Mathematics curriculum reform in the United States: a historical perspective

GEORG M. A. STANIC
JEREMY KILPATRICK*

Abstrat
In the United States by the turn of 20th century, the basic precollege mathematics curriculum of arithmetic, algebra, and geometry was firmly in place. Since then, there have been changes but no substantial reform. The two most significant reform efforts during this century have been the move toward unified and applied mathematics as the century began and the modern mathematics movement of 1950s and 1960s. Neither of these efforts had it intend effect on the school curriculum, though both left residues. In each case, however, the movement had a profound effect on the mathematics education community, particularly at the post-second level. Viewing curriculum reform as the technical rather then a moral and ethical process has led reformers to neglect the basic issues of curriculum discourse.
Key-words: *reform curriculum; unified mathematics; new math; standards accountability; rhetoric.*

Resumo
Nos Estados Unidos por volta do século XX, o currículo básico do ensino pré-universitário de aritmética, álgebra, e geometria estava firmemente estabelecido. Desde então, havia reformas, mas não substanciais. Os dois esforços de reforma mais significantes durante esse século foram o movimento em direção à matemática unificada e aplicada quando do início do século e o movimento da matemática moderna nas décadas de 50 e 60. Nenhum desses esforços teve efeito no currículo escolar, embora ambos deixaram resíduos. Em cada caso, no entanto, o movimento teve um profundo efeito na comunidade da educação matemática, particularmente no ensino básico. A visão da reforma curricular como um processo técnico mais que um processo moral e ético faz com que reformadores negligenciem as abordagens básicas do discurso sobre o currículo.
Palavras-chave: reforma currículo; matemática unificada; nova matemática; *standards accountability*; retórica.

Introduction

There has been constant reform rhetoric but little actual reform of school mathematics curriculum since the establishment of mathematics

* University of Georgia. E-mail: jkilpat@coe.uga.edu

education as professional field of study in the United States at the turn of 20th century. Before that time, there clearly were significant changes in school mathematics as, for example, arithmetic and then successively algebra, geometry, trigonometry, and even, in a few schools, calculus became part of the curriculum. These subjects did not so much reform the mathematics curriculum as provide the original curriculum form which was to become the topic of much discussion and a source of frustration for reformers. Furthermore, despite concerns about school mathematics expressed by educators such as Warren Colburn during the early 19th century, curriculum reform was not a crucial issue as long as very few school-age children actually went school. As the fraction of the school-age population attending school rose dramatically beginning in the middle of the 19th century, curriculum reform became an important part of the professional and public agendas. These new students were more a source of fear than excitement for most educators, who, in the words of Granville Stanley Hall (1904), saw them as a "great army of incapables". It was in this context that attempts to reform the mathematics curriculum took on more urgency, especially since mathematics had become one of the greatest sources of failure in school. And it was in the context that our professional forebears set the standard of failure of curriculum reform that is our legacy.

Among the constant calls for mathematics curriculum reform, two historical moments stand out – the first at the turn of the century when a curriculum of unified and applied mathematics was the focus, the other coming during the 1950s and 1960s when modern mathematics was the core of reform efforts. Others citing these same moments (Wheeler, 1989; Wojciechowska, 1989) have suggested that they occurred when the gap between the mathematics taught in school and mathematics as a scientific discipline seemed particularly large and when the views of mathematics on how to close the gap seemed to set the terms of debate. Mathematicians did indeed play a central role during each reform effort. That role, however, was not the same, and in both cases mathematicians were only part of a constellation of forces. The two eras represent "fault lines" in the field of mathematics education – not because of the changes they wrought in the curriculum, but because of the consequences they had for community of professional mathematics educator. Both reform efforts left certain residues in the curriculum, but it was not fundamentally reformed. In fact, both efforts perpetuated the elite status of those who already benefited from school experience and had their most distorted effects on low-achieving students.

In the end, the story of mathematics curriculum reform is not the story of continual progression toward a curriculum that is best for students, teachers, and society nor even the story of different ideologies cyclically replacing each other's influence on school mathematics; instead, it is the story of a developing community preoccupied with a limited and ill-defined agenda.

