

Recent publications on the early history of Chinese mathematics (essay review)

Recentes publicações sobre a origem da história da matemática chinesa (revisão crítica)

ALEXEI VOLKOV¹

Abstract

The author discusses the earliest extant Chinese mathematical treatises, Jiu zhang suan shu 九章算術 (Computational procedures of nine categories), compiled no later than the first century AD, and three recently unearthed mathematical texts, Suan shu shu 算數書 (Scripture on computations [to be performed] with counting rods), Shu 數 (Numerical [procedures (?)]), and Suan shu 算術 ([Computational] Procedures [to be performed with] Counting Rods), all dated of the late first millennium BC. The main question addressed in the review is that of the origin and early history of the Jiu zhang suan shu. Some modern historians suggested that this treatise was considered a “canonical” text by its commentator of the third century, Liu Hui 劉徽 (fl. AD 263). The author suggests that this claim is based solely on one phrase found in the commentator’s preface. Instead, the recent findings suggest that a number of mathematical texts were produced, copied and studied in China from the 3rd century BC onwards, and none of them had the status of a “canonical scripture”; these texts, the author suggests, most likely were used for educational purposes.

Keywords: ancient Chinese mathematics, Jiu zhang suan shu, Suan shu shu 算數書 (Scripture on computations [to be performed] with counting rods)

Resumo

O autor discorre sobre os primeiros e remanescentes tratados da matemática chinesa, Jiu zhang suan shu 九章算 (Procedimentos de conta das nove categorias), compilados não depois do primeiro século d.C., e sobre três textos matemáticos recentemente descobertos, Suan shu shu 算數書 (Registro de contas [a serem realizadas] com barras de contar), Shu 數 ([Procedimentos (?)] numéricos), e Suan shu 算術 (Procedimentos [de contagem] [a ser realizada com] barras de contar), todos datados do final do primeiro milênio a.C. A principal questão desta revisão crítica é a origem e a história de Jiu zhang suan shu. Alguns historiadores modernos sugerem que este tratado era considerado um texto “canônico” por seu comentador do terceiro século, Liu Hui 劉徽 (fl. 263 d.C.). O autor sugere, neste artigo, que essa afirmação está baseada em uma frase encontrada no prefácio do comentarista. Em vez disso, descobertas recentes sugerem que uma série de textos matemáticos foi produzida, copiada e estudada na China a partir do século 3 a.C., e nenhum desses textos tinham o estatuto de “escritura canônica”. O autor sugere que esses textos, provavelmente, eram destinados para fins educacionais.

¹ National Tsing Hua University – alexei.volkov@gmail.com

Palavras-chave: *matemática antiga chinesa; Jiu zhang suan shu, Suan shu shu 算數書 (Registro de contas [a serem realizadas] com barras de contar).*

The term “Chinese mathematics” in this essay review refers to the mathematical tradition that existed in China since the first millennium BC until the introduction of the Western mathematical tradition represented by the partial translation of the *Elements* of Euclid published by Matteo Ricci and Xu Guangqi 徐光啟² in 1607.³ This tradition, no longer extant, featured computations with counting instruments (counting rods and abacus) performed according to algorithms recorded in mathematical treatises organized, with a few exceptions, as collections of mathematical problems. In some treatises the algorithms were accompanied by commentaries providing their justifications. These commentaries, sometimes offering serious difficulties of interpretation, contain rich material for analysis of mathematical thought of the ancient authors and later commentators. This is why one of the central events in recent Western historiography of Chinese mathematics was the publication of two annotated translations of the ancient mathematical treatise *Jiu zhang suan shu* 九章算術 (Computational procedures of nine categories) with the commentaries of Liu Hui 劉徽 (fl. AD 263) into English (SHEN et al., 1999) and French (CHEMLA; GUO 2004).⁴ The strong and weak sides of these two translations, as well as their comparison with the Japanese translation published earlier by Kawahara Hideki 川原秀城 (1980a) were discussed by the author of the present essay (VOLKOV, 2010).⁵ The translation of

² In this paper I use the transliteration system *pinyin* adopted in Mainland China and in European sinology. The titles of texts and names of authors in Chinese, Taiwanese, and Japanese are provided in traditional versions of Chinese characters supplemented, in the case of publications of Mainland Chinese and Japanese authors, by their simplified versions, if necessary.

³ For arguably the best study of this translation project available in Western languages see Engelfriet (1998).

⁴ The treatise (but not the commentaries) was earlier translated into Russian (BEREZKINA, 1957), German (VOGEL, 1968) and Japanese (ŌYA, 1975; SHIMIZU, 1975-6).

