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Towards new documentation systems for mathematics teachers?[1]

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ABSTRACT

We study in this article mathematics teachers' documentation work: looking for resources, selecting/designing mathematical tasks, planning their succession, managing available artifacts, etc. We consider that this documentation work is at the core of teachers' professional activity and professional development. We introduce a distinction between available resources and documents developed by teachers through a documentational genesis process, in a perspective inspired by the instrumental approach. Throughout their documentation work, teachers develop documentation systems, and the digitizing of resources entails evolutions of these systems. The approach we propose aims at seizing these evolutions, and more generally at studying teachers' professional change.

Keywords: *Artifacts; Curriculummaterial; Digital resources; Documents; Documentational geneses; Instruments; Operational invariants; Teacher beliefs; Professional development.*

1. Introduction

The generalized availability of digital resources for mathematics teachers entails a complete metamorphosis of curriculum material (Remillard 2005). It also yields a deep change in teachers' professional knowledge and development. We propose here a theoretical approach aiming at illuminating the consequences of this phenomenon. However, the scope of this approach goes beyond capturing the changes brought by digital resources. We have indeed chosen, in order to analyze these changes, to consider the sets of resources used by teachers, not restricted to digital resources. We want to introduce here a general perspective for the study of teachers' professional evolution, where the researcher's attention is focused on the resources, their *appropriation* and *transformation* by the teacher or by a group of teachers working together. Similar issues have already been investigated by Adler (2000), who claims that, "in mathematics teacher education, resources in practice in context need to become a focus of attention" (p. 221). We fully agree with this claim, and mention throughout this paper many connections with Adler's conceptualization of resources. We maintain nevertheless a specific interest in digital resources. This interest is not directed towards the promotion of such resources; it is meant to shed light on the use of resources as a whole, including digital and non-digital

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pieces, and on teachers' professional evolution de Adler's conceptualization of resources. We maintain nevertheless a specific interest in digital resources. This interest is not directed towards the promotion of such resources; it is meant to shed light on the use of resources as a whole, including digital and non-digital pieces, and on teachers' professional evolution.

Studies on the integration of new technologies in the classroom have highlighted the need for holistic research approaches, suitable for capturing the wholeness of teachers' practices (Monaghan 2004). Lagrange, Artigue, Laborde and Trouche (2003), in their multidimensional study of research on technology in mathematics education, have observed a growing interest in the articulation between technology and other resources. Several intertwined features of the classroom context must be considered in studying integration issues. Ruthven (2008) distinguishes five such relevant dimensions: working environment; resource system; activity format; curriculum script ("a loosely ordered model of relevant goals and actions which serves to guide [the teacher's] teaching of the topic," p. 61); and time economy. We consider, as does Ruthven, that each resource must be viewed as a part of a wider "set of resources" (used here instead of "resource system" which suggests an a priori structure of the resource sets). Ruthven claims that resources and curriculum script interact. This perspective is close to Adler's view of resources, and to the one developed in a type of research on teachers' use of curriculum material, which Remillard (2005) in her literature review names, "Curriculum use as participation with the text" (p. 221). The evolution of the curriculum material actually used and the teacher's professional development are two intertwined processes. Our approach is situated within a similar perspective; we want nevertheless to emphasize the following specificities:

– Weassume that examining teachers' activity requires a consideration of their working environment (Cohen, Raudenbush & Ball 2003). We thereby devote particular attention to three environmental factors likely to influence teachers' work: institutional conditions and constraints (Chevallard 2005), use of ICT, and involvement in professional collectives (we do not analyze here teachers' collective documentation work, which deserves specific study and belongs to our work in progress);

– Westudysets of resources not limited to curriculum material, but including everything likely to intervene in teachers' documentation work: discussions between teachers, orally or online; students' worksheets, etc. According to Adler (2000), "resources for school mathematics extend beyond basic material and human resources to include a range of other human and material resources, as well as mathematical, cultural, and social resources" (p. 210). Adler proposes a distinction among material, human, and socio-cultural resources. Cohen et al. (2003) also propose a comprehensive approach to resources: "Conventional resources include teachers' formal qualifications, books, facilities, class size, and time. Personal resources include practitioners' will, skill, and knowledge. Environmental and social resources include state guidance for instruction, academic norms, professional leadership,

and family support" (p. 127). We do not use the distinctions introduced by these authors; however, we too adopt a broad perspective on resources and consider resources to include all the categories mentioned in the above quotations;

– Wedevotespecific interest to teachers' activity outside the class. As Ball, Hill and Bass (2005) state in their study of 'mathematical knowledge for teaching', teaching is not reduced to the work in class, but also includes planning, evaluating, writing assessments, discussing with parents, etc. Thus we concentrate on what Remillard (2005) terms 'the curriculum mapping arena' and 'the design arena'. These arenas are naturally strongly connected with what happens in class ('the construction arena', according to Remillard), and we often mention it, but our attention is directed uppermost towards out-of-class activity. We focus more precisely on what we term the teachers' documentation work: looking for resources, selecting/designing mathematical tasks, planning their succession and the associated time management, etc. This work is considered central by several authors, in particular when a new curriculum is proposed (Christou, Eliophotou-Menon & Philippou 2004), or within the perspective of professional development programs grounded in the work with curriculum material, or in studies related to selecting, adapting, and refining mathematical tasks (Ball & Cohen 1996, Arbaugh & Brown 2005). We consider here that, even outside a particular reform or professional development program context, documentation work is at the core of teachers' professional activity and professional change.

– Weretain a broad view of professional change. We naturally regard, as Ball and Cohen (1996) suggest, the adoption of new material as one component of professional development; but we do not limit our study to that kind of change. We retain all kinds of professional practice evolution. In the approach we develop, change of practice and change of professional knowledge or beliefs are connected (in a specific manner, as exposed in Section 3); thus we consider both as simultaneous processes (as, for example, in Cooney 1999, 2001). In his conceptualization of teachers' knowledge, Shulman (1986) distinguishes between subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Documentation work obviously modifies curricular knowledge; but it can also yield evolutions in the other kinds of knowledge. In our study, we consider teachers' knowledge as a whole, without sorting out its different kinds. This can be a further refinement of the theory, but deserves specific study. Finally, in our analyses of professional evolution, we distinguish among integration of new material, other change of practice, and change of knowledge or beliefs.

