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Remarks on Tarski's Nominalism

Observações sobre o nominalismo de Tarski

Daniel Arvage Nagase*
dan.nagase@gmail.com

Abstract: In this article, I will offer some remarks about Tarski's nominalism. First, I will show that, even though it was mainly developed in private conversation and lectures, Tarski did try to develop a rigorous nominalistic theory, which shows that the issue was of some importance to him. In particular, I show how Tarski's formulation is based on the idea of a humanly understandable language and show how he tried to develop this idea throughout his career. Unfortunately, even though his formulation is interesting, it seems to face an insurmountable obstacle, which I examine in detail in the article. Finally, I offer some remarks about what went wrong with Tarski's nominalist program.

Keywords: Nominalism. Ontological Commitment. Pragmatism. Tarski.

Resumo: *Neste artigo, farei algumas observações sobre o nominalismo de Tarski. Primeiro, mostrarei que, embora tenha sido desenvolvida principalmente em conversas e palestras privadas, Tarski tentou desenvolver uma teoria nominalista rigorosa, que mostra que a questão tinha alguma importância para ele. Em particular, mostro como a formulação de Tarski se baseia na ideia de uma linguagem humanamente compreensível e mostro como ele tentou desenvolver essa ideia ao longo de sua carreira. Infelizmente, embora sua formulação seja interessante, parece enfrentar um obstáculo intransponível, que examino detalhadamente no artigo. Finalmente, apresento algumas observações sobre o que correu mal com o programa nominalista de Tarski.*

Palavras-chave: *Comprometimento ontológico. Nominalismo. Pragmatismo. Tarski.*

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1 Introduction

It is well-known that Taski always sympathized with an austere physicalist outlook, one that sometimes led him in the direction of finitism.[1] For one who was merely acquainted with his mathematical work, and who did not pay much attention to his off-hand philosophical remarks, this is surprising, since Tarski actively pursued research involving large cardinals, which is plainly at odds with this finitistic tendency. Moreover, he routinely employed in his investigations, when it suited him, higher-order logic, and even infinitary logic, which obviously run against any finitistic scruples. Nonetheless, once one becomes acquainted with his

* Departamento de Filosofia - FFLCH-USP.

1 Cf., in particular, (Mancosu, 2010a; 2010b) and (Frost-Arnold, 2013). Tarski's outlook seems closer even to ultrafinitism, as Givant told Rodríguez-Consuegra, and as it appears in Tarski's conversations with Carnap. Cf. (Rodríguez-Consuegra, 2005, p. 255) and (Frost-Arnold, 2013, p. 153).

private remarks, be it in conversation with Carnap[2] or in lectures,[3] a coherent nominalist undercurrent seems to appear.

My aim in this paper is to bring to the fore this undercurrent and examine its development in Tarski's career. In the first section, I examine his *linguistic nominalism*, and the thesis that the only humanly understandable languages are nominalist ones. In the second section, I examine how Tarski attempted to develop this idea by developing a nominalistic language for mathematics, paying specific attention to how his attempts failed. Finally, in the conclusion, I offer some brief remarks on what we can learn from this failure.

2 Tarski's Linguistic Nominalism

Tarski himself was quite forthright about his nominalism, and even joked about its conflict, noted in the Introduction, between this nominalism and his research agenda:

> I happen to be, you know, a much more extreme anti-platonist. [...] So, you see, I am much more extreme; I would not accept the challenge of Platonism. You agree that continuum hypothesis has good sense; it is understandable. No, I would say, it's not understandable to me at all. [...] I represent this very rude kind of anti-Platonism, one thing which I could describe as materialism, or nominalism with some materialistic taint, and it is very difficult for a man to live his whole life with this philosophical attitude, especially if he is a mathematician, especially if, for some reasons, he has a hobby which is called set-theory, and worse – very difficult. (Tarski, 2007, p. 259-260).

