

Editorial from Issue 4, Volume 12, Year 2025 of the journal Teaching Mathematics in Debate

The teaching of mathematics has never been static, but we rarely experience so many simultaneous epistemological recompositions. This issue of Teaching Mathematics in Debate emerges precisely in this scenario of movement of knowledge and practices, functioning as a mutiny that composes the act of teaching and learning mathematics today. This academic "mutiny" is not due to disorder, but to the multiplicity of fronts that question the traditional boundaries of knowledge. Issue number 4 of volume 12 of 2025 converges on a multiplicity that materializes in research themes that outline new trajectories for the field of mathematics education.

It includes two sections: the usual section with 17 articles, whose authors focus on diverse phenomena in the teaching and learning of mathematics, and a special section with 8 articles, constituting the Thematic Dossier: Mathematics, Discourse and Languages: Contributions to Mathematics Education.

Thus, issue number 4 comprised a total of 25 articles. We begin by describing the varied articles, the first of which is the French (original) and Portuguese versions of the article, "Artificial intelligence and mathematics teaching: continuities, ruptures and epistemological recompositions" by Trouche, which discusses the impact of generative artificial intelligence on mathematics teaching, situating it within the continuity of historical instrumental evolutions, but highlighting ruptures in the production of discourse and reasoning. The second article, "Quadratic function and the catapult game" by Dantas Lira et al., analyzes the contributions of a didactic sequence to the teaching of quadratic functions through equation competitions, the history of mathematics, and playful experiments with catapults in a public school in Alagoas. Moraes Brum et al., in their third article, "Artificial Intelligence and Station Rotation in Mathematics Teaching: Students' Perceptions," report on an innovative experience in a public school in Rio Grande do Sul, integrating the Station Rotation methodology with the use of Artificial Intelligence tools (such as Chat GPT and Copilot) in mathematics teaching. They reinforce that these technologies renew pedagogical practice, highlighting the importance of teacher training for the critical and intentional use of these tools in the final years of elementary school.

Zontini and Marcondes, in "Modeling in Mathematics Education and the Formation of a Complex Subject: Reflections on Practices," the 4th article, presents the connections between Mathematical Modeling and Complexity Theory in a practice with 9th-grade students on the theme of the World Cup. The qualitative analysis reveals that this methodology favors the formation of a "complex subject," developing skills that go beyond content, such as critical thinking, creativity, and collaboration. The 5th article, "Equivalent Isoperimetric Figures, Pentominoes, and Geoboard," discusses the use of recreational and manipulative geometry, specifically through pentominoes and the geoboard, as a strategy for teaching isoperimetric and equivalent polygonal figures. For the authors Fadigas and Soares, the articulation of these resources favors the visualization and analysis of metric properties, allowing students to understand in a practical way the relationships between area and perimeter.

In Ramos study, the 6th article, "Financial Education in Mozambique: Paths to Developing Financial Intelligence", the urgency of financial education in Mozambique is observed, identifying the necessary skills to develop financial intelligence from childhood. The research reveals that the lack of financial literacy and low access to banking services limit economic inclusion and sustainable development in the country. The 7th article, "Planning Lessons Based on Exploratory Mathematics Teaching for Teaching First-Degree Polynomial Equations", by Maieski and Baniak, investigates lesson planning involving these equations from the perspective of Exploratory Mathematics Teaching (EEM) for the 7th grade. Through a rigorous process that included the (re)elaboration of tasks, student tests, and the construction of anticipation charts, the study highlights the challenging nature and time required for this teacher preparation. The results confirm the viability of the methodology, offering practical support for the initial and continuing training of teachers seeking to implement investigative approaches in the classroom. From a neuroscience perspective, the work of Médice Júnior and Abar, article 8, "Neuroscience in the Training of Mathematics Teachers: A Bibliographic Review of Theses and Dissertations," presents a bibliographic review analyzing how Brazilian research articulates Cognitive Neuroscience with the training of mathematics teachers. The study reveals that concepts of memory, attention, and emotion are used as structuring foundations for new pedagogical practices; however, they note that despite academic advances, this knowledge is still scarce in teacher training curricula. They conclude that there is an

urgent need for systematic integration between neuroscience and education to underpin teaching practices more aligned with learning processes.

