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DOSSIÊ PEIRCE E A LÓGICA / *DOSSIER PEIRCE AND LOGICS*

Peirce and modal logic: delta existential graphs and pragmaticism

Peirce e a lógica modal: grafos existenciais delta e pragmaticismo

Jon Alan Schmidt*
JonAlanSchmidt@gmail.com

Abstract: Although modern modal logic came about largely after Peirce's death, he anticipated some of its key aspects, including strict implication and possible worlds semantics. He developed the Gamma part of Existential Graphs with broken cuts signifying possible falsity, but later identified the need for a Delta part without ever spelling out exactly what he had in mind. An entry in his personal Logic Notebook is a plausible candidate, with heavy lines representing possible states of things where propositions denoted by attached letters would be true, rather than individual subjects to which predicates denoted by attached names are attributed as in the Beta part. New transformation rules implement various commonly employed formal systems of modal logic, which are readily interpreted by defining a possible world as one in which all the relevant laws for the actual world are facts, each world being partially but accurately and adequately described by a closed and consistent model set of propositions. In accordance with pragmaticism, the relevant laws for the actual world are represented as strict implications with real possibilities as their antecedents and conditional necessities as their consequents, corresponding to material implications in every possible world.

Keywords: Existential Graphs. Modal logic. Peirce. Possible worlds semantics. Pragmaticism.

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Resumo: *Embora a lógica modal moderna tenha surgido em grande parte após a morte de Peirce, ele antecipou alguns de seus principais aspectos, incluindo implicação estrita e semântica de mundos possíveis. Ele desenvolveu a parte Gamma dos Gráficos Existenciais com cortes quebrados significando possível falsidade, mas depois identificou a necessidade de uma parte Delta sem nunca soletrar exatamente o que tinha em mente. Uma entrada em seu Caderno de Lógica pessoal é um candidato plausível, com linhas grossas representando possíveis estados de coisas onde proposições denotadas por letras anexadas seriam verdadeiras, em vez de sujeitos individuais aos quais predicados denotados por nomes anexados são atribuídos como na parte Beta. Novas regras de transformação implementam vários sistemas formais comumente empregados de lógica modal, que são prontamente interpretados definindo um mundo possível como aquele em que todas as leis relevantes para o mundo real são fatos, cada mundo sendo parcialmente, mas precisamente e adequadamente descrito por um conjunto de modelos de proposições fechado e consistente. De acordo com o pragmaticismo, as leis relevantes para o mundo real são representadas como implicações estritas com possibilidades reais como seus antecedentes e necessidades condicionais como seus consequentes, correspondendo a implicações materiais em todos os mundos possíveis.*

Palavras-chave: *Grafos existenciais. Lógica modal. Peirce. Pragmaticismo. Semântica dos mundos possíveis.*

* Independent Scholar.

1 Introduction

As an extension of classical logic, modal logic adds operators for possibility and necessity that are commonly interpreted as analogous to the existential and universal quantifiers, respectively. The idea is that rather than quantifying predicates over subjects, they quantify propositions over "possible worlds". C. I. Lewis is widely credited with launching this now-standard approach by drawing a distinction between material implication and strict implication, and then insisting that the latter is the "correct" formalization of a conditional proposition (Lewis, 1912). However, decades earlier, Charles Sanders Peirce anticipates Lewis's position as well as Saul Kripke's later formulation of possible worlds semantics (Zeman, 1997, p. 416 n. 2; Copeland, 2002, p. 99), albeit with a different motivation:

> A hypothetical [if-then] proposition, generally, is not confined to stating what actually happens, but states what is invariably true throughout a universe of possibility. (CP 3.366, W 5:166, 1885).[1]
>
> The question is what is the sense which is most usefully attached to the hypothetical proposition in logic? Now, the peculiarity of the hypothetical proposition is that it goes out beyond the actual state of things and declares what would happen were things other than they are or may be. The utility of this is that it puts us in possession of a rule, say that "if *A* is true, *B* is true," such that […] throughout the whole range of possibility, in every state of things in which *A* is true, *B* is true too. […] If, then, *B* is a proposition true in every case throughout the whole range of possibility, the hypothetical proposition, taken in its logical sense, ought to be regarded as true, whatever may be the usage of ordinary speech. If, on the other hand, *A* is in no case true, throughout the range of possibility, it is a matter of indifference whether the hypothetical be understood to be true or not, since it is useless. (CP 3.374, 1885).
>
> The quantified subject of a hypothetical proposition is a *possibility*, or *possible case*, or *possible state of things*. (CP 2.347, W 5:169-170, c. 1895).
>
> The consequence *de inesse* [material implication], "if *A* is true, then *B* is true," is expressed by letting *i* denote the actual state of things, A_i mean that in the actual state of things *A* is true, and B_i mean that in the actual state of things *B* is true, and then saying "If A_i is true then B_i is true," or, what is the same thing, "Either A_i is not true or B_i is true." But an *ordinary* Philonian conditional [strict implication] is expressed by saying, "In *any* possible state of things, *i*, either A_i is not true, or B_i is true." (CP 3.444, 1896).[2]
>
> In the first place, according to *Modality* or *Mode,* a proposition is either *de inesse* (the phrase used in the *Summulae* [*Logicales* of Petrus Hispanus]) or *modal.* A proposition *de inesse* contemplates only the existing state of things – existing, that is, in the logical universe of discourse. A modal proposition takes account of a whole range of possibility. According as it asserts something to be true or false throughout the whole range of possibility, it is *necessary* or *impossible.* According as it asserts something to be true or false within the range of possibility (not expressly including or excluding the existent state of things), it is *possible* or *contingent*. (CP 2.323, 1903).

