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Linking proportionality of arithmetic, algebra and geometry domains in Indonesian lower secondary textbooks

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Résumé. Le but de cette étude est utilisé la notion de praxéologie, développée dans le cadre de la théorie anthropologique du didactique (TAD), pour l'analyse de manuels scolaires. On se centre dans le traitement de la proportionnalité dans les domaines d'arithmétique, algèbre et géométrie dans l'enseignement secondaire inférieur indonésien. Les résultats montrent que la TAD peut être utilisée pour construire un modèle de référence pour analyser les types de tâches proposées dans les manuels. Cette analyse permet en outre d'établir des liens possibles entre ces trois domaines.

Abstract. The aim of this study is to use the notion of praxeology, developed in the framework of the anthropological theory of the didactic (ATD), for textbook analysis. Our focus is the treatment of proportion in arithmetic, algebra and geometry domains in Indonesian lower secondary textbooks. The results show that ATD can be used to build a reference model to analyse the type of tasks proposed in textbooks. Furthermore, the analysis is useful to capture a link between these three domains.

1. Introduction

The use of the anthropological theory of the didactic (ATD) brings a new approach in terms of epistemological mathematics perspective in analysing textbooks. The aim of this study is to investigate the description of proportion in arithmetic, geometry, and algebra domains in Indonesian lower secondary textbooks. We can say that proportion has always been part of the mathematics to be taught. At the most elementary level, it appears just after the introduction of the four arithmetic operations, with the classic technique of the "rule of three". It is part of geometry in the treatment of similarities of plane figures. In today's curriculum, it also appears as an example of elementary function (the linear or proportional function), in the domain of algebra or functional relationships. It is one of the few cases where the same concept and some related techniques appear in different school mathematical domains with not always the same designation or mathematical environment. Moreover, this research also aims to investigate how these three mathematical organisations which appear in three different domains are connected to each other in textbooks.

The next section discusses the problems and research questions on how textbooks are used in Indonesia secondary schools. We also discuss related papers focusing on textbook analysis using ATD in section 2. Then, the methodology used is described in section 3 and followed by a discussion on context and data analysis. The results are divided into two categories: a discussion about the praxeological analysis of the three domains; a discussion about how these three domains are related based on the praxeological analysis performed.

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2. Textbooks use in Indonesia

The fact that 48 110 859 students from 7-18 years old went to school in 2013 (Statistic Indonesia, 2013) constituted a huge need of textbooks in Indonesia. These conditions also motivate a competition among publishing companies to focus on textbooks at school level as a marketing target. Furthermore, students' cultural and institutional diversity encourages government to initiate an authorized organization that is called *badan standar national pendidikan* (BSNP, national education standard organization). One of the responsibilities of this organization is to evaluate textbooks' feasibility. Moreover, it has been found that only 37% of teachers have a bachelor degree as required qualification to teach (The World Bank, 2011). Even though textbooks are not the only resource used to teach, the lack of teacher qualification might be a reason for a teacher to depend on a textbook. This dependency, together with the authorization process, turns Indonesian textbooks into an object of study of special interest.

There are many authorized textbooks for teachers (and schools) to choose. Do they really assure students' needs? What kind of didactic approach do they use? In this paper, we try to propose a textbook analysis based on praxeology organizations.

This research will only focus on the way textbooks deal with the notion of proportion that can be defined as a relationship between two quantities. Here, quantity can be conceived as amount, length, volume, weight, etc. Thus, two quantities (Q_1, Q_2) are said to be proportional if there exists a correspondence between them that, given any two elements $q_{1.1}$, $q_{1.2}$ in Q_1 and the corresponding $q_{2.1}$, $q_{2.2}$ in Q_2, we have $\frac{q_{1.1}}{q_{1.2}} = \frac{q_{2.1}}{q_{2.2}}$.

In Indonesian lower secondary schools, proportion is located in the arithmetic domain and it includes quantities such as time, distance, weight, price, etc. In this case, the proportion term often appears as a subsection of the chapter "ratio and proportion". Furthermore, proportion can also be found in the geometry domain to describe a relation between two figures: similarity. In the algebra domain, proportion could be pictured as a relation between two variables that often appear as a linear function and the equation of a straight line containing the origin. However, in this last case, the name "proportion" rarely appears.