⁵ Another Japanese translation of the *Jiu zhang suan shu* (not discussed in Volkov (2010)) is being published by a team of Japanese scholars led by professor Ohkawa Toshitaka 大川俊隆 in a series of papers in the *Journal of Osaka Sangyo University (Social sciences and Humanities Series)* 大阪産業大学論集 人文・社会科学編 starting from 2008; pdf versions of these publications are available at <http://pal.las.osaka-sandai.ac.jp/~suanshu/j/publications2.html>; the

Chemla and Guo is currently the most reliable interpretation of the texts of the treatise and of the commentaries among those available in Western languages.

The early textual history of the treatise, however, remains obscure. The preface to the treatise credited to the authorship of Liu Hui depicts it as follows: the treatise stemmed from *Jiu shu* 九數, literally “nine [kinds of] computations”, the mathematical curriculum used for instruction of the sons of aristocratic families introduced by the Duke of Zhou (Zhou gong 周公, fl. ca. 11 c. BC), a semi-legendary ruler of the Zhou 周dynasty (1046-256 BC); Liu Hui maintains that the term *jiu shu* was not referring to a mere set of mathematical techniques but was the title of a prototype of the *Jiu zhang suan shu*. Later, Liu Hui reports, the book was somewhat damaged during the famous “burning of books” performed on the order of the First Emperor (Shi Huangdi 始皇帝, r. 246-210 BC) of the Qin 秦dynasty (221-206 BC), yet its (incomplete?) copy (or copies?) was (or were) available to Zhang Cang 張蒼 (253-152 BC) and Geng Shouchang 耿壽昌 (fl. 57-52 BC), two high-rank officers of the Early Han 前漢dynasty (206 BC –9 AD). Those two individuals, according to Liu Hui, worked one after another to produce a new edition of the treatise; due to the substantial editorial work of these two scholars, the resulting compilation included a number of anachronistic terms noticed by Liu Hui in the version of the treatise he commented upon. The textual history of the treatise, as presented by Liu Hui, stops here. Later, the treatise went through an editorial work done by a team of scholars working under direction of Li Chunfeng 李淳風 (602-670 AD); it is generally believed that the earliest extant version of the treatise, a block-print of the early 13th century, is identical with, or at least close to, Li Chunfeng’s edition. In this review I will concentrate on the early history of the text and on its interpretation by Liu Hui, as well as on its interpretation by modern historians in the context of recent discoveries of mathematical texts of the 1st millennium BC.

same team produced a Japanese translation of the recently unearthed mathematical treatise *Suan shu shu* 算數書 (see below).

Until the end of the 20th century the historiography of the early Chinese mathematics focused almost exclusively on the *Jiu zhang suan shu*.⁶ Since all other extant mathematical texts were written at least several centuries later than the presumed time of compilation of this treatise, it was impossible to advance any hypotheses concerning the origin of mathematical methods it contained. However, a major event that happened in 1983 changed this situation dramatically: a mathematical treatise written on bamboo strips was found in a tomb sealed in 186 BC. The title of this treatise, *Suan shu shu* 算數書, literally means “Scripture (*shu* 書) on computations (*shu* 數) [to be performed] with counting rods (*suan* 筭)”; it remains unclear though whether this title (written on the *verso* of one of the strips) was indeed given to the text by its compilers or was given to it solely on the basis of its contents some time later by its owner, or a librarian, or a copyist. The treatise was published for the first time in 2000 (SSS, 2000a) and later reproduced photographically (SSS, 2001, 2006); it was translated into Japanese (JOCHI, 2001; OHKAWA et al., 2006),⁷ modern Chinese (HORNG et al., 2006) and English (CULLEN, 2004; DAUBEN, 2008). A wealth of publications on the *Suan shu shu* was produced in Mainland China, in Taiwan, in Japan, and in the West,⁸ and several international conferences and symposia devoted to it were held recently.⁹

Meanwhile, two other ancient mathematical texts were unearthed. One of them, titled *Shu* 數 (Numerical [procedures (?)]) and dated of the Qin dynasty (221-206 BC), was

⁶ “Mathematical” excerpts found in the philosophical treatise *Mo zi* 墨子 (compiled in the late 5th – early 4th century BC; relevant chapters were a later addition of the fourth and early thirds century BC) as well as some other texts of the first millennium BC did not allow to produce any coherent picture of the mathematical landscape of that time.

