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Frequentist probability in Japanese school curricula

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Resumen. Muchos maestros de matemáticas de la escuela japonesa, políticos e investigadores creen que los contenidos probabilísticos son difíciles de entender para la mayoría de los estudiantes. En este estudio, identifico varias razones para la dificultad a través de un análisis ecológico que es parte de un análisis didáctico. Esta tarea se logra a través de tres técnicas de investigación: (a) construcción de un modelo epistemológico de referencia de actividades probabilísticas en términos de praxeología, (b) análisis de contenidos probabilísticos de libros de texto de matemáticas escolares japonesas a partir del modelo de referencia y (c) Los contenidos utilizando la escala de niveles de codeterminación didáctica. En las matemáticas corrientes de la escuela japonesa, la probabilidad de frecuencia no se menciona, mientras que la probabilidad laplaciana comprende una gran parte del plan de estudios de probabilidad, aunque algunas condiciones genéricas hacen viable la probabilidad frecuencial. Este hecho está relacionado con las siguientes tres limitaciones: determinismo, teoricismo y desmatematización de los aleatorizadores.

Abstract. Many Japanese school mathematics teachers, policy-makers and researchers believe that probabilistic contents are difficult for most students to understand. In this study, I identify several reasons for the difficulty through an ecological analysis that is a part of a didactic analysis. This task is achieved through three research techniques: (a) constructing a reference epistemological model of probabilistic activities in terms of praxeology, (b) analysing probabilistic contents of Japanese school mathematics textbooks from the reference model and (c) identifying institutional conditions and constraints on the contents using the scale of levels of didactic codetermination. In current Japanese school mathematics, frequentist probability is hardly mentioned, whereas Laplacian probability comprises a large part of the curriculum of probability, although some generic conditions make the frequentist probability viable. This fact is related to the following three constraints: determinationism, theoricism and demathematisation of randomisers.

1. Aim, method and theoretical background

Previous studies in didactics of mathematics have reported several misconceptions about probability, which is a traditional topic of research in mathematics education (cf. Batanero & Sanchez, 2005; Savard, 2014; Shaughnessy, 1992). These studies appear to have been devoted to examine the misconceptions in the *cognitive research programme* (Gascón, 2003) from "psychological" perspectives. Many papers within this programme seem to imply a large-scale dissemination of irrelevant probabilistic knowledge around the world. From this viewpoint, the "cognitive" phenomenon of the probabilistic misconception can be translated into a theme in the *epistemological research programme* (ibid.) in which the didactics of mathematics are regarded as the science of the dissemination of mathematical knowledge. My study involves approaching the "didactic" phenomenon of *ill-attained dissemination of probabilistic knowledge* on the basis of "anthropological" approaches. In this study, within the *Anthropological Theory of the Didactic* (ATD) (cf. Bosch & Gascón, 2006; Chevallard & Sensevy, 2014), I aim to identify certain institutional constraints hindering the required dissemination of the probabilistic

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knowledge, with a special focus on the notion of frequentist probability in Japanese school mathematics curricula.

I have selected the frequentist probability as the focus of this study for the two following reasons: (a) This concept seems to be a pivotal tool for relevant *didactic transpositions* (cf. Chevallard & Bosch, 2014) from educational systems and classrooms to groups of learners. Many probabilistic misconceptions involve numerical calculations or verbal estimations of probability. For example, there is the notorious misconception of the Monty Hall problem related to conditional probability, which can be overcome, at least temporarily, by many simulations based on the frequentist definition of probability and the law of large numbers. (b) In spite of the didactic importance of *frequentist probability*, this concept may be unviable in the final transpositive stage, i.e. within the social institution of learners' groups, in the Japanese didactic transposition process. This fact has been demonstrated by the results of the national achievement tests in Japan: many Japanese students cannot relate the numerical values of probability with the results of repeated trials.

This study consists of three methodological steps. First, I construct a specific epistemological model of the frequentist probability using *praxeology* (cf. Chevallard, 2006; Chevallard & Sensevy, 2014; Chevallard et al., 2015) as a model for analysing any human activity. "Praxeology" is a central notion within the ATD framework. The reason why didacticians within ATD pay considerable attention to the praxeology is deeply related to its fundamental assumption: any human activity can be described as amalgamations of knowledge (*logos*) and know-hows (*praxis*). Roughly speaking, a given activity consists of a *praxis*, which is the part of a problem and method ($[T/\tau]$), and a *logos*, which are two-layered descriptions and justifications of the *praxis* ($[\theta/\Theta]$).