We have chosen to set our study in the context of secondary school teaching because of our focus on digital resources: secondary schools in France are much better equipped with computers than primary schools (French Education Ministry 2007). We have set up a series of teacher interviews; moreover, we draw upon previous research (concerning the use of e-exercises and a particular

approach to teacher training) for additional data. We present these data and our methodology in Section 2.

Acentral reference for the theoretical elements we develop is the instrumental approach in mathematics (Guin, Ruthven & Trouche 2005) and the work of Rabardel (1995) grounding it. We recall the principles of this approach, and expose the first steps of a generalization we propose in Section 3. We introduce in particular a distinction between resources and documents, and the notion of documentational genesis. Documents are developed throughout these documentational geneses. For a given teacher, these documents are organized in a documentation system, and the geneses are deeply interconnected with the teacher's professional development. Analyzing the documentation system and itsevolution permits the study of the teacher's professional development. We focus on these topics in Section 4.

2. Data collection and methodology

We propose here the seeds of a new approach to teachers' documentation work and professional development. This theoretical elaboration draws on previous theoretical elements, provided in particular by the instrumental approach, and on field data. More precisely, the data and the theoretical development we propose articulate in two directions. On the one hand, analyses of the data are used in the article as examples to display and illustrate the use of the theory. On the other, these analyses provide evidence of the theory's consistency and relevance in the realm of teachers' documentation work. The data also contributed to shaping the concepts, by leading us to focus on particular aspects of Rabardel's theory (1995, 2005), or to introduce new refinements. These field data have three distinct origins, the first two corresponding to previous research we engaged in:

– We investigated for several years the consequences of the use of e-exercise bases in class (Gueudet 2006). We studied in particular teachers' usage of a piece of software called 'Mathenpoche' ('Maths in the pocket', shortened to MEP1 in what follows), designed by a team of voluntary teachers, and proposing exercises covering the whole curriculum from grades 6 to 9 (Bueno-Ravel & Gueudet 2008). The teachers who were registered as 'MEP users' could choose among the exercises to program their own MEP classroom sessions. We participated in several research projects involving MEP[4], in particular the GUPTEn[5] project (Lagrange, Bessières, Blanchard, Loissy & Vanderbrouck 2007) devoted to teachers' uses of ICT. In this context, we observed classroom uses of MEP4, and collected teachers' descriptions of classroom use for a period of 3 years;

[4] http://mathenpoche.sesamath.net

[5] GUPTEn stands for, in French: Genesis of Professional Use of Technologies by Teachers. This French national research project is headed by Jean-Baptiste Lagrange.

– The SFoDEM7[6] (Guin & Trouche 2005) is a distance training organization for secondary school mathematics teachers, set up from 2000 to 2006 to provide continuous support for teachers in the conception, appropriation, and experimentation of resources integrating ICT. It was grounded on the idea that collaborative work is necessary to overcome the difficulties raised by the integration of ICT. Groups of teachers were gathered to design and experiment with various resources. Their work took the form of continuous communication, with one-day workshops three times a year, discussion and files exchanges the rest of the time via a shared platform. We were thus able to follow these discussions, as well as the evolution of resources on this platform;

– The last kind of data, central to this study, was a series of interviews. All the cases presented hereafter come from these interviews. We met nine teachers to ask them about the resources intervening in their out-of-class work. They received us in their homes, and the discussion took place where they usually work, next to their computer, books, binders… The one-hour interview was recorded and transcribed. The complete interview frame, which is presented in Appendix 1, is comprised of three main parts:

– A general description of everything judged useful by the teacher for her documentation work, with the identification of what was the most important;

– A detailed presentation of three selected items: a book, or a web site, or a lesson plan, etc. (at least one personal production), with an explanation of their history and use;

– A reflection on the experienced evolutions, and the expected ones (what was used 10 years ago, what they think they will use, or what they would like to use 10 years from now).

Weselected the teachers to be interviewed according to several specifications. First to be considered was their age and professional experience: most of them were aged between 40 and 50, and had at least 10 years of professional experience (this enabled them to respond to the third part of the interview). But the most important for us was to constitute a panel of teachers diverse enough with respect to the three important factors we emphasized in Section 1: institutional aspects, collective involvements, use of ICT. A summary of these teachers' profiles is provided in Appendix 2; we give here a short description. collective involvements, use of ICT. A summary of these teachers' profiles is provided in Appendix 2; we give here a short description.

Regarding institutional aspects, five of them teach in 'collège' (students from grades 6 to 9), and four in 'lycée' (students from grades 10 to 12). Moreover they have diverse institutional responsibilities: none for some of them, teacher trainers for others, national examiners, etc. Their involvements in teachers' communities vary, someofthembeingin particular involved in communities related withMEP, or withtheSFoDEM.Theyalsopresentvariousdegrees of ICT integration (Assude

[6] SFoDEM stands for, in French: Distance Training Support for Mathematics Teachers.

2008). Two of them work only with paper and pencil (degree zero), two scantily integrate ICT in class (low degree), and five to a strong degree. This last point in particular indicates clearly that these nine teachers must not be considered representative of French teachers at large. Because of our focus on digital resources, we selected a majority of teachers familiar with ICT use.

The data we gathered are the transcribed interviews, observations of the organization of teachers' workplaces at home, of the organization of the digital files on their computers, and selected materials (students' worksheets, textbooks extracts, agenda, etc., some on paper and some digital). We retained from the interviews all the relevant information: list of resources mentioned, evocation of interactions with colleagues and of participation in professional development programs, and for the three resources selected as the most important, description of their use in class, their history, their evolutions, and the causes of these evolutions. Capturing the work done by a teacher out of class is difficult for material reasons. We are very grateful to the teachers who accepted to receive us in their homes. Naturally, the corresponding results are grounded on reconstructions of their work and the evolution of this work, and on their declarations and collected resources, and not on direct observations of in-class or out-of-class activity. These reconstructions were naturally controlled by our classroom observation experience and by the knowledge we have built throughout the years regarding MEP users and SFoDEM participants; nevertheless, this methodological aspect creates an obvious limitation to our work. However, it also yields consequences of interest for our purpose in that it can lead to balancing in-class and out-ofclass work, and even bring the focus a little more to the latter, which is too often neglected by researchers.