⁷ The translation Ohkawa et al. (2006) was published by the above-mentioned team of Japanese scholars “Chō ka san Kan kan *San sū sho* kenkyukai 張家山漢簡『算數書』研究会 (Society for study of the *Suan shu shu* [written on] the Han [dynasty bamboo] strips from Zhanjiashan) working under direction of professor Ohkawa Toshitaka 大川俊隆; the list of the team members is available at <http://pal.las.osaka-sandai.ac.jp/~suanshu/j/roster.html>; the list of articles on the *Suan shu shu* published by this team (some of them available online) is found at <http://pal.las.osaka-sandai.ac.jp/~suanshu/j/publications.html>.

⁸ For references to publications devoted to the *Suan shu shu* see Ohkawa (2006, p. 168-169), Dauben (2008, p. 172-177), and Zou (2008, p. 95-98).

⁹ See, for example, the materials of the “International Symposium on the *Suan Shu Shu*: Appraisals and Appreciations” held on August 23-25, 2006, at National Taiwan Normal University, Taipei, published in the *HPM Newsletter* (*HPM* 通訊), vol. 9, no. 9 (available online at <http://math.ntnu.edu.tw/~horng/letter/909.pdf>).

found during illegal excavations, purchased in Hong Kong in 2007 and is currently preserved in the Yuelu Academy 嶽麓書院/岳麓书院 (Hunan University, Changsha, Hunan Province, China). The other mathematical text titled *Suan shu* 算術([Computational] Procedures [to be performed with] Counting Rods) was found during the excavations at Shuihudi 睡虎地 (Hubei Province, China) in 2008. The former book became the topic of several publications¹⁰ and of the international conference held in Changsha in 2010;¹¹ for the time being, the text remains unpublished, yet it is circulating within a small international community of scholars working on the history of Chinese mathematics. A short excerpt from the latter book, *Suan shu* 算術, was published in 2008¹² and studied by Karine Chemla and Ma Biao (2011).

The contents of the *Suan shu shu* to a certain extent resemble those of the *Jiu zhang suan shu*, and since the time when the former treatise was compiled (presumably prior to the early second century BC) was close to the time of the legendary revisions of the latter treatise by Zhang Cang and Geng Shouchang, the question of the possible connections between the two texts emerged naturally and was discussed in virtually all publications on the early history of Chinese mathematics starting from 2000. For instance, in her introductory chapter to the critical edition and translation of the *Jiu zhang suan shu*, Karine Chemla discussed the possible connections between the *Suan shu shu*, *Jiu zhang suan shu*, and the astronomical treatise *Zhou bi suan jing* 周髀算經 completed prior to the first century AD and featuring a large number of mathematical matters, such as Pythagoras theorem. When discussing the relationships between the three texts, Chemla insists that the *Jiu zhang suan shu* had a special status, that of a “Classic”, by the time when the commentary of Liu Hui (fl. 263 AD) was completed (CHEMLA; GUO, 2004, p. 5-8). To assume that the text had this particular status, according to Chemla, will help the reader avoid unjustified interpretations of the treatise portraying it as belonging to the category of “school textbooks”.

¹⁰ Xiao and Zhu (2009a-c); Zhu and Xiao (2009).

¹¹ Zou (2011).

¹² SS (2008).

Chemla's claim that the book was perceived by its contemporaries as a "Canon" is based on only one sentence in the Preface (traditionally credited to the authorship of the above-mentioned commentator Liu Hui) which reads: 往者暴秦焚書。經術散壞,¹³ that may be rendered as "in the past, the cruel [ruler] of the Qin [that is, the first Emperor of the Qin empire, Qin Shi Huangdi] 'burnt books', [and as result of that] the canonical texts (*jing*) and [technical] recipes (*shu*) were dispersed and damaged". This phrase opens a new paragraph devoted to the legendary reconstruction of the *Jiu zhang suan shu* by Zhang Cang and Geng Shouchang (see above) and explains why the edition commented upon by Liu Hui diverged considerably from the "ancient" one and contained "numerous contemporaneous terms": 故校其目則與古或異。而所論者多近語也。¹⁴ The opening phrase can be understood as a general reference to the legend of "books burning" in the third century BC used by Liu Hui to explain why the version of the treatise he had at hand was apparently a post-Qin text; in this case, the mentioned *jing shu* (canonical [texts] and [technical] recipes, most likely in plural) refer to the unidentified bulk of texts burnt, according to the Han dynasty historians, by the first Qin emperor. However, translation of Chemla and Guo suggests a rather different interpretation of the phrase of Liu Hui; they suggest: "Autrefois, le cruel Qin (shi huangdi) brûla les livres. Les procédures du Classique furent dispersées et endommagées" (CHEMLA; GUO, 2004, p. 127), that is "Formerly, the cruel Qin (Shi Huangdi) burnt the books. The procedures (*shu*) of the *Classic* [that is, according to Chemla and Guo, *Jiu zhang suan shu*. – A.V.] were dispersed and damaged."¹⁵ It is important to stress here that this is the *only* evidence found in Liu Hui's commentary on the *Jiu zhang suan shu* supporting Chemla's thesis of the special status of the *Jiu zhang suan shu*, that of a "Classic" (or "Canon" in her publications in

¹³ JZSS (2001, p. 83).