The 9th article, "Interlocution between the guidelines of curricular documents and speech acts of mathematics teachers in the light of Habermasian discursive ethics," by Guimarães and Palanch, discusses the convergence between institutional documents (PPI and PDI) and the speech acts of mathematics teachers regarding the inclusion of Vocational Education. Based on Habermas's discursive ethics, the research identifies how expressive and evaluative externalizations of teachers dialogue with official guidelines from the perspective of communicative rationality. The study highlights thematic affinities between norms and practical concerns, proposing methodological advances for future investigations that articulate discourse analysis with institutional policies. In the systematic review by Araújo Filho and Monteiro Júnior, the 10th article, "Systematic Literature Review: a search for applications of Fuzzy logic in education", investigated the application of fuzzy logic in Brazilian basic education over the last decade. Although this logic is an essential mathematical tool for high technology, such as Artificial Intelligence and Automation, the results indicate that its implementation in education is still preliminary.

The study "Fedathian Thought" by Sales and Menezes, the 11th article, presents the concept of Fedathian Thought, defined as a pedagogical mental structure that guides the teacher in organizing investigative practices based on the Fedathi Sequence. Through a review of the "state of the art", the research systematizes how the internalization of this methodology favors intentional planning and informed decision-making in the classroom. The article "Reflective Writings of a Future Mathematics Teacher and the Development of Professional Teacher Identity" by Teixeira and Dos Santos, the 12th article, analyzes the reflective writings of a future teacher during her participation in the Pedagogical Residency Program, investigating how these narratives reveal the development of her professional identity. The results highlight the strengthening of awareness about planning, the management of unforeseen events, and the sense of agency in the face of classroom challenges. The 13th article, "Learning probability through the game 'The Wall", by Araújo Junior and Lima Neto, investigates the use of the game "The Wall" and simulators in teaching probability to high school students. The results indicate that playful and technological methodologies spark student interest and facilitate the construction of mathematical concepts, promoting more dynamic and effective learning.

Kovacevic's study, the 14th article, "From the 19th Century to the Present: The Evolution of Astronomy in Brazilian Curricula", analyzes the evolution of the teaching of spherical geometry and astronomy in Brazilian curricula, highlighting the historical French influence since the 19th century. Through the theory of Didactic Transposition, it discusses how these themes, although interdisciplinary, remain present in national guidelines as fundamental tools of modern mathematics. The 15th article, "Proposal of an Ergonomic Analysis Model to Support the Identification of the Causes of Mathematical Anxiety", by Ferreira and Reis, investigates Mathematical Anxiety from an Ergonomics perspective, proposing a model that transcends factors intrinsic to the subject. The research culminates in a tool adapted to identify the multi-causality of the phenomenon in the relationship between student and school community.

The 16th article, "3D Printing in Geometry Teaching in Elementary School: A Survey of Academic Productions," by authors Moraes and Tabuti, maps the use of 3D printing in geometry teaching in the final years of elementary school between 2016 and 2024. The results reveal that, although the technology boosts spatial visualization, Brazilian academic production is still incipient, demanding more systematic studies in the area. "Topics in the History and Philosophy of Mathematics: discussions and reflections" is the 17th article of the first phase. It was written by doctoral students from the PPG in Mathematics Education at PUC-SP, with revisions made and signed by the class professor. The names of the student authors are listed in alphabetical order by first name: Adriana Santos Morgado, Barbra Candice Southern, Bernardo Fernandes Cruz, Debora Ferreira Ricardo, and Verônica Freires da Silva, accompanied by Sonia Barbosa Camargo Igliori. This article presents and analyzes discussions developed throughout the course "Topics in the History and Philosophy of Mathematics." The text aims to reflect on the historical, philosophical, and epistemological constitution of mathematical knowledge and its implications for Mathematics Education, articulating different theoretical frameworks and formative experiences. The analyses are primarily based on the works "Fundamental Concepts of Mathematics" by Bento de Jesus Caraça and "Introduction to the Philosophy of Mathematics" by Bertrand Russell, as well as texts by Ubiratan D'Ambrosio and contemporary contributions discussed in class.

In the second part, we present the words of the guest editors, Ana Lúcia Manrique and Walber Christiano Lima da Costa, describing the thematic dossier and the general coordinators of the 5th SENALEM, introducing the special articles.

The relationship between language, discourse, and the teaching-learning of mathematics has been consolidating, in recent decades, as one of the most fertile and challenging axes of research in Mathematics Education. The understanding that mathematics is not reduced to a set of abstract symbols and rules, but is constituted as a social practice, traversed by multiple forms of expression and signification, has demanded profound revisions in conceptions of teaching, teacher education, and assessment. This thematic issue, entitled "Mathematics, Discourse and Languages: contributions to Mathematics Education", brings together a set of articles selected from the conferences and round tables of the V National Seminar on Language and Mathematics Education (V SENALEM), held at the Pontifical Catholic University of São Paulo (PUC-SP), from December 3 to 5, 2025. The event, organized by the ForProfMat group (Mathematics Teacher: Education, Profession, Knowledge and Teaching Work), maintained the historical commitment of SENALEM to promoting dialogue among researchers, teachers, and students around the theme of language, a legacy initiated by Prof. Dr. Marisa Rosani Abreu da Silveira (in memoriam) and her group, GELIM-UFPA.

