Japanese and Swedish Mathematics Teacher Educators' Pedagogical Content Knowledge – An Institutional Perspective

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Résumé. L'objectif de ce travail est d'étudier quel genre de conditions et de limites constituent le contenu de la connaissance didactique du contenu dans les formateurs d'enseignants japonais et suédois (PCK). Nous décrivons comment les formateurs organisent leurs pratiques d'enseignement en relation avec la détermination de la zone pour analyser les praxéologies des leçons. Notre étude montre que le PCK des formateurs d'enseignants japonais est partagé plus explicitement par la communauté des formateurs par rapport à l'homologue suédois. En outre, un programme de formation japonais plus détaillé ainsi que l'approche structurée de résolution de problèmes illustrent comment créé de riches organisations mathématiques et didactiques pour les futurs enseignants.

Abstract. The aim of this paper is to investigate which kind of conditions and constraints affect Japanese and Swedish teacher educators' pedagogical content knowledge (PCK). We analyse the praxeologies of the lessons in which the educators teach area determination. Our study shows that the Japanese teacher educators' PCK are more explicitly shared by the community of the teacher educators compared to the Swedish counterpart. Also, the detailed Japanese curriculum and the structured problem solving approach promote to illustrate how to construct rich mathematical and didactical organisations for prospective teachers.

1. Introduction

1.1. Swedish primary teacher education program

The latest incarnation of Swedish teacher education programme is called "Top of the class". It was introduced in March 2010 and the first students began their education in autumn 2011 in Sweden. The programme for primary school teachers is extended with half a year compared to earlier requirements. The students have to study at least 30 credits (one semester) in Mathematics, which are 15 credits more than before. The background is, according to The Ministry of Education and Research, the sharp criticism of the quality of the teacher education that has been formulated. For example, it is said that the requirements on students are too low; the degree is too easily obtained (The Ministry of Education and Research, 2010).

Swedish students' achievements have declined over the past decade in the report of OECD survey. The Ministry of Education and Research refers to recent research showing that the teacher competence is a crucial factor for students' performance: "highly educated and experienced teachers having subject-didactical competence is a key factor for a successful education system" (ibid., p.18).

This is a part of our study comparing elementary mathematics teacher education in Japan, Finland and Sweden. This paper is an extended version of that presented at CERME 9 (Asami-

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Johansson & Attorps, 2015), where our focus was to analyse a lesson by a Japanese teacher educator for prospective elementary school teachers. In this paper, we focus on the comparison between the Japanese lesson mentioned above and a Swedish lesson of area determination for prospective elementary school teachers. As a result, we hope the study could expose the institutional conditions for the construction of each country's lessons.

1.2. Teacher knowledge

The three areas of teacher's knowledge, which can be seen as the cornerstones among various components of the definitions, are: subject matter knowledge (SMK), general pedagogical knowledge, and pedagogical content knowledge (PCK) (e.g. Shulman, 1986). SMK is "the amount and organization of knowledge per se in the mind of the teacher" whereas PCK consists of "the ways representing and formulating the subject that is comprehensible to others… [it] also includes an understanding of what makes the learning of specific topics easy or difficult…" (Shulman 1986, p. 9). Thus, PCK can be defined as the special knowledge used by the teacher to transform his/her SMK to benefit students' learning (Grossman, 1990). There is no universally accepted conceptualization of PCK. In this paper, we consider the PCK as integrated knowledge of content knowledge, pedagogical knowledge and didactical knowledge (Liljedahl et al., 2009).

Researchers also (e.g. Abell, 2008) argue that there is a need of more research concerning the relation of PCK to student learning. Kind (2009) argues that it is difficult to investigate PCK in teaching practices. He stresses that using Content Representation (CoRe), developed by Loughran, Mulhall and Berry (2006), as a methodological tool offers a unique insight into teachers' PCK and practices relating to specific topics and areas.

1.3. The aim

The aim of this paper is to investigate what kind of conditions and constraints influence the teacher educators' PCK concerning the designing of a lesson of area determination. In this paper, we try to clarify the components of the teacher educators' PCK within the method courses in primary teacher education from the perspective of the theory of didactic transposition and the anthropological theory of the didactic. The main reason we chose to compare the Japanese teacher educator's practice to the Swedish counterpart is that Japanese teaching practice appears to be collectively organized in a manner that is quite distinct from the Western teaching culture (Winsløw, 2012).