Although the above paragraph is clearly meant to be in part provocative, there are two ideas that we can extract from it that are crucial for understanding Tarski's nominalism:

i) **Materialism:** Tarski only accepted an ontology of *material* objects, or, more negatively, he rejected *abstract* objects.

ii) **Linguistic Thesis:** Tarski's rejection of Platonism is connected with the idea that a Platonist language is not understandable.

These are, of course, rather rough ideas. First, it is not clear what kind of criteria Tarski is employing to demarcate concrete from abstract objects, so it is not clear what is the precise content of his materialism.[4] Second, it is not entirely clear what is Tarski's criterion for a language to be "understandable". Since Tarski ever dealt directly with the first issue, and it is in any case a very thorny issue,[5] I propose to leave it here to the side and focus on the latter issue, namely what makes a language humanly understandable.

The idea that any acceptable language must be "understandable" also appears in the conversations with Carnap and Quine that Tarski held during his stay at Harvard in the early 1940's, as documented by Frost-Arnold (Frost-Arnold, 2013). There, Tarski says that he only "understands" certain types of finitistic, nominalist language, intending a contrast between an uninterpreted calculus, on the one hand, and a language to which we can attach concrete meanings. Unfortunately, none of the participants of

2 This valuable material has been edited and published by (Frost-Arnold, 2013).

3 See, for example, the recent lectures edited by Rodriguez-Consuegra (Tarski, 2007).

4 This would be an innocent omission, had there been a standard way of drawing this distinction; the problem is that how we draw it may have consequences for this type of proposal. For example, are sentence *types* abstract objects? Or books? Or proofs, for the matter? Cf. Wetzel (2009), for a discussion of the metaphysical status of types and tokens that bear directly on the issue of nominalism.

5 For a detailed discussion of how to demarcate concrete from abstract entities, cf. Cowling (2017).

those conversations is explicit about what constitutes an "understandable" language.[6] Nevertheless, Tarski himself points to at least four criteria that a language must meet in order to be "understandable":[7]

(1) It must mention only a finite number of individuals (or, in a weaker formulation, it must not presuppose that there are infinitely many individuals).

(2) Related to (1), it must contain only finitely many symbols.

(3) The individuals in (1) are assumed to be concrete physical things.

(4) Related to (3), it must not refer to universals or classes.[8]

One thing to note about these four criteria is that at least three of them are connected with the semantics of the language. That is clear enough about (1) and (3), since they are essentially saying how we should interpret the domain of discourse of the language. (4) is also best understood in this spirit: Tarski is alluding to the standard interpretation of higher-order variables, according to which they must be interpreted as ranging over classes or universals. If this is so, then (4) is really a consequence of (1) and (3), since it is basically saying that universals or classes must not be included in our domain of discourse.[9] As for (2), there are two complementary readings of it. One, if a symbol is taken as an object, then it should be a concrete object, since only those (per Tarski's materialism) exist. But then, if there were infinitely many symbols, there would be infinitely many concrete objects, and we are not allowed (per (1)) to assume this in advance. So, there should be only finitely many symbols. Two, given the usual assumption that the human mind has finite memory, this seems a reasonable assumption about any humanly learnable language.[10] Note that this second reading is also tied to semantic issues, since learning a language is presumably connected with learning its semantics.

In summary, Tarski posits as minimal requirements for an "understandable" language that it be humanly learnable and, moreover, that its semantics be both finitary and materalistic. The first requirement seems relatively non-controversial. But what about the second? In absence of direct textual evidence, it is difficult to work out what Tarski could have meant with this requirement. So, the answer here is a bit speculative. However, elsewhere (Tarski, 2007, p. 263) he puts in the mouth of an imaginary speaker the following sentence: "I don't know what the fundamental term of group theory means here. I can understand it only if you tell me which model you have in mind." This is flimsy evidence on which to base an interpretation, but it does seem to point to the idea that understanding an expression (in this case, the symbol for addition) is not merely (as suggested by Creath (2015)) to grasp formal rules governing its use. Rather, one must also know the *intended model* of the language. Now, presumably, in the case of a humanly understandable language, one that is normally used by ordinary humans, the *intended model* is just the world. Since, according to his materialism, the world consists of just finitely many concrete individuals, it follows that a language which we understand must only make reference to those individuals.