The national character of our study induces another limitation. France is a developed country; the teachers we interviewed work in schools equipped with computer laboratories connected to the Internet, and most of them also have one or several computers at home. Another French characteristic is strong national instructional guidance, with a national curriculum complemented by additional commentary and advice published in 'curriculum guides'. But the ministry does not control the textbook market, and a great variety oftextbooks exist. The mathematics teachers' association, and the IREMs (Institutes for Research on Mathematics Teaching) are quite influential, and their publications are very popular. But collective work is still scarce. Most teachers stay at school only for their courses, and go back home for their out-of-class work. All these national features certainly influence our results, because documentational geneses and their outcomes naturally depend on the available resources, on national culture (Leung, Graf & Lopez-Real 2006), institutions, and collectives. We hypothesize that the concepts exposed in Sections 3 and 4 are likely to illuminate documentation work in very diverse situations; however, further studies in other countries are certainly needed to render more precise the influence of national characteristics.

3 Resources and documents: a dialectical relationship

3.1 Distinguishing between resources and documents

We introduce here a distinction between resources and documents, extending the distinction introduced by Rabardel (1995) between artifact and instrument.

According to Rabardel, an artifact is a cultural and social means provided by human activity, offered to mediate another human activity. A bicycle, a computer, are artifacts, and a given language is an artifact too. The subject engaged in a goal-directed activity develops an appropriation of the artifact. Rabardel distinguishes the artifact and the instrument, the latter of which is built from the artifact by a subject through his/her activity. An instrument results from a process, named instrumental genesis, through which the subject builds a scheme of utilization of the artifact, for a given class of situations. A scheme, as Vergnaud (1998) defined it from Piaget, is an invariant organization of activity for a given class of situations. It comprises goals and subgoals, anticipations, rules of action, of gathering information and exercising control, and possibilities of inferences. It is structured by operational invariants, which consist of implicit knowledge built through various contexts of utilization of the artifact. We represent this distinction by the formula:

Instrument =Artifact + Scheme of Utilization

Instrumental geneses have a dual nature. On one hand, the subject guides the way the artifact is used and, in a sense, shapes the artifact: this process is called instrumentalization. On the other hand, the affordances and constraints of the artifact influence the subject's activity: this process is called instrumentation. As Noss and Hoyles (1996, p. 58) note regarding computer artifacts: "Far from investing the world with his vision, the computer user is mastered by his tools".

The work of Rabardel has grounded the development, within mathematical didactics, of the instrumental approach (Guin et al. 2005). This theoretical framework has been used in a great number of research studies, most of them considering, as components of instrumental geneses, students as subjects, and ICT tools as artifacts. The long-term observations we carried out on the uses of MEP, and within the SFoDEM project, support our interpretations concerning geneses for teachers. Teachers who integrated MEP in their practice organize more individualized teaching than they did before, because MEP offers the possibility to program different sessions for different students (instrumentation: influence of the affordances of MEP on teachers' activity). In one of the SFoDEM groups, while working on problem solving, teachers introduced students' digitalized productions on the platform, a new form of sharing information (instrumentalization=appropriation of the platform). We also observed stabilization processes, regularities appearing in the teachers' activity across different contexts consistent with the development of schemes. And another consequence of this

previous research was evidence for the need to take into account not only one given artifact but also a wide range of artifacts of different types. It led us to propose a theoretical approach inspired by the instrumental approach, with distinctive features and a specific vocabulary that we will detail hereafter.

We use the term resources to emphasize the variety of artifacts we consider: a textbook, a piece of software, a student's sheet, a discussion with a colleague, etc. A resource is never isolated; it belongs to a set of resources. The subjects we study are teachers. A teacher draws on resource sets for her documentation work. A process of genesis takes place, producing what we call a document. The teacher builds schemes of utilization of a set of resources for the same class of situations across a variety of contexts. Adler (2000) suggests "think[ing] of a resource as the verb re-source, to source again or differently" (p. 207). We agree with this suggestion; the documents can be similarly thought of as the verb document: to support something (here the teacher's professional activity) with documents. Let us consider, for example, the class of professional situations (Rabardel & Bourmaud 2003): 'propose homework on the addition of positive and negative numbers'. For this class of situations, a given teacher gathers resources: textbooks, her own course, a previously given sheet of exercises… She chooses among these resources to constitute a list of exercises, which is given to a class. It can then be modified, according to what happens with the students, before using it with another class during the same year, or the next year, or even later. The document develops throughout this variety of contexts. The operational invariants can be very general, like 'the homework must be extracted from the textbook', or more precisely linked with the mathematical content, like: 'the additions proposed must include the cases of mixed positive and negative numbers, and of only negative numbers,' etc. These operational invariants can be inferred from the observation of invariant behaviors of the teacher for the same class of situations across different contexts. They are teacher beliefs, and are both driving forces and outcomes of the teacher's activity, instrumented by a set of resources. Thus the document is much more than a list of exercises; it is saturated with the teachers' experience, just as a word, for a given person, is saturated with sense in a Vygotskian perspective (Vygotsky 1978). The formula we retain here is:

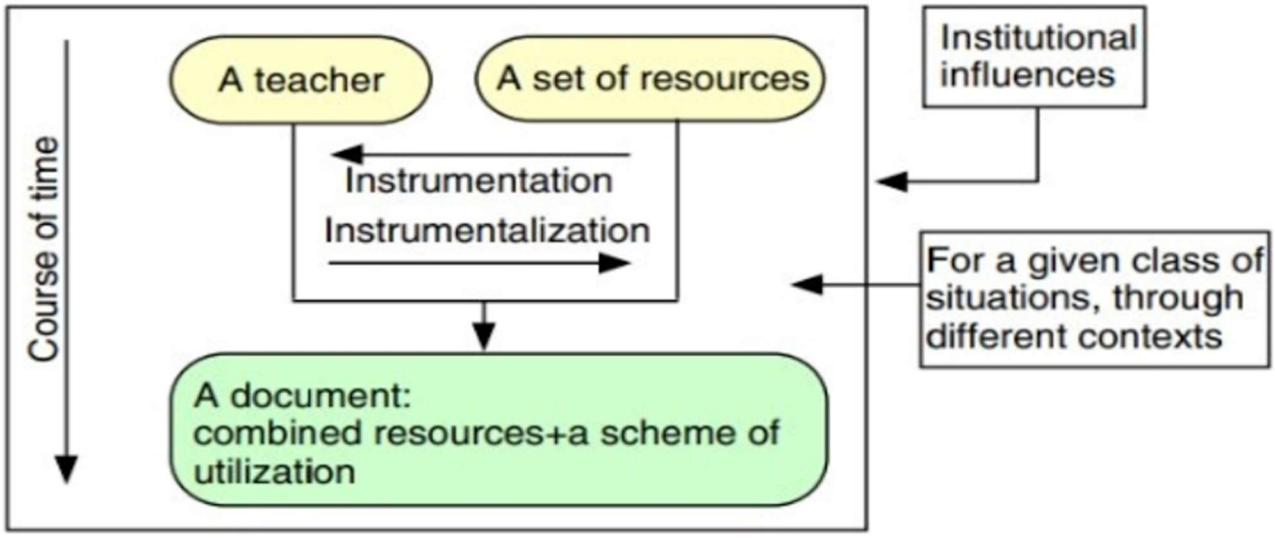
$$Document = Resources + Scheme\ of\ Utilization$$

