C. S. Peirce and Aristotle on Time

C.S. Peirce e Aristóteles sobre o Tempo

Demetra Sfendoni-Mentzou

Aristotle University of Thessaloniki - Greece sfendoni@edlit.auth.gr

Abstract: The question concerning the nature of time is intimately related to the traditional antithesis between the static a-temporal Parmenidean universe and the dynamic model of becoming. This is wonderfully illustrated in Peirce's theory of the flux of time. In his rejection of the atomistic worldpicture, Peirce treated time in close relation to the physical processes. He thus propounded an extremely interesting theory which bears a kinship to contemporary theories of the *arrow of time*. However, what makes his approach extremely interesting is not only its air of modernity, but also its striking similarities with Aristotle (*Physics*, book IV, esp. chs. 10-14).

My purpose, therefore, in this paper is to reconstruct Peirce's theory of time in the light of Aristotle's philosophy. My starting point will be Peirce's analysis of *continuity* in relation to *infinitesimals* (6.109), through the use of which he could treat time as a "continuum par excellence, (6.86, 1898) which is not a static collection of discrete instants, (see MS 137, p. 4-5, 1904), but a collection of real possibilia (see NE, 360), thus making possible the understanding of the *flow of time* (6.11). I will then proceed to the examination of Peirce's connection of time, as a real *continuum*, with the idea of the infinite. In this respect, I will focus on his rejection of actual infinity - in the Zenonean-atomistic-Cantorian sense - and his adoption of the Aristotelian idea of potential infinite (Physics, book III, chs. iv-viii). I will, thus, argue that what led Peirce to pass from a logico-mathematical analysis of continuity to an *ontological theory* was his appeal to *potentiality* in his treatment of continuity-infinity-time. This, I believe, was the result of the influence he had received from Aristotle, who was also deeply concerned with giving to change, motion and becoming their proper place in nature. Thus, both Peirce and Aristotle were able to built a dynamic theory of time, intimately related to the idea of *potential infinity*, which expresses a physical process progressively being actualized, so that it can never exist as a realized whole.

Key-words: Peirce. Aristotle. Time. Continuity. Infinitesimals.

Resumo: A questão concernente à natureza do tempo está intimamente relacionada à antítese tradicional entre o universo parmenidiano a-temporal estático e o modelo dinâmico do vir-a-ser. Isso está maravilhosamente ilustrado na teoria peirciana do fluxo do tempo. Nesta rejeição da visão de mundo atomista, Peirce tratou o tempo em relação íntima com os processos físicos. Ele então propôs uma teoria extremamente interessante que tem certo parentesco com teorias contemporâneas da flecha do tempo. Entretanto, o que torna essa abordagem extremamente interessante é não apenas seu ar de

Cognitio, São Paulo, v. 9, n. 2, p. 261-280, jul./dez. 2008

modernidade, mas suas notáveis semelhanças com Aristóteles (Física, livro IV, esp. caps. 10-14). Meu propósito, portanto, neste trabalho é reconstruir a teoria do tempo de Peirce à luz da filosofia de Aristóteles. Meu ponto de partida será a análise de continuidade em relação aos infinitesimais (6.109), por meio da qual ele pode tratar o tempo como um "continuum par excellence" (6.86, 1898), que não é uma coleção estática de instantes discretos (ver MS 137, p. 4-5, 1904), mas uma coleção de possibilia reais (ver NE, 360), tornando assim possível a compreensão do fluxo do tempo (6.11). Então prosseguirei com o exame da conexão do tempo de Peirce, como um continuum real, com a idéia de infinito. A este respeito, focalizarei a sua rejeição da infinidade verdadeira no sentido zenoneano-atomístico-cantoriano – e sua adoção da idéia aristotélica de potencial infinito (Física, livro III, caps. iv-viii). Argüirei, portanto, que o que levou Peirce a passar da análise lógico-matemática de continuidade para uma teoria ontológica foi seu apelo à potencialidade no seu tratamento de continuidade-infinidade-tempo. Isto, creio, foi o resultado da influência que ele recebera de Aristóteles, que estava também profundamente preocupado em dar à mudanca, movimento e vir-a-ser, seu lugar apropriado na natureza. Assim, tanto Peirce quanto Aristóteles puderam construir uma teoria dinâmica do tempo, intimamente relacionada à idéia de infinidade potencial, que expressa um processo físico sendo progressivamente atualizado, de tal modo que ele jamais poderá existir como um todo imaginado.

Palavras-chave: Peirce. Aristóteles. Tempo. Continuidade. Infinitesimais.

Introduction*

The question concerning the nature of time and its intimate relation to change and becoming in the physical world has occupied the minds of philosophers since the time of the pre-Socratics. What is the nature of time? Is the flow of time real or is it only an illusion? This is a question that in its longstanding history remains essentially the same and brings into light the traditional antithesis between the static a-temporal Parmenidean¹ universe, and the dynamic model of becoming. The latter is wonderfully illustrated in Peirce's theory of the flux of time. In his rejection of the Newtonian-atomistic world-picture, Peirce treated time in close relation to the physical processes. He thus propounded an extremely interesting theory which bears a kinship to contemporary theories of the *arrow of time*, such as that of Ilya Prigogine². However, what makes Peirce's theory extremely interesting is not only its air of modernity, but also its striking similarities with Aristotle.

^{*} A substantial part of the present work is the product of my research in the context of a project carried out within the framework of the "Operational Programme for Education and Initial Vocational Training (O. P. "Education"), "Herakleitus", co-financed by the European Union and the European Social Fund.

¹ The Parmenidean attitude towards time marked an important stage in the history of philosophical thought and at the same time Parmenides' theory of time has become, as Hans Reichenbach has pointed out, the "historical symbol of a negative emotional attitude towards the flux of time" [H. Reichenbach, *The Direction of Time*, ed. Maria Reichenbach (Berkeley, California, 1956)].

² See Ilya Prigogine, *The End of Certainty* (New York: The Free Press, 1997).

My purpose, therefore, is to reconstruct Peirce's doctrine of time in the light of Aristotle's philosophy. What both thinkers share in common, as is my hope to show, is their deep concern to give to *change-motion-becoming* its proper place in nature. They were, therefore, both opposed to the static model of reality, each of his own time: Aristotle to the Eleatic school and the atomists, whereas Peirce to Newtonian mechanics and atomism of his time. To provide an answer to those theories, respectively, they both saw that it was necessary to defend continuity, as "an indispensable element of reality".

Let us, now, set the historical stage, starting with Aristotle.