2. Theoretical framework

2.1. The didactic transposition and the anthropological theory of the didactic

The process of adapting knowledge for the purpose of being taught within a given institution, is called "*didactic transposition*" (Chevallard, 1992). It is a transposition of the scholarly knowledge into praxeologies that are adapted to different levels within the teaching system (Bosch and Gascón, 2006).
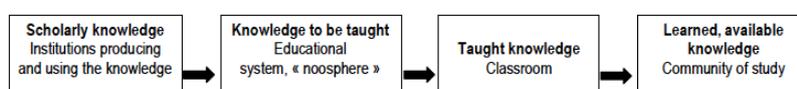
Figure 1: The didactic transposition process (Bosch & Gascón, 2006, p.56)

Chevallard extended the concept to study the mathematical knowledge in an institutional context into the anthropological theory of the didactic (ATD) (ibid.). He modelled mathematics learning as the construction within social institutions of mathematical praxeologies (ibid.). A praxeology provides both methods for the solution of a domain of problems (praxis) and a structure (the logos) for the discourse regarding the methods and their relations to more broad settings. A praxeology that describes activities of learning mathematics is also called a mathematical organisation (MO) (ibid.). In the same way, a didactical organisation (DO) (ibid.) is a praxeology that describes the activities to support achieving the goal of the MO, used by teachers.

To study the intricacy of different praxeologies, the mathematical praxeologies are classified into increasing complexity as: specific, local and regional praxeologies. A specific praxeology consists of a single type of tasks, where a specific technique is applied, thus the technology usually is implicit. A local praxeology is characterised by the integration of several specific praxeologies that are connected by a common technology. A regional praxeology is a coordination of local praxeologies where a common theory is framed.

2.2. Research questions

Our question in this paper is two-fold. First, what is the teacher educators' main emphasis when they use teachers' PCK to design the lessons of method courses? Second, from what kind of conditions are those emphases originated in each country?

3. Methodology

3.1. Analytical method

As Liljedahl et al. (2009) point out, what is unique within teacher education is that what educators teach is also how educators teach, and what the prospective teachers learn is also how they are learning (p. 29). In that sense, we state that one of the most important characteristics of teacher educators' PCK is to be able to teach the mathematical teaching contents (or the SMK) and the teaching practice of the contents *at the same time* for prospective teachers. From the viewpoint of the praxeology, this purpose is interpreted as the *type of tasks* of the DO. Teacher educators' DO (which is constructed by using their PCK) promotes prospective teachers to learn how to construct the praxeologies of the lesson of school mathematics where the MO and the DO are mutually interwoven. In that reason, we denote the construction of the DO in the teacher education which includes the school praxeology (MO/DO) as *double praxeologies*; we call the didactic organisation of teacher educators and prospective teachers as the DO_{te}, and the mathematical/didactic organisation of, say, a class of grade five, as the MO_{sc} (the MO of school class) and the DO_{sc} (the DO of school class). Our hypothesis is that, by studying the constructions of the double praxeologies of the lessons, e.g. in what way the lesson is designed to support prospective teachers to learn both the mathematical contents and how to teach those contents, we can reveal the main components of teacher educators' PCK in each country.

3.2. Data collection

In an attempt to study the didactic transposition between the knowledge to be taught and the taught knowledge, concerning the measurement and area determination, we have made small-scaled study on Japanese and Swedish curricula and textbooks. We consider that to investigate

the curricula/textbooks may lead us to understand the ecology of praxeologies of the lessons concerning area determination in method courses. Besides, we have used classroom observations with video recordings and questionnaires, inspired by Content Representation (CoRe) format, developed by Loughran et al. (2006) as a reflection document. In this study, we let the teacher educators write down the answers to the items on the questionnaire to reflect and to consider on the content of the lesson. The questionnaire consists of eight questions to help to identify different components of the didactic organisation of the method course and the mathematical /didactic organisations of the school lessons. For instance, Q1: "What do you intend the students to learn on this content (e.g. area of rectangles)?" is related to the first research question to identify the elements of the MO_{sc} and what is prioritized in the DO_{te}. Also the questions Q4: "What kind of difficulties are connected with teaching the content" and Q7: "What are your teaching procedures and particular reasons for using these to engage your teaching?" help to identify the conditions on the praxeology, which is relevant to the second research question to investigate the wider explanations for the differences between the two countries.