Our choice of vocabulary was intended to match the terminology of document management research. According to Pédauque (2006), "A document is not anything, but anything can become a document, as soon as it supplies information, evidence, in short, as soon as it is authoritative" (p. 12, our translation).

3.2 Documentational genesis: an ongoing process

Figure 1 represents a process of documentational genesis. The instrumentalization dimension conceptualizes the appropriation and reshaping processes, well known by researchers studying the design and diffusion of instructional sequences: “an instructional sequence developed by one group is necessarily reshaped and transformed while others use it” (Cobb et al. 2008, p. 117). The instrumentation dimension conceptualizes the influence on the teacher’s activity of the resources she draws on. Let us give a first example coming from our interviews. Frédéric has taught for 15 years to students from grades 6 to 10.

Figure 1 - *Schematic representation of a documentational genesis*

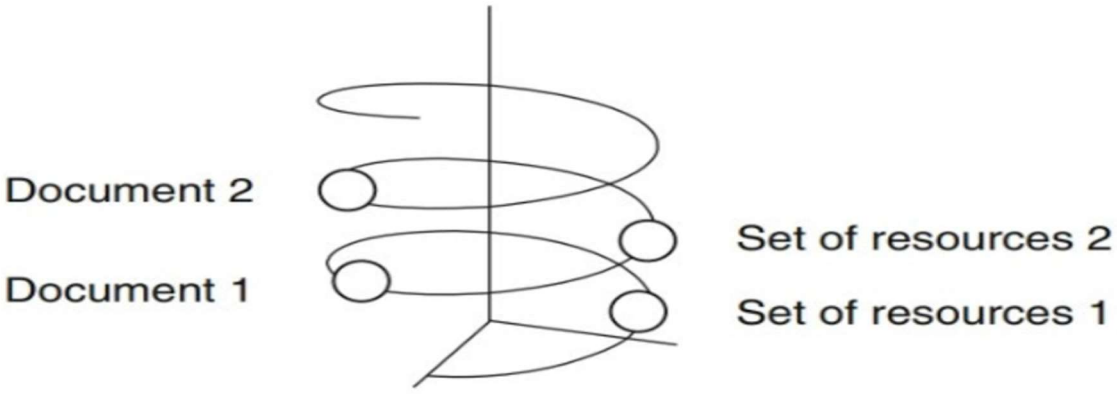


Source: G. Gueudet, L. Trouche-Educ Stud Math, 2009, p.206

During the interview, he presented to us a mathematical task designed to introduce the square root in grade 9. This task deals with the areas of squares: several side-lengths are given, students must compute the areas, then place the points with coordinates (length, area) on a graph, draw a curve through these points, and use the curve to find approximate values of the side-length for given areas. These approximate values are then compared with the values obtained with a calculator, using the square root key. Frédéric declared that he had used this task for more than 10 years (it had changed along the years, and was initially used in grade 8 before a curriculum change; but the features described above were always present). For the class of professional situations: ‘Preparing the introduction of the square root in grade 9’, Frédéric now draws on a set of resources comprising the original textbook extract; the student sheet proposed the year before, with notes on it about changes he thought of when using it in class; a slide with the points and the curve joining them; but also the

students' calculators. We claim that Frédéric developed a scheme of utilization of this set of resources for this class of situations. This scheme entails general operational invariants: 'a new notion must be introduced through a mathematical task that yields evidence of the meaning of this notion'; and invariants linked with the mathematical content: 'searching for the side-length of a square for a given area gives evidence of the meaning of square root'; 'the square root is the reverse process of squaring'; 'the calculator square root key supports the introduction of the symbol'.The analysis of the interviews, and our previous research, led us to notice a very important aspect of the geneses that we want to emphasize here. A documentational genesis must not be considered as a transformation with a set of resources as input, and a document as output. It is an ongoing process. Rabardel and Bourmaud (2003) claim that the design continues in usage. We consider here accordingly that a document developed from a set of resources provides new resources, which can be involved in a new set of resources, which will lead to a new document etc. Because of this process, we speak of the dialectical relationship between resources and documents. We find convenient to represent it by a helix, wrapped around an axis representing time (Figure 2).

Figure 2 - The resource/document dialectical relationship



Source: G. Gueudet, L. Trouche-Educ Stud Math, 2009, p.206

In the example of Frédéric, the document developed to introduce the square root in a given year provided an annotated text that became a resource for the following year. Let us give another example. Marie-Françoise works with students from grades 10 to 12. She organizes for them 'research narratives': problem-solving sessions, where students work in groups on a problem and write down their solutions and their research processes. Thus one class of situations for Marie-Françoise is that of 'elaborating problems for research narrative sessions'. For this class of situations, she draws on a set of resources comprising various books and websites, ideas communicated by colleagues; but as she told us: "There is the problem and the way you enact it, because students are free to invent things,

and afterwards we benefit from the richness of all these ideas, and you can build on it." We did not observe Marie-Françoise in class; however, it appears clear from this quotation that the research narrative session depends on the students' ideas and propositions. Thus the design goes on in class. But the helix represents more than the evolution brought by enactment; the time axis suggests a consideration of long-term processes, along the years. Marie-Françoise collects the students' productions, she makes copies of selected productions ("most of the time, I copy, when there are interesting things in it, sometimes all of them, sometimes some of them"), and keeps digital versions of students' productions on her computer in a folder entitled 'research narratives' for each class level. Thus the document produced in a given year provides new resources: a problem text; students' productions, which can also comprise new versions of the problem; historical texts found on a web site and related to the problem, etc. And a further step in the process of genesis leads to a new document for the next year, which will later yield resources for a new elaboration. The time axis of the helix is an important matter. The resources evolve, are modified, combined; documents develop according to the processes of genesis and bear new resources… Long-term evolutions, but also more limited events, must be taken into account.