¹⁴ Literally, "This is why, [if one] compares their [i.e., those of the pre-Qin and Han versions] tables of contents, then [one can see that the Han version] probably diverged from the ancient one. At the same time, as far as the [matters] discussed [in the Han version] are concerned, there are numerous recent terms/statements." (JZSS, 2001, p. 83). Chemla and Guo translate this phrase as follows: « C'est pourquoi, quand on examine ses sections, par endroits elles diffèrent des anciennes, et ce qui est discuté l'est pour beaucoup en termes modernes » (CHEMLA; GUO, 2004).

¹⁵ In an endnote Chemla admits that her interpretation is not the only one, and suggests the second rendering of the term *jing shu*, namely, "la connaissance des Classiques confucéens"(knowledge of Confucian Classics) which, however, according to her opinion, will make the text of the preface less coherent: "Si tel était le cas, le texte, me semble-t-il, perdrait en cohérence" (CHEMLA; GUO, 2004, p. 752, n. 14).

English). In her later publications, Chemla, however, considers this rather obscure phrase of Liu Hui as a definitive proof of the special status of the *Jiu zhang suan shu* and makes a number of statements concerning the early textual history of the treatise and of the commentarial work of Liu Hui that would automatically stem from its assumed “canonical” status.¹⁶ But is the interpretation of the statement of Liu Hui suggested by Chemla and Guo supported by historical evidence?

The standard description of the “books burning” by the First Emperor of the Qin is found in Chapter 6 of the *Shi ji* 史記 (Records of the [Grand] Astrologer) by Sima Qian (ca. 145 or 135 BC – 86 BC); the relevant excerpt reads:

丞相李斯曰。[...]臣請史官非秦記皆燒之。非博士官所職。天下敢有藏詩、書、百家語者，悉詣守、尉雜燒之。[...]。所不去者，醫藥卜筮種樹之書。

Chancellor Li Si said: "[...] [I, your] servant, demand the personnel of the [office of] astrology/history that all the records other than those of Qin's [state] be burned. With the exception of the [books] used for duties by the teaching staff (*boshi*), those in the empire who dare to hold [copies] of the [*Classic of*] *Poems*, the [*Classic of*] *History*, or the teachings [lit. “sayings”. – A.V.] of the “hundred [philosophical] schools,” in all cases [must] submit [the books] to the governors or commandants to be burnt indiscriminately. [...] The [only books that do] not [have to] be removed are those on medicine, drugs, divination of various kinds, agriculture and forestry.¹⁷

The motion thus was targeting the books that might have represented an ideological threat to the new regime. Books used with official authorization to perform teaching duties, as well as technical manuals were not supposed to be destroyed. Mathematical texts most likely fell into the latter category and therefore were not in danger; the recent findings suggest that a number of mathematical texts freely circulated in the Empire during and after the “books burning”. A problem is posed by the term *shu* 術 in Liu Hui's Preface: it can mean “computational rules” or “algorithms” in mathematical context, and it was precisely how Chemla and Guo chose to render it. However, in his translation Kawahara Hideki made a suggestion that was supposed to solve the problem differently. The relevant excerpt quoted by Kawahara (again from the *Shi ji*) reads:

¹⁶ See, for example, Chemla (2008, 2010).

¹⁷ *Shi ji* 史記, juan 卷6, section “Qin Shi Huang benji” 秦始皇本紀; in this and other cases, if the name of translator is not specified, the translation is mine.

焚詩書。阮術士。六藝從此缺焉。¹⁸

[When the Qin authorities] burnt the [Classic of] Poems and the [Classic of] History, and buried [alive] the “experts in techniques”, the “Six arts” [= traditional educational curriculum] were damaged. (KAWAHARA, 1980a, p. 81, n. 12).