The inclusion of proportion into different school mathematical domains (arithmetic, geometry, functions) shows how school mathematics is a connected body of knowledge even if it is not so easily visible for the students, who are immerse in a pedagogical paradigm where they are invited to visit a theme after another without necessity of connecting them. Chevallard (2015) describes this condition as the "*visiting monument*" phenomena, in which students are introduced to mathematics without any contact to its *raisons d'être* who have originated its construction or put in use.

Proportion is a vintage discussion in mathematics worlds, but it is still used in today's curriculum in some countries, for example Indonesia. This research focuses on proportion in Indonesian lower secondary textbooks from grade 7^{th}, 8^{th}, and 9^{th} (12-15 years old). Based on this background, we will formulate three research questions for this study:

1. How can the mathematical praxeologies related to proportion in arithmetic, algebra and geometry domains be described?
2. What are the explicit connections among these three domains (arithmetic, geometry and algebra)?

3. What can this praxeological analysis tell us about the dominant epistemological models related to proportionality in these three domains when we look at textbooks?

3. Textbooks analysis research using the ATD

Textbook analysis using ATD has been discussed in some domains in relation to proportionality. Hersant (2005) conducted a research about how missing value tasks are discussed in six different periods from 1977-2004. She found six different techniques that were presented differently in each period of time, for example rule of three, unit value, ratio, etc. We can say that she considers a point-praxeology, which is a praxeology build around a single type of tasks, through different periods of time. In the geometry domain, a research was conducted by Miyakawa (2017) considering mathematical proof in plane geometry as a subject taught in Japan and France. He found that the nature of geometry proof in Japanese textbooks is influenced by Euclidean geometry while it is affected by the emphasis of the transformational geometry from the *New Math Era* in the French case. Research by Miyakawa is about geometry proof tasks focusing on plane geometry focuses on a regional praxeology formed by different types of tasks and techniques, organised around different technologies but sharing a common theory. Even though the two focus research are different in terms of the "size" of the praxeology, they present an example of textbooks analysis focusing on a single type of task: rule of three and proof.

In an analogous way, González-Martín, Giraldo and Souto (2013) conducted a research about how Brazilian high school textbooks introduce real numbers. By using the praxeological analysis, they distinguish six different types of tasks. Additionally, the results also revealed two dominant tasks in textbooks: (1) students are asked to determine if a given number is rational or irrational, (2) students are asked to find a finite decimal approximation from a given irrational number (application of rational number). These authors present a number of types of tasks that can be analysed using the same technological elements, which still belong to a local praxeology.

A narrower research on how the notion of praxeology is used to analyse different types of tasks in textbooks was conducted by Wijayanti and Winsløw (2017) who found two themes in the arithmetical domain that were divided into seven specific types of tasks. Additionally, this research also showed that there were some dominant tasks that appeared in the textbooks. The fact that there is a dominant task in the textbooks confirms a tendency among authors to use particular types of task. Again these authors confirmed that the notion of praxeological organization in ATD can be used not only to analyse a specific type of tasks or technique, but also to analyse many types of tasks in the textbooks, that is, to move from point to local praxeologies.

Generally, textbooks include many topics that students have to face in a particular period of time. Naturally, they are related to each other. However, research on how these topics link to each other has not been conducted yet intensively. An remarkable study on how proportion in algebraic and arithmetic domain can be linked was carried out by García (2005). He found that the connection between the arithmetic and algebraic domains was quite poor in Spanish mathematics education at lower secondary level. García's work gives a picture on how ATD was used to describe a regional praxeology, thus capturing the link between the arithmetic and the algebraic domain and exploiting it in the teaching of elementary functions. These results encourage us to expand the link and include the geometrical domain.

4. Methodology

In this research, we focus on how the praxeological organization of proportion in arithmetic, algebra and geometry domains is treated and how the praxeological analysis can be used to see the link between these three domains in textbooks and to characterise the dominant epistemological models about proportionality described in the textbooks. In this research, we focus on the specific levels that can be described using the elements of mathematical praxeologies: "sectors" that can be characterised by a shared theory (Θ); "themes" that are defined by a shared technology (θ), and "subjects" that correspond to one or few techniques (τ) and types of task (T). For example, proportion in geometry domain is called similarity. This similarity sector can be defined by polygon similarity. Additionally, some types of task and techniques can be constructed related to polygon similarity (see table 4).