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1 Peirce's published writings are cited as CP with volume and paragraph number(s) for (Peirce, 1931-1958), NEM with volume and page number(s) for (Peirce, 1976), W with volume and page number(s) for (Peirce, 1982-2010), EP with volume and page number(s) for (Peirce, 1992-1998), and LF with volume and page number(s) for (Peirce, 2021-2024), except that page numbers are not yet available for LF 3/2. His unpublished writings, including some that will appear in that forthcoming volume, are cited as R with manuscript number in accordance with (Robin, 1967) and page number(s) corresponding to the microfilm sequence as reproduced in the scanned images made available online by the Digital Peirce Archive (https://rs.cms.hu-berlin.de/peircearchive) and the Scalable Peirce Interpretation Network (https://fromthepage.com/collection/show?collection_id=16), followed by the page numbers that appear in the images themselves [in square brackets] where different. The year of original publication or composition is that assigned by (Robin, 1967) unless subsequent investigation has produced an updated estimate as documented at Commens: Digital Companion to C. S. Peirce (http://www.commens.org).

2 By referring to a strict implication as a "Philonian conditional", "Peirce may be reading his modal distinctions back into Philo" (Starr, 2014, p. 1 n. 2).

Peirce reiterates near the end of his life that some propositions purportedly describing a general universe – namely, conditionals with antecedents that would not be true in any possible state of things – are neither true nor false, but meaningless:

> A conditional proposition, – say "if A, then B" is equivalent to saying that "*Any* state of things in which A should be true, *would* (within limits) *be* a state of things in which B is true." It is therefore essentially an assertion of a *general* nature, the statement of a "*would-be*." But when the antecedent supposes an *existential* fact to be different from what it actually is or was, the conditional proposition does not accurately state anything […]. A historian simply tells nonsense when he says "If Napoleon had not done as he did before the battle of Leipzig (specifying in what respect his behaviour is supposed different from what it was,) he would have won that battle." Such historian may have meant something; but he utterly fails to express any meaning. (R L477:9-10, LF 3/2, 1913).

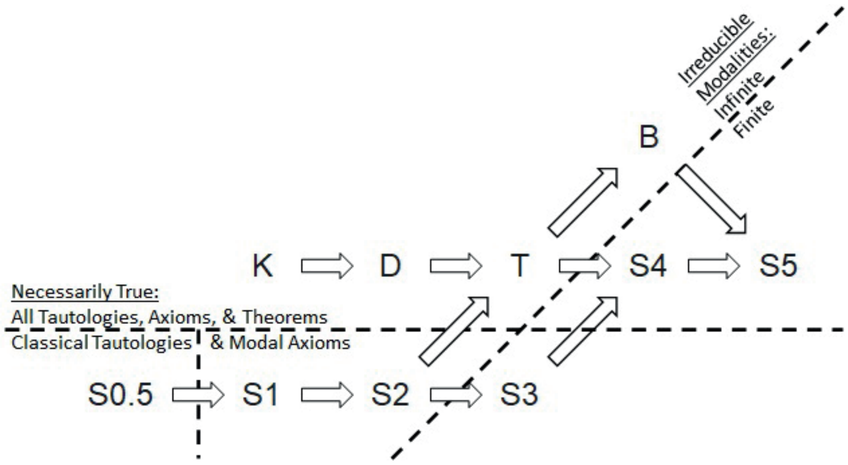
Peirce thus challenges the standard interpretation of conditionals with false antecedents as material implications that are always true: "it can no longer be granted that every conditional proposition whose antecedent does not happen to be realized is true" (CP 4.580, LF 3/1:281, 1906). He also dismisses conditionals whose antecedents are "existential" and "counter to fact" because those alternative events are no longer real possibilities.[3] The upshot is that, in accordance with his versions of pragmatism (Schmidt, 2020) and the "growing block" theory of time (Schmidt, 2022a), classical logic is for reasoning about actualities as both antecedents and consequents in material implications, so it properly applies to the past and present; while modal logic is for reasoning about real possibilities as antecedents and conditional necessities as consequents in strict implications, so it properly applies to the future, but only to the extent that it is genuinely accessible from the present:

> [T]he mode of the Past is that of Actuality… [while] everything in the Future is either *destined*, i.e., necessitated already, or is *undecided*, the contingent future of Aristotle. In other words, it is not Actual, since it does not act except through the idea of it, that is, as a law acts; but is either Necessary or Possible… [The present] is the Nascent State of the Actual. (CP 5.459&463, EP 2:357-359, 1905).
>
> That a possibility which *should* never be actualized, (in the sense of having a bearing upon conduct that might conceivably be contemplated,) would be a nullity is a form of stating the principle of pragmaticism. One obvious consequence is that the potential, or really possible, must always *refer* to the actual. The possible is what *can become actual*. A possibility which could not be actualized would be absurd, of course. (R 288:69[134-135], 1905).