3.3 Components, schemes, and usages

We introduce here additional concepts to render precise several aspects of documentational genesis. We first expose an example, whose analysis illustrates the theoretical distinctions we propose.

3.3.1 Components: material, mathematical, didactical

Considering a set of resources, or a document, requires taking into account three intertwined components:

– The material component: paper, computer, USB key, ring binder...

– The mathematical content component: notions involved, mathematical tasks and techniques;

– The didactical component: organizational elements, ranging from mapping over the whole year to planning a single one-hour session. Let us consider, in Marie-Pierre's case, the class of situations: 'prepare a lesson on the formula for area of a parallelogram for grade 7' (the paperboard extract in Figure 3 corresponds to that class of situations; during the preceding session, the teacher proposed to her students a task that would provide evidence for this formula, and the link between the area of a parallelogram and the area of a rectangle). She draws on a set of resources for this class of situations, and develops a document.

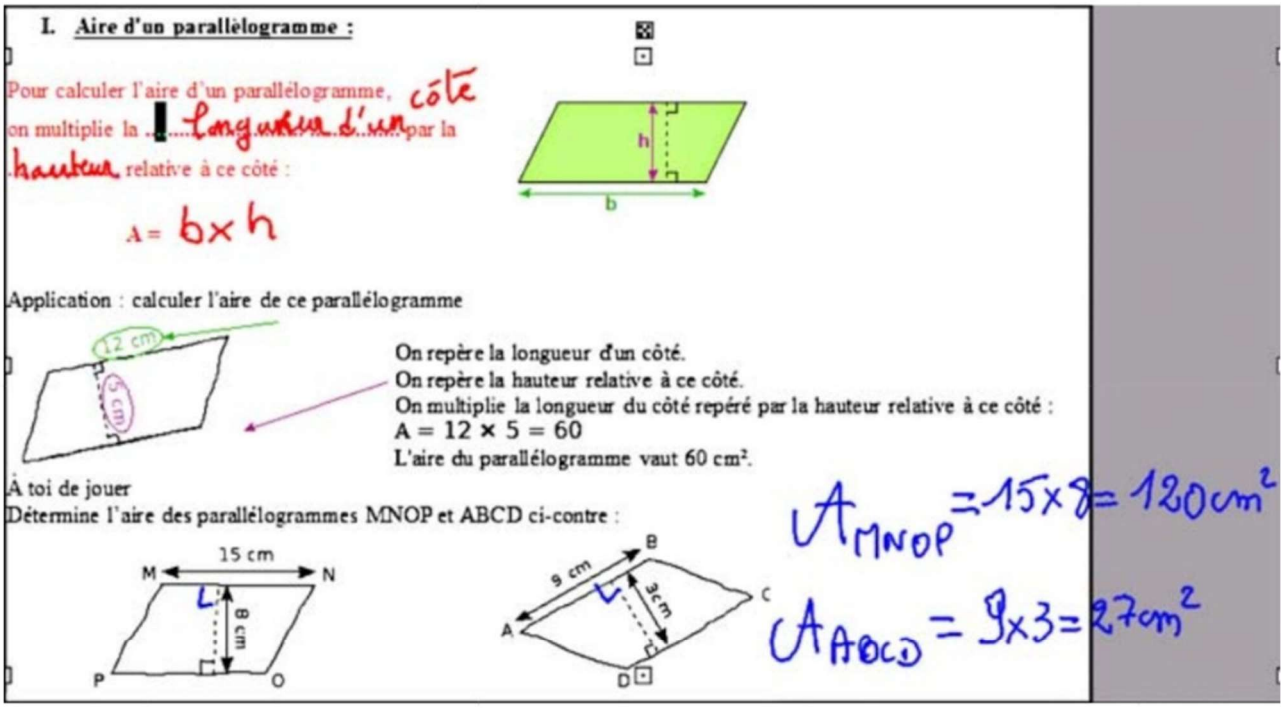


Fig. 3 Marie-Pierre's case. Elements of documentation work and example of interactive whiteboard display. Translation of the screen: 1. Area of a parallelogram. To compute the area of a parallelogram, we multiply the length of one side by the associated altitude. Application: compute the area of this parallelogram. We identify the length of one side. We identify the corresponding altitude. We multiply the length of the side by the altitude: $A=12\times5=60$. The area of the parallelogram is 60 cm^2. Your turn to play. Compute the area of the parallelograms MNOP and ABCD

Two aspects of the data indicate here a regularity. The first one is the paperboards; in the case of Marie-Pierre, the interactive whiteboard permits following through the paperboards the successive states of the board during all her sessions. For many different mathematical contents, she prepares lesson frames comprising texts with empty slots, in particular when a new formula is presented. The second one is more linked with the content: Marie-Pierre declared using the same lesson for 3 years (most of it comes from her digital textbook). The material component of this set of resources comprises word processing software, several web sites, the digital textbook, the interactive whiteboard, and a paper form to be filled by students. The material aspect of the produced document

is a file prepared for display on the whiteboard, and a paper sheet to be filled by each student. The mathematical component of this document comprises several properties, and mathematical tasks, related to the area of a parallelogram: the area formula, the computation of areas in several cases… A part of the organization planned for the session, belonging to the didactical component, can be observed in the document elaborated by Marie-Pierre: she will write the missing elements of the formula on the whiteboard, while the students do the same on their paper form. These missing elements will probably be determined during a class discussion. Then the students must apply the formula in direct application exercises (Figure 3), and then in more complicated ones.