This textbook analysis research is located in the *knowledge to be taught* of the didactic transposition process. However, this analysis does not try to approach the bridge between the *knowledge to be taught* and the *knowledge actually taught* or between the *knowledge to be taught* and the *scholarly knowledge*. Even so, the analysis of a whole sector of proportionality, as the *knowledge to be taught* and captured through textbooks, can reveal important characteristics, not only of the textbooks considered but of the didactic transposition process itself.

The data for our analysis are Indonesian online textbooks that correspond to the 2006 curriculum (http://bse.kemdikbud.go.id/) of lower secondary school. The chapter chosen are those that include proportion in some way: "Ratio and proportion" from grade 7 (Nuharini & Wahyuni, 2008a; Wagiyo, Surati, & Supradiarini, 2008; Wintarti et al., 2008), "Function" from grade 8 (Agus, 2008; Marsigit, Erliani, Dhoruri, & Sugiman, 2011; Marsigit, Susanti, Mahmudi, & Dhoruri, 2011; Nugroho & Meisaroh, 2009; Nuharini & Wahyuni, 2008b) and "Similarity" in ninth grade (Agus, 2007; Djumanta & Susanti, 2008; Dris & Tasari, 2011; Marsigit, Susanti, et al., 2011; Masduki & Budi Utomo, 2007; Wagiyo, Mulyono, & Susanto, 2008). We develop a description of the mathematical praxeologies in each domain. We focus on examples and exercises to identify the types of task and techniques. Even though the theory level is limited, we try to highlight the informal technological elements that are presented in the textbooks or, at least, need to be mobilised to solve the types of tasks using the proposed techniques. In this theory analysis, we also analyse textbooks from the 2013 curriculum.

In relation to the second question, we try to analyse the connexion of types of task among the three domains. Then, we focus on the technology elements by classifying the similar notions of "proportion" used. Lastly, we observe a relation at the level of the theory. Here, we concentrate on how the term of proportion is used /and explained in the upper grade. For example, how is the proportion term from the 7^{th} grade introduced when working with linear functions (8^{th}) and similarity (9^{th})?

5. Results

This section will describe the mathematical praxeologies that appear in each domain (section 5.1.). Additionally, in section 5.2., we will discuss how these three domains are connected.

5.1. Mathematical praxeologies of proportion in each domains

Mathematical praxeologies of proportion

In arithmetic, proportion is defined as two pairs (x_1,x_2) and (y_1,y_2) that satisfy the relation $x_1 \cdot y_2 = x_2 \cdot y_1$ and is often symbolised as $(x_1,x_2) \sim (y_1,y_2)$. More generally, two n-tuples $(x_1,\ldots,x_n)$ and $(y_1,\ldots,y_n)$ are said to be proportional if $(x_i, x_j) \sim (y_i, y_j)$ for all $i, j = 1, \ldots, n$ (Wijayanti & Winsløw, 2017).

In the textbooks, the introduction of proportion starts with the definition of a ratio between two similar quantities. Then, it is developed with comparison of two ratios from similar quantities or different quantities. Quantities that are normally used are length, mass, time, etc. and it is generally wrapped with daily life situations. The authors also introduce the algebraisation of the proportion technique, that is, the calculation with expressions of the form $\frac{x_i}{x_j} = \frac{y_i}{y_j}$.

Question: A car needs 3 litres of gasoline to get 24 km. How many kilometres can the car reach with 45 litres of gasoline? [Translated from Indonesian]	Sebuah mobil memerlukan 3 liter bensin untuk menempuh jarak 24 km. Berapa jarak yang ditempuh mobil itu jika menghabiskan 45 liter bensin?
Solution: 3 litres of gasoline can reach 24 km, thus 1 litre gasoline can reach = $\frac{24}{3}$ km = 8 km. Distance reached with 45 litres of gasoline : 45 x 8 km= 360 km [Translated from Indonesian]	**Penyelesaian:** *Cara 1* 3 liter bensin menempuh jarak 24 km, sehingga 1 liter menempuh jarak = $\frac{24}{3}$ km = 8 km. Jarak yang dapat ditempuh dengan 45 liter bensin = 45 × 8 km = 360 km.

Table 1. Tasks that exemplify type of task in arithmetic proportion (Nuharini & Wahyuni, 2008a, p. 152).