3 For more on Peirce's theory of the conditional, see Zeman (1997a).

2 Formal systems of modal logic

Figure 1: Standard formal systems of modal logic



This philosophical framework, which Peirce calls "pragmaticism" to distinguish it from other varieties of pragmatism (CP 5.414, EP 2:334-335, 1905), has bearing on which of the many different formal systems of modal logic is most appropriate for such reasoning, as differentiated by which axioms and inference rules they add to classical logic. Figure 1 presents the standard ones, arranged from weakest (left) to strongest (right) and showing how they are contained within each other. As indicated, those in the bottom row treat only classical tautologies and (for **S1-S3**) their added modal axioms as necessarily true, while the others extend this to all their theorems; in fact, that is the sole difference between **S0.5** and **T** (sometimes called **M**). Lewis himself introduced **S1-S5**, and the two strongest of these are now generally considered to be the "best" for most purposes:

> The question Which is the right validity logic? has been answered at the sentential level, which is the only level that will be considered here: it is the system known as **S5**. This result is essentially established already in Carnap (1946).
>
> The question Which is the right demonstrability logic? goes back to the earliest days of modern modal logic. […] To the extent that there is any consensus or plurality view among logicians today, I take the view to be that the right demonstrability logic is **S4**. (Burgess, 1999, p. 82).

On the other hand, John L. Pollock proposes a "basic modal logic" and calls it **B** (Pollock, 1967), although it is very different from the system commonly named **B** after L. E. J. Brouwer and shown in Figure 1 as containing **T** and contained in **S5** but independent of **S4**. Pollock's approach is to eliminate all "iterated" modalities where at least one modal operator is within the scope of another, thus corresponding to compounded expressions like "possibly possible" and "necessarily possibly necessary". He describes his motivation and results as follows:

> [W]hen philosophers and logicians *apply* modal logic to concrete problems, they rarely need principles which involve iterated modalities. For most practical purposes, principles involving only one layer of modalities are all that are needed. This suggests that if we try to construct a theory of modal logic in which there are no iterated modalities, we can avoid most of the controversy and still have a theory that is strong enough for all of the normal uses to which modal logic is put. (Pollock, 1967, p. 355).

> [A]ll of the controversy over which theory of propositional modal logic is correct stems from disagreement about principles involving iterated modalities. **S1**-**S5** and **M**[=**T**] are the most common theories of propositional modal logic, and […] we see that they all give exactly the same theorems not involving iterated modalities. Thus there is a very real sense in which **B** is *basic* modal logic. **B** gives us the core theorems that everyone accepts. (Pollock, 1967, p. 363).

"Indeed, **S1-S5** and Feys' **T** all have the same theses of modal degree at most one and, among the systems which have this property, **S5** is the strongest, and **S0.5** is the weakest" (Porte, 1980, p. 672). In other words, all theorems of **S5** in which no propositional variable is within the scope of more than one modal operator, which together comprise Pollock's **B**, are also theorems of **S0.5-S4** and **T**, as well as the Brouwer-inspired **B**. Moreover, "A formula of modal order greater than one is a thesis of **S0.5** if and only if it is a substitution instance of a first-order thesis" (Porte, 1980, p. 675).

Accordingly, Pollock's **B** matches the original specification for **S0.5** (Lemmon, 1957, p. 180-181), except that such substitution is precluded. Lemmon even anticipates this in a footnote, referencing Parry's brief suggestion of the same idea, which he calls **S.1** (Parry, 1953, p. 328). As indicated in Figure 1, without such a limitation, infinitely many higher-degree modalities are irreducible in all the systems shown except **S3-S5**. Even in **S3** and **S4**, there are 42 and 14 non-equivalent modalities, respectively.

By contrast, "Every formula of higher than first degree is reducible in **S5** to a first-degree formula", and there is "an effective procedure for reducing any wff [well-formed formula] of higher than first degree to one of first degree by equivalence transformations" (Hughes and Cresswell, 1996, p. 98-101). "Due to this, every thesis of **S5** may be related to a thesis being its translation in [Pollock's] **B**", although "different theses of **S5** may have the same corresponding thesis of the first modal degree" (Gardies, 1998, p. 32).

This approach is appealing in some respects. After all, what does it really mean to assert, "There is some possible state of things where there would be some possible state of things where the proposition *p* would be true"? Why and how is it significantly different from merely asserting, "There is some possible state of things where the proposition *p* would be true"? Again, a fundamental principle of pragmaticism is that there are *real* possibilities, namely, those accessible from the *actual* state of things. Accordingly, since **B** is already taken, perhaps Pollock's "basic" system should instead be named **P** for him along with Parry, Peirce, and pragmaticism.