4. The treatment of the concept of measurements in Japanese and Swedish curricula

4.1. "Quantity and Measurements" in the Japanese national curriculum

In the Japanese national curriculum "The Course of Study for Elementary School" (MEXT, 2008a), the determination of length, area and volume is described in a chapter "Quantity and Measurements", between the chapters of "Arithmetic" and "Geometry". In "the Elementary School Teaching Guide for the Course of Study" (MEXT, 2008b), the contents for each grade are described in the detail, and the goal of the section and guidelines for the teaching methods are proposed in more than six pages. Miyakawa (2010) describes that the introduction to "Quantities and Measurements" usually consists of four phases in Japanese elementary schools: 1. Direct comparison of two objects. 2. Indirect comparison of two objects with a third object, having the same kind of quantity. 3. Comparison of two objects with arbitrary object as a unit (e.g. a pencil). 4. Comparison using standard units (e.g. meters). This order is clearly followed by Japanese textbooks (e.g. Seki & Hashimoto, 2008; Shimizu, Hunakoshi, Negami & Teragaki, 2014). In Japan, the contents of the textbooks are controlled by MEXT (The Ministry of Education) before publishing. Some of textbook authors often are researchers who have influence to design the Japanese national curriculum. Consequently, the contents of the textbooks may be treated nearly as the knowledge to be taught.

4.2. The Swedish national curriculum

The core contents regarding quantities, units and measurement in the Swedish "Curriculum for the compulsory school, preschool class and the recreation centre 2011" are described for grades 1-3 and for grades 4-6. Unlike the Japanese curriculum, the Swedish curriculum does not give these contents own caption and they are placed in the domain of Geometry. Contents regarding quantities, units and measurement are shortly described in the curriculum for grades 1-3 (Skolverket, 2011a, p. 61) as below:

- Comparisons and estimates of mathematical quantities.

- Measurement of length, mass, volume and time in common contemporary and older measurement units.

The contents for grades 4-6 are (ibid., p. 62):

- Methods for determining and estimating (the amount of the length of a given) circumference and areas of different two-dimensional geometrical figures
- Comparing, estimating and measuring length, area, volume, mass, time and angles using common units of measurement.
- Measurements using contemporary and older methods.

The Commentary Material to the Swedish curriculum for primary school (Skolverket, 2011b) describes that teaching in geometry should treat the basic geometric objects (e.g. points, lines, distances, quadrilaterals, triangles, circles, spheres, cones, cylinders, cuboids) and their mutual properties. The Commentary Material in principle only retells the national curriculum and does not give practical guidelines for teachers.

The presentation of the concept of quantities, units and measurements is randomly placed in Swedish textbooks for grade one. Several textbooks (e.g. Falck, Picetti & Elofsdotter Meijer, 2011; Brorsson, 2013) introduce arbitrary units, direct comparison and standard unit (cm) simultaneously. Also, the problem that corresponds to the indirect comparison is not addressed in most textbook. In addition, the direct comparison is sometimes presented after the introducing of comparison with standard units. Also, these presentations in the textbooks for grades 1-3 are placed in sections covering Arithmetic, although the Swedish curriculum for grades 1-3 present this concept in Geometry.

4.3. Comparison of the national curricula and textbooks

Comparing these two educational contexts, the Japanese curriculum and the textbooks give concrete ideas of a teaching approach for school teachers to work with the transition from arbitrary to standard units in the lower grades. The broad description of the concept of quantities, units and measurements in the Swedish curriculum for grades 1-3 make the progression of the concept to become unclear. Additionally, we assume the reason that many Swedish textbook authors place the content of measurements in the domain of arithmetic, against the curriculum, is that they consider that introducing the measurement directly after the contents of addition/subtraction enables a natural connection between length/weight/area determination and the basic arithmetical operations. This indicates that the Swedish curriculum does not give sufficiently clear guidelines for the teaching of this content. Consequently, different textbooks provide different teaching approaches in Sweden. The transposition between the knowledge to be taught and the taught knowledge in the Swedish context is therefore not as predetermined as in the Japanese context.