Forming the curriculum

Arithmetic was taught but was not universal in the colonial elementary school; the only real universals were reading and religion. Gradually, however, arithmetic moved into the elementary school to become part of education of the literate citizen.

> In secondary schools, "ciphering" was taught during colonial times, which consisted generally in drilling students in the manipulation of integral numbers. He was an exceptional teacher who possessed a fair knowledge of "fraction" and the "rule of three", and if some pupil or rare genius manage to master fraction, or even pass beyond the "rule of three", then he was judged a finished mathematics. (Cajori, 1890, p. 9)

The curriculum of Latin grammar school did not at first include arithmetic at all, but some was introduced after 1700 in response to popular demand. As the 18th century progressed, the curriculum of the growing number of academies and those private schools with a more practical orientation came to include arithmetic but algebra, geometry, trigonometry, and even a little calculus. " In many cases, the mathematical teaching in the private school was good as if not better than that found in some of the colleges" (Butts, 1947, p.375). Arithmetic and geometry had been taught at Harvard since the 17th century "but sometimes [were] rather neglected because they were looked upon as practical subjects" (Butts, p. 303). Beginning in the mid-1700s colleges required arithmetic for admission (Jones & Coxford, 1970, p. 18; Sigurdson, 1962, p. 7). As colleges continued to raise their admissions requirements, arithmetic gradually lost its favored position in the secondary school, with algebra and geometry replacing it as the centerpiece of mathematics instruction.

Algebra was required for college entrance as early as 1820, followed by geometry several decades later (Jones& Coxford, 1970, pp. 18-19; Sigurdson, 1962, p. 9). It is not obvious why algebra was required at entrance so much earlier than geometry. In 1786, arithmetic and geometry were in the Harvard college curriculum some time prior to the end the junior year, and algebra apparently came in the senior year (Cajori, 1890, p. 57). In 1787, Harvard's curriculum was organized to put arithmetic in the freshman year and "algebra and other branches of mathematics" (Cajori, p. 57) in the sophomore year. Furthermore, the proliferation of textbooks in algebra, while geometry remained with few alternatives to Euclid, may account for some of the delay in marking geometry a high school course.

Like algebra and geometry, trigonometry was being taught in American colleges in the first quarter of the 18th century (Breslich, 1950, p. 39), but unlike them, it never became a college entrance requirement after moving into the high school curriculum. In both high school and college, trigonometry sometimes appeared as a separate course and sometimes as a part of an advanced algebra course. Analytic geometry and calculus began as college courses that gradually moved to the borderline between high school and college. Although some preparatory school had long taught the rudiments of these subjects to their more advanced students, analytic geometry and calculus remained college subjects. Many high schools today teach an introductory course in calculus in the senor year, but calculus it still viewed as a college course. In fact, students can receive college credits for calculus as part of the College Board's Advanced Placement program.

The United States did not follow Germany and other European countries, under the influence of Felix Klein, in a reform movement around 1900 that established analytic geometry and calculus firmly in the secondary school (Schubring, 1988). Instead, inspired by the 1902 presidential address of Eliakim Hastings Moore (1903/1926) to the American Mathematical Society and affected by the work of John Perry in England, the reform effort in the United States was limited to breaking down the isolation of the separate mathematics courses in high school from one another and from other subjects. As Ernst R. Breslich (1950) observed in the 50th anniversary volume of the Central Association of Science and Mathematical Teachers:

> Traditionally the mathematical content {of the high school curriculum } was organized for teaching purposes into separate courses: arithmetic, algebra, geometry and trigonometry, each being studied to the exclusion of the others. During the first decade {of the 20th century} a movement was stared to break down the traditional organization by correlating these subjects with each other…Another movement which developed simultaneously with the first aimed to correlate mathematics with other school subjects, such as physics, biology, shop work and drawing. (p. 59)

Nonetheless, the traditional separation of branches within mathematics and mathematics from other school subjects was to remain an enduring feature of the 20th century mathematics curriculum in American schools.