The first step forms a *Reference Epistemological Model* (REM) (cf. Bosch & Gascón, 2006; Chevallard & Bosch, 2014) of probabilistic activity for the *praxeological analysis* (Chevallard & Sensevy, 2014) of the frequentist probability to be taught in Japan. REMs are specific models of different bodies of school mathematical knowledge. The construction of the REM is an important methodological act not only for didacticians within ATD, but for any research programme. REMs allow didacticians to "detach scientifically" from prevalent models of given bodies of mathematical knowledge of the scholarly mathematics and in the didactic *noosphere* (cf. Chevallard, 1992a), that is, social institution of mathematics teachers, policy-makers, mathematicians and so on who "think" about teaching (cf. Bosch, 2015). In fact, different didactic research-programmes already have REMs according to their own perspectives even if they are not specifically called REM (cf. Ruiz-manzón et al., 2013). REMs express working hypotheses about natures-and-functions of different mathematical works-and-activities in didactic contexts. Within the framework of ATD, they are usually described as praxeologies (e.g. Barbé et al., 2005) or as parts of them (e.g. García et al., 2006). In other ATD-cases, they are represented as tools for constructing and building more sophisticated mathematical praxeologies (e.g. Ruiz-manzón et al., 2013). Similarly, in this paper, a REM of probability is created as a *reference praxeological model*.

Secondly, I analyse popular junior and senior secondary school mathematics textbooks in terms of the REM that is constructed in the first step, in order to identify the nature of probabilistic knowledge to be taught. In Japan, every elementary and secondary school textbooks conform to the national curriculum, the *course of study*. In Japanese national curricula, probability is usually a subject taught in secondary school mathematics and not in elementary school mathematics. At the secondary school mathematics, there are "implicitly" the two notions of *Laplacian or classical probability*, and *frequentist*. The notion of Laplacian probability regards the probability as the ratio of the number of all favourable outcomes to the number of all possible outcomes of the trial, while the probability is a stabilized value of relative frequency after a large number of identical trials for the notion of frequentist probability (cf. Batanero & Díaz, 2007; Borovcnik & Kapadia, 2014). In Japanese school mathematics, they call by the same name, "probability". In this step, I identify some negative epistemological characteristics of the frequentist probability within the Japanese probabilistic curricula.

Third, I indicate certain institutional *conditions* and *constraints* through an *ecological analysis* (Chevallard, 1992a, b) using the *scale of levels of didactic codetermination* (Humankind, Civilization, Society, School, Pedagogy, Discipline, Domain, Sector, Theme and Question) (cf. Bosch, 2015; Chevallard & Sensevy, 2014), which is a methodological tool for clarifying the desirable scope of didacticians' investigation of institutional ecology. All mathematical and didactic activities live within multiple institutional conditions and constraints: contents to be taught, students' readiness, classroom infrastructure and so on. Moreover, many conditions and constraints are usually invisible in classroom because they are like "air" and "gravity" for our daily life: school system, language, epistemology and so on. The codetermination framework allows us to focus on entities, material or not, of more generic levels and to find invisible conditions and constraints on the teaching and learning of mathematics.

My ecological analysis in this paper is linked with the praxeological analysis of didactic organisations for probabilistic activities to be taught. I believe that ATD's inquiry processes include both implicit and explicit analyses of both mathematical and didactic praxeologies (e.g. Barbé et al., 2005). This study is not an exception, even if the nature and ecology of school probabilistic praxeologies are the primary focus. In the following section, I attempt to create an REM involving frequentist probability.

2. Reference praxeological model of school probability

In the Japanese school mathematics curriculum, the term "probability" [*Kakuritsu* in Japanese] belongs to at least two categories. The first is the category of the *concept* in which probability is regarded as a mathematical notion about uncertainty. In other words, this word refers to a magnitude expressed by quantities such as 1/2, 0.5 and 50%. The second curricular meaning of probability is as a *theory* of mathematical phenomena in relation to the concept of probability. This second category uses the label of probability as a sector or domain in school mathematics curricula. Here, I regard the term "probability" as a mathematical *activity*, more precisely as a praxeology, which is the third characterisation for didacticians. For this, a REM concerning probability as a whole is constructed, and its frequentist aspect is identified subsequently. In this study, the REM is modelled after the *regional* praxeology $[T_{ij}/\tau_{ij}/\theta_i/\Theta]$ with $i = 1, \ldots, n$ and $j = 1, \ldots, m_i$, which is the praxeological organisation unified by a theory Θ (cf. Barbé et al., 2005; Chevallard et al., 2015).