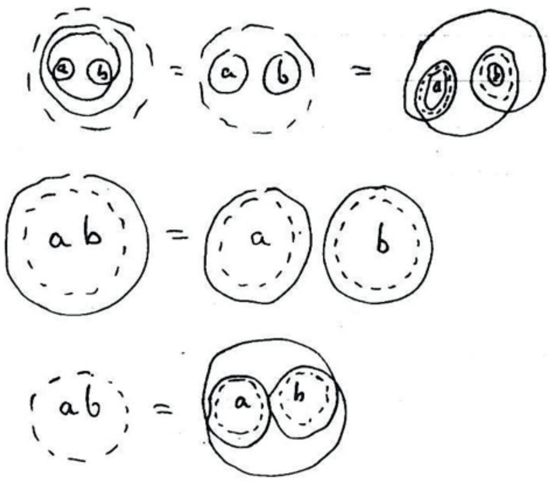
These specific formal systems of modal logic were not developed until long after Peirce's death, but which of them best reflects what he had in mind? His original presentation of the Gamma part of Existential Graphs (EG) during the 1903 Lowell Lectures includes various innovative features that go beyond classical propositional logic as implemented by the Alpha part and first-order predicate logic as implemented by the Beta part. For example, both Alpha and Beta include the solid cut, signifying that a graph enclosed by it is *actually* false, while Gamma adds the broken cut, signifying that a graph enclosed by it is *possibly* false, thus introducing modality.[4]

Maintaining all the regular transformation rules with broken cuts implements the unusual four-valued Ł-modal system of Jan Łukasiewicz (Zeman, 1967, p. 149-160; Schmidt, 2022b). Although according to Peirce, "The Rule of Iteration and Deiteration does not apply to the broken cut" (LF 2/2:362, 1903), it turns out that allowing iteration and deiteration of only certain kinds of subgraphs across broken cuts implements **S4** and **S5**, as well as an intermediate system **S4.2** (Zeman, 1967, p. 160-175). Such specification of strictly graphical permissions and restrictions is faithful to Peirce's original purposes and methods when he invented and developed EG, but it does not seem feasible to implement the weaker

4 Explaining the different parts of EG in greater detail is beyond the scope of this paper. In addition to the references cited in the main text, see (Pietarinen, 2015) for a concise introduction to Alpha and Beta, (Roberts, 1973) for a detailed exposition of them along with Gamma, and LF for a comprehensive compilation of Peirce's relevant manuscripts, including many that had not been published previously.

standard systems in Gamma without stipulating at least one modal axiom as an *ad hoc* inference rule (Van den Berg, 1993; Ramharter; Gottschall, 2011; Ma; Pietarinen, 2015). Nevertheless, Peirce's own initial experiments with the new notation include three key entries (LF 2/2:396-400, 1903):

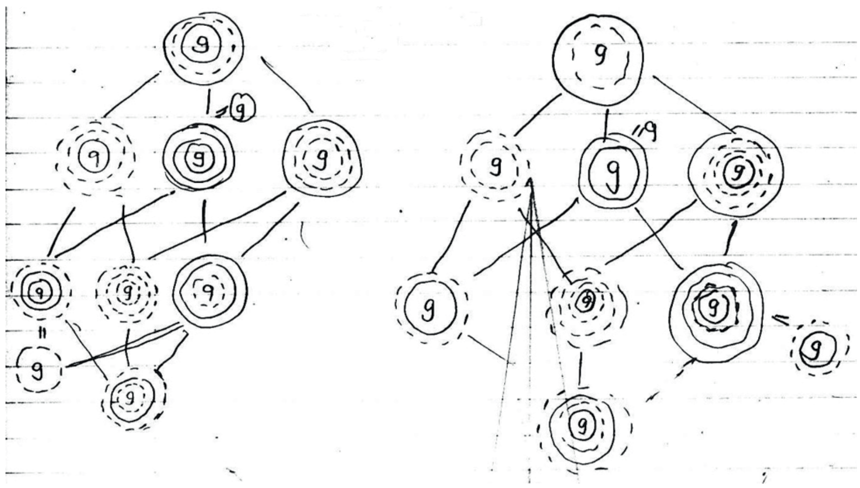
Figure 2: Modal equivalences in Gamma EG



(LF 2/2:400)

- Figure 2 shows graphs for several distributive modal equivalences: "*a* or *b* is possibly true" is equivalent to "*a* is possibly true or *b* is possibly true", "*a* and *b* is necessarily true" is equivalent to "*a* is necessarily true and *b* is necessarily true", and "*a* and *b* is possibly false" is equivalent to "*a* is possibly false or *b* is possibly false". Each of these is an axiom or theorem of every system included in Figure 1, as well as **P**.