Tarski then held the following thesis:

Linguistic nominalism: The only humanly understandable languages are those that fulfill conditions (1)-(4) above.

6 Cf. Frost-Arnold (2013, p. 153) for the Tarski quotation and (*Idem*, p. 27-37) for an analysis. Cf. also the recent discussion around Frost-Arnold's book, some of which revolves around this specific point: Creath (2015) and the reply by Frost-Arnold (2015).

7 Cf. Frost-Arnold (2013, p. 153) and the introduction to Frost-Arnold (2015).

8 Tarski takes this to be equivalent to the requirement that the language be first-order. Here, he anticipates Quine's criterion of ontological commitment and his discussion of second-order logic.

9 In the next section, I will show that there is a way, suggested by Tarski, to quantify over higher-order variables in what appears to be a semantically innocent manner. Crucially, it will reject the standard interpretation of higher-order variables.

10 A classic paper discussing (2) as a necessary condition for a humanly learnable language is (Davidson, 1984). Ironically, Davidson (1984, p. 9-11) goes on to argue that Tarski's own account of quotation marks violates (2). For a more thorough discussion of this paper and its central argument for this finitary requirement, cf. (Lepore; Ludwig, 2005, chap. 2).

This linguistic nominalism may seem at first innocuous, or even common sensical. Nonetheless, as we will see in the next section, it has a crippling effect on mathematical practice. Indeed, none of Tarski's own attempts at formulating a language for mathematics that adhered to these strictures bore fruit.

3 Implementing Tarski's nominalism: problems and prospects

Given the tension between Tarski's linguistic anti-platonism and his own mathematical work, it is not surprising that he had a consistent interest in developing nominalist strategies for avoiding commitment to abstract entities. In particular, he had a consistent interest in developing a humanly understandable language for mathematics, one that would satisfy the above strictures.[11] A first sketch towards this development can be found in his conversations with Carnap, in which he makes the following observation:

> The tendencies of Chwistek and others ("Nominalism") to talk only about designatable things are healthy. The only problem is finding a good implementation. Perhaps roughly of this kind: in the first language numbers as individuals, as in language I, but perhaps with unrestricted operators; in the second language individuals that are identical with or correspond to the sentential functions in the first language, so properties of natural numbers expressible in the first language; in the third language, as individuals those properties expressible in the second language, and so forth. Then one has in each language only individual variables, albeit dealing with entities of different types. (Frost-Arnold, 2013, p. 141).

According to this strategy, our Tarskian nominalist starts with a nominalistically acceptable language L_0 and a nominalistically acceptable theory T_0 (closed under logical consequence) stated in L_0.[12] For simplicity, suppose the logical vocabulary of L_0 consists of the standard first-order quantifier "$\exists$", and two propositional connectives, say "&" (conjunction) and "~" (negation), with the other symbols defined in their usual way. We then introduce a new language, L_1, with a new quantifier, say "$\exists_1$", and new propositional connectives, say "$\&_1$", "$\sim_1$",[13] along with infinitely many variables not in L_0, such as "x_0^1, x_1^1". These variables will range over the formulas of L_0. The intuitive idea is that, e.g., $\exists_1 x_i^1 x_i^1 (t^0)$ is true iff there is a formula φ from L_0 such that $\varphi(t^0)$ is true (t^0 is a term from L_0); (Kripke, 1976) shows that this intuitive idea can be rigorously formulated so that truth for L_1 is well defined.

