Setting a new curriculum in a classroom: variability and space of freedom for a teacher

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Abstract

This paper presents an analysis of teachers' activity according to a double point of view: an institutional analysis in terms of constraints and space of freedom, and an analysis of classroom practices in terms of mathematical and didactical organizations, and in terms of a study of teachers' discourse.

Key-words: arithmetic; teachers' practices; teacher's space of freedom; ecology and praxeology.

Resumo

Este artigo apresenta uma análise de atividades de professores sob um dublo ponto de vista: uma análise institucional, em termos de exigências e espaço de liberdade, e uma análise de práticas de sala de aula, em termos da organização matemática e didática e em termos do estudo do discurso de professores.

Palavras-chave: *aritmética; prática docente; espaço de liberdade do professor; ecologia e praxiologia.*

Introduction

Our research work studies teachers' activity, in the context of the teaching of arithmetic[1] in the class of "terminale S spécialité mathématiques" (17-18 year-old pupils) in France. In this paper we will briefly present the main results of the first stage of our work, consisting of an institutional analysis, based on a study of curricula, textbooks and a questionnaire for teachers. These enabled us to explicate the constraints a teacher faces and the space of freedom left for him [or her] when preparing a lesson. With this information in hand, we then studied the variability of teachers' practices. Our methodology is based on a naturalistic observation

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1 This corresponds to elementary number theory or abstract algebra.

of two teachers and includes a comparative analysis of exercises given by each of those teachers and an analysis of their discourse. We used several theoretical frameworks, that we will present in this paper. This stage of our work is still in process; in this paper we will present the general framework and the first results along with some brief concluding remarks.

TEACHERS' constraints and space of freedom: institutional analysis

We refer to the teacher's activity model in terms of institutional constraints and space of freedom as proposed by Coulange to analyse teacher's activity when preparing a course:

> We consider that (mathematics) teachers' activity is submitted to constraints both generic (related to general didactical settings) and specific (linked to epistemological aspects of the mathematics to be taught and to the organisation of official curriculum). These constraints result partly from the fact that a teacher belongs to several different institutions. Nevertheless, there remains within this set of institutional constraints a space of freedom for a teacher. (Coulange 2001, p.66, our translation)

Clearly when teaching any mathematical concept a teacher makes certain choices and decisions. In our research we address the following questions:

- What are the choices available to a teacher when setting a new arithmetic course in his [or her] classroom?
- What system of constraints does a teacher face when making his [or her] choices for the arithmetic course?
- What is a teacher's space of freedom? To what extent do all teachers "invest" this space of freedom, left by the institution, in the same way or not?

To answer these questions, we wanted to know how teachers react and place themselves in relation to official curricula[2] and textbooks. Indeed, a curriculum specifies the concepts that teachers have to teach

2 In France, for curricula, there is a stated policy objective by the Ministry of Education.

when new objects of knowledge are introduced, but curricula and textbooks are neither monolithic nor exhaustive; there is still room for interpretation within the institutional constraints they impose. So we undertook a comparative analysis of the new curriculum in arithmetic (1998) and the previous one (1971) and analysis in terms of ecology and praxeology (Chevallard, 1991; Artaud, 1997) of four textbooks.

Before discussing the constraints faced by the teachers, we note that arithmetic was re-introduced to emphasise the areas of algorithms and algorithmic reasoning. However, we found little evidence of this orientation in the four textbooks we studied. On the other hand, teachers have an important state of freedom to organize a course that takes into account the algorithmic aspect of arithmetic. Indeed, they have at least three possible ways to take it into account:

- give constructive proofs of arithmetical theorems,
- integrate programming and the use of computers into their courses,
- propose exercises in which an algorithm is either the subject of the study or an efficient tool for solving problems.

We designed a questionnaire which focused on the choices that the teachers made when planning classroom activities and sequences of lessons. The questionnaire was divided into four areas: questions about the materials and sources teachers use, questions about the proofs[3] of arithmetical theorems that teachers choose to present in their classroom, questions about greatest common divisor (gcd) [4] and questions about the use of calculators and computers in the classroom. The analyses in this section are based upon the 43 questionnaires we received. Teachers' answers suggest that they do not emphasise algorithmic aspect of arithmetic. Some teachers did consult sources emphasising algorithmic aspects of arithmetic and some did frequently use constructive proofs on their teaching. However, very few introduced the gcd in an algorithmic way or made any use of calculators and computers during these arithmetic courses.

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3 For example, questions directed at whether they preferred very "theoretical" proofs or more constructive ones.

4 For example, one focus was on whether teachers favour the use of the Euclidian algorithm to calculate the gcd of two-integers?