5. Results from the lesson observations

In this section, we present the result of pre-service teacher education lesson observations from each country, and our analysis of the praxeologies of these lessons.

5.1. The lesson "Quantities and Measurements" in a course Arithmetic Education in Japanese teacher education

The course Arithmetic Education for grades 1-6 teachers in a state university located in the middle of Japan focuses on the content of primary school mathematics and the teaching methods for those contents. The lecturer of this course, Mr. Matsui, has worked as a mathematics teacher in lower secondary school for 14 years and as a teacher educator for 12 years. We observed 55 prospective teachers in this lesson.

Mr. Matsui instructs the lesson by explaining the four phases in the process of pupils learning about quantities and measurement, referring to the "Teaching Guide for the Course of Study". He clarifies the direct comparison and indirect comparison of two quantities. Thereafter, Mr. Matsui describes how to use an arbitrary object as a unit to measure. Finally, he discusses how the above mentioned four phases are treated in digital textbooks for grades one to five.

Mr. Matsui then applies a short version of the structured problem solving approach (Stigler & Hiebert, 199) in his class. The approach emphasises to create students' active participation to mathematical activities by challenging problems and collective reflections through whole-class discussions. Shimizu (1999) explains some professional languages which are used daily by Japanese teachers within the structured problem solving; *hatsumon*: asking a key question, *kikan-shido*: teachers' instruction at students' desks, *neriage*: whole-class discussion, *matome*: summing up. These common didactical terms indicate that Japanese school teachers have a shared perception about the teacher's didactical role in the classroom.

Mr. Matsui lets the students find out several different methods for the determination of the area of parallelograms aiming to teach pupils of grade five. He reminds that grade five pupils have learned the methods to determine the area of rectangles and squares but not of triangles. So that pupils cannot apply the formula for the area of triangles yet to determine the area of parallelograms. The students are given several minutes to reflect, and Mr. Matsui starts to walk between the students' desks (kikan-shido) and gives them short advices.

Four chosen students draw pictures of parallelograms and explain their solutions on the blackboard.
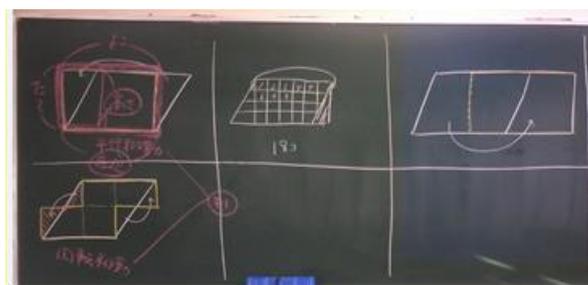
Figure 2. Students' presented methods for determination of area of parallelogram.

Mr. Matsui points out the different kinds of "shifts" used by students and categorises them. Student A has combined some incomplete grid squares with the corresponding incomplete squares on the opposite side of the parallelogram (see figure 2 in the middle). Mr. Matsui remarks to the class that student A is using an arbitrary unit to determine the area. Student B divides the parallelogram into a triangle and a trapezium, then shifts the right triangle beside to the left side of the trapezium to make a rectangle (see figure 2 in the top left). Mr. Matsui compares this method to student D's method. Student D divides the parallelogram in the middle

and shifted one of the two trapeziums to the other side (figure 2 in the top right). He remarks this method is called "*parallel translation*". Student C has divided the parallelogram into two small rectangles, two trapeziums and two triangles and rotates the triangles (see figure 2 in the bottom left). Mr. Matsui remarks this method is usually called "*rotation*". Further, Mr. Matsui let the class to compare the student B's method (figure 2 in the top left) and also the student C (figure 2 in the bottom left) and explains that those methods use a "*same area transformation*", while the student D (figure 2 in the top right) uses "*double area transformation*". Mr. Matsui writes down all those didactical terms on the blackboard.