The works gathered here reflect the theoretical and methodological diversity that characterizes the field, while at the same time pointing to convergent concerns: the need for an effectively inclusive mathematics education, critical and multimodal teacher education, reflection on the linguistic challenges inherent to mathematical learning, and the examination of the impacts of new technologies and communicational paradigms on the teaching of the discipline.

To present the works that compose this collection, we have chosen an organization that seeks to reflect a logical progression of the debates in the field: starting from a macro reflection on the trajectory and state of the art, moving through analyses of specific research foci and contemporary challenges, and finally arriving at case studies and applications that confront issues of inclusion and new educational realities.

Opening the issue, Valdomiro Pinheiro Teixeira Junior provides the necessary panorama in "Perspectives of studies in Language in Mathematics Education: where we came from, where we are, and where we are going". His text, of a meta-scientific character, revisits the philosophical and epistemological foundations of the field, analyzes its consolidation, and points to future trends, functioning as an intellectual map for the other articles.

Next, two works highlight the institutional consolidation and productivity of Brazilian research in this area. Paulo Vilhena da Silva and Marlon Augusto das Chagas Barros systematize the "Research on mathematical language in GELIM-UFPA: themes, approaches and contributions", demonstrating the breadth and depth of the production of one of the pioneering groups. Sueli Cunha, in turn, presents a specific and fundamental aspect of these studies by discussing the "Grammar of Mathematical Language", revealing the advances in describing the internal structure of this language and its implications for teaching.

The core of the discussion on teacher education is addressed by Ana Lúcia Manrique in "For an Inclusive Teacher Education: Mathematical Language as a Transversal Axis in the Construction of Equitable Practices". The author vigorously defends the need for a multimodal and critical approach in teacher education, positioning language as a central element for overcoming barriers and promoting equity.

This commitment to inclusion is tested and detailed on two challenging fronts. Walber Christiano Lima da Costa and Ana Lucia Manrique revisit, ten years later, the issue of "Teaching mathematics X deaf students". Their bibliographic mapping reveals partial advances, but highlights persistent obstacles, such as the lack of standardized mathematical signs in Libras (Brazilian Sign Language), demanding urgent political and pedagogical actions. In the same vein, Elton de Andrade Viana discusses the "Advances in scientific communication stimulated by neurodiversity", arguing, from the neurodiversity paradigm, that a terminological change is a crucial ethical and methodological step for a mathematics education that respects and enhances distinct forms of thinking, such as in autism.

The issue also confronts a contemporary challenge. Cristiane Maria Cornelia Gottschalk, in "The learning of mathematical certainties in the age of social networks and AI", draws on Wittgenstein's philosophy to problematize the uncritical use of generative artificial intelligence in mathematics education. The reflection warns of the risk of replacing the construction of thought and meaning with the search for immediate answers, emptying the learning process.

Finally, as a fitting tribute and conclusion, the article "Marisa: A Legacy for Mathematics Education", by Carlos Evaldo dos Santos Silva, Janeisi de Lima Meira, and Ocsana Sônia Danyluk, celebrates the trajectory of the professor who was the cornerstone for the consolidation of these studies in Brazil. The text not only pays homage to her theoretical contribution – especially the articulation between

Wittgenstein and language games in teaching – but also reaffirms the vitality of her intellectual project, which echoes in all the previous discussions.

Together, the articles in this issue bear witness to a field in vibrant motion, capable of connecting sophisticated theoretical investigation to pressing ethical and political commitments, such as inclusion, equity, and social justice. They demonstrate that thinking about language in Mathematics Education is, above all, thinking about the conditions of access, participation, and production of meanings by all subjects involved in the educational process. It is the privilege of the journal Ensino da Matemática em Debate to foster and disseminate this fundamental debate, in the hope that these contributions will inspire new research and, above all, concrete transformations in classrooms across the country.

Finally, we express our gratitude to the authors who, through their research, have made this edition a space for dialogue. We also thank the reviewers and the editorial team, whose meticulous work ensures excellence and the dissemination of scientific knowledge in the field of Mathematics Education.

We dedicate this volume to you, the reader, hoping that the reflections presented here will serve as inspiration and support for the continuous transformation of your pedagogical and scientific practices in strengthening quality education.

Sonia Barbosa Camargo Igliori Joanderson de Almeida Reis Ferreira

Editors of the Journal Teaching Mathematics in Debate