Apparently, then, Tarski's idea is to extend this construction further, so that L_2 has its own existential quantifier $\exists_2$ ranging over formulas of L_1, L_3 has $\exists_3$ ranging over formulas from L_2, etc. The idea behind this strategy is clear: to trade an ontology of abstracta by a presumably concrete ontology of language inscriptions. Indeed, this actually allows us to understand the quantification employed in this language quite literally, with the quantifiers of the higher languages varying over a restricted domain of individuals, namely the actual sentences that are formed in the previous levels of the hierarchy. That is, one starts with a basic language, which talks about individuals. In order to capture mathematics, it should be sufficient to capture some form of set theory. The above hierarchy does this by mimicking the iterative construction of sets but using formulas as surrogates for sets. That is, instead of talking about the set $\{x: \varphi(x)\}$, one would talk directly about the formula $\varphi(x)$. Similarly, a formula such as $x \in A$ would be

11 There are interesting parallels between Tarski's nominalism and Hilbert's finitistic proof theory. Both assume as a given a humanly understandable language which is finitistic and try to show that the rest of mathematics can be somehow reduced to this finitary basis. In Hilbert's case, the means was syntactical: he wanted to show that any detour through "transcendent" mathematics could be in principle eliminated. Tarski, however, opted for a semantical approach, formulating an alternative reading of the quantifiers. I regret that I do not have the space here to develop these parallels in detail. For more on Hilbert, cf. especially (Sieg, 2013).

12 In their conversations, Carnap, Quine, and Tarski apparently opted for a very weak form of arithmetic as a "toy" theory, preferably one that couldn't decide even whether the domain was finite or infinite. This means that the theory would be weaker than the theory R studied by (Tarski; et. al. 2010).

13 Generally, the new quantifier is denoted by Σ or some other symbol; it's important that the chosen symbol be different from the symbol chosen in the original language. Since we will be working with a hierarchy of languages, I thought it simpler to add a subscript.

replaced by one such as φ(x), where φ would be a formula that defined A. Quantification over sets would then be replaced by quantification over formulas, thus replacing one's ontological commitments to sets by commitments to formulas.[14]

Here, however, we begin to run into a couple of problems. The first one is that it is not entirely clear how far we should go in this hierarchy. Presumably, given his finitistic tendencies, Tarski would not want to consider L_ω, which is defined as the union of all L_n for every natural number n. So, there must presumably be some n which would be a stopping point. Which n is this, however? There does not seem to be any natural choice for such an n. Indeed, it seems that the best Tarski could hope for is a nominalist *program* for formalizing mathemathics, along the following lines. We try to formalize all of mathematics inside a language L_n, for a specific n. If we succeed, wonderful! If not, we try to extend L_n to L_{n+1}, which is possible, since we have a neat recipe for doing so, outlined in the preceding paragraph. If we now succeed with L_{n+1}, again, wonderful. Otherwise, try L_{n+2}, etc., until we find an m such L_{n+m} succeeds in formalizing our mathematical theories. A program, however, is no guarantee of success, and, indeed, there appears to be insurmountable obstacles to the completion of such a program.

Grant that we have executed the program and now have a given L_n which formalizes mathematics, with L_0 satisfying (1)-(4) of the previous section. Is it the case that, for a given n >0, L_n will satisfy (1)-(4)? This is not obvious, since it is not clear what the domain of quantification of, say, L_1 is. Is it the set of all sentences from L_0? Presumably, this set is infinite, so L_1 would not satisfy (1), since its domain would mention infinitely many individuals, and it would not satisfy (2), since it would contain infinitely many terms (one term for each sentence of L_0). Perhaps this could be blocked by insisting that, in L_1 (and above) we are quantifying over *inscriptions*, instead of over sentences types.[15] As Tarski himself remarks (Tarski, 1983, p. 174), inscriptions are "the products of human activity", and, from this point of view, "the supposition that there are infinitely many expressions appears to be obviously nonsensical". Treating, then, the quantifiers as ranging over inscriptions would then make each L_n acceptable by Tarski's standards.

This introduces a new problem, however: the hierarchy collapses to L_0, i.e. there is no expressive advantage of moving to higher languages, since every quantification involving higher types would have an equivalent in L_0 by taking disjunctions or conjunctions of the appropriate formulas.[16] Since the hierarchy was introduced precisely to rectify the poor expressive power of L_0, this makes the whole exercise otiose. In fact, this seems to pose Tarski a dilemma: either one accepts only finite domains, but then the hierarchy is rendered otiose, or else one accepts infinite domains, but then one violates his finitistic scruples.