Establishing a community

The reform effort in the United States at the turn of the century served to legitimate the field of mathematics education as an area of study at college and universities (Jones, 1970; Kilpatrick, in press; Stanic, 1986). Yet neither of the two people usually cited as the most important forebears of presented-day mathematics educators, David Eugene Smith and Jacob William Albert Young, was leading advocate for fundamental reform of school mathematics. Smith, a faculty member at Teachers College, Columbia University, played the role of apologist for the traditional curriculum. Young was at the University of Chicago, where E.H. Moore was building a powerful mathematics department. Chicago, an institution at the center of the unified mathematics movement, was the home of John Dewey'lab school; and, according to Parshall (1984), Moore was influenced by Dewey's ideas about pedagogy. Mathematics educators at Chicago such as Ernest R. Breslich and George W. Myers were responding to Moore'call for reform of school mathematics (Senk, 1981). Working in this reform environment, Young was willing to give serious consideration to suggested changes, but in the end he was closer to Smith than to the reformers.

Mathematics education, therefore, is a field whose two most important forebears in the United States were not members of the reform movement that served to legitimate the field. Ironically, during this era when mathematics education was established as a professional field of study at institutions of higher education, the role of mathematics in the secondary school curriculum was seriously threatened, with steady decreases in both requirements and enrollments in algebra and geometry (Stanic, 1986). The problem became so severe that by the 1920s, states such as Ohio had "ruled that an approved high school nee not any longer require a unit of mathematics" ("Mathematics", p. 641). The number of students in the last four years of high school who we enrolled in algebra went from 57 percent in 1910 to 40 percent in 1922; geometry suffered a similar reduction in enrollment.

The larger picture of events in the early years of the 20th century helps to explain both the contradictory effects of the reform movement on mathematics education and the position of Smith and Young. E. H. Moore's speech of 1902 occurred against a background of other educational reform ideas – ideas that threatened the strong and seemingly secure position of mathematics in the school curriculum of the United States. As educational psychologists such as Edward Thorndike called into question the theory of mental discipline which had justified the place of mathematics in the school curriculum, a number of educators and sociologists called for schools to be more efficient and school subjects more useful, reflecting the apparently different needs, interests, abilities, and future destinations of the many students who were staying longer in schools. American education was witnessing a huge increase in students attending and graduating from high school. In the 50-year-olds graduating increased from less than 5percent to almost 50 percent (James § Tyack, 1983). Many of those interested in reforming school mathematics saw their work as distinct from other school reform efforts and describe Moore´s address as their inspiration (see, e.g., Betz, 1908), but they could not escape the effects of the larger educational reform context and the changing school population. Furthermore, the unified mathematics movement was not itself unified; significant differences of opinion existed among those people who were calling for mathematics to be organized into correlated, fused, unified, or parallel courses (Sigurdson, 1962).

D. E. Smith – proud of historic role of mathematics in the school curriculum, not yet ready to give up on mental discipline theory, not trusting the ideas of educational reformers who did not have his appreciation of the beauty of mathematics, and worried that any change would be for the worse – rejected unified mathematics and argued against too much applied mathematics educators suggesting change in the school curriculum, his perception of need to defend the role of mathematics in the school curriculum made him, too, way of any changes that might adversely affect that role. By 1920, when the National Council of Teachers of Mathematics was established, the mathematics education community as a whole was on the defensive:

> Mathematics courses have been assailed on very hand. So-called educational reformers have tinkered with the courses, and they, not knowing the subject and its values, in many cases have thrown out mathematics altogether or made it entirely elective….To help remedy the existing situation the National Council of Teachers of Mathematics was organized. (Austin, 1921, pp. 1-2)

The community had grown considerably since the turn of century and contained within itself contradictory impulses coming from those who saw them selves as defenders of traditional mathematics, those who shared a love for mathematics but wanted to move the school curriculum forward, those who ultimately became part of the community because of the research they had done on "useful" mathematics, and the teachers who had to deal with the rapidly changing population of students (Kilpatrick, in press; Satanic, 1984).

Contrasting movements

Mathematics educators at the turn of the century who were willing to reconsider school mathematics and its relationships to the rest of curriculum were rewarded with declining secondary school enrollments.