3.3.2 Scheme of utilization and usages

A scheme of utilization of a set of resources entails both an observable part and invisible aspects. The invisible aspects are the operational invariants, the cognitive structure guiding the action. The observable part corresponds to the regularities in the teacher's action for the same class of situations through different contexts. This part is what we call usages. We distinguish between utilizations: when a teacher draws on a resource one, or a few times, but without developing a stable behavior for a given class of situations; and usages, which correspond to a stable organization of activity, and are part of a scheme. Thus we can render our formula more precise by writing:

$$Document = Resources + Usages + Operational\ Invariants$$

In order to begin to describe a scheme, the teacher's action should be observed on a long-term basis. The regularities identified across several events corresponding to a given class of situations permit describing the usages, the rules of action in particular. Then the researcher can try to infer operational invariants from the usages. In the work we present here, as said before (Section 2), we reconstructed the regularities in the teachers' documentation work from our data. Thus our analyses in terms of the scheme must always be seen as possible interpretations; we propose in particular plausible operational invariants. Let us go back to Marie-Pierre and the class of situations: 'prepare a lesson on the formula for area of a parallelogram for grade 7'. We observed that she constituted a file for the interactive whiteboard, with a figure and the incomplete text: "To compute the area of a parallelogram, we multiply the by the associated". Thus the usages, for this class of situations, include the following action rules: 'propose a text with empty slots, each one corresponding to only one word or expression'; 'propose immediate application exercises' (general rules); 'propose a figure where the lengths intervening in the formula can be observed'; 'propose different positions of the parallelogram and of the heights' (specific rules). And we infer from the data that the scheme comprises in particular the following operational invariants: 'students understand and memorize better a formula when a

class discussion is organized before the teacher writes the formula on the board'; 'immediate application exercises support the understanding of a formula' (general operational invariants); 'the association side-horizontal and height-vertical must be avoided'; 'students must make precise the unit when they compute an area' (specific operational invariants).

Marie-Pierre's documentation work entails professional evolution of several types. One evolution is naturally the integration of the interactive whiteboard. But it also led to more general evolution in her practice. In particular, the interactive whiteboard led Marie-Pierre to use in class paperboards corresponding to previous lessons, thus to make many more links with previous sessions. As Cohen et al. (2003) write: "Learning depends on students and teachers making bits of lessons develop and connect" (p. 126). Marie-Pierre has expanded her pedagogical practice, and in particular the way she connects bits of lessons. What we want to emphasize here is that the teacher's documentation work is strongly linked with her professional development; therefore both issues must be studied together.

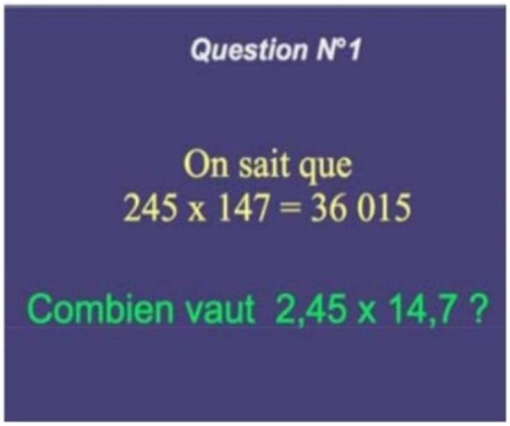
4. Professional evolution and documentation systems

4.1 The productive/constructive dialectical relationship Adler (2000) concludes her study of resources by declaring: "Our attention shifts away from unproblematised calls for more [resources] and onto the inter-relationship between teacher and resources and how, in diverse, complex contexts and practices, mathematics teachers use the resources they have, how this changes over time, and how and with what consequences new resources are integrated into school mathematics practice" (p. 221). We study here likewise not only the way teachers use resources but also the more or less longterm evolution yielded by these uses in their professional practice. Rabardel (2005) introduces the productive and constructive dialectical relationship, essential for understanding the dynamics of teachers' professional development. Professional activity has a productive dimension: the outcome of the work done. But the activity also entails a modification of the subject's professional practice and beliefs, within a constructive dimension. Naturally, this modification influences further production processes, thus the productive/constructive relationship has a dialectical nature. In previous research, and likewise in the analysis of the interviews, we observed that the productive dimension, for example, the design and enactment of a session, was associated with evolution in practice, not limited to the integration of a new resource. We already observed this dialectical relationship at the end of the preceding section in Marie-Pierre's case. Let us give another example. Benoîte uses with her students mental arithmetic slide shows. She started elaborating these slide shows 2 years ago, after a discussion with a colleague (in Benoîte's school, the team of mathematics teachers is used to working

together). This discussion led Benoîte to a productive activity: the elaboration and utilization of mental arithmetic slide shows.

The example displayed in Fig. 4 is related to the class of situations: ‘organize mental arithmetic sessions in class on the product of decimal numbers’. The resources involved in the preparation work have material components: the slide show, a computer and an associated projection device. They also entail mathematical components: a list of the products of two decimal numbers, a list of the products of the corresponding integers; and didactical components: the planned organization involves the whole class looking at the projected slide and then answering the question on paper, within a limited amount of time (programmed by Benoîte within the slide show). The document produced includes the resources selected; rules of action like ‘prepare a precise schedule for the slides’, ‘propose a task on deducing decimal number products from the results of the corresponding integer products’; and operational invariants like ‘computing in limited time enhances mental arithmetic procedures’ and ‘students must be able to recognize and use the property: if *a x b= c, then m x a x n = b x m x n x c*. Benoîte’s students appreciate these slide shows; she uses them regularly, and even practices more mental arithmetic than before. Thus beyond the obvious change of practice involving the utilization of slide shows in mental arithmetic, the process of genesis also entails a constructive dimension. It induced indeed a more general change of practice: organization of more mental arithmetic. It also corresponds to a change in Benoîte’s beliefs. Three years ago, she was reluctant to practice mental arithmetic in class because of behavioral problems with students who found it uninviting.

Fig. 4 A slide used by Benoîte for mental arithmetic



Translation

Question 1

We know that

245 × 147 = 36 015

How much is 2,45 × 14,7?

Source: Gueudet e Luc, p.210

Now she is convinced that it is for mental arithmetic possible to motivate the students, to raise their interest for mental arithmetic. Studying the evolution of a teacher’s documents contributes to the study of her professional evolution. Naturally, such a study must not be limited to the material aspect of documents, but must also investigate the evolution of usages (for example, Arnaud, another

teacher whom we interviewed, now uses with whole classes the 'help sheets' he elaborated years ago for students encountering specific difficulties) and operational invariants.