Then, he explains the formula for the area of parallelograms as height times length since the geometric transformations shows that the width (or height) and length of the parallelograms corresponds to those of rectangles. He notes that the operations of translation and rotation will be treated in more detail in grade seven in the "Course of Study".

Lastly, Mr. Matsui gives a second problem to find out methods for determining the area of trapeziums, using the same didactical approach. This time, he chooses 7 students to demonstrate their different solutions. One student doubles the trapezoid so as to transform it into a big parallelogram. Mr. Matsui uses this solution and concludes the formula for the area of a trapeziums, $(a + b)h/2$.

To finish the lesson, Mr. Matsui shows an article written by an in service teacher about a case study of teaching the area of trapeziums using the structured problem solving approach. In this way, he institutionalises the conception of the structured problem solving as a general didactical approach.

We now describe in more detail, the praxeology of the last demonstrated lesson above.
The didactic organisation of the educator in relation to the prospective teachers (DO_{te}):

- Tasks (***T***): (a) To help the PTs learn a model of the MO_{sc}/DO_{sc}
- (b) To illustrate how to link the previously experienced MOs.
- (c) To help the prospective teachers to anticipate pupils' way of solving problems, and examine the viability of the different solutions.
- Technique (**τ**): (a) To address some solution methods and related issues in specific terms described in the Teaching Guide for the Course of Study (e.g. *arbitrary object*, *parallel translation*, rotation, *same area transformation* and *double area transformation*) (b) to make the student participate in an a short version of an example lesson using the structured problem solving approach.
- Technology (***θ***): statement of previous knowledge, mathematical textbook and curriculum used as reference.
- Theory (**Θ**): The shared statement: 1. the specific didactical terms described in the Teaching Guide should be taught within the method course. 2. The structured problem solving should be introduced to prospective teachers.

The mathematical organisation of a teacher and pupils of grade five (MO_{sc}):

- Types of tasks (**T**): to calculate the area of a parallelogram/trapezoid and to derive a formula for these geometrical figures
- Technique (**τ**): transformation of shapes, using formulas for rectangles
- Technology (***θ***): figures, parallel shift, rotation
- Theory (**Θ**): figures and area, Euclidean geometry

The didactic organisation of a teacher of grade five (DO_{sc}):

- Types of tasks (**T**): making the pupils participate in the lessons and to reason the determination of area of parallelogram and trapezoids
- Technique (**τ**): questioning, giving the task (hatsumon), and using graph paper (grid of 1cm) to draw their ideas on, whole-class discussion (neriage)
- Technology (***θ***): statement of previous knowledge, mathematical textbook
- Theory (**Θ**): Structured problem solving

The didactical task of this lesson is to help the prospective teachers learn the construction of the MO_{sc}/DO_{sc} of the section Quantities and Measurements and letting them explore how pupils would reason about such problems concerning area determination. The technique Mr. Matsui uses is referring to the Teaching Guide for the Course of Study, and demonstrating a short version of the structured problem solving approach. When he explains the process of establishing the method of area determination, the specific terms from the Guidelines are described: *arbitrary object*, *parallel translation*, rotation, *same area transformation* and *double area transformation*, which are components of the MO_{sc} technology. School pupils do not have to master to apply these terms, but the prospective teachers do, in order to understand the whole construction of the MO_{sc} better. Thus the DO_{te} technique of discussing the use of the different terms makes the technology of the MO_{sc}/DO_{sc} explicit.

The complexity of the mathematical organisation becomes at least local, since several techniques based on different kinds of transformations of the figures are generated by a common technology, due to the nature of the whole-class discussion.

In the digital textbook, which Mr. Matsui used in the lesson, the determination of the area of rectangles is treated as an initial task where some techniques are justified by algebraic reasoning. The theory, which justifies this technology, is both algebra and geometry. The praxeology of the section Quantities and Measurements in the textbook is at least local, since it features multiple methods for solutions and constructs technological discourse of arithmetic and algebra. The textbook clearly follows the specified praxeology in the "Course of Study".