School mathematics was still feeling the effects of the first effort at reform at the outbreak of Second World War. After the war, the debate over the curriculum intensified dramatically. The modern mathematics reformers, consciously or not, had learned some lessons. Despite calling for curriculum movement, they were reluctant, for example, to tamper with the established structure of secondary mathematics courses, thereby limiting both negative and positive outcomes from the beginning.

Like the unified mathematics movement, the modern mathematics movement of the 1950s and 1960s was affected the views of mathematicians and larger social and educational conditions, although in quite different ways. Both movements encompassed a variety of not-always-consistent activities. Perhaps most importantly, the two movements had much more effect on the reformers themselves than on the school curriculum.

Views of mathematicians

In each movement, mathematicians perceived a gap between the scholarly discipline of mathematics, especially as it was being taught in universities, and mathematics as it was taught in school. At the turn of the century, Moore was not the only American mathematician unhappy with the school curriculum. Florian Cajori, for example, observed in 1890 that students were spending far too much time on arithmetic. He endorsed the recommendation of T. H. Safford of Williams College that students study arithmetic (including algebra) in a course running parallel to one that would include geometry and the conic sections. "Geometry, like arithmetic, should be taught sparingly at a time, but for many years in succession" (Safford, quoted in Cajori, p. 295). Cajori's and Safford's criticisms notwithstanding, mathematics as a group did not unite behind Moore in a reform effort.

By the time of the new math, university mathematicians were better organized, and more of them were willing to get involved in reforming precollege mathematics. When the 1959 report of the Commission on Mathematics of the College Entrance Examination Board noted a discrepancy between what students were doing and what mathematics was, mathematicians were already beginning to offer suggestions. The commission expressed the discrepancy in terms of the "explosive development of mathematics" (p. 1) and the "reorganization, extension, and transformation of parts of the older mathematics (p. 2) such as algebra. The subtitle of the report's first chapter indicated the commission's concern: na urgent need for curriculum revision." As that revision began, curriculum development projects such as the School Mathematics Study Group (SMSG) gave a particular attention to providing the mathematical content needed for the study of mathematics in college. "The corrections to be made were simply those which manifested themselves to university mathematicians as obvious shortcomings of the systems at the time. Questions of method were largely ignored: indeed a previous overemphasis on method was by many held responsible for the neglect of content" (Howson, Keitel, § Kilpatrick, 1981, p. 133). Because the university mathematicians who dominated the modern mathematics movement tended to specialists in pure rather than applied mathematics, they saw pure mathematics, with an emphasis on set theory and axiomatics, not only as the content that was missing from school curriculum but also as providing the framework around which to reorganize that curriculum. In contrast, the reform movement at the turn of the 20th century, although it too sought reorganization, aimed at bringing more application into the curriculum rather than adding new topics.

Social and educational conditions

The two reform movements were also differently affected by societal pressures on the schools. The call for a unified mathematics curriculum at the turn of the 20th century was not a response to the urbanization, industrialization, and immigration that expanded and changed the school population. Reform efforts in the general curriculum field, on the other hand, had been strongly motivated by these changes. The failure to deal adequately with societal pressures on teachers and the curriculum had much to do with the unified mathematics reformer's lack of success.

The issue for the new math reformers was that the schools appeared to be preparing too few students to study advanced mathematics in college, and people were worried that the nation would suffer a serious shortage of mathematically trained personnel. Unlike the unified mathematics movement, which was too often blind to societal pressures, the new math reform was in part born from those pressures. The first newsletter of the

SMSG expressed the reformers´ concern about the growing gulf between what society demanded and what the schools were providing:

> The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; an understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them.

The threat posed in the late 1950s by Soviet Union's achievements in outer space fueled the arguments for reform. The federal government launched a number of curriculum development projects designed to improve the teaching of science and mathematics so that American could regain its perceived dominance in engineering and scientific research. The SMSG was one of the earliest, easily the largest, and perhaps the best known of these projects, but many others were to follow in the next decade.