The example of table 1 illustrates the type of tasks T_6. Wijayanti and Winsløw (2017) found seven types of tasks that are divided into two themes: 1. ratio and scale, and 2. direct and inverse proportion (table 2).

Ratio and proportion
T_1: Given x_1 and r, find x_2 so that $(x_1, x_2) \sim (1,r)$. T_2: Given x_2 and r, find x_1 so that $(x_1, x_2) \sim (1,r)$. T_3: Given x_1 and x_2, find r so that $(x_1, x_2) \sim (1,r)$.
Direct and inverse proportion
T_4: Given numbers a, b and given that $x > y$ are relations with a, b. Can it be true that $(x, y) \sim (a,b)$? T_5: Given (x_1, x_2) and (y_1,y_2), compare internal ratios. T_6: Given (x_1, x_2) and y_1 find y_2 so that $(x_1, x_2) \sim (y_1, y_2)$. T_7: Given x_1, x_2, y_1 find y_2 such that (x_1,x_2) and (y_1, y_2) are in inverse proportion.

Table 2. Type of tasks related to proportion

Mathematical praxeologies of linear function

Proportion in algebra appears as a relation between variables: the linear function. In this case, a linear function is defined as a polynomial function of degree zero $f(x) = ax$. This definition does not include any reference to linear maps, that is, functions satisfying the relationships $f(x + y) = f(x)$ and $f(ax) = af(x)$.

In all examined textbooks, linear function is located in a chapter called "Functions". The authors discuss function by presenting a set of data in an arrow diagram (or Cartesian diagram) that fulfil the function condition. Then, the authors describe the definition of function such as "function (mapping) from a set A with set B is a particular relation where one value from the set A associates with exactly one value from the set B" (Nuharini & Wahyuni, 2008b, pp. 37, our translation). The formalisation such as the use of the 'f' notation and the notions of domain, codomain and linear function are presented right after the definition. In terms of the linear function definition, we found that textbooks mention a different form of linear function which is $f(x) = ax + b$. Most of the tasks use numbers and variables. However, we found a few numbers of linear functions tasks that use quantities.

Wijayanti (2017) identified five types of tasks in the textbooks regarding linear function that are presented in table 3. For example, a specimen of T_1 can be the following:

Given function $f: x \to 2x - 2$ defined on integer numbers. Determine a. $f(1)$, b. $f(2)$ (Agus, 2008, p. 30).

T_1	Given a function $\boldsymbol{f(x)}$ on integers, find the image of the function at specific integers.
T_2	Given the closed form expression $f(x) = ax + b$ (where a and b are given), and given $f(x_0)$ for a given x_0, find x_0.
T_3	Some values of a linear function are given at certain points, and students are asked to determine the correct function expression from a list.
T_4	For a linear function $x \mapsto ax + b$, where a and x are given, along with one value of f at a point. Determine (the expression defining) f.
T_5	Given the algebraic expression of a linear function $f(x)$ and another algebraic expression E (depending on x, so $E = E(x)$). Compute $f(E(x))$.

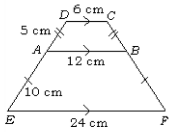
Table 3. Type of tasks about linear functions

We decide to consider that these type of tasks correspond to "linear function" ($f(x) = ax$) even though the definition is different, because all the types of task found can be applied to the linear function form.

Mathematical praxeologies of similarity

Proportion in geometry connects with the notion of "similarity" of figures. We can summarize that the six textbooks considered use the same definition about similarity. Two polygons P and Q as well as their angles $\angle a_1, \ldots, \angle a_n$ in P and $\angle b_1, \ldots, \angle b_n$ in Q and sides $p_1, \ldots, p_n$ in P and $q_1, \ldots, q_n$ in Q are defined as similar if $\angle a_1 = \angle b_1, \ldots, \angle a_n = \angle b_n$ so that $\frac{p_1}{q_1} = \frac{\cdots}{\cdots} = \frac{p_n}{q_n}$.

In all examined textbooks, discussion about similarity always goes together with congruence and is related to length as quantities. Additionally, the authors used "scale" to introduce similarity. Since the cross product technique is also found in this domain, the use of unknowns and equations are also central technical elements in similarity, even if they do not explicitly appear at the technological level. Furthermore, we found that algebra manipulation is also used to solve some types of task, e.g T_8 : find a side of a triangle similar to another one (see table 5).