Aristotle and the Parmenidean Static Universe

An essential aspect of Aristotle's philosophy was his antithesis to the philosophers of South Italy, Parmenides and Zeno, as well as to Democritus from Abdera. What these philosophers had in common was their attempt to prove that there is no real motion in the world, no real change of qualities, no generation and corruption. According to the Eleat philosopher Parmenides, if a real becoming were taking place in nature, then there should be a passage from being to non being. However, this was something impossible from a logical point of view, because only being exists. He thus rejected the idea of becoming and with this, the idea of the reality of time. According to the Eleat philosopher, the world in its totality is an unchanging whole in which nothing is moving, in which the flux of time is an illusion.³

On the basis of this theory and having as his goal to show the a-temporality of being and the impossibility of motion, his successor Zeno developed his paradoxes, the most notorious of which is that of *Achilles and the tortoise*. In simple terms, this argument goes as follow⁴:

In a race between Achilles and a tortoise, in which the tortoise starts before Achilles, the latter will never be able to catch up the tortoise. This will happen, according to Zeno, because before Achilles reaches the tortoise he will have to traverse half of the distance that separates him from the tortoise. Therefore, when Achilles is at point A and the tortoise at point B, Achilles must first traverse the $\frac{1}{2}$ of the distance between the two points. Then he will have to traverse half of the remaining distance, i.e. the $\frac{1}{4}$ and then the $\frac{1}{8}$ and so on. As a result, he must traverse an infinite number of parts of the distance, each one of which represents a distance that has to be traversed. Since motion from one point to the other takes some time – no matter how short – and since an infinite number of distances must be traversed in order that Achilles can move from one point to the other, the movement from point A to point B would take an infinite time. Thus, Zeno arrives at the conclusion that Achilles can never reach point B.

³ I have dealt with this issue in more detail in D. Sfendoni-Mentzou, "The 'Frozen Passage' and Aristotle's 'Becoming' of the Physical World" (in Greek), In Vita Contemplativa. Essays in Honour of Demetrics N. Koutras (in Greek), ed. Athanasia Glykofrydi-Leontsini (Athens: University of Athens, 2006), 469-485.

⁴ For an instructive presentation of Zeno's paradoxes see, W. MacLaughlin, "Resolving Zeno's Paradoxes", *Scientific American* (1994): 66-71.

The next most famous *paradox* is that of *the arrow*. This is used by Zeno with the purpose of proving that movement itself does not exist. Imagine an arrow at the moment that it is moving to its target. The arrow, which has a length – no matter how small – occupies a part of space equal to its length. Exactly at that moment, it is not moving. And since this holds true for each separate instant of time, we must come to the logical conclusion that the arrow is never in motion.

As opposed to the Eleats and Zeno, Aristotle tried to build a dynamic model of the physical world with a fundamental temporal structure. For the Stagerite, every process has inherent temporal parts and relations, all of which are essential elements of its reality, as I will try to show. But, before that, let us have a look at Peirce.

Peirce and the 19th Century Atomism

The way Peirce developed his objections to the 19th century proponents of the atomistic model reveals an interesting analogue between Peirce and Aristotle. Peirce's opposition to Newtonian mechanistic determinism pervaded all aspects of his philosophy and gave rise to such important doctrines, as that of *tychism* – or theory of *probability* and *chance*⁵ – and his *synechism* – or theory of *continuity*.⁶ As he claims in 1902,

Synechism is not an ultimate and absolute metaphysical doctrine; it is a regulative principle of logic, prescribing what sort of hypothesis is fit to be entairtained and examined. The synechist, for example, would never be satisfied with the hypothesis that matter is composed of atoms. (CP 6.174, *Baldwin's Dictionary*, 1902)⁷

We must, though, say that Peirce's opposition to the atomists started with his opposition to the logico-mathematical model of Richard Dedekind (1831-1916)⁸ and Georg Cantor (1845-1918), a model which was accepted by the majority of mathematicians in the

⁵ For a detailed examination of Peirce's Tychism see, Demetra Sfendoni-Mentzou, *Probability and Chance in C. S. Peirce and Contemporary Science* (doctoral thesis, in Greek, Thessaloniki: Aristotle University of Thessaloniki, 1980), "The Role of Potentiality in C. S. Peirce's Tychism and in Contemporary Discussions in Q.M. and Micro-Physics", in *Charles S. Peirce and the Philosophy of Science: Papers from the 1989 Harvard Conference*, ed. Edward Moore (Tuscaloosa: University of Alabama Press, 1993), 246-261.

⁶ I have discussed the issue of Peirce's doctrine of continuity in relation to his theory of laws of nature in D. Sfendoni-Mentzou, "Peirce on Continuity and Laws of Nature," *Transcactions of the Charles S. Peirce Society*, vol. 33, No 3 (1997): 646-678.

⁷ References of this form are to the *Collected Papers of Charles Sanders Peirce* (vols. 1-8), eds., C. Hartshorne, P. Weiss and A. Burks (Cambridge, Mass: Belknapp Press of the Harvard University Press, 1931-58). The *Collected Papers*, hereafter *CP*, will be referenced by volume and paragraph number.

⁸ "Personally," says Peirce, "I agree entirely with James, against Dedekind's view: and hold that there would be no actually existent points in an existent continuum, and that if a point were placed in a continuum it would constitute a breach of the continuity" (CP 6.128, 1892).

nineteenth century⁹. The idea on which this model was built was the same as that of Zeno: The continuum was a collection of actual, discrete, independent points.¹⁰

The Continuity of a Line

In *The New Elements of Mathematics*, Peirce gives the following description of Cantor's idea of a continuum:

Cantor, in effect, defines continuity of a line as consisting in that line's containing *all* its points. $(3.58)^{11}$

This is something that Peirce characterizes as a pseudo-continuity:

This consideration renders it easy to define a pseudo-continuum. It is in the first place a collection of objects absolutely distinct from one another. (CP 6.176, 1906)

Peirce was perfectly aware of Aristotle's views on continuity. So, he explicitly refers to Aristotle's definition of a continuum as "something whose parts have a common limit" (CP 6.122, 1892). "What is required, therefore", he claims,

is to state in non-metrical terms that if a series of points up to a limit is included in a continuum the limit is included. It may be remarked that this is the property of a continuum to which Aristotle's attention seems to have been directed when he defines a continuum as something whose parts have a common limit... This property, which may be called the Aristotelicity of the series, together with Kant's property, or its Kanticity, completes the definition of a continuous series. (CP 6.122-123, *The Monist*, 1892)

⁹ This holds true, at least until the appearance of Robinson's non-standard analysis.

¹⁰ To this idea, both Kronecker and Peirce were opposed, as we shall see in more detail. Today the debate is still ongoing between the intuitionists and the formalists, i.e. between the followers of Kronecker and Brower on the one hand and Cantor and Hilbert on the other.

¹¹ This is also how Peirce describes Cantor's continuum in his *Monist* series: "Cantor defines a continuous series as one which is *concatenated* and *perfect* [...] It must be granted that Cantor's definition includes every series that is continuous; not can it be objected that it includes any important or indubitable case of a series not continuous. Nevertheless, it has some serious defects. In the first place it turns upon metrical considerations; while the distinction between a continuous and a discontinuous series is manifestly non-metrical. In the second place, a perfect series is defined as one containing "every point" of a certain description. But no positive idea is conveyed of what all the points are: that is definition by negation, and cannot be admitted[...] Finally, Cantor's definition does not convey a distinct notion of what the components of the conception of continuity are" (CP 6.121, *The Monist*, 1892).

And he explains:

The property of Aristotelicity may be roughly stated thus: a continuum contains the end point belonging to every endless series of points which it contains. An obvious corollary is that every continuum contains its limits. (CP 6.123. *The Monist*, 1892; cf. 6.174, 4.121, 4.642)

To understand this, we must have in mind that, according to Cantor, there is an isomorphism between the system of points on a line and the system of real numbers.¹² So, if a line is divided into two parts, what will happen to the line is analogous to what will happen to the real numbers. This idea is expressed by the "Dedekind Cut Theorem", according to which, if we make a division of real numbers at point P into two sections L (left) and R (right), known as a "Dedekind Cut", then every number belongs to exactly one of the two sections. So, no matter how the "Dedenkind Cut" is made, it is always the case that the two halves created by the division cannot be mirror images of each other. If this be applied to the line, when we divide the line AD at point P (fig.1), we must either include point P in the R half of the division, or include point P in the L half of the division. In either case one of the two half parts of the line must remain without an end point, so that the two half lines will not be mirror images.