First, the reference *types-of-tasks* (T), problems as starting points of construction of any praxeology, must be constructed. We can distinguish two subtypes of types-of-tasks: the *determination type* ($_{\mathrm{D}}T$) and the *authentication type* ($_{\mathrm{A}}T$). The determination types are for finding or constructing numerical values of the probability of different events. The determination type is central for pragmatic situations of probability such as decision-making in gambling. A typical task-form is represented as “calculate the probability of ____”. In contrast, the authentication types are for confirming the numerical values or verbal representations of probabilities that are found or constructed in a particular manner. In other words, this type of task is “to verify whether a given value of probability ia true or not”. A typical case of this type is problem about explanation of meaning of the value of probability. Mainly, I assume “use of other frameworks of probability” as the method for tackling to this type of tasks. For example, if someone determines a value of probability within the Laplacian probability framework, she can authenticate the value using the frequentist probability framework and vice versa. This circularity is in the nature of the definition of the concept of probability taught at school (Steinbring, 1991). In other cases, if someone determines a value of probability by intuition, he can authenticate the value by the frequentist and/or Laplacian probability frameworks. The authentication type is crucial for reflective situations of probability, such as studies of the foundations of probability[2].

Based on this distinction of probabilistic task types (determination type $_{\mathrm{D}}T$ and authentication type $_{\mathrm{A}}T$), I would like to propose in this paper, as a tentative basis for a REM, four reference types of probabilistic tasks. The first and second types-of-tasks are $_{\mathrm{D}}T_1$ and $_{\mathrm{A}}T_1$. T_1 represents a type-of-tasks involving two characteristics of probabilistic modelling. The first is that, any problematic situation of T_1 can be modelled by a relevant sample space as a set of symmetric or homogeneous sample points. Such a space is called, here, a “*flat*” *sample space* because “equally-likely”, “equally-possible” and “equiprobable” are adjectives used not for the sample space and points, but for the elementary events. The second characteristic is that trials for T_1 can be simulated multiple times. For example, one can think of tasks that question the probabilities of elementary events {1}–{6} in a toss of a symmetric dice. As for other types of tasks in the model, T_1 is separated into two subtypes of tasks: determination T_1 ($_{\mathrm{D}}T_1$) and authentication T_1 ($_{\mathrm{A}}T_1$).

The third and fourth types are $_{\mathrm{D}}T_2$ and $_{\mathrm{A}}T_2$. T_2 represents the type-of-task that cannot be modelled by a relevant flat sample space, and many trials can be simulated. For example, this includes tasks that question the probabilities of elementary events {1}–{6} in a toss of an *a*symmetric dice. T_2 is also composed of a determination type, $_{\mathrm{D}}T_2$, and an authentication type, $_{\mathrm{A}}T_2$.

Of course, for example, we can find T_3, which cannot be modelled by a relevant flat sample space and for which many trials cannot actually be simulated (at least in classrooms) as is the case for meteorite impacts on the Earth. However, this study does not include such types-of-task in its reference model. Table 1 summarises the reference types-of-tasks.

<table>
<tr><th></th><th>States of types-of-tasks</th><th>Relevant flat sample space</th><th>Large-scale simulation</th></tr>
<tr><td>$_{\mathrm{D}}T_1$</td><td>determination</td><td rowspan="2">○</td><td rowspan="2">○</td></tr>
<tr><td>$_{\mathrm{A}}T_1$</td><td>authentication</td></tr>
</table>

[2] The authentication type has twofold function. The first is a role as a starting point of the construction of a *praxis* part of a given praxeology, as we have already saw. In addition, this type-of-tasks can be related to the growing of the *logos* parts of probabilistic praxeology, because it could be to *justify* a result of works for the determination type-of-tasks A similar case is reported about the limit of function by Barbé et al. (2005).

${}_{\mathrm{D}}T_2$	determination	–	○
${}_{\mathrm{A}}T_2$	authentication		

Table 1. Reference types-of-tasks.