4.2 Documentation systems

According to Rabardel and Bourmaud (2005), the instruments developed by a subject in his/her professional activity constitute a system, whose structure corresponds to the structure of the subject's professional activity. We hypothesize here similarly that a given teacher develops a structured documentation system, and that this documentation system and the teacher's professional practice evolve together. From a research point of view, the observation and analysis of the documentation system permits a better understanding of the teacher's professional development, and in particular allows for capturing the evolution introduced by digital resources. We do not detail the structure of a documentation system, which deserves specific study (in progress).

We only provide here some evidence of the existence of such a structure, by analyzing Céline's case (Fig. 5). This description offers only a very incomplete view of Céline's documents. It permits nevertheless a perception of the structured organization of these documents, which corresponds to the structure of the class of professional situations she encounters. Céline uses MEP (Section 2) to organize drill on technical exercises for a half-class. It permits her to propose a research task on paper to the other half-class, and to stay with the students in order to help them in their research processes.

Figure 5 Céline's case. Documentation work and documentation system

Celine is 36 years old, and has taught for 10 years students from grades 6 to 9. She has a computer at home, with a modem connection to the Internet, a scanner and a printer. Her computer is equipped with mathematics software (spreadsheet, dynamic geometry software…). She has used MEP for four years, most of the time to propose technical exercises to a half-class while she works with the other half-class. Celine uses the official curriculum texts (downloaded on the Internet) to map her teaching over the year. She dissects this curriculum into lists of precise mathematical competences. She uses these lists to choose the mathematical content of her courses, and to program out-of-class MEP sessions for the preparation of assessments. Celine follows many training sessions. She buys various publications recommended by the teacher trainers. She selects in these publications ideas for mathematical tasks and designs her own tasks from these sources. She works with her colleagues only to prepare common evaluations twice a year for the grade 9 students who undertake an examination at the end of the year, and for the organization of personal monitoring for students with special needs. Celine has several ring binders for each class component, containing planning elements, courses, exercises, and assignments. In most cases, corresponding digital files can be found on her computer.

Thus the 'organize drill on technical exercises' and the 'organize research on rich mathematical tasks' professional situations in class are connected, as are the associated documents. Céline also sets up outof-class MEP sessions to help her students to prepare for the assessment at the end of the teaching of a given topic. Thus, both of the situations, 'help the students in their reviewing'

and ‘organize the assessment’, are also strongly connected, and the corresponding documents are connected as well. Moreover, the MEP exercises are organized according to precise mathematical abilities that often coincide with Céline’s dissection of the curriculum. Thus the mathematical content in MEP fits the planning she writes up, because this planning is grounded in her curriculum dissections; it is thus easy for Céline to organize MEP sessions. MEP as a resource is involved in many different documents developed by Céline, and these documents are connected with others within her documentation system. This indicates the important degree of integration of MEP by Céline. Conversely, the integration of MEP by Céline depends on her possibility to associate it with other resources available to her, and in particular with resources stemming from previous documents. The integration of a new resource corresponds to a process of genesis, developing a document from it and other resources. And this document must have its place within the documentation system. We want to emphasize in this example the following essential dimensions of our approach:

– The processes of genesis apply to a complex set of resources;

– They entail productive and constructive aspects;

– Thereasons for the involvement of a new resource in the development of a document (we call this process the integration of a resource in a document) are intricate, but the study of the documentation system permits an enlightening of some of these reasons.

4.3 Integrating new resources

The evolution of the resources used and of the documents developed by a teacher must be considered for different relevant time scales. The school year has naturally a specific importance: a mathematical task organized in class a given year yields resources for another year where the teacher encounters again the same class level. A shorter time can nevertheless intervene: the teaching planned for a given topic can be modified according to what happened in class. And longer periods of time can also bring important changes, like curriculum reforms, or a change of school for the teacher. Whatever the time scale, the integration and appropriation of new resources is a complex issue. A study of documentation systems and their evolution can enlighten this issue. We illustrate here with two examples (Fig. 6). Sonia develops for the class of situations, ‘maintaining course continuity for a missing student’, a document involving several resources. Its material component comprises the school’s VLE and Sonia’s course and exercises as PDF files. The VLE is also used by Sonia to collect students’ productions for correction after spreadsheet sessions. In this case the VLE is involved in a document developed for the class of situations: ‘correcting students’ productions obtained with computer’. Sonia has integrated the VLE resource: it has been involved with other resources in the

processes of genesis leading to a document. This document is strongly connected with others within Sonia's documentation system: her course, the mathematical tasks for spreadsheet sessions.

Figure 6 - The case of Sonia and Céline: integration or non-integration of new resources?

Sonia (aged 50) works in a 'collège' (grades 6 to 9) equipped for two years with a Virtual Learning Environment (VLE). When a student misses class, Sonia puts her course and exercises (as PDF files) on the VLE for him/her. The student loads the files, solves the exercises and sends his/her solutions back. Sonia corrects the student's production, and sends it back. Sonia also uses the VLE to collect students' productions after spreadsheet sessions. Celine uses MEP, and the general website hosting it. Several resources can be found on this website, in particular, lists of exercises and problems called 'Mathenligne' (online mathematics) presented as PDF files. Celine never uses these lists; she declares in the interview: "I never found in them anything that was satisfying from beginning to end".

The existence of these documents played a crucial role in Sonia's integration of the VLE: it was easy for her to put her courses on the VLE because all of them were in digital form; and she was already searching for several years for ways to collect students' digital productions. The integration of the VLE contributed to developing the idea of possible work-at-a-distance with her students, which is now part of Sonia's beliefs about teaching. In Céline's case, several explanations for her refusal of 'Mathenligne' can be proposed. She judges its mathematical content unsatisfying. Moreover, the corresponding files are PDF, thus cannot be easily modified. Even the structure of the mathematical content makes a 'Mathenligne sheet' difficult to modify: it proposes a mixture of technical exercises and more complicated problems designed to be used as a whole. 'Mathenligne sheets' can neither serve for drill, nor for research tasks. They cannot combine with resources provided by Céline's documents. And they collide with Céline's beliefs about her preparation work: she prefers to design very personal productions from ideas found in diverse publications. This is a very general belief; we do not interpret it in terms of operational invariants because it extends beyond the context of a given class of situations, even a general one. Céline's interview clearly indicates that this belief shapes her documentation work. All the teachers we interviewed expressed similar general beliefs. Arnaud declared: "I have never been such a good student in mathematics, so my question has always been: what can I do to avoid my students' dropping their pencils, to motivate them to do something… Maths must be fun, thus you have to go off the beaten track, give up textbooks and try to create things. It is what I like doing, I like to create things".

