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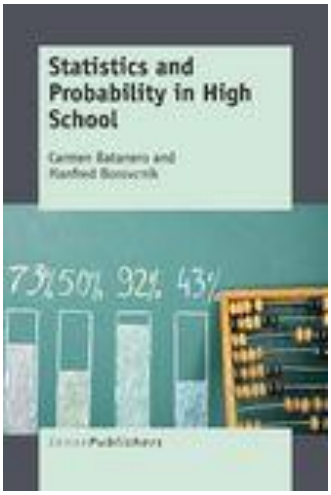

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PhD Carmen Batanero, is a university researcher in the Department of Mathematics Didactics at the University of Granada and coordinator Research Group on Statistical Education. PhD Manfred G. Borovcnik researcher at the Institute of Mathematics, University Klagenfurt, Austria. Both are renowned researchers who have contributed a lot to the field of Statistical Education with a vast production on teaching and learning of Statistics and Probability.

In this way, they bring together their fields of research and teaching experience in the book entitled "Statistics and Probability in High School" which aims to present

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guidelines, recommendations and activities about the teaching and learning the concepts of statistics and probability.

The guidelines and recommendations were built based on the authors' extensive experience in research and teaching, as well as the results of investigations into the analysis of these concepts in the curriculum of High School in several countries. What makes this book special is precisely the discussion and reflection on the main statistical and probabilistic concepts. This was possible due to the rich review of the researches done by the various researchers cited throughout the text.

With this publication, the authors hope to assist both the professors responsible for training the professors of High School Mathematics and also the professors of this level in the teaching of concepts of statistics and probability. Thus, in each of the chapters, the concept development is focused in on a statistical study or a real-world problem that requires data to be answered so that the application of these concepts helps the student to find a solution that is meaningful for him. All chapters have a section in which the authors present research results that point out the student's reasoning and the potential difficulties in understanding the proposed concepts. To do so, the authors reviewed relevant research on the main mistakes made by the students in understanding the concepts and synthesize the results in order to assist the teachers in the development of the activities.

In the first chapter Batanero and Borovcnik present the educational principles that reflect their understanding about the teaching of statistics and probability in High School. In the other chapters the authors present the concepts they consider fundamental for this level of education. All chapters have a common structure that makes reading easier. Each one begins with a brief summary followed by a historical review of concepts, teaching activities, potential student difficulties and resources to support the teaching and learning process.

The book begins by emphasizing the importance of a statistically literate society so that every citizen is able to understand random phenomena and make appropriate decisions when confronted with uncertainty. The purpose of this first chapter is to clarify what is meant by literacy, thinking and statistical reasoning and decision making. In addition, the authors present a discussion about the relationship between mathematics and statistics and the contribution of different areas of knowledge in the researches about Statistical Education.

Another important aspect is the contribution of digital technologies in the teaching and learning the probabilistic and statistical concepts. With the creation of computer programs, specially designed for the teaching of statistics and probability, the data analysis became accessible to researchers and professionals from various areas of knowledge. However, the authors emphasize that teachers should be careful not to emphasize the use of technology more than statistical and probabilistic concepts. Following, throughout the book, technology is presented as a complement rather than a substitute for statistical thinking.

The second chapter presents a brief discussion of how exploratory data analysis can be incorporated into the high school curriculum. The authors highlight the importance of formulating teaching situations where students need to collect, summarize and compare a set of data to investigate and provide potential solutions to a research question. Subsequently, they present as a proposal, the collection of physical measures that students can collect from themselves or from the classmates, such as weight, height, gender, shoe size, waist circumference, among others. Furthermore, the students will be able to explore and relate center measures, variability, and ways of data distribution in a context in which they are familiar. This approach allows the exploration of the concepts of qualitative and quantitative variables, the ways of data distribution, graphical representation (sectors, box plot, histogram, bar chart), data comparison, central position measurements (average, median, mode), variability (variance and standard deviation) and the relationship between two variables (scatterplot). In the synthesis section of learning objectives the importance of understanding the difference between the qualitative and quantitative variables by the students is highlighted, as well as the difference between the statistical measures and the graphs that are most adequate to represent them., barras

Additionally, it is presented some basic strategies in the exploration of the data and difficulties, and reasoning of the students in the application of the techniques of exploratory data analysis. In particular, about the second aspect, research in the area of education warns that students have a low level of competence regarding the production, reading and comprehension of statistical graphs. Another difficulty that appears in several studies concerns the understanding of statistical measures such as those of central position, variability and ways of data distribution. However, in some measures such as quartiles and percentiles, students' greatest difficulty is not in the concept but in the algorithm used to obtain the values.