This excerpt makes it clear that the “experts in techniques” (*shu shi* 術士) buried alive were not experts in mathematical algorithms but rather those scholars who maintained doctrines opposed to the official ideology. It thus appears more plausible to understand Liu Hui’s mention of the dispersed and damaged *jing* and *shu* as a reference to the events of the Qin times well known to all educated contemporaries and not as a very specific mention of “mathematical procedures (*shu*) of the Canon (*jing*)” that thus would have been assigning to the *Jiu zhang suan shu* a status of “Canon”.¹⁹ Liu Hui’s claim looks as an attempt to explain to the contemporaneous reader why the text he was commenting upon did not look “ancient enough”, and a reference to the legendary “books burning” at his time may have been considered a passable explanation. In his paper C. Cullen (2007) discussed in detail Liu Hui’s statements and arrived at a similar conclusion:

[t]o sum up the position on Liu Hui’s attempt to write history of mathematics, we may say that where he makes a claim not supported elsewhere, as in his theory that the *Nine Chapters* [i.e., the *Jiu zhang suan shu*. – A.V.] was a pre-Qin text, lost and later reconstituted, his views seem to be based on a mixture of uncritical acceptance of historical cliché and straightforward conjecture (CULLEN, 2007, p. 34).

Returning back to the part played by the *Jiu zhang suan shu* in the contemporaneous academic environment, one may advance at least four hypotheses: (1) it was a collection of applied problems related to taxation, construction works, etc., designed mainly for the low-level state functionaries directly involved in these activities; (2) it was a work on “theoretical mathematics” designed within, and addressed to, the community of professional mathematicians, a text in which purely theoretical mathematical statements were presented in disguise of “applied mathematics”; (3) it was seen mainly as a

¹⁸ *Shi ji* 史記, section “Ru lin lie zhuan” 儒林列傳 (Ordered biographies of Confucian scholars).

¹⁹ One more interpretation of the words *jing shu* 經術, “canonical methods”, was suggested by C. Cullen (2007, p. 32). This rendering makes it unclear what might have been the “non-canonical methods” that presumably remained intact during the purges; it apparently does not take into consideration the above-mentioned quotations from the *Shi ji*.

“classic”, that is, a starting point of an exegetical (commentarial) tradition, as Chemla and Guo suggested, and (4) it was a “school textbook” used mainly for educational purposes. The first hypothesis stems directly from the very contents of the treatise. As far as the second hypothesis is concerned, one cannot exclude the possibility that at least some pre-modern mathematical texts, even though styled as collections of problems, did contain new methods devised by their authors and therefore were addressed, at least to a certain extent, to peers and not exclusively to the learners. However, it can be argued that a considerable number of pre-modern Chinese mathematical texts, especially those styled as collections of arithmetical problems including the *Jiu zhang suan shu*, even though produced in a variety of historical settings, were either originally compiled or adapted at subsequent stages to be used in an educational context and thus are much closer, as far as their contents and presumed applications are concerned, to mathematical textbooks rather than scholarly monographs. Even when the information about the use of a given text for educational purposes is insufficient or unavailable, its very structure, and, in some cases, the numerical parameters of the problems, may provide sufficient material for a sensible reconstruction of at least a hypothetical use of the text in an educational context, while the same elements would appear unnecessary if the text was designed and used exclusively for professional mathematicians or for practical applications.²⁰

The following example from the *Jiu zhang suan shu* may provide an illustration of the latter statement. Chapter 8 of the treatise contains a list of problems equivalent, in modern terms, to simultaneous linear equations.²¹ Problems 1 and 2 of the chapter do not involve any operations with negative numbers, that is, only non-negative coefficients appear at all subsequent stages of calculations prescribed by the algorithm presented in the beginning of the chapter. However, in the process of solution of problem 3 (which in modern terms can be written down as the system of simultaneous equations $\{2x + y = 1; 3y + z = 1; x + 4z = 1\}$) negative numbers do appear: according to the algorithm, the third equation is supposed to be multiplied by 2, thus yielding $2x + 8z = 2$, and the first equation is required to be subtracted from it to eliminate the term $2x$. At this point the reader is facing the problem of physically removing one counting

²⁰ See Volkov (2012, 2013).

²¹ For a translation of the chapter and interpretation of the methods, see, for example, Chemla and Guo (2004, p. 614-701, 861-877).

rod (the coefficient of the term y in the first equation) from an empty cell on the counting surface. This problem is solved with the use of the negative unit (represented with a counting rod of a different color or shape). At the next step, the resulting equation $-y + 8z = 1$ is to be multiplied by 3 (the coefficient of y in the second equation), and the resulting equation, $-3y+24z = 3$, is supposed to be *added* to the second one. The term with a negative coefficient thus will disappear, and in the remaining part of the process of solution no other negative coefficient will appear again. Despite the “applied” appearance of the problem (dealing with grain of three kinds), the choice of numerical parameters is clearly made on the basis of educational considerations: when the negative numbers appear for the first time, it is certainly didactically sensible to provide an example in which the smallest (in absolute value) negative number appears, and appears only once. This example demonstrates that even the early mathematical treatises may have been compiled for educational purposes.²²

This short presentation cannot be complete until the treatises found in 2007-2008 become available to the entire scientific community. However, as far as the first publications on the book titled *Shu* by Xiao Can and Zhu Hanmin, as well as the analysis of a relatively short published excerpt from the other book, *Suan shu*, undertaken by Karine Chemla and Ma Biao, already allow to judge, the mathematical texts that circulated in China in the late first millennium BC had a large number of connections with the earliest extant mathematical treatises still awaiting for future investigation.