A possible fix to this problem would be to treat each domain of quantification introduced as being *indefinitely extendable*.[17] Very roughly, a domain is indefinitely extendable if it is finite, but, for each n, there is a possible world accessible to it such that the domain of this world consists of the original domain plus n entities of a given type (in our case, inscriptions). In other words, a domain is indefinitely extendable if it is always possible to extend it by adding entities to it. This would still be finitistically

14 This strategy for reducing ontological commitment is similar to what Burgess and Rosen (Burgess; Rosen, 1997) call a substitutional strategy. Interestingly, they connect this strategy with Tarski's teacher, Leśniewski. For some support for this attribution, Tarski himself mentions Leśniewski in this connection in a 1953 lecture on nominalism, if we are to trust Beth's report. Cf. the quotation from Beth in (Mancosu, 2010b, p. 406). It seems, however, that this strategy may have originated with Russell. For discussion about this point, cf. Landini (1998), Klement (2010; 2017), Hodes (2015), and Soames (2014, chap. 10). Do notice that Russell accepted propositions as objects, so he is not exactly a full-blooded nominalist.

15 Which would make this type of quantification reminiscent of the valuated sentences from Kaplan (1986).

16 This was already noticed by Quine: "If substitutional quantification is not to resolve to mere finite conjunction without quantifiers, the supply of substitutible terms must be infinite or indefinite" (Quine, 1976, p. 319). I will comment on the indefiniteness suggestion next.

17 One of the first discussions of indefinite extendable domains, and certainly an influential one, is Dummett (1978). Cf. also Parsons (1983) and Hellman (2006) for a formalization of this approach that informed our commentary here. Linnebo (2018, chap. 3) presents a dynamic approach that could be useful in this context. Finally, closer to the spirit of Tarski's discussion, Hook (1983) and Nelson (1986) develop a predicative system of mathematics that may have some appealing features from a nominalist perspective. Again, I regret that I do not have here the space to pursue these parallels.

acceptable, since there would be no domain with infinitely many entities in it. Unfortunately, there seems to be other problems with this proposal. First, there is the problem of making sense of quantification relative to an indefinite domain. Suppose the domain of quantification for L_1 is indefinitely extendable. What are we quantifying over, then, when we write a formula prefixed with "$\exists_1$"? Is it over all the entities in each possible domain? But this seems to be merely another way of saying that we are quantifying over the *union* of each possible domain, which will be infinite. If we take the domain to be one of the possible domains, then we fall back into the dilemma of the preceding paragraph.

One option would be to introduce modal operators into the language, and then formulate whatever existential sentence we needed by prefixing it with a possibility operator. Now, however, we are faced with the problem of interpreting this modal operator. Is it a quantifier over possible worlds? But then, what *are* possible worlds? If they are concrete entities, then we seen to be faced with an infinity of concrete entities, again contravening Tarski's finitism. If they are not concrete entities, we are contradicting Tarski's materialism. Either way, it seems that it is not possible to satisfy Tarski's strictures with this kind of maneuver.

A final attempt at a solution to this problem can be found in a talk Tarski gave in the early 1950s. As told in (Mancosu, 2010b), in the summer of 1953 Beth organized a meeting in Amersfoort to discuss "Nominalism and Platonism in Contemporary Logic", whose main speakers were Tarski and Quine. Although apparently no extant typescript of Tarski's lecture survives,[18] it's possible to extract the content of such lecture from Beth's writings (Beth, 1970, p. 94-96);[19] and, from what we can gather from Beth, in particular, Tarski not only outlined again a construction remarkably similar to the one sketched above, but also proposed one more solution to the problem of the size of the initial domain.