Peirce's objection to this idea was this: If you divide a line at point P the two half parts created by the division will be mirror images. This is exactly where Peirce meets Aristotle. The basic idea for both thinkers is that a line is an irreducible geometrical object and not a collection of more elementary objects.¹³ Thus, if a line is divided into two parts, it makes no sense to ask to which half the point of division belongs, because points do not belong to lines although they lie on them. A line does not consist of actual, discrete, independent points.

Hence, the end-points A and D are not to be regarded as members of the line segment AD, but simply as points that are located there because of the fact that the line we have constructed ends there. Moreover, it must be noted that in fig.2 the L half AB of the original segment AD still has two endpoints and this is, because the idea of an

¹² For the understanding and presentation of Peirce's analysis of continuity, as opposed to that of Cantor and Dedekind, I have greatly benefited from my reading *Reasoning and the Logic of Things: Charles Sanders Peirce*, eds., K. Ketner & H. Putnam (Cambridge, Mass: Harvard University Press, 1992), and Hilary Putnam, "Peirce's Continuum", in *Petrce and Contemporary Thought. Philosophical Inquiries*, ed. K. L. Ketner (New York: Fordham University Press, 1995). I also owe a lot to my reading K. Eisele, *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce* (The Hague: Mouton Publishers, 1979), 1-22.

¹³ Geometry for Peirce is the study of continuity. In the geometric analysis Peirce was led to topology and modes of connection with the continuum. In this respect, his logic is a modal logic.

"open" line interval has no meaning at all; a line interval defines its endpoints. The same holds true for the line interval CD. Taking this as a starting point, we can unroll the threats of thought of our two thinkers and shed light on their deep relationship as regards the ideas of *continuity*, *infinity*, *infinite divisibility*, and finally their *theory of time*.¹⁴

Aristotle and Infinite Divisibility of a Continuum

Aristotle, like Dedekind, Cantor and Peirce, starts with the notion of a straight line. In book Viii of his *Physics*, the Stagerite is dealing with the idea of a *continuum*. His definition is the following:

The 'continuous' ($\sigma \nu \nu \epsilon \chi \dot{\epsilon} \zeta$) is a subdivision of the contiguous ($\delta \pi \epsilon \varrho \dot{\epsilon} \chi \dot{o} \mu \epsilon \nu \dot{o} \nu \tau \iota$); for I mean by one thing being continuous with another that those limiting extremes of the two things in virtue of which they touch each other become one and the same thing, and (as the very name indicates) are 'held together' ($\sigma \nu \nu \epsilon \chi \eta \tau \alpha \iota$), which can only be if the two limits do not remain two but become one and the same. From this definition it is evident that continuity is possible in the case of such things as can, in virtue of their natural constitution, become one by touching. And the whole will have the same sort of union as that which holds it together, e.g. by rivet or glue or contact or organic union. (Phys. 227a 10-17)¹⁵

In book VI.i he adds one more extremely important idea: the continuum cannot consist of indivisible monads, e.g. points on a line: Points cannot be distinct, because between two points there is always a line:

Again, one point, so far from being continuous $(\sigma \upsilon \upsilon \epsilon \chi \epsilon \varsigma)$ or contiguous $(\dot{\alpha} \pi \tau \dot{\sigma} \mu \epsilon \upsilon \upsilon \upsilon)$ with another point, cannot even be the *next-in-succession* $(\dot{\epsilon} \varphi \epsilon \xi \tilde{\eta} \varsigma)$ to it, [...] for things are 'next' to each other when there is nothing of their own sort between them, and two points have always a line (divisible at intermediate points) between them... (Phys. 231b 6-10)¹⁶

¹⁴ For an illuminating analysis of the relationship between the ideas of continuity and time in Peirce, see Sandra B. Rosenthal, *Charles Peirce's Pragmatic Pluralism*. (Loyola University, New Orleans: State University of New York Press, 1994), in particular chapter 3: "Habit, Temporality and Peirce's Proofs of Realism" pp.63-76 and "Continuity, Contingency and Time: The Divergent Intuitions of Whitehead and Pragmatism", *Transactions of the Charles S. Peirce Society*, Vol. 32, No.4 (1996): 542-566. See also, S. H. Levy, "C. S. Peirce's Theory of Infinitesimals", *International Philosophical Quarterly*, vol.31, No2 Issue No 122 (1991): 127-140.

¹⁵ "τὸ δὲ συνεχὲς ἔστι μὲν ὅπεϱ ἐχόμενόν τι, λέγω δ' εἶναι συνεχὲς ὅταν ταὐτὸ γένηται καὶ ἕν τὸ ἑκατέۅου πέۅας οἶς ἄπτονται, καὶ ὥσπεϱ σημαίνει τοὕνομα, συνέχηται. τοῦτο δ' οὐχ οἶόν τε δυοῖν ὄντοιν εἶναι τοῖν ἐσχάτοιν. τούτου δὲ διωρισμένου φανερὸν ὅτι ἐν τούτοις ἐστὶ τὸ συνεχές, ἐξ ῶν ἕν τι πέφυκε γίγνεσθαι κατὰ τὴν σύναψιν. καὶ ὥς ποτε γίγνεται τὸ συνέχον ἕν, οὕτω καὶ τὸ ὅλον ἔσται ἕν, οἶον ἢ γόμφῷ ἢ κόλλῃ ἢ άφῆ ἢ προσφύσει" (Phys. 227a 10-17).

¹⁶ "ἀλλὰ μὴν οὐδὲ ἐφεξῆς ἔσται στιγμὴ στιγμῆ...ἐφεξῆς μὲν γάǫ ἐστιν ὧν μηθέν ἐστι μεταξύ συγγενές, στιγμῶν δ' αἰεἰ [τὸ] μεταξύ γǫαμμὴ..." (Phys. 231b 6-10).

Aristotle's conclusion, therefore, is that the continuum is that which can be divided into what can also be divided indefinitely:

I mean by continuous 'capable of being divided into parts that can in their turn be divided again, and so on without limit'.

λέγω δὲ συνεχὲς τὸ διαιρετὸν εἰς αἰεὶ διαιρετά. (Phys. 232b 24-25)¹⁷

A similar idea is expressed in book I of *De caelo*:

Now a continuum is that which is divisible into parts always capable of subdivision, and a body is that which is in every way divisible.

Συνεχὲς μὲν οὖν ἐστι τὸ διαι
οετὸν εἰς ἀεἰ διαιρετά, σῶμα δὲ τὸ πάντῃ διαιρετόν (De cael. 268
a6-7)

(Εἰδ' ἐστὶ συνεχὲς καὶ ἀπτόμενον καὶ ἐφεξῆς, ὡς διώǫισται πǫότεǫον, συνεχῆ μὲν ὧν τὰ ἔσχατα ἕν, ἀπτόμενα δ' ὧν ἅμα, ἐφεξῆς δ' ὧν μηδὲν μεταξὺ συγγενές...) (Phys. 231a21-23).

