace that determines the internal domain of that which one intends existentially to attribute as a predicate to the subject. The same shall occur in the situation in which the point that designs the subject of the existential attribution is already inserted in a certain domain, when inside the latter, a new domain is added. Finally, the text reminds us that nothing shall prevent existential subjects graphed in the same domain from being identical, if nothing else distinguishes them, when the impossibility to decide, if any, must apply to all of them.

In *Lectures on Pragmatism* (1903), Peirce will expose his three categories in the third conference entitled "The Categories Continued." On that occasion, he will take to the consideration of the audience two degenerated forms of Third.²⁸ In relation to the first degenerated form, Peirce recalls the case of binary classifications of subdivision processes, referring to Tree diagrams. When seeking to exemplify twice degenerated forms of the third, he gave the example of a map that, due to its infinite succession of approaches, became a map for itself. This would be a similar case, Peirce would say, of a self-consciousness statute. In both cases, the approaches would tend to surpass the representation limit, reaching the represented thing through itself. Non-predicable, for it is no longer representative, one would reach the boundary element, which, according to Peirce, as it is known, is an infinitesimal pertaining to the Third continuum, to which any determined predication and, consequently, the principle of contradiction shall not apply. It can be concluded that the existing object only exists in relation to another object, it is what it is in itself, it is so only as a potentially affirmative quality.

In a long explanatory text, Peirce intends to present the fundamentals of the simplest mathematics, or, as he will designate, dichotomic Mathematics. The available

²⁷ CP 4.349.

²⁸ CP 5.70-71.

text, as the title itself suggests, is a sketch, but carefully develops several concepts, frequently seeking the Aristotelian roots, the concepts used on the date of creation of this sketch. The manuscripts are dated from 1902 or 1903.²⁹ The conceptual approach shown with Aristotle's to deal with issues clearly related to the constructive procedures of the Existential Graphs are of great interest to this research. This interest increases to the extent that it takes from Aristotle the resources to understand boundary elements well.

Two definitions and an axiom derived from the former insert the reader within the context of the Existential Graphs constructive principles and the Semiotics proposed by the author. He defines the symbol as predicable and the sheet where the register is done as an assertion sheet.

Thus, one may read:

Defintion 1. Any Blank is a symbol that could not be vaguer than it is (although it may be so connected with a definite symbol as to form with it, a part of another partially defined symbol), yet which has a purpose.

Axiom 1. It is the nature of every symbol to be blank in part.

Definition 2. Any Sheet of Paper would be that element of an entire symbol which is the subject of whatsoever definiteness it may have, and any such element of an entire symbol would be the Sheet.³⁰

Thence, such concepts approach those about Matter and Form, for Aristotle. At the moment that the text introduces a third mediatory element that enables to forward the matter to be informed, Peirce acknowledges it as the Sign and compares it to Entelechy.

The sheet of paper, as he will designate in other texts as the Assertion Sheet,³¹ is an incomplete element of knowledge, just as matter, and so is the graph, that represents form. The act of writing on a sheet of paper and the habit that leads in the puncture will be the Entelechy, which completes the knowledge.

Throughout a patient presentation of such components, the text gradually gives place to a consideration on boundary elements. The indeterminate statute in terms of predication of these elements will prove, according to Peirce's standpoint, that the Form as such is a possibility, and the matter responsible for the existence is vague, and according to the text, there is no "aversion to any contradiction." For being exactly on the border and not in its neighborhood, the boundary, punctual elements are purely existent and not liable to the predication of a certain quality or its negative. The adjacent surfaces, in their turn, when it comes to colors, will have one or another color, because they will not be pure matter, and there will be within the limits "some general regularity or law." The point, as individual, is only an existence.³²

²⁹ The text, transcribed by Carolyn Eisele, NEM4, p. 284-300, seems to correspond to one of the following manuscripts: 302 or 431, according to Robin's catalog numeration; the first, dated 1903 and the second one, 1902.

³⁰ P. 292.

³¹ Matter that formerly for Peirce, seemed to be constituted in the presence of Firstness, is now constituted, if I am not mistaken, in the element responsible for the establishment of Secondness, effectively limiting the affirmative potentiality of the Form it is, such as it is, regardless of anything else.