Secondly, the reference *techniques* (τ), which are actual procedures for working on types-of-tasks, have to be explitated. Reference techniques are roughly divided into the two following categories: the calculation within the framework of Laplacian probability (Hereafter, the *Laplacian calculation*) and the experimentation within the framework of frequentist probability (Hereafter, the *frequentist experimentation*). The Laplacian calculation, ${}_{\mathrm{L}}\tau$, is a procedure based on the notion of Laplacian probability; it consists of sample space constructions, calculations of proportion between a partial and a whole event, combinatorial calculations and so on. Likewise, frequentist experimentation, ${}_{\mathrm{F}}\tau$, is a procedure based on the notion of frequentist probability. ${}_{\mathrm{F}}\tau$ consists of simulations using a randomiser like a coin or a dice, recordings of frequency, calculations of relative frequency and so on.

The technique for T_1 can consist of the techniques in the Laplacian calculation, ${}_{\mathrm{L}}\tau$, and of the frequentist experimentation, ${}_{\mathrm{F}}\tau$. By contrast, The technique for T_2 can consists exclusively of the technique in the frequentist experimentation, ${}_{\mathrm{F}}\tau$, because these problematic situations cannot be modelled by relevant flat sample spaces. From the above, the *praxis* of a REM of probabilistic praxeology is referred to by six couples of types-of-tasks and techniques.

Then, *technologies* (θ) in the REM must be described; they are first-level discourses for explaining and justifying the *praxis*. Main references to the technological dimension that are assumed in this study, are the definitions of Laplacian probability and frequentist probability, because they each explain the techniques: Laplacian calculation and frequentist experimentation.

Finally, the *theory* of the regional praxeology constituting our REM must be described. The *theory* (Θ) is constituted by second-level discourses for explaining and justifying technologies. The *law of large numbers* is a central piece of the reference theory presumed in this study because it can connect the two aspects of probabilistic technologies, Laplacian probability and frequentist probability. This does not, however, necessitate the use of the law as a mathematical notion to be studied; rather, I intend that it is considered as an instrumental or *paramathematical* notion (cf. Brousseau, 1997), ${}_{\mathrm{I}}\Theta$. Such a law means that a value of Laplacian probability of an event is equal to a value of frequentist probability of the event, and is obtained and justified experimentally. Figure 1 is a summary of the REM used in this study. The grey cells indicate the frequentist aspects of REM.

Types-of-tasks / Techniques	Technologies	Theory
${}_{\mathrm{D}}T_1/{}_{\mathrm{L}}\tau$	${}_{\mathrm{L}}\theta$	${}_{\mathrm{I}}\Theta$
${}_{\mathrm{A}}T_1/{}_{\mathrm{L}}\tau$		
${}_{\mathrm{D}}T_1/{}_{\mathrm{F}}\tau$	${}_{\mathrm{F}}\theta$	
${}_{\mathrm{A}}T_1/{}_{\mathrm{F}}\tau$		
${}_{\mathrm{D}}T_2/{}_{\mathrm{F}}\tau$		
${}_{\mathrm{A}}T_2/{}_{\mathrm{F}}\tau$		

Figure 1. Reference regional praxeology.

3. Nature of frequentist praxeology to be taught

This section analyses two Japanese school mathematics textbooks using the REM. One is *Mirai e Hirogaru Sūgaku 2* (*MHS*) (Mathematics Extending Towards the Future 2, Okamoto et al., 2016), which is a popular lower-secondary school textbook. The other is *Koutougakkou Suugaku A* (*KS*) (Mathematics in Upper Secondary Schools A, Okabe et al., 2015), which is a popular upper-secondary school textbook. Almost all school probabilistic content in Japan is taught using these two textbooks or similar[3] textbooks. I analyse probabilistic contents of these textbooks using the REM.

For analysis of types-of-tasks, techniques and technologies, I focus on examples and exercises included in the textbooks. I examine each example-and-exercise; identify categories of their types-of-tasks, techniques and technologies; and count the number. Thus, I postulate that categories of example-and-exercise subsume types-of-tasks, techniques and technologies all at once. In fact, there is no situation in which the frequentist practice are described and justified by the Laplacian technology and vice versa, because the Laplacian calculation and the frequentist experimentation are completely different techniques. The techniques can be connected not in the technological but in the theoretical dimensions in our REM framework. For the theoretical analysis, I focus on the presence or absence of the law of large numbers in each textbook and, on the descriptive form of it.