In order to defend his thesis, Aristotle develops three arguments: According to the first, an indivisible cannot have any limits, since the limit must be something else than that of which it is a limit. Consequently, that which has limits must consist of parts, and therefore must be divisible. "If these definitions are accepted, it follows that no continuum can be made up of indivisibles, as for instance a line out of points, granting that the line is continuous and the point indivisible. For two points cannot have identical limits, since in an indivisible there can be no distinction of a limit from some part other than the limit; and (for the same reason) neither can the limits be together, for a thing that has no parts has no limit, since a limit must be distinct from what it limits

(ἀδύνατον ἐξ ἀδιαιφέτων εἶναί τι συνεχές, οἶον γφαμμὴν ἐκ στιγμῶν, εἴπεφ ἡ γφαμμὴ μὲν συνεχές, ἡ στιγμὴ δὲ ἀδιαί φετον. οὔτε γὰφ ἓν τὰ ἔσχατα τῶν στιγμῶν οὐ γάφ ἐστι τὸ μὲν ἔσχατον τὸ δ' ἄλλο τι μόφιον τοῦ ἀδιαιφέτου, οὔθ' ἅμα τὰ ἔσχατα οὐ γάφ ἐστιν ἔσχατον τοῦ ἀμεφοῦς οὐδέν· ἕτεφον γὰφ τὸ ἔσχατον καὶ οῦ ἔσχατον) (Phys. 231a24-29).

According to the second argument, the indivisibles would not be able to form a continuum in case they are contiguous for the following reason: If a point were in touch with another point, since it is indivisible, it would be in touch as a whole, which means for Aristotle that it should occupy exactly the same space. However, this kind of touch is not possible to create a continuum, because continuum must be divided in parts, each one of which occupies a different space: "But since the indivisible has no parts, if two indivisibles touched each other at all it must be in their entirety. But if they were touching in their entirety, they could not make a continuum, for a continuum is divisible into parts which are distinguishable from each other in the sense of being in different places"

(ἐπεὶ δ' ἀμερὲς τὸ ἀδιαίρετον, ἀνάγκη ὅλον ὅλου ὅπτεσθαι. ὅλον δ' ὅλου ἀπτόμενον οὐκ ἔσται συνεχές. τὸ γὰρ συνεχὲς ἔχει τὸ μὲν ἄλλο τὸ δ' ἄλλο μέρος, καὶδιαιρεῖται εἰς οὕτως ἕτερα καὶ τόπῳ κεχωρισμένα) (Phys. 231b 3-6).

Cognitio, São Paulo, v. 9, n. 2, p. 261-280, jul./dez. 2008

¹⁷ "The terms 'continuous', 'contiguous', and 'next-in-succession' have been defined above as follows: things are 'continuous' if (while they are themselves distinct in the sense of occupying different places) their limits are one, 'contiguous' if their limits are together, 'next in succession' if they have nothing of the same nature as themselves between them..."

Peirce and Aristotle: Continuity-infinity

Thus for Aristotle and for Peirce, as we shall see, the idea of *continuity* involves that of *infinite divisibility*: Any attempt, therefore, to provide a definition of continuity involves the idea of infinity. It is the belief of both thinkers that without a clear understanding of infinity there can be no clear answer to the issue of continuity and *vice versa*. In *Physics* book V, Aristotle makes clear that,

[I]t is in connexion with continuity that we first encounter the concept of the 'infinite'. And this is why in definitions of continuity this concept of the 'infinite' frequently occurs, as when we say that the continuous is that which is susceptible of division without limit. (Physics, 200b 18-22)¹⁸

So then, (a) if a single *continuum* is what is meant by 'one', it follows that 'the One' is many, for every *continuum* is divisible without limit.

εἰ μὲν τοίνυν συνεχές, πολλὰ τὸ ἕν εἰς ἄπειρον γὰρ διαιρετὸν τὸ συνεχές. (Phys. 185b 10-12)

This is exactly the idea of Peirce, who remarks:

[c]ontinuity involves infinity in the strictest sense, and infinity even in a less strict sense goes beyond the possibility of direct experience. (CP 1.166, c. 1897; cf., 1.165)

Here we have the introduction of an extremely interesting Aristotelian idea: Infinity goes "beyond the possibility of direct experience." To understand the meaning of this in its full depth, we need to make an appeal to Peirce's theory of *infinitesimals*.

Infinitesimals-Potential Aggregate

Let me first say, that the theory of *infinitesimals*¹⁹ implies the possibility of non standard points on a line,²⁰ as opposed to Cantor's idea of a continuum, built on the assumption of standard points.

^{18 &}quot;διὸ καὶ τοῖς ὁριζομένοις τὸ συνεχὲς συμβαίνει προσχρήσασθαι πολλάκις τῷ λόγῳ τῷ τοῦ ἀπείρου, ὡς τὸ εἰς ἄπειρον διαιρετὸν συνεχὲς ὄν" (Phys., 200b 18-22).

¹⁹ For a very interesting and illuminating analysis of the idea of infinitesimals see, S. H. Levy, op. cit.; J. Dauben, "Abraham Robinson and Nonstandard Analysis," *Minnesota Studies in the Philosophy of Science*, vol. XI (1988): 177-200; W. MacLaughlin and S. L. Miller, "An Epistemological use of Nonstandard Analysis to Answer Zeno's Objections against Motion," *Synthese*, 92 (1992): 371-384.

²⁰ In Robinson's non-standard analysis, infinitesimals are line intervals whose length lies between zero and every positive standard number, or, in other words, an infinitesimal is greater than zero, but less than any possible length whatsoever.

This is expressed by Peirce as follows: "within a single point one can find at least c different point parts whose c is the power of the set of real numbers and the cardinal number of points on the line is not only greater than c, but greater than the cardinal of all sets." As a result, Peirce arrived at the idea of a *continuum* as consisting of the standard points together with all the nonstandard points in their monads, or as he himself remarks, a continuum is "merely a discontinuous series with additional possibilities" (CP 1.170, c.1897).

There is no doubt that Peirce's treatment of continuity is deeply Aristotelian, as he introduces the idea of *potentiality*. This is exactly what Aristotle himself had done in order to provide an answer to Zeno's arguments: In referring to the division of a line, he claimed that the two halves of a line *do no exist actually* but only *potentially*. This was also expressed in book III of *De Anima* (430b 10), where the Stagerite remarked that length is not actually divided, since its parts exist not actually but only potentially. And in *Metaphysics* he found a most interesting way to express this thesis by using an analogy between the half of a line and the statue of Hermes:

'Actuality' means the presence of the thing, not in the sense which we mean by 'potentially'. We say that a thing is present potentially as Hermes is present in the wood, or the half-line in the whole... (*Met.* 1048a 32-4)²¹

What Aristotle is trying to make clear here, is that a line consists, both of *potential* and *actual* parts, or points. The potential parts, or points, are those in which the line *could be* divided, whereas the actual are those in which it is *actually* divided. So, for Aristotle, points have a potential being on the line and pass to actuality only when the division is actually made.