Figure 3: Modal derivations in Gamma EG

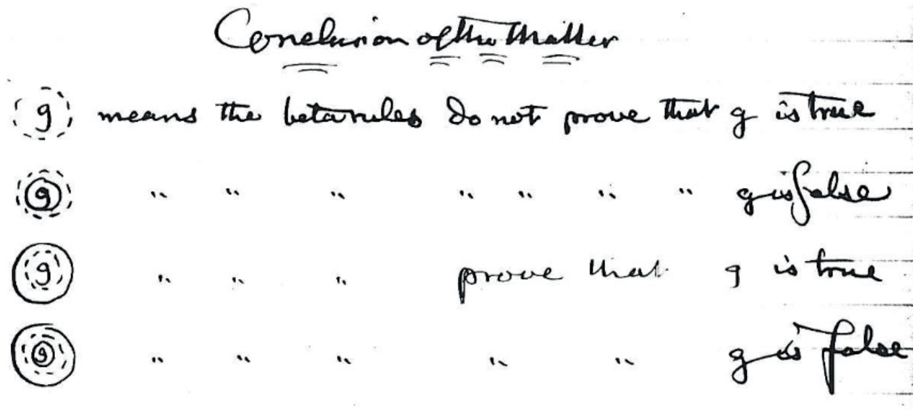


(LF 2/2:398-399)

- Figure 3 shows derivations of graphs from top to bottom, including several for iterated modalities, using the transformation rules for the two kinds of cuts: an evenly enclosed solid cut may be converted into a broken cut (erasure), and an oddly enclosed broken cut may be converted into a solid cut (insertion). Accordingly, being *necessarily* true or false implies being *actually* true or false

– the characteristic axiom of system **T** – and being *actually* true or false implies being *possibly* true or false.[5]

Figure 4: Modal definitions in Gamma EG



(LF 2/2:396-397)

- Figure 4 shows that according to Peirce, being *necessarily* true or false in Gamma means being *proved* true or false in Beta. This seems to require that the graph *g* represents a *non-modal* proposition, such that only classical tautologies are treated as necessarily true, which is precisely what distinguishes system **S0.5** from **T**.

Taken together, these clues suggest that Peirce might have preferred a formal system of modal logic along the lines of **S0.5**, not **T** nor an even stronger system like **S4** or **S5**.

3 Delta existential graphs

In any case, Peirce eventually replaces broken cuts with various tinctures within cuts, and he ultimately disavows those in favor of simple shading:[6]

> In this ["Prolegomena to an Apology for Pragmaticism," CP 4.530-572, 1906] I made an attempt to make the syntax [of Existential Graphs] cover Modals; but it has not satisfied me. The description was, on the whole, as bad as it well could be, in great contrast to the one Dr. Carus rejected [in 1897]. For although the system itself is marked by extreme simplicity, the description fills 55 pages, and defines over a hundred technical terms applying to it. The necessity for these was chiefly due to the lines called "cuts" which simply appear in the present description as the boundaries of shadings, or shaded parts of the sheet. The better exposition of 1903 divided the system into three parts, distinguished as the Alpha, the Beta, and the Gamma, parts; a division I shall here adhere to, although I shall now have to add a *Delta* part in order to deal with modals. (R L376, R 500:2-3, LF 3/2, 1911).

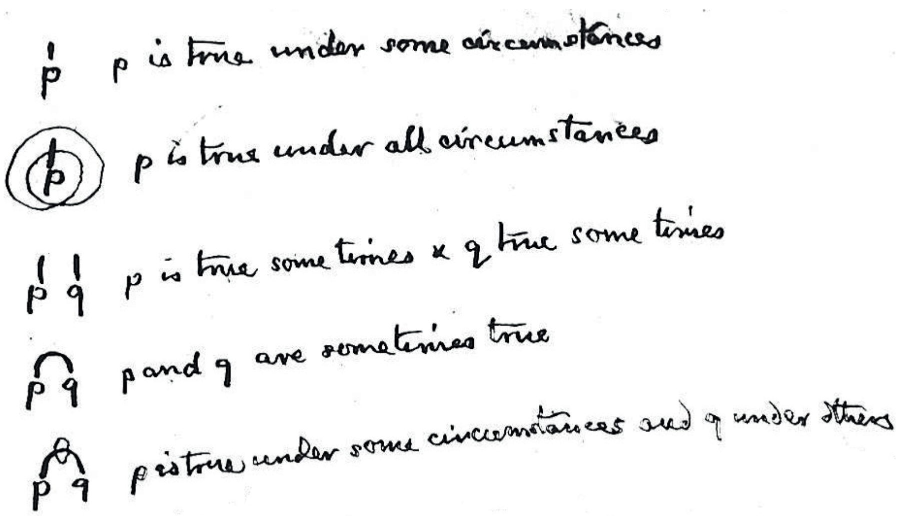
Unfortunately, this letter goes on to meander through various other topics before breaking off without ever specifically discussing the different parts of EG, including Delta. Moreover, Peirce does not mention

5 Peirce's evident endorsement of this modal principle is unsurprising since it can be understood as a formalization of the logical relations among his three universal categories: possibility (Firstness) can be directly inferred from actuality (Secondness), which can be directly inferred from necessity (Thirdness).