References

A. Primary sources

JZSS (2001). *Jiu zhang suan shu* 九章算術 (Computational Procedures of Nine Categories). In GUO SHUCHUN; LIU DUN (Eds.). *Suan jing shi shu* 算經十書. Taipei: Jiu Zhang Publishers.

SS (2008). *Hubei sheng wenwu kaogu yanjiusuo* 湖北省文物考古研究所 and *Yunmeng xian bowuguan* 雲夢縣博物館. “Hubei Yunmeng Shuihudi M77 fajue jianbao 湖北雲夢睡虎地M77發掘簡報 (Short report on the excavations of the tomb M77 at

²² On a possible function of the commentarial tradition formed around the *Jiu zhang suan shu*, see Volkov (2012).

Shuihudi in Yunmeng, Hubei province)”. In *Jiang Han kaogu* 江漢考古, 109, 31-37, plates 11-16.

SSS (2000a). *Jiangling Zhangjiashan Hanjian zhengli xiao zu* 江陵張家山漢簡整理小組 (Restoration Team for the Han [dynasty] bamboo strips [found at] Zhangjiashan in Jiangling). “Jiangling Zhangjiashan Hanjian ‘Suan shu shu’ shiwen” 《江陵張家山漢簡〈算數書〉釋文》 (Transcription of the Suan shu shu of the Han [dynasty written on] bamboo strips [found at] Zhangjiashan in Jiangling). In *Wenwu* 文物, September, 78-84.

SSS (2000b). SU YI-WEN 蘇意雯 et al. (Eds.). “Suan shu shu jiaokan 《算數書》校勘” (Collation of the *Suan shu shu*). In *HPM Tongxun* 通訊 (HPM Newsletter), N. 11, v. 3, 2–20.

(<http://math.ntnu.edu.tw/~horng/letter/hpmlletter.htm>).

SSS (2001) *Zhangjiashan Han mu zhujian (erbai sishi qi hao mu)* 張家山漢墓竹簡〔二四七號墓〕 (Bamboo strips from the Han [dynasty] tomb number 247 in Zhangjiashan). Beijing: Wenwu Chubanshe.

SSS (2006). OHKAWA (= Ōgava), Toshitaka 大川俊隆 et al. *Kan kan “San sū sho”*. *Shashinban* 漢簡『算數書』. 写真版 (Photo reproduction of the bamboo strips of the *Suan shu shu* on the Han [dynasty]). Kyoto: Hōyū shoten 朋友書店 [appendix to Ohkawa 2006].

B. Translations

BEREZKINA, Élvira I. (Березкина, Эльвира И.) (tr., 1957). 九章算術 Математика в девяти книгах (Matematika v devyati knigakh = Mathematics in nine books [= *Jiu zhang suan shu*], Russian annotated translation), *Историко-математические исследования* (Istoriko-matematicheskie issledovaniya = Studies in the history of mathematics), v. 10, 427-584.

CHEMLA, Karine; GUO Shuchun (2004). 九章算術。 *Les Neuf Chapitres: le classique mathématique de la Chine ancienne et ses commentaires. Edition critique bilingue traduite, présentée et annotée par Karine Chemla et Guo Shuchun. Glossaire des termes mathématiques chinois anciens par Karine Chemla. Calligraphies originales de Toshiko Yasumoto*. Préface de Geoffrey Lloyd. Paris: Dunod.

CULLEN, Christopher (2004). *The Suan shu shu ‘Writing on reckoning’: A Translation of a Chinese Mathematical Collection of the Second Century BC, With Explanatory Commentary*. Needham research institute working papers 1. Cambridge, Needham Research Institute.

_____. (2007). The *Suàn shù shū* 算數書, ‘Writings on reckoning’: Rewriting the history of early Chinese mathematics in the light of an excavated manuscript. In *Historia mathematica*. N. 1, v. 34, 10-44.

DAUBEN, Joseph W. (2008). 算數書 *Suan Shu Shu*: A Book on Numbers and Computations. In *Archive for History of Exact Sciences*. v. 62, 91-178.