Are figures *ABCD* and *ABFE* in the picture similar?
(Wagiyo, Mulyono, et al., 2008, p. 4)

Table 4. Example of similarity task

We identify in textbooks eight types of tasks that can be divided into two themes, namely polygon similarity and triangle similarity (Wijayanti, 2016). The eight types of tasks can be seen in table 5. And the example of T_1 can be seen in table 4.

Polygons similarity
T_1: given two figures of polygons *P* and *Q*, with given side lengths and given angles, determine if the two polygons are similar. T_2: given two similar polygons *P* and *Q* with given angles and given sides, identify what angles and sides correspond to each other. T_3: given two similar polygons *P* and *Q* as well as one side p_1 in *P* and two sides q_1, q_2 in *Q* with p_1 and q_1 being in correspondence, find the side p_2 in *P* that corresponds to q_2. T_4: given two similar polygons *P* and *Q* as well as sides $p_1, \ldots, p_n$ in *P* and $q_1, \ldots, q_n$ in *Q*, determine the scale factor of *P* and *Q*.
Triangles similarity
T_5: given two triangles *S* and *T* with two or three angles known, , determine if *S* and *T* are similar. T_6: given two triangles *S* and *T* as well as sides s_1, s_2, s_3 in *S* and t_1, t_2, t_3 in *T*, determine if they are similar. T_7: given two triangles *S* and *T* as well as sides s_1, s_2, and $\angle a_1$ that is located between s_1 and s_2 in *S* and sides t_1, t_2, and $\angle b_1$ that is located between t_1 and t_2 in *T*. Determine if they are similar. T_8: given a figure like Figure 1 with $\Delta ADE \sim \Delta ABC$, DE//BC, and given the length of three of four sides AE, AC, AD, AB. Find the remaining length.

Table 5. Types of tasks about similarity in the geometry domain

5.2. Relations among the three domains

Firstly, we will discuss a connection in the level of the subject, especially we will present a commonality in the types of tasks and techniques. From the description above, we can notice that there are two types of tasks that share similar techniques. In ratio and proportion, we will find T_6 'given (x_1, x_2) and y_1 find y_2 so that $(x_1, x_2) \sim (y_1, y_2)$' and in similarity we will find T_3 "given two similar polygons *P* and *Q* as well as one side p_1 in *P* and two sides q_1, q_2 in *Q* with p_1 and q_1 being in correspondence, find the side p_2 in *P* that corresponds to q_2".These two types of tasks can be solved by using $\boldsymbol{y_2 = \frac{x_2 . y_1}{x_1}}$. Thus, this condition is a potential fact to connect proportion and similarity.

Furthermore, we found a task in linear function that corresponds to the "missing value problem":

Diketahui harga sebuah pensil Rp1.200,00, harga 2 pensil Rp2.400,00, dan harga 5 pensil Rp6.000,00. Fungsi yang menunjukkan pemetaan tersebut adalah ....

a. f : x → 1200x
b. f : x → 2400x
c. f : x → 1000x + 200
d. f : x → 1300 – 100

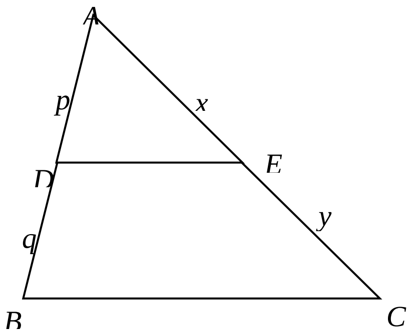
(Marsigit, Erliani, et al., 2011, p. 63)

The price of a pencil is Rp. 1.200,00, the price of two pencils is Rp. 2.400,00, and the price of 5 pencils is Rp. 6.000,00. Which of the following functions describe this? a. … [as above]. (Transleted to English by the author)

Based on the example above, we see that a typical proportion task also appears in both linear function and similarity themes. This situation can be used as evidence that the three themes are connected at the level of the praxis.

Secondly, we will focus on the theoretical and technological elements that are used in the three domains. Here, we observe on how proportion is used in similarity and linearity. We find proportion term is used to define similarity, even though they do not explain how proportion can be connected to similarity. Meanwhile, we do not find that the term proportion is used in linear function.