6 For a further development of Gamma EG using tinctures instead of broken cuts to implement **S4**, see Zeman (1997b).

the prospect of a Delta part anywhere else. However, nearly three years earlier, as shown in Figure 5, he scribes five graphs and gives their translations in his personal Logic Notebook (LF 1:624, 1909). There are heavy lines as in Beta EG, but instead of individual subjects to which predicates denoted by names attached to them are attributed, they represent circumstances under which propositions denoted by letters attached to them are true. Although it apparently has never been thoroughly explored in previous secondary literature – only briefly mentioned in (Roberts, 1973, p. 99-100) and (Pietarinen, 2006, p. 353) – this seems like a plausible candidate for Delta EG, which can then be fully specified as follows.

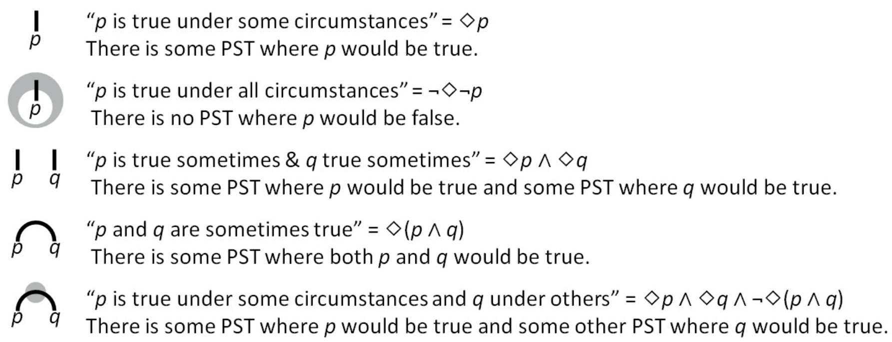
Figure 5: Delta EG in Peirce's Logic Notebook



(LF 1:624)

As in all parts of EG, a blank sheet represents the *actual* state of things in a recognized universe of discourse as the inexhaustible continuum of all propositions that *are* true of it. Any graph scribed on the sheet asserts one such proposition, and its constituent subgraphs may include instances of three other primitive signs as shown in Figure 6:

Figure 6: Delta EG with Shading



- A letter denotes an atomic non-modal proposition.
- Shading of oddly enclosed areas signifies negation.
- A heavy line of compossibility (LoC) represents some *possible* state of things (PST) as the inexhaustible continuum of all propositions that *would be* true of it if it *were* to be actualized, and any subgraph attached to it in the same area asserts one such proposition.

A ring-shaped scroll consists of a shaded area surrounding an unshaded area or vice-versa. It signifies double negation with an empty outer area, material implication with subgraphs for the antecedent and consequent in its outer and inner areas, or strict implication with an LoC attached to both these subgraphs. An LoC crossing the boundary between areas represents a PST where the proposition asserted by the subgraph in the inner area would be false, like a broken cut in Gamma, so the LoC must be attached to every letter within that boundary, branching as needed. This precludes unrestricted iteration or deiteration of subgraphs through such a boundary, and of LoCs through any boundary, unlike lines of identity in Beta. Interpretation is *endoporeutic* – proceeding from the outside inward – as "and" ($\wedge$) for each subgraph in the same area, "not" ($\neg$) for each boundary, and "possibly" ($\Diamond$) for each LoC, including multiple LoCs attached to the same subgraph for iterated modalities.

With these conventions, the only Delta axiom besides the blank sheet is an otherwise empty scroll with an unattached LoC crossing its inner boundary, asserting that there is no PST where tautology would be false ($\neg\Diamond\neg\top$) in accordance with the rule of necessitation ($\vdash\alpha \Rightarrow \vdash\Box\alpha$). Most Alpha permissions are maintained for individual subgraphs: insertion in any shaded area, erasure in any unshaded area, iteration or deiteration through any boundary not crossed by an LoC, and addition or removal of a double negation scroll in any area as long as any LoC crossing one of its boundaries also crosses the other. However, only a subgraph already attached to an LoC crossing a boundary may be iterated or deiterated through that boundary, i.e., any subgraph attached to an LoC may be duplicated on another branch of that LoC in the same area or a more enclosed area, and any such duplicate subgraph may be removed.

Each Delta permission for LoCs themselves corresponds as follows to a standard modal axiom for extending the classical propositional calculus, with each LoC classified as shaded or unshaded to match the area where its outermost portion is located:

1. Insertion, axiom K = $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$: Any LoCs may be joined in a shaded area, and an LoC in the surrounding unshaded area may be extended inward across the boundary to be joined with a shaded LoC already attached to every subgraph in that area.
2. Erasure, axiom D = $\Box p \rightarrow \Diamond p$, or $\Diamond\top$: Any LoC may be broken in an unshaded area, and upon being broken, an LoC crossing the boundary from the surrounding shaded area must be retracted outward if attached to other subgraphs or removed if unattached.
3. Attachment/Release, axiom T = $\Box p \rightarrow p$, or $p \rightarrow \Diamond p$: A new LoC may be added simultaneously to any subgraphs in the same unshaded area, and any shaded LoC may be removed simultaneously from every subgraph to which it is attached.
4. Iteration/Deiteration, axiom 4 = $\Box p \rightarrow \Box\Box p$, or $\Diamond\Diamond p \rightarrow \Diamond p$: A new LoC may be added to any subgraph with an LoC by being attached to all the same letters and crossing all the same boundaries, and any such duplicate LoC may be removed.
5. Equivalence, axiom 5 = $\Diamond p \rightarrow \Box\Diamond p$, or $\Diamond\Box p \rightarrow \Box p$: A new shaded LoC may be added to any subgraph with an LoC, and an unshaded LoC not attached to other subgraphs may be removed from any subgraph unless it is the sole innermost LoC.