HORNG Wann-Sheng 洪萬生; LIN Cang-Yi 林倉億; SU Hui-Yu 蘇惠玉; SU Jun-Hong 蘇俊鴻 (2006). *Shu zhi qi yuan: Zhongguo shuxue shi kaizhang* ‘*Suan shu shu*’ 數之起源:中國數學史開章 “算數書” (The Origin of the Numbers: *Suan shu shu*, the Opening Chapter of the History of Mathematics in China). Taipei, Taipei shangwu yinshuguan.

JOCHI, Shigeru 城地茂 (tr.) (2001). “*San sū sho nihongo yaku*算數書日本語譯” (Japanese translation of the *Suan shu shu*). In *Wasan kenkyuso kiyo* 和算研究所紀要, v. 4, 19-46.

KAWAHARA, Hideki 川原秀城 (tr., 1980a). *Ryū Ki chū Kyū Shō San Jutsu* 劉徽註九章算術 (The *Jiu zhang suan shu* commented by Liu Hui). In YABUUTI, Kiyosi 藪内清 (Ed.). *Kagaku no meicho* 科學の名著, Tokyo: Asahi, v. 2, 75-264.

[OHKAWA (=Ōgava), Toshitaka 大川俊隆 et al.] (tr., 2006). *Chō ka san Kan kan* “*San sū sho*” kenkyukai 張家山漢簡『算數書』研究会 (Society for study of the *Suan shu shu* of the Han [dynasty written on bamboo] strips from Zhanjiashan). *Kan kan* “*San sū sho*”: *Chugoku saiko-no sugakusho* 漢簡『算數書』: 中国最古の数学書 (The *Suan shu shu* of the Han [dynasty written on bamboo] strips from Zhanjiashan: the most ancient Chinese mathematical book). Kyoto: Hōyu shoten 朋友書店.

ŌYA, Shinichi 大矢真一 (tr., 1975). *Kyū Shō San Jutsu* 九章算術. In YABUUCHI, Kiyoshi 藪内清 (Ed.). *Sekai meicho* 世界名著. Tokyo: Chūōkōronsha 中央公論社 (Series 1: *Chūgoku no kagaku* 中國の科學 (Chinese science), 97-164.

PENG Hao 彭浩 (2001). *Zhangjiashan Hanjian* “*Suan shu shu*” zhushi 張家山漢簡《算數書》註釋 (Annotated “Scripture on computations with counting rods” [found among] the Han [books] on bamboo strips in Zhangjiashan). Beijing: Kexue Chubanshe.

SHEN Kangshen; CROSSLEY, John N.; LUN, Anthony W.-C. (tr., 1999). *The Nine Chapters on the Mathematical Art. Companion and commentary*. Oxford: Oxford University Press, and Beijing: Science Press.

SHIMIZU, Tatsuo 清水達雄 (tr., 1975-1976). *Chūgoku sūgaku no koten – Kyū Shō San Jutsu* 中國數學の古典 – 九章算術. In *Sūgaku seminā* 數學せセミナー, February 1975 - April 1976.

VOGEL, Kurt (tr., 1968). *Chiu chang suan shu: Neun Bücher arithmetischer Technik, Ein Chinesisches Rechenbuch für den praktisschen Gebrauch aus den Frühen Hanzeit*, Braunschweig: Friedrich Vieweg Verlag (Ostwalds Klassiker).

C. Secondary works

CHEMLA, Karine (2008). Classic and commentary: An outlook based on mathematical sources (preprints of Max Planck Institut für Wissenschaftsgeschichte, available online at <http://www.mpiwg-berlin.mpg.de/Preprints/P344.pdf>).

CHEMLA, Karinie (2010). A Chinese Canon in Mathematics and Its Two Layers of Commentaries: Reading a Collection of Texts as Shaped by Actors. In BRETELLE-ESTABLET, Florence (ed.). *Looking at It from Asia: The Processes that Shaped the Sources of History of Science*. Boston: Studies in the Philosophy of Science, v. 265, 169-210.

_____.; MA Biao (2011). Interpreting a newly discovered mathematical document written at the beginning of the Han dynasty in China (before 157 B.C.E.) and excavated from tomb M77 at Shuihudi (睡虎地). In *Sciamvs*. v. 12.

Cullen, Christopher (1993). ‘*Chiu chang suan shu* [= *Jiuzhang suanshu*]’, In LOEWE, M. (Ed.). *Early Chinese Texts*. Berkeley: The Society for the Study of Early China and The Institute of East Asian Studies, University of California (Early China Special Monograph Series no. 2), 16-23.

ENGELFRIET, Peter (1998). *Euclid in China: The Genesis of the First Chinese Translation of Euclid's Elements, Books I-VI (Jihe Yuanben, Beijing, 1607) and Its Reception Up to 1723*. Leiden: Brill.