Tables 2 and 3 contain the results of analyses of *MHS* and *KS* in terms of the types-of-tasks, techniques and technologies. Each number expresses the frequency of each item. *KS* has only one type of probabilistic practices [${}_{D}T_1/{}_{L}\tau$], while *MHS* includes several types.

Types-of-tasks / Techniques		Technologies	
${}_{D}T_1/{}_{L}\tau$	21	${}_{L}\theta$	21
${}_{A}T_1/{}_{L}\tau$	0		
${}_{D}T_1/{}_{F}\tau$	1	${}_{F}\theta$	5
${}_{A}T_1/{}_{F}\tau$	1		
${}_{D}T_2/{}_{F}\tau$	3		
${}_{A}T_2/{}_{F}\tau$	0		

Table 2. Result of analysis of *MHS*.

Types-of-tasks / Techniques		Technologies	
${}_{D}T_1/{}_{L}\tau$	51	${}_{L}\theta$	51
${}_{A}T_1/{}_{L}\tau$	0		
${}_{D}T_1/{}_{F}\tau$	0	${}_{F}\theta$	0
${}_{A}T_1/{}_{F}\tau$	0		
${}_{D}T_2/{}_{F}\tau$	0		
${}_{A}T_2/{}_{F}\tau$	0		

Table 3. Result of analysis of *KS*.

As for theory, *KS* does not contain any description of the law of large numbers. As mentioned above, every problem in the textbooks demand the use of the Laplacian probability framework.

[3] In Japan, some publishers make different textbooks. However, the textbooks are more or less "similar", because they are reviewed by the government in viewpoint of the national course of study.

In contrast, in *MHS*, there is a single description of the law of large numbers (Figure 2). Let me emphasise here that, in that textbook, probability is introduced through frequentist probability; the description of Figure 2 provides an opportunity for introducing Laplacian probability as a new concept. Regarding this description, we observe that the law of the large numbers is expressed in an implicit form in Figure 2 because two definitions of the Laplacian probability and the frequentist probability have different functions here. In this description, the frequentist probability is a "meaning" of probability, and the Laplacian probability is related to a "method" of identifying values of probability, although the law of large numbers represents a connection between the two meanings. In other words, the law of the large numbers works as a *protomathematical* notion (cf. Brousseau, 1997) or a *theorem-in-action* (cf. Vergnaud, 2009) rather than a mathematical theorem at school.

> When you toss a dice, what is the probability that "1" turns up?
> If you toss a dice many times, you will know that probability of getting a "1" is a value close to 1/6.
> This probability can be approached as following:
> I. The total number of outcomes is six: 1, 2, 3, 4, 5 and 6.
> II. Each outcome has an equal likelihood.
> III. The number "1" as an outcome occurs once.
> Then,
> the number of outcomes in III/the total number of outcomes in I = 1/6.
> This value is nearly equal to the result obtained in experiments.
> It is called "equally-likely" outcome; the possibility of any outcome is the same.
> If all outcomes are equally-likely, then the probability can be calculated by proportions between numbers of outcomes.

Figure 2. The law of large numbers in *MHS* (Okamoto et al., 2016, p. 154). (Translated by the author.)

In summary, the probabilistic praxeology to be taught in Japanese secondary school mathematics is drawn in Figure 3. The signs "–" expresses none, and the parentheses indicate that there are only few items in each category, that is to say, they are "red-listed". From this, we can identify two characteristics of frequentist praxeology to be taught in Japanese schools.

<table>
<tr><th>Types-of-tasks</th><th>Techniques</th><th>Technologies</th><th>Theory</th></tr>
<tr><td colspan="2">${}_{D}T_1/{}_{L}\tau$</td><td rowspan="2">${}_{L}\theta$</td><td rowspan="6">$({}_{I}\Theta)$</td></tr>
<tr><td colspan="2">–</td></tr>
<tr><td colspan="2">$({}_{D}T_1/{}_{F}\tau)$</td><td rowspan="4">$({}_{F}\theta)$</td></tr>
<tr><td colspan="2">$({}_{A}T_1/{}_{F}\tau)$</td></tr>
<tr><td colspan="2">$({}_{D}T_2/{}_{F}\tau)$</td></tr>
<tr><td colspan="2">–</td></tr>
</table>

Figure 3. Probabilistic praxeology to be taught in the *MHS* and *KS*.