5.2. The lesson concerning area and perimeter in the course "Geometry" in Swedish teacher education

The course "Mathematics and Learning for Primary School, grades 4-6 teachers II, Geometry", in a state university located in middle part of Sweden, treats the knowledge in mathematics and mathematical education in relation to the current Swedish national curriculum. The contents of the course are, for example, the mathematical symbol language and mathematical terminologies in geometry, or analysis of pupils' knowledge in geometry. The lecturer, Mrs. Nilsson, has worked as a primary school teacher for 13 years and as a teacher educator for 12 years. 24 prospective teachers participate in this lesson. The duration of the lesson is 2 hours and 30 minutes.

Mrs. Nilsson begins the lesson by asking prospective teachers to reflect on their own perception of the notion of area. The first exercise is to measure the area of the chair seats using ice cream sticks. The students present two different techniques. In this exercise, Mrs. Nilsson lets the prospective teachers discover the concept of area by using arbitrary units and techniques for area determination.

Thereafter, Mrs. Nilsson refers to two examples of Swedish pupils' erroneous solutions in TIMSS 2007 regarding area determination. Then she briefly shows what the Swedish curriculum for grades 1-3 and 4-6 emphasises in Geometry.

Further, four group-exercises concerning concepts of area and perimeter are done. The last exercise consists of determining the area of different geometrical figures by using a Geo-board. Mrs. Nilsson demonstrates a method for area-determination of an isosceles triangle (See Figure 3a) by using a rubber band around the triangle (Figure 3b). She divides the rectangle into two squares which are in turn divided into two halves (Figure 3c). The half of the area of the squares is subtracted from each side. She explains that this gives the area $2–½–½ = 1$.

a   *b*   *c*
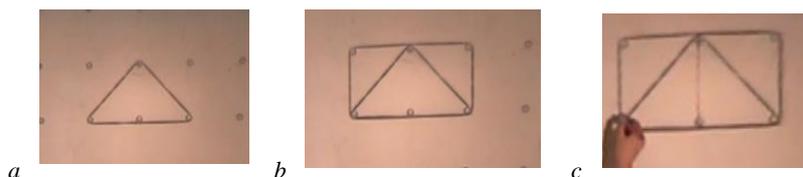
Figure 3. a: area determination of a triangle. b: with a frame of rubber band around the triangle. c: the divided triangle in a half.

The students ponder the method for area-determination of another isosceles triangle in groups. Mrs. Nilsson then demonstrates one student's solution where the same method is applied as the one she explained (Figure 4a). She writes $4 – 1 – 1 – ½ = 1½$ on the white board.


a   b   c
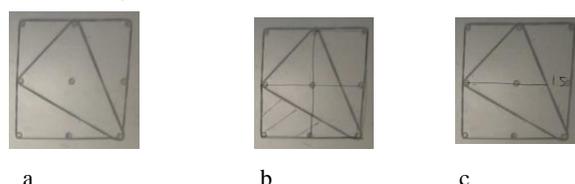
Figure 4.a: an isosceles triangle. b. with an auxiliary line. c. student E's figure

Student E asks if he can apply the formula of the area determination for a triangle; first, dividing the original triangle into two triangles with the base of 1.5 length units (see Figure 4c), and then adding the area of the two triangles. This gives the area, $(1.5 \cdot 1)/2 + (1.5 \cdot 1)/2 = 0.75 + 0.75 = 1.5$. Some of the students express that they do not grasp directly how it works. Mrs. Nilsson comments "One can understand (this method) if one has more mathematical skills".

The common didactical goal of the six exercises is to give the prospective teachers opportunities to explore some teaching methods for area and perimeter determination. In that sense, Mrs. Nilsson's DO_{te} also include MO_{sc} and, DO_{sc}. We now sketch the praxeology of the last exercise, the determination of the area of an isosceles triangle.