It is well known that "idealization of the professional autonomy leads to the view that good teachers do not follow textbooks, but instead make their own curriculum" (Ball & Cohen 1996, p. 6). Our purpose here is not to identify this trend among the teachers we interviewed (even if such a trend certainly strongly influences the documentation work of some teachers). We want to draw attention to a kind of professional sensitivity that contributes to shaping the documentation work, and the documentation system, conditioning in particular the phenomena of integration or non-integration. This is certainly close to what Ruthven (2008) calls the curriculum script (Section 1). In a more general study on professional practices, Béguin (2005) proposed the notion of world. A world is a

structure, piloted by professional characteristics, offering a particular cutting of reality. It shapes a subject's point of view, his/her perception of the surrounding reality, within his/her activity in a given context. Our interviews evoked expressions of sensitivity, both professional and personal, that we interpret as constituent of the teacher's world. The discussion with Ingrid indicates such sensitivity:

– What would be your dream resource, ideal for your work?

– A basis for exercises and tasks. I am always looking for non-standard exercises, which is difficult to find. It must be possible to adapt the tasks, according to the students you work with. […] For a course, on the other hand, I think there can be no ideal resource. Each teacher has his own sensitivity, and tries to express it.

The worlds of the teachers we interviewed (or, at least, the parts of it we were able to observe or infer) are all very different from each other. They result from their own professional, social, personal background. Their documentation systems are very different too. And the underlying processes of genesis must be taken into account because they are part of the teacher's memory. Each teacher has a particular attachment to his/her own documentation system. A teacher's world influences the development of his/her documentation system. And the documentation system reveals features of the teacher's world.

5. Conclusion

We have developed in this paper a documentational approach, focusing on the documentation work of the teacher outside the class (even if this documentation work goes on in class). A first crucial distinction in this approach was introduced, that between a resource and a document, the latter of which is generated from a set of resources within a process of genesis. It can be represented by a first equation:

$$Document = Resources + Scheme\ of\ Utilization$$

Three intertwined components must be considered for the analysis of a resource, or a document: the material component, the mathematical component, and the didactical component. A document is never isolated: it belongs to a teacher's documentation system, evolving through documentational geneses. These processes are central to the teacher's professional development, and closely related with the teacher's world. We only presented here, and illustrated, the elementary concepts of a documentational approach. Further studies are needed to complement and render precise these concepts. Proposing categories of operational invariants permitting refinement of the analysis of schemes; investigating the structure of the documentation systems for individuals; examining collective documentation work; testing the consistency of the approach in otherteaching contexts: these are all part of our work in progress. Our research projects naturally include the use of this

approach to study teachers' documentation work, in and out of class, and the continuity and ruptures between both, but also more generally to study teachers' professional development. What we propose here can be considered as a perspective change on teachers' activity and professional development. Instead of locating the essential part of teachers' activity in class, we regard the documentation work (in and out-of-class) as the core of their activity and the driving force of their professional development. This raises delicate methodological issues. It necessitates the observation of long-term phenomena and processes, set in different places, out of class and at the teacher's own place in particular. We consider nevertheless as fundamental for mathematics education research the development of this documentational perspective, in particular because of the need for theoretical tools that permit capturing the current evolutions brought by the general availability of digital resources.

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Appendix 1: interview guidelines

Teachers are interviewed at their own place, where they keep their resources. It is most of the time in a specific room, their office at home, with a computer connected to the Internet. A one-hour interview is planned, in a rather informal manner, but following the guidelines exposed here. The discussion is recorded, and photos of the office are taken. The interviews took place between April and June 2007, thus at the end of the school year. First part: inventory, rationale of the documents used this year. The questions are of the following form: "For your teaching, from the beginning of the year, which documents (book, personal documents, web site…) did you use? Which has been the most important?" Second part: detailed presentation of three documents. We ask the teachers to present in detail three of the most important documents of the year, and their history (the teacher proposes him/herself the documents, with at least one personal production):

– If it is not a personal production: how was it encountered, chosen, modified, used...

– Ifit is a personal production: which sources were used; was it elaborated by the teacher on her own, or with colleagues; how was it used, was it modified after use; what is planned for it in the future: communication to colleagues in particular.

Third part : past and future.

– Tenyears ago, how would you have answered the first part ? Have you been influenced by specific resources ? Which ones ?– What do you think you would answer in 10 years ? Which sources will you use, how would you access these sources ? Will you work on their elaboration on your own,

or with colleagues ? Will these documents be broadcast, and how ? What would be for you a dream resource ?

Appendix 2 : profiles of the teachers interviewed

Table 1 - Explanations regarding the content of the table: Collège: grades 6 to 9, students aged 11 to 16; Lycée: grades 10 to 12, students aged 16 to 18; APM: Association of mathematics teachers; CAPES: Teaching certificate (national competition); IREM: Institute for Research on Mathematics Teaching; INRP: National Institute for Pedagogical Research

Teacher	Age	School	Communities institutional responsibilities ICT degree of integration
Arnaud	47	Lycée	– Teacher trainer Degree zero
Anaïs	57	Lycée	SFoDEM trainee, responsibilities in APM CAPES examiner Low degree
Benoîte	52	Collège	– Low degree Registered MEP user
Céline	36	Collège	Trainer at the IREM, registered MEP user ICT responsibilities in the local education administration Strong degree
Frédéric	50	Collège	Trainer at the IREM – Degree zero
Ingrid	27	Lycée	Trainer at the IREM, member of an INRP – ICT groupStrong degree
Marie-Pierre	40	Collège	Trainer at the IREM, APM member, registered MEP user – Strong degree
Marie-Françoise	54	Lycée	Ex-SFoDEM trainer, member of an INRP ICT group, IREM trainer Teacher trainer Strong degree
Sonia	50	Collège	Member of an INRP ICT group Former ICT responsibilities in the local education administration Strong degree

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