KAWAHARA, Hideki 川原秀城 (1980b). *Kyū Shō San Jutsu keisatsu* 九章算術解説 (An analysis of the *Jiu zhang suan shu*). In YABUUCHI, Kiyoshi 藪内清 (Ed.) *Kagaku no meicho* 科学の名著. Tokyo: Asahi, v. 2, 47-73.

VOLKOV, Alexei (2010). Commentaries upon commentaries: The translation of the *Jiu zhang suan shu* 九章算術 by Karine Chemla and Guo Shuchun (essay review). In *Historia Mathematica*. v. 37.

_____. (2012). Argumentation for state examinations: demonstration in traditional Chinese and Vietnamese mathematics. In CHEMLA, K (Ed.). *The History of Mathematical Proof in Ancient Traditions*. Cambridge University Press.

_____. (2013). Didactical dimensions of mathematical problems: ‘weighted distribution’

in a Vietnamese mathematical treatise. In PROUST, C; BERNARD, A. (Eds.). *Scientific sources and teaching contexts: problems and perspectives*. Boston Studies in Philosophy of Science, New York etc: Springer (forthcoming).

XIAO Can 肖燦/肖灿; ZHU Hanmin 朱漢民/朱汉民 (2009a). Yuelu shuyuan cang Qin jian Shu shu zhongde tudi mianji jisuan 嶽麓書院藏秦簡《數書》中的土地面積計算/岳麓书院藏秦简《数书》中的土地面积计算 (Computation of areas of terrains in the *Book Shu* of the Qin [dynasty on bamboo] strips held in Yuelu Academy). In *Hunan Daxue Xuebao (Shehui kexue ban)* 湖南大学学报 (社会科学版). N. 2, v. 23.

_____. (2009b). Yuelu shuyuan cang Qin jian Shu de zhuyao neirong ji lishi jiazhi 嶽麓書院藏秦簡《數》的主要內容及歷史價值 / 岳麓书院藏秦简《数》的主要內容及历史价值 (Historical value and main contents of the [book] *Shu* of the Qin [dynasty on bamboo] strips held in Yuelu Academy). In *Zhongguo shi yanjiu* 中國史研究/中国史研究. N.3.

_____. (2009c). Zhou Qin shiqi guwu cesuanfa ji bishong guannian – Yuelu shuyuan cang Qin jian Shu de xiangguan yanjiu 周秦時期穀物測算法及比重觀念 - 嶽麓書院藏秦簡《數》的相關研究 / 周秦时期谷物测算法及比重观念——岳麓书院藏秦简《数》的相关研究 (Methods of measuring cereals and the concept of proportion in the periods Zhou and Qin: A study of the [book] *Shu* of the Qin [dynasty on bamboo] strips held in Yuelu Academy). In *Ziran kexue shi yanjiu* 自然科學史研究 / 自然科学史研究. N. 4, v. 28.

ZOU Dahai 鄒大海/邹大海 (2008). Chutu jiandu yu Zhongguo zaoqi shuxueshi 出土簡牘與中國早期數學史 (Unearthed texts on bamboo strips and history of ancient Chinese mathematics). In *Renwen yu shehui xuebao* 人文與社會學報. N.2, v. 2.

_____. (2011). Guanyu Qin jian shuxue zhuzuo Shu de guoji huiyi zai Yuelu shuyuan juxing -- ‘Yuelu shuyuan cang Qin jian (di er juan) guoji yanjiuhui’ jianjie 关于秦简数学著作《数》的国际会议在岳麓书院举行 -- “《岳麓书院藏秦简》(第二卷)国际研读会”简介 (On the international meeting in Yuelu Academy devoted to the mathematical work *Shu* on [bamboo] strips [dated of] the Qin [dyansty] - a brief introduction of the Second International Workshop on Qin dynasty bamboo strips held in Yuelu Academy). In *Ziran kexue shi yanjiu* 自然科學史研究 / 自然科学史研究. N.1.

ZHU Hanmin 朱漢民/朱汉民; XIAO Can 肖燦/肖灿 (2009). 從嶽麓書院藏秦簡《數》看周秦之際的幾何學成就/从岳麓书院藏秦简《数》看周

秦之际的几何学成就 (Achievements of the science of geometry of the [late] Zhou – [early] Qin, as seen [on the basis of] the [book] *Shu* of the Qin [dynasty on bamboo] strips held in Yuelu Academy). In *Zhongguo shi yanjiu* 中國史研究/中国史研究. N. 3.

Artigo recebido em 03 de setembro de 2012