We remember that this is exactly what Peirce, too, claimed concerning the "Dedekind's cut". For both thinkers, therefore, points do not exist as points, unless they are marked on the line. When we are, thus, dealing with a collection of material numbers, what we are really considering is its possibility of being and not an existent particular object. In other words, we are dealing with a "collection of possible individuals" and this is exactly what happens with our line, as well. Accordingly, we could also claim that a line should be viewed not as a collection of points, but as a collection of possible points, i.e. a collection of *possibilia*.²²

²¹ "ἕστι δὴ ἐνέργεια τὸ ὑπάρχειν τὸ πρᾶγμα μὴ οὕτως ὥσπερ λέγομεν δυνάμει λέγομεν δὲ δυνάμει οἶον ἐν τῷ ξύλῳ Ἐρμῆν καὶ ἐν τῷ ὅλῃ τὴν ἡμίσειαν" (Met. 1048a 32-4).

²² See Caroline Eisele, *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce.* (The Hague: Mouton Publishers, 1979), 212.

In an extremely interesting manuscript, titled *Topical Geometry*, Peirce remarks:

the points on a line are not a collection of discrete objects. Their being is welded together, so that no one can logically be removed alone. That is as much as to say that there really exist no points upon a continuous line... they have a potential being; but they do not exist until something happens which marks them... therefore those intermediate points, being possible, are already there in the only sense there is in speaking of unmarked points. Such is the notion of continuity.... (MS 137, 1904)²³

Potential Aggregate-Possibilia: Peirce and Aristotle

This is, therefore, how Peirce arrived at his idea of a "*potential aggregate*".²⁴ A "*potential aggregate*" is defined by Peirce as a collection of possible individuals which have not been made actual yet, "a collection of which the individual units have no distinct identity" (CP 4.172).²⁵ On this view, the aggregate of all abnumeral collections finally leads to such a dense field of possibility that the units of the aggregate lose their individual identity. The aggregate ceases to be a collection (in the Cantorian-atomists' sense) and becomes a continuum.

Hence, the reason that a line is a collection of points that "lack distinct individuality" is that it is not a collection of mathematical objects but a collection of *possibilia*.²⁶ On the basis of this idea, the definitions of continuity given by Peirce in various places take the following form: "the doctrine of continuity is that *all things* so swim in continua" (CP 1.171, c. 1897), or "a continuum" is merely a discontinuous series with additional possibilities" (CP 1.170, c.1897), or a true continuum is something whose possibilities of determination "no multitude of existent things could exhaust" (CP 5.103); "Continuity is fluidity, the merging of part into part" (1.164, c. 1899).

²³ "Let part of a surface be painted green while the rest remains white. What is the color of the dividing line; is it green or not? I should say that it is both green and not. ' But that violates the principle of contradiction, without which there can be no sense in anything'. Not at all; the principle of contradiction does not apply to possibilities". This view is also connected with the boundary problem in Peirce's topological studies. (See, C. Eisele, *op.cit.* p. 212)

²⁴ For a detailed analysis of how Peirce's idea of a "potential aggregate" is connected with his theory of generality, and how both are related to the scholastic idea of universals, see my "Peirce on continuity and laws of nature," *Transactions of the Charles S. Peirce Society*, Vol. 33, Nº.3 (1997): 646-678. Peirce explicitly connected continuity with generality; see e.g., "...that continuity is the absence of ultimate parts in that which is divisible; and that the form under which alone anything can be understood is the form of generality, which is the same thing as continuity" (CP 6.173, 1902).

²⁵ See also, "It is vague, but yet with such a vagueness as permits of its accurate determination in regard to any particular object proposed for examination" (CP 6.186, 1898), or else, " [i]t is a *potential* collection indeterminate yet determinable" (*ibid*.).

²⁶ In this sense Peirce's theory is similar to that of L. E. J. Brouwer. See K. Ketner and H. Putnam, *op.cit.*, p. 50.

The Doctrine of Time: Peirce and Aristotle

All the above analyses concerning the *continuum* and the relevant ideas of *infinite divisibility*, absence of actual points, *potential aggregate* and *infinitesimal intervals*, offer an extremely useful tool for dealing both with the Dedekind theorem and with Zeno's paradox. Remember that Zeno's aim was to reject motion/change and time. According to Zeno, the arrow is at rest at discrete points of time, and thus it can never be said that there is some motion. We can have an analogue of this, if we imagine, as Peirce puts it,

a series of instantaneous photographs to be taken. Then, no matter how closely they follow one another, there is no more motion visible in any one of them than if they were taken at intervals of centuries. (NE 3.59)²⁷

Thus Peirce – following Aristotle – thinks that the only way to provide an answer to this static model of time is to defend *real continuity* which has all the characteristics that have already come to light, through his analysis of a *continuum*²⁸.

Points on a Line-Instants of Time

He thus associates points on a line sequent with instants in an interval of time in order to show that,

between any two instants of time or between any two points on a line, there is room for any multitude of instants or of points, whatsoever – not merely room for any enumerable multitude, but room for any one of the single denumeral series of abnumerable (abzahlbor) multitudes.

In this passage we have two fundamental aspects of time: The first is *infinite divisibility*: "continuity", claims Peirce, "is the absence of ultimate parts in that which is divisible" (6.173, 1902), and "A true continuum is something whose possibilities of determination no multitude can exhaust" (6.170, *Baldwin's* 1902). The second is the idea that "the instants of time are so close together as to merge into one another" (NE, 3.60). "The very word continuity implies that the instants of time or the points of a line are everywhere welded together" (NE, 3.61),²⁹ so as to loose their identity.³⁰ Hence, instants of time as points on a line, become a collection of possible points, a collection of *possibilia* which, as *possibilia*, lack distinct individuality but are nonetheless real.

²⁷ Charles S. Peirce, *The New Elements of Mathematics*, 4 vols., ed. Caroline Eisele (The Hague: Mouton, 1976, 3.60). *The New Elements*, hereafter *NE*, will be referenced by volume and paragraph number. See also, "It would be in the general spirit of synechism to hold that time ought to be supposed truly continuous in that sense. The term was suggested and used by C. S. Peirce in 1892" (CP 6.170, *Baldwin's Dictionary*, 1902).

²⁸ Time, says Peirce, is "a continuum *par excellence*, through the spectacles of which we envisage every other continuum" (CP 6.86, 1898).

²⁹ See also, "[t]heir being is welded together" (MS 137, p. 4-5, 1904).

 $^{^{30}}$ *Cf.* [the instants of time are not distinct, but rather] "are so close together as to merge into one another..." (NE, 3.60).

Time-Potential Infinite

This view of time is essentially interwoven, both for Peirce and Aristotle, with the idea of a potential infinite, which gives to time its dynamic character. The actual infinite is that whose infinitude exists or is given all at the same time. The *potential infinite* is that whose infinitude is given over time and is never present as a whole. Of extreme interest here is the Stagerite's remark in his *Physics*, book III. vi that:

For it [the infinite] is not that 'beyond which there is nothing', but 'what is always beyond'

ού γà
ρ οὖ μηδὲν ἔξω, ἀλλ' οὖ ἀεί τι ἔξω ἐστί, τοῦτο ἀπει
φόν ἐστιν. (Phys. 207a 1-2)

This can only be understood, if we make an appeal to the Aristotelian idea of the *potential infinite*: "But since we can always make another division of a magnitude into two", claims Aristotle,

however many divisions you have already made to get it, you can always conceive a higher number of divisions than any given number however great; consequently the 'possibility of more' is inexhaustible and incapable of completion, but can be carried on through a greater than any assignable number of steps. $(Pbys. 207b10-13)^{31}$

And in book III.v, he also claims,

It is further manifest that infinity cannot exist as an actualized entity and as substance or principle

φανεφὸν δὲ καὶ ὅτι οὐκ ἐνδέχεται εἶναι τὸ ἄπειφον ὡς ἐνεφγεία ὄν καὶ ὡς οὐσίαν καὶ ἀρχήν. (Phys. 204a20-21)

The Flux of Time: Peirce and Aristotle

The idea of *potential infinite*, seen in connection with that of *continuity* and Peirce's theory of *infinitesimals*, can contribute an essential element for our understanding of the *flux of time* and of *time's arrow*. According to Peirce "the present is connected with the past by a series of real infinitesimal steps" (CP 6.109, *The Monist*, 1892). But, as he makes clear, "this is only true if the series be continuous. Here, then, it seems to me, we have positive and tremendously strong reason for believing that time is continuous" (CP 1.169, 1897). Time can in no way be understood in terms of our indefinite succession of discretes, but rather as a continuous flow.