(DO_{te}):

- Tasks (**T**): to help the prospective teachers experience a model of specific MO_{sc}/DO_{sc}
- Techniques (**τ**): group discussions about using manipulatives (Geo-board)
- Technology (**θ**): rules and terminology regarding the use of manipulatives in lessons
- Theory (**Θ**): theory of learning by adaption: by experiencing a suggested teaching approach, one acquires the method (our assumption: since it was not recognised any explicit theory of the DO_{sc} in the observed lesson)

(MO_{sc}):

- Types of tasks (**T**): to determine the area of an isosceles triangle

- Techniques (τ): division of figures and subtraction of area
- Technology (θ): additivity of area, formula for area determination of rectangles
- Theory (Θ): figures and area, Euclidean geometry

(DO_{sc}):

- Types of tasks (**T**): to give the pupils opportunities to explore some methods for area and perimeter determination
- Techniques (τ): questioning, giving the task, using manipulatives (Geo-board), group discussion (not stressed as in DO_2)
- Technology (θ): absent
- Theory (Θ): absent

Compare with the observation of the Japanese lesson, the components of the DO_{sc} is not clearly shown to the prospective teachers. Mrs. Nilsson's intention is to train students' algorithmic skills and establish their own perceptions of area and perimeter. In the last exercise, Mrs. Nilsson let the student explain his alternative method. However, she did not validate it by verifying that the base is 1.5 length units as stated. Her intention was not to discuss the viability of different mathematical technologies for the grade five class, but to establish a certain technique which is possible for all students to manage. Thus, the praxeology of this lesson remains to be specific.

6. Result from the questionnaire

6.1. Mr. Matsui's reflections

Mr. Matsui intends that his students will realise that area of polygons can be determined in various ways by using pupil's previously learned knowledge. He stresses also that the prospective teachers should be able to use the specific terms. The terms describe the various ways for determination and help them in understanding the pattern of the different methods. Mr. Matsui also aims that the prospective teachers learn the process of finding out the formulas for area determination of geometrical figures, rather than memorising the formulas.

Mr. Matsui remarks: "a teacher educator should stress the importance of taking children's perspective into account". It is one of the key components in the teaching guidelines of Japanese national curriculum. Pupils' previous knowledge has a strong influence when choosing techniques of the MO.

These answers indicate, in line with the lesson observations, that Mr. Matsui's DO focuses on making the PTs to learn how to relate the local MO_{SCS} of individual lessons on a larger time-scale and thus to construct a complex MO_{sc}. Also, his remarks about the importance of acquiring the use of the specific terms, point to that Mr. Matsui's purpose is to make the theory block of the MO_{sc}/DO_{SC} explicit for the prospective teachers. It also indicates that the statement – prospective teachers should learn the teaching methods using the established specific terms, which are prescribed in the teacher's guide for the Course of Study; – is a crucial component of the theory block of the DO_{te}

6.2. Mrs. Nilsson's reflections

Mrs. Nilsson highlights the importance of understanding mathematical concepts and their definitions. She points out that the prospective teachers should be aware of their own perceptions of the concept.

Compared to Mr. Matsui's answer to this question, Mrs. Nilsson's answer may be interpreted as that she deals with her prospective teachers' content knowledge and school pupils' contents knowledge almost on a same way, since her concern at this point is to establish a specific technique of the MO_{sc} which all her prospective teachers can manage.

Regarding prospective teachers' difficulties and limitations in this content area, she mentions that some of them have learnt formulas for area determination by heart without a deeper understanding and sometimes incorrectly. Also, their perception that "Geometry is a difficult subject" results in a blockage of their learning process. Furthermore, she comments that the students have not developed mathematical terms that allow them to explain their solutions.

The fragmental mathematical knowledge of the prospective teachers and their anxiety for learning mathematics influences Mrs. Nilsson's teaching procedures. She describes her various didactical strategies: to use manipulatives, working in small groups, discusses pupils' misconceptions of area and perimeter to let the prospective teachers realise their own misconceptions of this content and so on.

Hemmi and Ryve's research (2015) suggests that the "Swedish discourse on classroom teaching builds on a rather extreme interpretation of constructivism" (p. 516). Ms. Nilsson's remark – her first didactic task is to make the PTs be aware of their own perceptions of the concept the area, indicates that a constructivist theory of learning underlies the justification of her DO_{te} technique. Also, her concern on the prospective teachers overcoming their anxieties of learning geometry indicates the influence from a psychological view of teaching, focusing on the development of students' self-efficacy (Bandura, 1977).