Finally, the authors suggest some resources to support the teaching and learning process, for example, books, magazines and scientific events that focus on Statistical Education. In particular, among the last two suggestions teachers can find research results on the main mistakes made by the students, the difficulties in relation to different concepts or procedures, as well as in the statistical and probabilistic reasoning.

For Batanero and Borovcnik as students get familiar with exploratory data analysis, they need some basic knowledge of probability to be able to begin the study of inferential statistics. Chapter two, in this way, begins with a brief historical account that excels by the understanding the development and advances of probability theory. In the next section, it is presented teaching activities related to the frequentist, subjectivist and classical (Laplace) approaches, seeking to interconnect these different approaches, through mathematical concepts, in order to help students achieve a broader understanding of probability.

Also in this chapter the authors seek to relate probability and descriptive statistics by exploring concepts such as random variables, binomial distribution, normal distribution, and the central limit theorem. The objective of some of the proposed activities is to help students build a clearer idea of how different situations can be modeled by probability, and how the various grasp of concepts influence models and their interpretations.

The authors also present the results of several surveys that helps to understand the difficulties and obstacles that individuals, in different age groups, find in understanding the concepts of probability.

Chapter four analyzes the concepts of association, regression and correlation in order to find relationship between variables and mathematical models that can be used to predict the values of a dependent variable (response) as a function of one or more independent (exploratory) variables. For that reason, it is presented two teaching situations that explore simple methods and cases that are taught in High School: 2x2 contingency table and linear regression. Also, it is discussed the underlying concepts that are used to model the statistical relationships between variables.

Once students are familiar with the linear regression model, the authors suggest introducing other types of regression often used to describe social, economic, environmental, or biological phenomena. Moreover, they extend the discussion, even in

an introductory way, which may help the students in the understanding and relevance of the models, since often linear regression does not provide an adequate description of the interactions among the variables involved. In addition, since students in high school study functions such as exponential, logarithmic, and polynomial, activities involving nonlinear regression enable them to identify examples in which these functions are used in real-life contexts.

Discussing about the difficulties that students may find in the interpretation of correlation, regression and contingency tables (association) the authors used researches in the field of psychology, since they have found little research done by mathematical educators. In the section of additional resources and contents it is presented the chi-square statistic that makes possible both to test if the variables are independent, and to measure the degree of association between them by the contingency coefficient.

In the chapter five the main concepts of statistical inference are analyzed, such as confidence intervals and hypothesis tests. Considering the complexity of these concepts the authors suggest a very elementary approach in order to adapt them to the level of teaching that the book proposes. Thus, they begin with a teaching activity inspired by a famous random experiment created by Fisher: The lady tasting tea. In this activity, it is introduced the main elements of a statistical test, sample distributions, and confidence intervals. Batanero and Borovcnik stimulate teachers to use computational resources in the estimation of the p-value of a hypothesis test, always considering the previous knowledge of the students. In the section of additional activities it is proposed to make the use of computational simulation in the study of the sample distribution.

In the synthesis of the learning objectives of this chapter, it is presented in detail each of the elements involved in the inference as the hypotheses, experiment, randomization, sample variability, sample parameters and statistics, sampling distribution, statistical significance, estimation interval, margin of error and confidence level.

Considering that statistical inference is based on many key concepts, as well as on complex reasoning, this is often misinterpreted or even misused. Moreover, the authors were careful to present a summary of research results on the understanding the inference and suggest some implications for the teaching method.

Batanero and Borovcnik recognize that statistical concepts are difficult and require a long period of learning. However, they argue that continuous teaching of statistics and probability at all levels of education can help students in building this knowledge progressively. Consequently, we will be contributing to the formation of citizens capable of understanding the random phenomena present in many situations of personal, social and economic life, as well as making appropriate decisions when confronted with uncertainty.

We consider that the contributions made by this book both for the training of teachers who teach statistics in elementary school, and also for those who teach at the university level are fundamental, since they stimulate the construction of the concepts from the effective understanding of their meaning and not only by the formula or algorithm associated with its calculation.

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