First, we find that there is only one kind of authentication type-of-tasks; almost all problems in these textbooks are in the form "calculate a probability". This nature of noospherians' probabilistic praxeology in Japan can be called the *small role of the authentication types-of-*

tasks. This characteristic is related to both the Laplacian parts and the frequentist parts of the probabilistic organization, because the authentication task is included within the Laplacian probability framework and the frequentist probability framework.

The small role of the authentication types-of-tasks is a general characteristic of the probabilistic praxeology. I focus the frequentist aspect of the praxeology. The second characteristic involves the determination types-of-tasks in the probabilistic praxeology. The first determination type, ${}_{D}T_1$, can be completed by either Laplacian or frequentist techniques; however, in the textbooks, ${}_{D}T_1$ problems are solved exclusively by Laplacian calculation and not by frequentist experimentation, though the latter could also be used to deal with ${}_{D}T_1$. In short, the frequentist probability is not regarded as a tool for solving problems of ${}_{D}T_1$. In other words, the technical and technological functions of the frequentist probability for ${}_{D}T_1$ are fewer than that of the Laplacian probability. Furthermore, there are only few *punctual* praxeologies around ${}_{D}T_2$ in the textbooks. Based on these observations, I call this praxeological characteristic the *small role of the frequentist technology*. Moreover, this characteristic implies that the law of large numbers has a limited role in textbooks, because almost all of probabilistic problems in the textbooks are solved within the Laplacian probability framework without connection with the frequentist probability.

4. Ecology of frequentist praxeology to be taught

In this section, I refer to institutional conditions and constraints on the frequentist probability at school, using the codetermination framework for identifying them. The two negative characteristics, discussed in the third section, imply three constraints hindering the emergence of the frequentist probability in Japanese school curricula.

The first constraint is related to the small role of the authentication type-of-tasks. Most Japanese school examinations of various disciplines are typically organised by *problems-to-find*, not by *problems-to-prove* (I use these words in more general terms than the original meanings intended by Polya [1957/2014]). This institutional constraint at the Pedagogy level in the codetermination framework, which I call *determinationism* (different from “determinism” in philosophical discourses), confines the authentication function of the frequentist probability, even if it may accelerate the determination function.

The second constraint is related to the small role of the frequentist technology. In the Japanese school system, elementary and secondary school mathematics have been called by different names. Elementary school mathematics is called *Sansū*, which roughly means arithmetic. Secondary school mathematics is called *Sūgaku*, which means mathematics. This indicates that Japanese noospherians have given the differences between *Sansū* and *Sūgaku* considerable thought. One of the differences between the two is that *Sansū* is more pragmatic, while *Sūgaku* is more discursive. In other words, the didactic organisation of *Sansū* is based on the *technicism* (Florensa et al., 2015), which emphasises practical exercises without many discursive tools, while the didactic organisation of *Sūgaku* relies on the *theoricism* (ibid.), focusing on logical structures without many pragmatic roots. In Japan, *Sūgaku* deals with probabilistic content about both Laplacian probability around set theory and deductive reasoning, and frequentist probability with empirical experiment and inductive reasoning. Thus, Laplacian probability is compatible with *Sūgaku*, whereas frequentist probability is not. Even though the frequentist probability fits into *Sansū*, which is more empirical, there is no frequentist content in *Sansū* because probabilistic contents are taught exclusively in *Sūgaku*. Under this second constraint, the theoricism at the Discipline level, the frequentist contents with the experimental features are neglected, while the Laplacian contents from flat sample spaces are regarded as important.

The third constraint is also related to the small role of the frequentist technology. In mathematics classrooms, there are many explicit or implicit rules of the teaching and learning of mathematics, that is, the *didactic contract* (Brousseau, 1997). The didactic contract includes rules by which a tool used by a student are legitimised as a "mathematical" tool. For example, in Japan, the ruler is a mathematical tool in elementary school, but is a didactic or auxiliary tool in upper secondary school. Similar analyses can be done about different tools: protractor, abacus, compass, pocket calculator and so on. How about the randomiser like coins and dice which support activities with respect to the notion of the frequentist probability? In Japan, it is not a mathematical tool but a didactic tool. This contract explicitly emerges in school examinations: students must not use coins and dices. This didactic contract, which I call the *demathematisation of randomisers*, is a strong constraint at the Domain level to the life of the frequentist technique and technology.