At first, the focus of curriculum development was the "college capable" student – the student who would be likely to enter college and who might be persuaded to pursue a scientific career if the school mathematics curriculum were more stimulating, intelligible, and mathematically elegant. During the early 1960s, several projects that had started by revising the secondary school curriculum began to tackle the elementary curriculum as well. Shortly thereafter, the federal government launched its War on Poverty, and the so-called disadvantage student became a new focus of curriculum development work.

These expansions of effort brought the reformers into contact with new constituencies in the school and outside it. Expectations were raised, with enthusiastic teachers and eager students in the high schools near universities often floundered when they were exported to less advantage schools. Critics such as Morris Kline (1973), who along with other applied mathematicians (Ahlfors *et al*., 1962) had long criticized the new math reform, began to find a more receptive audience for their complaints that the new math was too abstract, impractical, and confusing. With the public, as well as educators themselves, increasingly convinced that the new math had failed, the wave of reform receded, and 'Back to Basics" became the hallmark of textbooks and instructional programs.

Variety beneath the slogans

Like "unified mathematics", "new math" is the label not so much for a cohesive set of reform proposals and activities as for an era during which a variety of reforms were undertaken. In each era, the need for change seemed obvious, but the direction was not always clear. For most new math reformers, the remedy lay in bringing advanced mathematics (e.g., topology, group theory) into the lower grades. For others, the key issue was how mathematics should be taught: Students should discover as many as possible of the mathematics principles they needed to know. The language used in discussing mathematics with students should be as precise possible. The goals of reform, although they could be expressed generally as to let students discover more mathematics, were diverse and were approached in various ways. As the National advisory Committee on Mathematical Education noted in its 1975 report, the new math was no monolith. The term refers to "two decades (1955-1975) of developments that had a general thrust and direction but sprang from many roots, took many different and even opposing forms, evolved and changed with facets disappearing and new ones arising" (p. 21). Once underway, reform movements provide a context in which multiple goals can be pursued by people who are not necessarily responding to same impulses and whose visions of needed change are likely to be quite different and may possibly conflict.

Unintended effects

Neither reform effort had its intended effect on the school curriculum. A rigorous unified mathematics curriculum for all students became, by the 1930s, the general mathematics taken by those students deemed incapable of doing higher level mathematics. A modernized mathematics curriculum that appealed to students' taste for structure and was "dumbed dow" to basic skills.

The dominant outcome of each movement was its effect on the mathematics education community. Professional mathematics educators in college and universities were the primary beneficiaries of the reform efforts. In the first fault line helped establish the field, the second rejuvenated it. Mathematics educators may have been on the sidelines as the new math era began, but they quickly joined the movement, and its energy drew even more people into the community. In particular, research done those who labeled themselves as mathematics educators increased dramatically during this era (Kilpatrick, 1992).

The paradox of curriculum reform

The two major reform efforts in mathematics education differed in important ways but were alike in their failure to achieve their intended outcomes. What makes *fault line* an appropriate term to each era is not the effect the reform effort had on the curriculum, but the effect it had on the mathematics education community, particularly the part of the community at institutions of higher education. The establishment and rejuvenation of the field of mathematics education could be seen as either remarkable accomplishments of selfless reformers or selfish outcomes that came at the expense of the teachers and students the movements reflects the fundamental paradox of educational reform: That is, reformers are in the position of having to believe and act as though they have the final answers for questions that can have no final answers.

The limits of educational reform have been explained in a variety of ways. Schools operate under severe structural constraints on what they can accomplish (Apple, 1979; Cohen, 1988; Cuban, 1990; Erickson, 1989; Kliebard, 1986; Lortie, 1975; Stephens, 1967).

The constraints work their way through, for example, state departments of education and the testing that is part of accountability within bureaucracies, as well as through the textbooks and test publication industries. Although specific mechanisms for constraining reform have developed considerably during the 20th century, the second reform movement did not differ from the first in its major consequences:

little effect on the intended audience, significant effect on the reformers themselves. The source of the problem is deeper than such mechanisms. The problem tying the two eras together and underlying all constraints on reform is that curriculum questions are moral and ethical, while reform efforts have had a technical character.