7. Discussion and conclusion

A significant characteristic of the DO_{te} of the Japanese teacher educator is to make the theory block of the MO_{sc} and DO_{sc} explicit for the prospective teachers, by applying specific didactical terms and referring to the teaching guide for the national curriculum. In Swedish national curriculum, these kind of didactical terms are not described, and the teacher educator does not give any explanation to make the theory block of the MO_{sc} and DO_{sc} explicit during the lesson. The use of the technical terms in Japan is described in Iwasaki and Miyakawa's study (2015) that the Japanese teachers in service begin to use the didactical terms at quite early stages of their career: "These terms principally allow teachers to draw attention to significant facts – the nature of mathematical problems, teachers' acts, students' acts, etc. – in the complicated teaching and learning situation, and apply some labels to them" (p. 91). Therefore, the use of these terms within the teacher education helps prospective teachers to see how the MO_{sc} and the DO_{sc} are mutually connected.

The Japanese teaching guide describes the different stages of the progression within the section "Quantities and Measurements" in detail, and does not give much space for different interpretations of its contents. This in turn, the Japanese textbook authors give homogeneous proposals for teaching approaches to aim for in the classrooms. In that sense, the transposition

between the knowledge to be taught and the taught knowledge becomes clear. The Swedish curriculum and the textbooks do not give teacher educators the same kind of practical support. Every teacher educator must interpret the transposition between the knowledge to be taught and the taught knowledge on her/his own. Also, the practice block of the DO_{te} is *individually* designed by the teacher educators, since a collectively shared and generally adapted theory block of the DO_{TE} is absent. If a different educator would be in charge of this course, the structure of the lessons could be fairly different even at the same university.

The short version of the structured problem solving approach used in the Japanese lessons has several advantages for the prospective teachers: firstly, this teaching approach illustrate how to design mathematics lessons with local and bigger praxeologies for the prospective teachers; the moment of the whole-class discussion promotes to create a mathematical activity where alternative techniques are assessed and a technological discourse is taking place. Secondly, this approach gives prospective teachers' possibilities to develop their ability to use the didactical terms in the context of the teaching practice. One of the Swedish teacher educator's techniques of the DO_{te} was also applying a short version of simulated lesson with problem solving. The actual intention with this lesson was primarily to change the prospective teachers' subjective conceptions of the area and perimeters, and secondly, to establish a specific technique of area determination. The Swedish teacher educator demonstrates a model of a DO_{sc} practice with the prospective teachers as a "pupils". This teaching discourse may also be confirmed from the previous study of Hemmi and Ryve (2015) on Swedish teacher educators' perception of "effective teaching", where "the Swedish discourse circulates a lot around basing teaching on students' thinking, ideas and interests" (p. 511).

Lessons with manipulatives in school mathematics are institutionally established in Sweden (see e.g. Rystedt & Trygg, 2010). The aim is to support pupils' conceptual understanding of mathematical operations by using hands-on materials, where the materials are used to concretise the operation (Löwing & Killborn, 2002). This approach in the Swedish teacher education exemplify a model of the construction of MO_{sc} and DO_{sc} for prospective teachers. However, it does not explicitly promote how to design the lessons which hold complex mathematical praxeologies.

8. Final remarks

One of the most crucial components of teacher educators' PCK is to be able to construct a DO_{te} which promotes prospective teachers to learn how to construct interwoven MO_{sc} and DO_{sc} in their future lessons. The Japanese teacher educators' PCK regarding this issue is collectively established. It is based on the national curriculum there the theoretical blocks of the MO_{sc}/DO_{sc} are described in large extent by using well-established technical terms, and the tradition of applying the didactical theories such as structured problem solving approach. In the Swedish case, teacher educators' PCK are based on broader psychological ideas such as students' self-efficacy and constructivism. The absence of an explicit theory of the DO_{te} to show the prospective teachers how to construct the theoretical blocks of MO_{sc}/DO_{sc} means that the PCK of Swedish teacher educators are individually piled up.

As Winsløw (2012) points out, the teaching practices are constructed by various institutional factors. Studies concerning them require "that we develop very precise and explicit

models of what we want to study and compare" (ibid., p. 291). In this paper, we have investigated the teacher educators' didactic praxeologies observed in the method courses in Japan and Sweden. Although the scale of this study is limited, and results cannot be generalised to the two countries, the study shows some alignments of teacher educators' PCK in the differences of the didactic organisations between the two lessons.

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