At this point, we encounter a new question. As mentioned above, the frequentist praxeology at school in Japan is hindered by several institutional constraints: determinationism, theoricism and demathematisation of randomisers. However, the frequentist praxeology is still present in Japanese school curricula. Why is it viable?

One explanation may be found in the effect of constructivist pedagogy as a teaching theory in didactic praxeologies of mathematics. In 1980–1990s, *constructivism* had been imported from America to Japan through the problem-solving approach. This import-phenomenon has occurred in not only academic institutions but also in the noosphere. One important tenet of the constructivist pedagogy is that teachers should conduct *adidactic situations* (Brousseau, 1997) if we explain daringly it within French perspectives. For probabilistic contents, the introduction of simulations by the use of dice or coins is a *naïve* didactic technique to promote adidactic situations. There is chemistry between these situations of simulations and frequentist probability. Thus, we can consider that the frequentist probability is taught in Japanese school mathematics to help teachers realise adidactic situations of probability following a condition at the Pedagogy levels of the constructivism.

However, the condition of constructivism is not an adequate explanation because frequentist probability had already been in Japanese school mathematics curricula, which have undergone reforms about every ten years, before the import-phenomenon of constructivism. To find more reasons, let us proceed to more generic levels of codetermination.

First (and unfortunately immediately rejected) candidate of a more generic condition is the *epistemological high status of statistics* in Japanese schools because statistics needs the frequentist probability as its theoretical base. In fact, the frequentist probability is usually called the "statistical" probability in Japan. However, the epistemological status of statistics in school has been lower than those of algebra, geometry and so on, even though we see some signs of statistical movement today. For example, there were no statistical contents in the 1998–2008 version of junior secondary school mathematics curriculum, in which there were probabilistic content together with frequentist probability. We should, therefore, discover other conditions.

One generic condition is the *empire of chance* (Gigerenzer et al., 1989) at the Civilisation level, which indicates that probability widely and deeply takes root in our culture. The notion of probability changed our perspectives for both science and everyday life through the 20th century. In everyday life, for example, we can easily find probabilities of various kinds of events in the Internet: rain, lottery, accident and so on. In science, the concept of probability revolutionised not only natural sciences. In social sciences, probability introduced the quantitative research methodology for studying a variety of social facts. Today, we can hardly imagine the world without the concept of probability. In other words, probability has become permeated many institutions.

In addition, the *duality of probability* is another generic condition at the Civilisation level by which the frequentist probability can survive in school mathematics. On the institutional condition of empire of chance at the Civilisation level, it is a matter of course that probability, especially its *nature*, becomes a content to be taught. What is the nature of probability? This is a perpetual question in the philosophy of science. However, we can say that the notion of probability has *duality* in its meaning, that is, *aleatory* and *epistemic* (Hacking, 1975/2006). In the aleatory interpretations, "probability" represents an entity in the real world. In contrast, the epistemic interpretations regard probability as a theoretical lens for cognition of actual entities. Typical concepts of each meaning are frequentist probability and Laplacian probability.

5. Conclusion

The point of departure of my study was the problem of an ill-attained dissemination of probabilistic knowledge. As an approach to solve this problem, I identified three institutional constraints in the didactic ecology of school probabilistic activities (i.e. determinationism, theoricism and demathematisation of randomisers), focusing on the frequentist probability and analysing the Japanese secondary school mathematics textbooks. These constraints can be regarded as reasons for the frequentist probability having a small role in Japanese school probabilistic praxeology in curricula, although some institutional generic conditions (i.e. constructivism, empire of chance and duality of probability) promote frequentist praxeologies.

Although Laplacian probability ranks high in mathematics textbooks, I believe that frequentist probability is equally important. The Laplacian aspect and the frequentist aspect of praxeological organizations about probability could help each other, because there is an interdependence between the concepts of probability and chance (cf. Steinbring, 1991). However, the Japanese curricula meet only few of the condition of dialectic of the Laplacian probability and the frequentist probability. I conjecture that the nature of probabilistic knowledge to be taught become a constraint on teaching activities of probability. In addition, the teaching of probability in classroom is affected by more conditions: for instance, teachers' praxeological equipment, didactic infrastructure, and didactic contract. Thus, a further study task will be to observe and analyse realised didactic situations around probability in order to validate this conjecture and find other constraints.

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