³¹ "ἐπὶ δὲ τὸ πλεῖον ἀεὶ ἔστι νοῆσαι ἄπειροι γὰρ αί διχοτομίαι τοῦ μεγέθους. ὥστε δυνάμει μὲν ἔστιν, ἐνεργεία δ' οὕ ἀλλ' ἀεὶ ὑπερβάλλει τὸ λαμβανόμενον παντὸς ώρισμένου πλήθους" (Phys. 207b10-13).

What, therefore, unites the 'before' and 'after' for both thinkers, is the 'now' ($\nu \bar{\nu} \nu$), which is not a distinct instant or point, but is always in a state of transition, being the end of the past and the beginning of the future. As Peirce remarks, "the present is half past and half to come" (CP 6.126). This is exactly what Aristotle, also, pointed out:

We have said that it is through the 'now' that time is continuous, for it holds time past and future time together; and in its general character of limit' it is at once the beginning of time to come and the end of time past. (Phys. $222a10-12)^{32}$

It seems that Peirce was perfectly aware of Aristotle's conception of the role that the "now" $(\nu \bar{\nu} \nu)$ plays in the flux of time:

Let us now consider an aspect of the Aristotelical principle which is particularly important to philosophy. Suppose a surface to be part red and part blue; so that every point on it is either red or blue, and, of course, no part can be both red and blue. What, then, is the color of the boundary line between the red and the blue?

Here is his answer:

...as the parts of the surface in the immediate neighborhood of any ordinary point upon a curved boundary are half of them red and half blue, it follows hat the boundary is half red and half blue.

And he continues,

In like manner, we find it necessary to hold that consciousness essentially occupies time; and what is present to the mind at any ordinary instant is what is present during a moment in which that instant occurs. Thus, the present is half past and half to come. (CP 6.126)

The Problem of Consciousness and Real Time

At this point, the problem of consciousness and real time makes its appearance. It is certainly true that we all have the experience of the flux of time, as an inner awareness of a flow from past to future. We all remember things that happened in the past and we anticipate future events. This is exactly how we are in a position to make the distinction of the 'before' and 'after', and thus have a feeling of the *arrow of time*, i.e. of time's *uni-directionality* or time's *asymmetry*. But is time merely that inner feeling we have of its flow? Shall we say that it is solely the product of our consciousness and, thus, not real,

³² "Τὸ δὲ νῦν ἐστιν συνέχεια χρόνου, ὥσπερ ἐλέχθη συνέχει γὰρ τὸν χρόνον τὸν παρεληλυθότα καὶ ἐσόμενον, καὶ πέρας χρόνου ἐστίν ἔστι γὰρ τοῦ μὲν ἀρχή, τοῦ δὲ τελευτή" (Phys. 222a 10-12).

but only an illusion? This is an extremely interesting problem, which occupied the minds of both Peirce and Aristotle.³³ So Peirce claims:

One of the most marked features about the law of mind is that it makes time to have a definite direction of flow from past to future. The relation of past to future is, in reference to the law of mind, different from the relation of future to past. This makes one of the great contrasts between the law of mind and the law of physical force, where there is no more distinction between the two opposite directions in time than between moving northward and moving southward. (CP 6. 127).³⁴

This was also a serious problem considered in depth by Aristotle. Thus, in Book V. xiv of his *Physics* the Stagerite remarks:

The relation of time to consciousness ($\pi \varrho \delta \varsigma \tau \eta \nu \psi \upsilon \chi \eta \nu$) deserves examination, and so does the question why we conceive of time as immanent in everything in earth, sea and sky. (Phys. 223a 16-8)³⁵

And he continues:

The question remains, then, whether or not time would exist if there were no consciousness; for if it were impossible for there to be the factor that does the counting (numbering, $\tau o \tilde{\nu} \, \dot{\alpha} \varrho_i \theta_\mu \eta \sigma o v \tau \sigma \varsigma$), it would be impossible that anything should be counted ($\dot{\alpha} \varrho_i \theta_\mu \eta \tau \delta v$)[...]and if nothing can count except consciousness[...]it is impossible that time should exist if consciousness did not. (Phys. 223a 21-26)³⁶

To understand what Aristotle has in mind when he refers to the "factor that does the counting", we must remember that in the same book ch. xi he defines time as the counting of motion:

³³ Aristotle starts chapter x of book IV of *Physics* as follows: "The subject of inquiry next in succession is *time*. It will be well to begin with the questions which general reflections suggest as to its being or non-being, and then as what is its nature" (Phys. 217b 30-33).

³⁴ See also, "Thus in respect to the direction of its flow, time seems to be, if not purely a psychological affair, at any rate not purely dynamical affair. Those physical phenomena which proceed in one direction and not in the reverse direction, and which seem to be well explained, such as the viscosity, diffusion, and conduction of gases, may all be explained by principles of probability" (CP 6.387, *Baldwin's*; *cf.* CP 1.493, 1896).

³⁵ "άξιον δ' ἐπισκέψεως καὶ πῶς ποτε ἔχει ὁ χρόνος πρὸς τὴν ψυχήν, καὶ διὰ τί ἐν παντὶ δοκεῖ εἶναι ὁ χρόνος, καὶ ἐν γῆ καὶ ἐν θαλάττῃ καὶ ἐν οὐρανῷ" (Phys. 223a 16-8).

³⁶ "πότερον δὲ μὴ οὕσης ψυχῆς εἰη ἂν ὁ χρόνος ἢ οὕ, ἀπορήσειεν ἄν τις. ἀδυνάτου γὰρ ὄντος εἶναι τοῦ ἀριθμήσοντος ἀδύνατον καὶ ἀριθμητόν τι εἶναι... εἰ δὲ μηδὲν ἄλλο πέφυκεν ἀριθμεῖν ἢ ψυχὴ καὶ ψυχῆς νοῦς, ἀδύνατον εἶναι χρόνον ψυχῆς μὴ οὕσης" (Phys. 223a 21-26)

For this is time, the numbering of motion ($\dot{\alpha}\varrho\iota\theta\mu\delta\varsigma\kappa\iota\nu\eta\sigma\epsilon\omega\varsigma$) in respect to before and after.