The essence of curriculum is the struggle to answer the question of what we should teach. "If human mental capacities were so great that everything could be know or if the amount of knowledge available were so small that it could be know to everyone, then would have no need for the study of curriculum" (Kliebard, 1977, pp. 2-3). But human mental capacities are not so great, and available knowledge is not small. Because the curriculum always represents a selected sample from an almost unlimited universe of knowledge, it will always grant a privileged status to some knowledge over other knowledge. The main questions that guide work in the curriculum field – What should we teach? Why should we teach one thing rather than another? Who should have access to what knowledge? – are, then, fundamentally moral and ethical questions. Decisions must be made about what knowledge is so valuable that it is worthy of being explicitly taught to children in schools.

The point is not that curriculum reform is inevitably doomed to failure because the questions are moral and ethical ones. The point is that reformers are doomed to failure who either neglect value dilemmas or assume that empirical evidence of one sort or another is sufficient to justify reform. Changes in society, in the economy, in the school population, or in the field of mathematics may be sufficient cause to reconsider what we teach to children in schools, but such changes do not in and of themselves justify particular decisions.

Value dilemmas make curriculum reform difficult; a failure to get people from the appropriate constituencies involved in decision making may make it impossible. The two fault lines in mathematics education were clearly linked by the passive role in which they cast teachers and students. It is not at all surprising that the reform movements had their most significant effect on the university mathematics educators who were most involved in making decisions. Viewing curriculum reform as a technical rather than a moral and ethical process causes reformers to neglect not just the basic questions but also the people who should be involved in answering them. Teachers, for example, may not be especially able to confront value dilemmas. They can be as stupid and short-sighted as anyone else. Their involvement is nonetheless essential.

Conclusion

The history of school mathematics in the United States during the 20th century shows that changes have occurred in the precollege curriculum. But when compared to the intended outcomes of the unified and modern mathematics movements, those changes have been limited.

The mathematics education community has traditionally acted from and defended a fundamental faith in the importance of mathematics for everyone. According to this faith, more people should study more mathematics in school, for their own sakes as well as the sake of our society. Ironically, the same faith that established and sustained the community gets in the way of seeing and dealing with the important differences that divide it and that hamper curriculum reform. Competing visions – that is, competing answers to the questions of what we should teach, why we should teach one thing rather than another, and who should have access to what knowledge – can be healthy, but only if the are recognized and dealt with. It is naïve, moreover, to assume that wide-raining reform in school mathematics will result from any effort that focuses only on schools and is not somehow linked to reform of the wider society.

The community of mathematics educators was born of arrogance and defensiveness. It need to acknowledge and overcome both the limitations of its history and the paradox of curriculum reform. Genuine reform requires a counting struggle with the moral and ethical questions that constitute curriculum discourse.

References

AHLFORS, L. V. *et alii* (1962). On the mathematics curriculum of the high school. *Mathematics Teacher,* 55, 191-194; *American Mathematical Monthly,* 69, 189-193.

APPLE, M. W. (1979). *Ideology and curriculum.* London, Routledge & Kegan Paul.

AUSTIN, C. M. (1921). The National Council of Teachers of Mathematics. *Mathematics Teacher*, 14, 1-4.

BEGLE, E. G. (1968). SMSG: The first decade. *Mathematics Teacher*, 61, 239-245.

BETZ, W. (1908). The teaching of geometry in its relation to the present educational trend. *School Science and Mathematics,* 8, 625-633.

BRESLICH, E. R. (1950). Mathematics. In W.G. Carnahan (Ed.), *A half century of science and mathematics teaching* (pp. 39-79). Oak Park, IL, Central Association of Science and Mathematics Teachers.

BUTTS, R. F. (1974). *A cultural history of education.* New York, McGraw-Hill

CAJORI, F. (1890). *The teaching and history of mathematics in the United States* (Bureau of Education Circular of Information No. 3). Washington, DC, Government Printing Office.

COHEN, D. K. (1988). *Teaching practice: Plus ça change…*(Issue Paper 88-30). East Lansing, Michigan State University, National Center for Research on Teacher Education.

COLLEGE ENTRANCE EXAMINATION BOARD, Commission on Mathematics. (1959). *Program for college preparatory mathematics.* New York, CEEB.