According to Beth, in this talk, the problem discussed above regarding the paucity of individuals in our domain is dealt with by means of what Beth (Beth, 1970, p. 95) calls a "cosmological hypothesis", which simply *postulates* the existence of countably many material bodies. Given that this is in flagrant contradiction to the spirit of Tarski's finitism, Beth appeals here to the deduction theorem, and mentions that every sentence φ derived with the help of such an hypothesis can be replaced by a conditional sentence having the hypothesis as an antecedent: "For all practical purposes '*if X then A*' is just as useful to us as A" (Beth, 1970, p. 96). Finally, in case anyone objects that mathematics needs more than just the countably infinite, Beth also mentions that one can appeal to the Löwenheim-Skolem theorem in order to obtain a countable model of set-theory and suppose that the world is such a model.[20]

This is clearly a desperate manoeuvre. Setting aside the (low) plausibility of this "cosmological hypothesis", there are, again, a number of problems with this proposal. Beth is not wrong when he proposes that mathematical theorems are really conditional in form. But what we normally consider the antecedents of these conditionals are *the particular axioms used in their demonstrations, not a "cosmological hypothesis"*. Indeed, why should the correctness of a particular theorem depend on a dubious *empirical assumption*? That is, suppose the cosmological hypothesis turned out to be *false*. Should we then withdraw any mathematical claim which depended on there being infinitely many objects? It seems, rather, that the most immediate reaction to this state of affairs would be to *seek an alternative interpretation of mathematics*, one that did not depend on this cosmological hypothesis. And since, as far as we know, the hypothesis *is* false, then it seems that we should indeed seek such an alternative interpretation.

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18 As Mancosu (2010b, p. 559) notes, it's likely that Tarski never wrote the text of his lecture. Cf. the letter from Tarski to Quine quoted by Mancosu, in which Tarski states that "it would be too late" for him to "prepare any formal talk". Interestingly, the excerpt quoted by Mancosu of this letter ends with Tarski saying to Quine that he wants to examine the "possibility of a semantic interpretation of quantifiers with variables of higher orders".

19 There, Beth proposes a program for developing a nominalistic acceptable reconstruction of mathematics, which he claims follows "in the main lines the exposition given by Tarski in Amersfoort" (Beth, 1970, p. 100).

20 This greatly simplifies the situation: an application of the Löwenheim-Skolem theorem to, say, ZFC will give us a countable model of the theory, but this model will not be standard. In fact, the statement that there is a standard model of set-theory is strictly stronger that the statement that there is a model of set theory. For a proof, cf. Takeuti and Zaring (1982, appendix).

4 Final Considerations

In Section 2 of this article, I presented Tarski's linguistic nominalism, the idea that the only languages we can understand are those which conform to his nominalistic strictures, so that a platonist interpretation of our mathematical practice would be literally unintelligible. In Section 3, I explored how Tarski developed this idea either in private conversations or in unpublished lectures, and how his attempts at rigorously developing this idea foundered on the same point, namely the need to account for the infinity of mathematical objects while at the same time respecting his finitistic scruples. Nonetheless, the idea that our mathematical language needs to be humanly understandable seems clearly correct. So where did Tarski go wrong?

I submit that Tarski's mistake was to think that, because our human mind is finite, it is limited to understanding language with finite domains. True, we can only grasp finitely many objects, so, if in order to understand a language, we need to grasp the denotation of each of its denoting terms, then we indeed cannot comprehend the language of mathematics.[21] But, since we do seem to comprehend the language of mathematics, this seems to be a good reason to apply modus tollens to the preceding conditional and *deny* that we need to grasp the denotation of each of the denoting terms of a language to understand it. Rather, I suggest that it suffices for us to *imagine* what this denotation is like, and that this is enough for our understanding.[22] Close attention to our mathematical *practice* shows that it consists not only of *demonstrating* theorems, but also of *imagining* models. But to *imagine* a model, one need not be acquainted with the objects which compose its domains. And this, I submit, is the beginning of an answer to Tarski's quandary.

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21 A similar reasoning, of course, underlies Benacerraf's reflections (Benacerraf, 1983) on the incompatibility between semantics based on the causal theory of reference and the epistemology of mathematics.

22 This role of the imagination in constructing models is central to the understanding of our general scientific practice. Cf. the essays collected by Levy and Godfrey-Smith (2020) for more details.

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