³² P. 293-294.

In the following year, in a text called “Topical Geometry,”³³ the critique created by Peirce focuses on the undue actuality attributed to entities that are solely potentialities. Such critique is expressed when Peirce denounces the fallacy of considering that a demonstration by absurd concludes for the impossibility of the assumption under consideration. With the absurd argument, Peirce stated, we only demonstrate “that the assumption does not have the defined character of a real fact.” It would not exclude the possibility of it being true in another context. The following examples imply the impropriety to apply to intersection points, in an exclusive manner, the properties of the adjacent places, if they are so with higher generality. It occurs on the point of intersection of two lines, the line besides the adjacent surfaces that composes it, etc. When asking which of the two predicates must be attributed to the intersection element, and when we are not allowed to deny them to both or affirm that the intersection element is half one and half the other, we must affirm them to both as true predicates at that moment. In the case of colors, or, in case of the temporal continuum instants, the same thing would certainly apply to other examples, the continuum is not broken, neither the constitution of the entirety is broken, nor compromises the elements in their indistinct potentiality, for the particular fact that one of them is existentially detached. Taking time as an example, and from it generalizing to points in a line or on the intersection of more than one line, from the line to surface, etc., the text states:

The ordinary instants of time, then, as long as that they are not actualified by events, are mere possibilities and are not actually in a lapse at all; and we shall see that it follows that the same thing must be true of the points of a line, the lines on a surface, and the surfaces within a space. Indeed, when they are actualified, they are not thoroughly independent of one another in a logical sense. For though that one that is actualified becomes independent, so far as it can be so in itself alone, yet those that are welded to it, and not being affected by the actualification of that one, remain welded to it.³⁴

As a variation to this text, Peirce will complement with the statement that for the true continua, the principle of contradiction does not apply to their potential parts. It only applies to the components of collections, provided that they are made up of elements which are independent from each other. It would certainly be the case that, if unduly generalized, would impose the well-known and challenging Zeno’s arguments and, why not, many current skeptical arguments, or a little attentive resource of the Cantorian series, in order to comprehend continuous entities.

In 1905, in the text called “Issues of Pragmaticism,” a certain paragraph fragment calls special attention.³⁵ After stating that the affirmative or negative of a predicate to a subject, whether the predicate consists of a quality or its indeterminate negative, or a couple of determined predicates, in such a way that one’s affirmative will correspond to the other’s negative, do not affect the logical and predicative properties of both, which seems to enable the understanding of the subject inclusion or exclusion from the predicate domain, the text focuses on the adjacent case in which this rule finds its limit, or, perhaps

³³ Manuscript 134, of Robin. Published in NEM 2, p. 529-529.

³⁴ P. 529.

³⁵ CP 5.450 and EP2, p. 352.

its origin. In such a case, the negative and the affirmative in the predicate attribution to the subject could not be concordantly applied. Thus, the texts states:

... but it is to be remarked that there are cases in which we can have an apparently definite idea of border line between affirmation and negation. Thus,, a point of a surface can be in a region of that surface, or out of it, or on its boundary. This gives us an indirect and vague conception of an intermediate, or nascent state, between determination and indetermination.³⁶

The exception strength brought by the statute of boundary elements is so strong to Peirce's view that, as we can be at the beginning of this explanation, in 1909, therefore, nineteen years after the first collected texts, and five years after this latest text we read, the author promises to devote a new class of values, in addition to the two classic ones – truth and falseness –, to a third value, which could be called “origin state” prior to the restriction brought by the principle of contradiction.

However, saying that Peirce was worthlessly concerned about the issue, well, the attentive reading of the texts collected here does not seem to confirm it. The problem exists, but its reality is categorical enough to require a proper evaluation. The boundary cases will only confirm in an extremely acute manner the importance given by Peirce to the true continuum theory, and the decisive contribution that the notion of potentiality, as an affirmative way of being so that a realistic view of the logic and the phenomena does not conceal the problems, but seeks solutions that reasonably determine the future behavior of science.

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³⁶ P.352.

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