Note that with #3, the Delta counterpart of broken cut erasure/insertion in Gamma, the blank sheet is no longer a distinct axiom since it is derivable from a necessitation scroll by removing its LoC and the resulting empty shaded outer area. Various combinations of these permissions implement different formal systems—**K** requires #1, **D** requires #1 plus #2, **T** requires #1 plus #3, **S4** requires #1 plus #3 plus #4, and **S5** requires #1 plus #3 plus #5. There does not seem to be a straightforward diagrammatic permission that corresponds to axiom B = $\Diamond\Box p \rightarrow p$, or $p \rightarrow \Box\Diamond p$; nor for limiting necessitation to non-modal tautologies as in **S0.5**, which is its only difference from **T**. However, **P** simply requires #1 plus #3 plus the restriction that no letter may be attached to more than one LoC.

Overall, in addition to being able to implement the standard systems that are weaker than **S4**, Delta EG has definite notational advantages over Gamma EG with broken cuts:

- The simplest modal graph is possibly true (attached to an LoC) vs. possibly false (enclosed by a broken cut).
- Shading may be employed to distinguish oddly enclosed areas from evenly enclosed areas vs. counting cuts.
- Iterated modalities correspond to multiple LoCs attached to a graph vs. multiple nested broken cuts enclosing it.
- Strict implication is expressed by a scroll with an LoC attached to the graphs in both closes vs. a scroll enclosed by a broken cut enclosed by a solid cut.
- As shown by the last example in Figures 5 and 6, a single graph represents two propositions being possibly true individually but not together (incompossible).
- The same graph represents a contingent proposition with not-*p* replacing *q*, which can be derived from the graphs for "*p* is possibly true" and "*p* is possibly false".
- Translations of graphs into possible worlds semantics, at least informally, are straightforward since an LoC always represents some PST.[7]

4 Modality and pragmaticism

An especially suitable version of possible worlds semantics for Delta EG is the laws and facts semantics (LFS) originally co-developed by Dunn (1973) and Goble (1973). The basic insight underlying LFS is that "the relation of relative possibility rests upon consistency relations between non-modal laws and non-modal facts" (Dunn, 1973, p. 99). This can be formalized in various ways (Goble, 1973, p. 155), but Sowa (2003) advocates using the model sets of Hintikka (1969).

A *model set* is a closed and consistent set of discrete propositions that need not be complete, corresponding to those that are explicitly scribed on the continuous sheet in EG. As such, "a model set is the formal counterpart to a partial description of a possible state of affairs", but it is "large enough a description to make sure that the state of affairs in question is really possible" (Hintikka, 1969, p. 59). After all, "reality may outrun the descriptive powers of a given language" (Dunn, 1973, p. 98), or as Peirce puts it, "The actual world cannot be distinguished from a world of imagination by any description" (CP 3.363, W 5:164, 1885). Elsewhere, Peirce offers the following helpful definitions:

> A *state of things* is an abstract constituent part of reality, of such a nature that a proposition is needed to represent it. There is but one *individual*, or completely determinate, state of things, namely, the all of reality. A *fact* is so highly a prescissively abstract state of things, that it can be wholly represented in a simple proposition, and the term "simple," here, has no absolute meaning, but is merely a comparative expression. (CP 5.549, EP 2:378, 1906).

Hence, propositions are not laws and facts themselves, they are *entia rationis* that represent laws and facts prescinded from the totality of reality.[8] The designated model set (DMS) is comprised of non-modal propositions representing facts about the actual state of things (AST), while every alternative model set

7 For additional improvements to EG that Peirce proposed in his late writings, which turn out to be very useful when implementing modal logic in Delta as specified herein, see Schmidt (2024).

8 Prescinding, or prescissively abstracting, "consists in supposing a state of things in which one element is present without the other, the one being logically possible without the other" (EP 2:270, 1903).

(AMS) is comprised of non-modal propositions representing facts about some PST. Together, they are the members of a *model system* serving as partial but accurate and adequate descriptions of the actual world and every possible world, and each is paired with a set comprised of non-modal propositions representing the relevant laws for that AST or PST. For iterated modalities, every AMS for the facts in a PST is the DMS in another model system whose other members are every AMS for the facts in a possible PST, each paired with a set of propositions for the relevant laws, and so on.