CUBAN, L. (1990). Reforming again, again, and again. *Educational Researcher,* 19 (1), 3-13.

ERICKSON, F. (1989, March). *Structure and agency in education, culture, and society.* Division G vice-presidential address at the annual meeting of the American Educational Research Association, San Francisco.

HALL, G. S. (1904). *Adolescence: Volume II.* New York, D. Appleton.

HOWSON, G., KEITEL, C., & KILPATRICK, J. (1981). *Curriculum development in mathematics.* Cambridge, Cambridge University Press.

JAMES, T., & TYACK, D. (1983). Learning from past efforts to reform to high school. *Phi Delta Kappan,* 66, 400-406.

JONES, P. S. (1970). Epilogue: Summary abd forecast. In JONES, P. S. (ed.), *A history of mathematics education in the United States and Canada* (32nd Yearbook of the National Council of Teachers of Mathematics, pp. 451-465). Washington, DC, NCTM.

JONES, P. S., & COXFORD, A. F., Jr. (1970). Mathematics in the evolving schools. In JONES, P. S. (ed.), pp. 9-89. *A history of mathematics education in the United States and Canada* (32nd Yearbook of the National Council of Teachers of Mathematics, pp. 9-89). Washington, DC, NCTM.

KILPATRICK, J. (1992). A history of research in mathematics education. In. GROUWS, D. A. (ed.), *Handbook of research on mathematics teaching and learning.* New York, Macmillan.

KLIEBARD, H. M. (1977, April). *Problems of definition in curriculum.* Paper presented at the annual meeting of the American Educational Research Association, New York.

KLIEBARD, H.M. (1986). *The struggle for the American curriculum: 1893-1958.* Boston, Routledge & Kegan Paul.

KLINE, M. (1973). *Why Johnny can't add: the failure of new math.* New York, St. Martin's.

LORTIE, D.C. (1975). *Schoolteacher: A sociological study.* Chicago, University of Chicago Press. Mathematics in the high school. (1921). *School Review.* 21, 641-646. National Advisory Committee on Mathematical Education. (1975). *Overview and analysis of school mathematics: Grades K-12.* Reston, VA: National Council of Teachers of Mathematics.

PARSHALL, K.H. (1984). Eliakim Hasting Moore and the founding of a mathematical community in America, 1892-1902. *Annals of Science,* 41, 313-333.

SCHUBRING, G. (1988, February). *The cross-cultural "transmission" of concepts – The first international mathematical curricular reform around 1900, with an appendix on the biography of F. Klein* (Occasional paper No. 92, corrected ed.). Bielefeld, Germany, University of Bielefeld, Institute for Didactics of Mathematics.

SENK, S. L. (1981). *The development of a unified mathematics curriculum in the University High School*: 1903-1923. Unpublished manuscript, University of Chicago, Department of Education.

SIGURDSON, S. E. (1962). The development of the idea of unified mathematics in the secondary school curriculum: 1890-1930. (Doctoral dissertation, University of Wisconsin-Madison, 1962). *Dissertation Abstracts International*, 23, 1997.

STANIC, G. M. A. (1984). Why teach mathematics? A historical study of the justification question. (Doctoral dissertation, University of Wisconsin-Madison, 1983). *Dissertation Abstracts International*, 44, 2347A.

STANIC, G. M. A (1986). The growing crisis in mathematics education in the early twentieth century. *Journal for Research in Mathematics Education*, 17, 190-205.

STEPHENS, J.M. (1967). *The process of schooling: A psychological examination.* New York, Holt, Rineahart & Winston.

WHEELER, D. (1989). Contexts for research on the teaching and learning of algebra. In WAGNER, S. & KIERAN, C. (eds). *Research on the teaching and learning of algebra* (Research Agenda for Mathematics Education, Vol. 4, pp. 278-287). Hillsdale, NJ, Erlbaum; Reston, VA, National Council of Teachers of Mathematics.

WOJCIECHOWSKA, A. (1989). Curriculum reform in mathematics: Beyond the impossible revolution? *Journal of Curriculum studies*, 21, 151-159.

Recebido em fev./2004; aprovado em mar./2005