τοῦτο γάρ ἐστιν ὁ χρόνος, ἀριθμὸς κινήσεως κατὰ τὸ πρότερον καὶ ὕστερον. (Phys. 219b 1-2, see also, 220a 25-26)

On the basis of this definition, he then asks: if time is the counting of motion, shall we then say that, if there is no consciousness to do the counting, there can be no real time, and therefore we can speak only of a subjective time? I believe that what follows, after Aristotle's posing the question, is illuminating:

And if nothing can count except consciousness and consciousness only as intellect ($\psi \nu \chi \tilde{\eta} \varsigma \nu o \tilde{\nu} \varsigma$), it is impossible that time should exist if consciousness did not: unless as the 'objective thing' which is subjectively time to us, if we may suppose that movement could thus objectively exist without there being any consciousness. For before and after are objectively involved in motion, and these, *qua* capable of numeration, constitute time. (Phys. 223a 25-29)³⁷

It is of extreme interest, I believe, to note how Peirce refers to Aristotle as regards this issue:

Aristotle is understood by modern critics to be in a childish naïve state of mind on this subject [the great difference in the logical status of the future and the past][...]I am ashamed to have to confess that I shared [this] general opinion[...]until the further progress of my own studies forced me to the very substance of what Aristotle says.

This very substance of what Aristotle says, according to Peirce, is this:

The past is ended and done; the future is endless and can never have been done.

What follows is an insightful remark:

Hence it might be inferred that the contrast Aristotle speaks of between the past and the future might be merely subjective, having to do with our different attitude toward them. But even a moderate appreciation of the Kantian argument will show that, besides being true in regard to our knowledge of time, it must also be regarded as true of *real* time; and time *is* real, whether we accept Kant's dubious view of it, which he is certainly far from making evident, as the form of the internal sense, or not. I do not question Time's being of a *form*, that is, being of the nature of a Law, and not an Existence. (CP 6.96, Sixth Lowell Lecture, 1903)

³⁷ "εἰ δὲ μηδὲν ἄλλο πέφυκεν ἀϱιθμεῖν ἢ ψυχὴ καὶ ψυχῆς νοῦς, ἀδύνατον εἶναι χϱόνον ψυχῆς μὴ οὕσης, ἀλλ' ἢ τοῦτο ὅ ποτε ὂν ἔστιν ὁ χϱόνος, οἶον εἰ ἐνδέχεται κίνησιν εἶ ναι ἄνευ ψυχῆς. τὸ δὲ πϱότεϱον καὶ ὕστεϱον ἐν κινήσει ἐστίν· χϱόνος δὲ ταῦτ' ἐστὶν ἦ ἀϱιθμητά ἐστιν" (Phys. 223a 25-29).

I believe that there should be no doubt that Peirce, as much as Aristotle, defends the reality of time:

Time is said to be the form of inward intuition. But this is an error of the sort just considered. It confuses what is evolved from the time-idea with what is involved in it[...] It is, in the first place, only real events that "take place" or have dates, in real time[...] It is, then, only existentially real events which the law of time represents really to have places in real time. (CP 1.492, 1896)

We can, thus, come to the conclusion, that for both Peirce and Aristotle, time is real and, what is even more important, time is intimately related to movement and change.

Time with its continuity logically involves some other kind of continuity than its own. Time, as the universal form of change, cannot exist unless there is something to undergo change and to undergo a change continuous in time there must be a continuity of changeable qualities. (CP 6.132, *The Monist*, 1892: cf. 6.110-11, $7.466)^{38}$

This is a spectacular meeting point of Peirce and Aristotle. Let us see how the Stagerite expresses this idea: In *Physics* book V.x he repeats times and again that,

... there is no time without movement and change.

... οὐκ ἔστιν ἄνευ κινήσεως καὶ μεταβολῆς χρόνος..

(Phys. 218b 33-219a 1)

and he makes clear that,

 \ldots time is neither identical with movement/change nor capable of being separated from it.

...
ὅτι μἐν οὖν οὖτε κίνησις οὖτ' ἀνευ κινήσεως ὁ χρόνος ἐστί, φανερόν.
 '(Phys.219a 2-3; cf. Phys.219b 12)

We could, thus, say that time is that immanent character of movement/change that permits the counting of successive states, which have a distinct orientation towards the future. Let us recall Aristotle's notorious definition of time:

For this is time, the counting of motion with respect to 'before' and 'after'.

τοῦτο γά
φ ἐστιν ὁ χρόνος, ἀριθμὸς κινήσεως κατὰ τὸ πρότερον καὶ ὕστερον. (Phys. 219
b1-2)

³⁸ As regards the fact that we only imagine time, Peirce's answer is this: "To imagine time, time is required. Hence, if we do not directly perceive the flow of time, we cannot imagine time" (NE, 3.60).

In the above passages two fundamental features of *time* are illuminated. The first is the internal relationship that connects *time* with *motion* and *change*, and the second, the distinction between *past* and *future*. Here are three extremely interesting definitions of motion (κ (ν η σ ς):

...we can now define motion or change as the progress of the realizing of a potentiality, qua potentiality.

....ή τοῦ δυνάμει ὄντος ἐντελέχεια, ἡ τοιοῦτον, κίνησίς ἐστιν. (Phys. 201
a 10-11)

 \ldots for primarily the movement is the realization of the thing's capacity for being in motion.

... διὸ ἡ κίνησις ἐντελέχεια τοῦ κινητοῦ, ἦ κινητόν. (Phys. 202a 7-8)

...for nature also is in the same genus as potentiality, because it is a principle of motion, although not in some other thing, but in the thing itself qua itself.

... καὶ γὰρ ἡ φύσις ἐν ταὐτῷ [γίγνεται: ἐν ταὐτῷ γὰρ] γένει τῷ δυνάμει ἀρχὴ γὰρ κινητική, ἀλλ' οὐκ ἐν ἄλλῷ ἀλλ' ἐν αὐτῷ ἦ αὐτό. (Met. 1049b 8-10)

The connection therefore of time with movement/change ($\kappa t v \eta \sigma \iota \varsigma$) in the Aristotelian sense, i.e. in the sense of a *transition* from *potentiality* to *actuality* leads directly to the heart of Aristotle's model of nature and reveals the *temporal* structure of the physical world.

All this has become the source of inspiration for Peirce's treatment of *time*. It is obvious that seen from this point of view *potentiality* becomes, both for Aristotle and Peirce, an indispensable ingredient of *continuity* and *infinity* and subsequently of *time*. Indeed, Aristotle's claim is that in this case we are to understand potentiality not in the sense in which we say that the potentiality of a statue exists in the bronze of a statue,

...for that implies that the whole of the bronze may actually become the statue, whereas it is not so with an infinite potentiality, since it can never become an unlimited actuality. As to this we must not be misled by the ambiguity of the word 'is' for the only sense in which the infinite is actualized at all is the sense in which we say that it 'actually is' such and such a day of the month, or that the games 'actually are' on; for in these cases, too, the period of time or the succession of events in question is not (like the statue-potentialities of the bronze) all actualized at once, but is in course of transit as long as it lasts. The Olympic games, *as-a-whole*, are a potentiality only, even when they are in process of actualization. (Phys.206a 18-25)³⁹

This is an extremely important point, because it stresses the fact that the *potential infinite* in real time is not the same as *logical divisibility*. It is rather a *physical process*