The specific kind of modality that is of interest – alethic, deontic, doxastic, dynamic, epistemic, temporal etc. – determines what counts as a relevant law and what are the appropriate mathematical properties of the binary relation between the AST and any PST: serial for axiom D, reflexive for axiom T, symmetric for axiom B, transitive for axiom 4, and/or euclidean for axiom 5. This alternativeness (or accessibility) relation is primitive and arbitrary in standard versions of possible worlds semantics (Sowa, 2003, p. 153-155), but LFS makes it more readily understandable by formalizing it as containment relations between the different sets of law-propositions and fact-propositions, including the stipulation that the entire set of law-propositions paired with the DMS is contained in every AMS of fact-propositions (Schmidt, 2024, appendix).

In other words, LFS defines a state of things as possible just in case the AMS representing its facts includes every proposition representing a relevant law for the AST, i.e., all relevant laws for the AST are facts in each PST, and therefore consistent with every fact in every PST. Accordingly, a proposition is *actually* true if it represents a fact in the AST, *possibly* true if it represents a fact in some PST, and *necessarily* true if it represents a relevant law for the AST and thus a fact in every PST. Hence, the criterion for a model set being "large enough" to describe a PST in LFS is that it includes all the propositions representing the relevant laws for the AST. In many cases, these are of the same form as the propositions representing facts, but not for alethic and temporal modality in accordance with pragmaticism:

> But what the answer to the pragmatist's self-question [how could law ever reasonably affect human conduct?] does require is that the law should be a truth expressible as a conditional proposition whose antecedent and consequent express experiences *in a future tense*, and further, that, as long as the law retains the character of a law, there should be possible occasions in an indefinite future when events of the kind described in the antecedent may come to pass. Such, then, *ought* to be our conception of law, whether it has been so or not. (CP 8.192, 1905).

This sort of law is properly represented by a *subjunctive* conditional proposition, which "means that, throughout this range of possibility, in whatsoever state of things the protasis, or antecedent, would be true, in that same state of things the apodosis, or consequent, would likewise be true" (NEM 3:408, LF 2/2:165, 1903). Moreover, "in our ordinary use of language we always understand the range of possibility in such a sense that in some possible case the antecedent shall be true" (NEM 4:169, 1898). In short, every relevant law for the AST is represented by a strict implication whose antecedent specifies a real possibility, i.e., a fact in some PST – formally, $\Box(p \rightarrow q) \land \Diamond p$. Peirce even anticipates LFS by suggesting that such a law is equivalent to a multitude of material implications, each representing a fact in every PST as well as the AST itself – i.e., the alternativeness relation is reflexive as in any formal system with axiom T:

> An ordinary Philonian conditional proposition [strict implication], having a range of possibility and asserting that "If *A* is true, *B* is true," that is, that in every possible state of things in which *A* is true, *B* is likewise true, may be regarded as a simultaneous assertion of a multitude of propositions each asserting that if [there is] a single state of things in which *A* is true, *B* is true [in that state of things], each therefore being a *consequentia de inesse* [material implication]. (NEM 4:278, c. 1895).

Put another way, "Conditionals universally quantify over possibilities, saying that the material conditional holds at each possibility" (Starr, 2014, p. 2) such that "a strict conditional describes the material conditional holding at all [possible] worlds" (ibid, p. 4 n. 5). What are the ramifications of this for EG?

> To say that there is a *connection* between one fact and another fact is to talk of *possibilities*. Since, therefore, the conditional *de inesse* does not refer to possibilities, but only to the actual state of things, it does not imply any connection between the facts expressed by antecedent and consequent. […]
>
> It is absolutely indispensable that our system of expression should be provided with a way of expressing a conditional *de inesse*; and if it can express this kind of conditional, other conventions will enable it to express all other kinds of conditional propositions without difficulty. (NEM 3:409, LF 2/2:165, 1903).

Figure 7: Material implication in Alpha EG

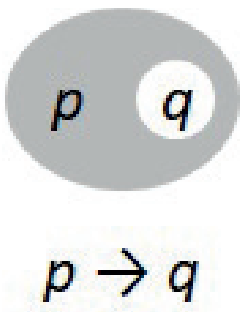
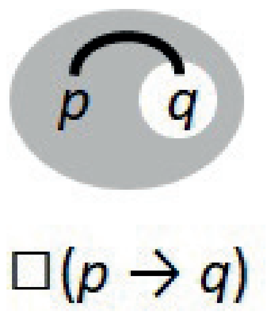


Figure 8: Strict Implication in Delta EG



As shown in Figure 7, a scroll with separate subgraphs in its outer and inner areas is Alpha EG's way of expressing a *material* implication that is or would be true of the AST or some individual PST, thus representing a *fact*. As shown in Figure 8, an LoC iconically signifies the *connection* between distinct facts that corresponds to a possibility, thereby serving as the additional convention enabling Delta EG to express a *strict* implication that is and would be true of the AST and every PST, thus representing a *law* – always a *conditional* necessity, such that the only *absolute* necessities are non-modal tautologies, consistent with formal system **S0.5**. Finally, the principle of pragmaticism that every *real* PST is accessible from the AST suggests dispensing with iterated modalities by prohibiting the attachment of multiple LoCs to any letter, thus implementing formal system **P**.

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