However, even if the theoretical blocks of the praxeologies is not explicitly presented, we can infer some of their elements. In table 6, we see an example of how Thales theorem is discussed in similarity. Here, students are given a figure with ΔADE~ΔABC, DE//BC and given the four sides AE, AC, AD, AB. Then, they are asked to find the proportion of the segments of the two lengths of triangle ABC ‘ $\frac{p}{q} = \frac{x}{y}$ ’.



Given *BC // DE*. The length of *AD = p* and *DB = q*. *ΔADE* and *ΔABC* are similar. Find the proportions of the segments of the two length of triangle ABC.

$\frac{AD}{AB} = \frac{AE}{AC}$.

$\frac{p}{p+q} = \frac{x}{x+y}$.

$p\,(x + y) = x\,(p + q)$

$px + py = px + qx$

$py = qx$

Therefore, the proportions of the segments of the two lengths of triangle ABC are: $\frac{p}{q} = \frac{x}{y}$ (Sulaiman et al., 2008, p. 12).

Table 6. Thales theorem proof

In order to solve the question above, students need to have prerequisite knowledge not only about similarity but also about quantity, variable, proportion, etc. Interestingly, some of this prior condition can also be found in linear function and proportion (table 7). Therefore, this evidence shows that these three domains are linked at the theoretical level.

Table 7 presents some of the elements that we consider as part of the theoretical level of the observed praxeologies. In this case, we notice that the word “proportion” is introduced with and similarity. For example, one textbook uses proportion term to define proportionality “Two triangles are similar if the corresponding sides are proportional and the corresponding angles are equal” (Djumanta & Susanti, 2008). There is even a book that declares explicitly that proportion is a prior knowledge needed to learn similarity. Conversely, we cannot find explicitly the term “proportion” in the introduction of linear functions.

Domain	“Arithmetic”	“Algebra”	“Geometry”
Sector	“Ratio and proportion”	“Function”	“Similarity”
Theme	Quantities (mass) Units (kg) Decimal numbers Ratio (scale) Direct proportion Indirect proportion Augmentation Reduction Table	Quantities (mass) Units (kg) Real numbers Set of numbers Variable Equation Relation function Linear function Domain Codomain Range Table Graph Cartesian diagram Arrow diagram Pair orders	Quantities (width, length) Units (cm, km) Decimal numbers Unknown Equation Ratio, scale proportion Polygon Congruent Augmentation and reduction Parallel line characteristics Pythagoras theorem Thales theorem Angles Sides Figures

Table 7. Theoretical elements of proportion, linear function, and similarity

6. Discussion

The study shows that praxeology organizations can be used to describe three important sectors of Indonesian school mathematics as it appears in textbooks: proportion, linear function and similarity. Furthermore, these praxeologies are useful to analyse the link between the three domains. It is found that there are at the same time some explicit connections and implicit relations. For example, we can find explicit connections at the theory level. Even though it is presented without explanation, one textbook clearly states that proportion is linked to similarity. However, many other possible theoretical links remain implicit. An example can be seen in the way textbooks share a type of tasks and some technologies without having a clear statement about their connectivity. Moreover, the fact that these three domains share some types of tasks and technologies (ratio and proportion, quantities, equation etc.) show that there is a possible link among domains (using type of tasks or techniques or others), but this link is not developed in textbooks. Additionally, it is also interesting to know the reason why this lack of explicit connectivity appears in the textbooks. The absence of connection in lower secondary textbooks raises a question on what proportion is about or what proportion is made of. These questions correspond to the analysis of the process of didactic transposition and the description of school mathematics that is important for further research.

Regarding the teacher’s point of view, the description of the types of tasks and techniques carried out in each domain might help them have a vivid description and common measure of the examples and exercises that are located in textbooks. Thus, the praxeological analysis also appears to be an efficient tool for teachers. Additionally, the example of connecting domains can be used to identify the quality of textbooks or, at least, their “mathematical coherence”. We acknowledge that we have mainly focused on the level of the *praxis* when analysing textbooks praxeologies. This decision is influenced by the fact that lower secondary textbooks are much more explicit in the praxis level elements than in the theory ones. However, it is also necessary to broaden our perspective to the theory level, for instance by using more advanced textbooks, teachers' and students' interviews or direct observation of teaching processes.

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