³⁹ "...ώς καὶ ἔσται τοῦτ' ἀνδριάς, οὕτω καὶ ἄπειρον ô ἔσται ἐνεργεία ἀλλ' ἐπεὶ πολλαχῶς τὸ εἶναι, ὥσπερ ἡ ἡμέρα ἔστι καὶ ὁ ἀγὼν τῷ ἀεὶ ἄλλο καὶ ἄλλο γίγνεσθαι, οὕτω καὶ τὸ ằπειρον καὶ γὰρ ἐπὶ τούτων ἔστι καὶ δυνάμει καὶ ἐνεργεία Ὁλύμπια γὰρ ἔστι καὶ τῷ δύνασθαι τὸν ἀγῶνα γίγνεσθαι καὶ τῷ γίγνεσθαι"(Phys. 206a.20-25).

progressively being actualized and never being able to be completed as a whole. This is beautifully stated by Aristotle as follows:

In all these cases, the 'infinite' may be regarded as the open 'possibility of more', the 'more' ($\delta\lambda\omega\varsigma\mu\dot{\epsilon}\nu\gamma\dot{\alpha}\varrho$ outure' $\delta\pi\epsilon_{1}\rho\nu$, the 'more' ($\delta\lambda\omega\varsigma\mu\dot{\epsilon}\nu\gamma\dot{\alpha}\varrho$) outure' $\delta\lambda\nu\epsilon_{1}\lambda\lambda\nu$ ($\delta\lambda\omega\kappa\dot{\alpha}\dot{\epsilon}\lambda\lambda\nu$) that is actually taken being always limited, but always different; but when this occurs in the case of magnitudes what is once taken remains; whereas in the case of time or of the human race the parts taken are constantly perishing in such a way that the succession never fails. (Phys. 206a 27-29)⁴⁰

Conclusion

In light of the preceding analysis, we can say that both for Peirce and Aristotle real time is an endless possibility, never existing actually in its totality. It thus has the characteristics of real continuity and potential infinity, which, in Aristotle's words is "always in the making and never made". And this is exactly what endows it with a dynamical character, since time is interwoven with perpetual motion and change, the very essence of which is the transition from a potential to an actual state. It thus has a mode of being which is open into an indefinite future, or in Peirce's words, "its mode of being is *esse in futuro*."

Bibliography

ARISTOTLE. *Physics*. 2 v. Translated by Philip H. Wicksteed and Francis M. Cornford. The Loeb Classical Library. Cambridge, Mass.: Harvard University Press, 1970.

— On the Heavens. Translated by W. K. C. Guthrie. Loeb Classical Library, 1939. Cambridge, Mass.: Harvard University Press, and London: William Heinemann, 1960.

———. *On the Soul, Parva Naturalia*, On Breath. Translated by W. S. Hett. Loeb Classical Library, 1936. Cambridge, Mass.: Harvard University Press, and London: William Heinemann, 1964.

— Metaphysics. 2 v. A revised text with introduction and commentary by W. D. Ross. Oxford: Clarendon Press, 1924.

DAUBEN, J. Abraham Robinson and Nonstandard Analysis. Minnesota Studies in the Philosophy of Science, v. XI (1988): 177-200.

EISELE, Carolyn. *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce*. The Hague: Mouton Publishers, 1979.

KETNER, K.L. & Putnam, H. eds. *Reasoning and the Logic of Things:* Charles Sanders Peirce. Cambridge, Mass: Harvard University Press, 1992.

⁴⁰ "άλλως δ' ἔν τε τῷ χρόνῳ δῆλον [τὸ ἄπειρον] καὶ ἐπὶ τῶν ἀνθρώπων, καὶ ἐπὶ τῆς διαιρέσεως τῶν μεγεθῶν. ὅλως μὲν γὰρ οὕτως ἔστιν τὸ ἄπειρον, τῷ ἀεὶ ἄλλο καὶ ἄλλο λαμβάνεσθαι, καὶ τὸ λαμβανόμενον μὲν ἀεὶ εἶναι πεπερασμένον, ἀλλ' ἀεί γε ἕτερον καὶ ἕτερον· ἀλλ' ἐν τοῖς μεγέθεσιν ὑπομένοντος τοῦ ληφθέντος [τοῦτο συμβαίνει], ἐπὶ δὲ τοῦ χρόνου καὶ τῶν ἀνθρώπων φθειρομένων οὕτως ὥστε μὴ ἐπιλείπειν" (Phys. 206a25-206b3).

Cognitio, São Paulo, v. 9, n. 2, p. 261-280, jul./dez. 2008

LEVY, S. H. C. S. Peirce's Theory of Infinitesimals. *International Philosophical Quarterly*, v.31, No2 issue No 122 (1991): 127-140.

MACLAUGHLIN, W. Resolving Zeno's Paradoxes. Scientific American (1994):66-71.

MACLAUGHLIN, W.; MILLER, S. L. An Epistemological use of Nonstandard Analysis to answer Zeno's Objections against Motion. *Synthese*, 92 (1992): 371-384.

MELBOURNE G. E. Aristotle, Newton and the Theory of Continuous Magnitude. *Journal of the History of Ideas*, 16 (1995): 548-557.

PEIRCE, C. S. *Collected Papers of Charles Sanders Peirce*. v. 1-8. Edited by C. HARTSHORNE, P. Weiss and A. Burks. Cambridge, Mass.: Belknapp Press of the Harvard University Press, 1931-58.

———. The New Elements of Mathematics, 4 v. Edited by C. Eisele. The Hague: Mouton, 1976.

. Manuscripts. MSS, Houghton Library, Harvard University, 1857-1913.

PUTNAM, Hilary. Peirce's Continuum. In: *Peirce and Contemporary Thought*. Philosophical Inquiries. Edited by K. L. Ketner. New York: Fordham University Press, pp. 1-22.

ROSENTHAL, Sandra B. *Charles Peirce's Pragmatic Pluralism*. Loyola University, New Orleans: State University of New York Press, 1994.

———. Continuity, Contingency and Time: The Divergent Intuitions of Whitehead and Pragmatism. *Transactions of the Charles S. Peirce Society*. V.32, No.4 (1996): 542-567. SEENDONI-MENTZOL Demetra. *Probability and Chance in the Philosophy of Charles*.

SFENDONI-MENTZOU, Demetra. *Probability and Chance in the Philosophy of Charles S. Peirce*. (Doctoral thesis in Greek). Thessaloniki:University of Thessaloniki, 1980.

———. The Role of Potentiality in C. S. Peirce's Tychism and in Contemporary Discussions in Q.M. and Micro-Physics. In: *Charles S. Peirce and the Philosophy of Science*: Papers from the 1989 Harvard Conference. Edited by Edward Moore, 246-261. Tuscaloosa: University of Alabama Press, 1993.

———. C. S. Peirce on Continuity and Laws of Nature. *Transactions of the Charles S. Peirce Society*, V. 33, No.3 (1997):646-678.

———. The "Frozen" Passage and Aristotle's "Becoming" of the Physical World (in Greek). In *Vita Contemplativa*. Essays in Honour of Demetrios N. Koutras. Edited by Athanasia Glykofrydi-Leontsini, 469-485. Athens: University of Athens, 2006.

———. Time and Becoming in Aristotle and Prigogine (in Greek). *Proceedings of the International Conference "Aristotle Today"*, 343-358. Naoussa-Mieza, 2002.

Address/Endereço

Demetra Sfendoni-Mentzou Department of Philosophy Aristotle University of Thessaloniki Thessaloniki 54006 Greece

Data de recebimento: 7/4/2008 Data de aprovação: 26/9/2008

Cognitio, São Paulo, v. 9, n. 2, p. 261-280, jul./dez. 2008