So, our hypothesis according to which integrating programming and the use of computers into arithmetic courses can be a mean to take into account the curriculum's orientation is invalidated by classroom's pratices. Moreover, our analysis suggests that teachers prefer to focus on the formal reasoning aspects of arithmetic rather than the algorithmic one. Indeed, proofs and exercises in arithmetic give students many opportunities to meet all kind of reasoning types including: exhaustive proof, reductio ad absurdum, consideration of all possible cases, proof by induction, and necessary and sufficient conditions, etc.

This change in orientation carried out by the teachers can be explained by several constraints and "ideological" choices that teachers have to deal with when they plan their course in arithmetic:

– Strong institutional constraints, for example: limited lesson time, exam preparation, limited access to computers. In short, although teachers may themselves want to make certain choice, these constraints may prevent them from doing so.

– Teachers think that they lack the training and skills to use computers in their classroom and they have to deal with students with very diverse programming abilities.

– Lack of resources in the institution of programming exercises.

– Teachers' conceptions of mathematics make them favour the reasoning more than the algorithmic aspect of arithmetic. The two following quotations highlight these "ideological" choices: "*I don't have a particular passion for computing tools and I always prefer what is obtained by thinking and reasoning without the necessary use of an 'heavy' machinery.*" and "*As for me, I insist on proof and reasoning. I use arithmetic for teaching them the rudiments of reasoning.*"

– Teachers' own perceptions of the needs and requirements of "terminale S spécialité mathématiques" students: students in this type of class are very likely to pursue mathematics orientated courses at university. As a result teachers feel that they must prepare them to these scientific studies by insisting on the reasoning aspect of mathematics for which arithmetic is a privileged subject.

These constraints allow us to understand why, on the one hand, the algorithmic aspect of arithmetic is not developed in actual teaching and why, on the other hand, the reasoning aspect is privileged. Here, institutional constraints and teachers' conceptions of mathematics and representations of their students run counter to the official orientation of the curriculum. Teachers permit themselves with a wide margin of freedom within the curriculum.

In this first stage of our research, our analyses focused on the preparation of a course. However, students' behaviour and several other parameters (which influence the course as it is taught in the classroom) must be taken into account when studying the teacher activity:

> Between general learning targets and everyday necessity, the teacher achieves a difficult balance that does not involve only students and is source of significant variation. (Robert 2001, p. 60, our translation)

Variability of teaching: analysis of practices in the classroom

In the previous part, we have seen that teachers create for themselves a wide margin of freedom within the curriculum to build their arithmetic course. To analyse the variability of practices in the classroom, we have then orientated our study towards analyses of specific cases. Our analyses are based upon a naturalistic observation of two teachers (P1 and P2) during one year[5]. Lessons were recorded on tape and each teacher was interviewed at the end of the year. We have chosen to take into account two aspects of the teaching: the mathematical contents introduced in the class and the discourse of the teacher in front of the pupils. Moreover, as an illustration, we present the analysis of the lesson about the Euclidean division[6].

Mathematical contents

To analyse how P1 and P2 introduce the Euclidean division in their classroom, we use the anthropological approach (Chevallard, 1999) and more precisely the notion of mathematical organisation (Matheron, 2000 and Bosch, 2002). P1 and P2's "problem" is to teach the Euclidean

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5 For P1: 13 lessons were observed (26 hours). For P2: 11 lessons were observed (22 hours).

6 Euclidean division or the division algorithm. Given $a \in \mathbf{Z}$ and $b \in \mathbf{N}^*$, there exists a unique pair (q,r), $q \in \mathbf{Z}$, $r \in N$ and $0 \leq r < b$ such that $a = bq + r$.

division to students. To solve this "problem", they set up a particular mathematical organisation (MO)[7].

In this paper, we will limit our analyses to the types of mathematical tasks that teachers propose to students during the lesson about the Euclidean division. The comparison of the types of mathematical tasks (T) proposed by P1 and P2, is summarised in the following table:

P1	P2
T_1: Calculate the quotient and the remainder of a given division.	**T_1**: Calculate the quotient and the rest of a given division.
T_2: Given {a,b,q,r}. Find 1 or 2 of these elements when knowing relations between the others.	
T_3: Prove that any n can be written: $n=\alpha k+\beta$ with $0\leq\beta\leq k\text{-}1$	**T_3**: Prove that any n can be written: $n=\alpha k+\beta$ with $0\leq\beta\leq k\text{-}1$
T_4: Find r when it is not possible directly compute the Euclidean division. Ex: What is the remainder of the Euclidean of n(n-3) by 5	**T_4**: Find r when it is not possible compute the Euclidean division.
T_5: Prove that b divides N(n) by distinguishing all possible remainders in the division of n by b. Ex: Prove that for every $a\in Z$, $a(a^2\text{-}1)$ is a multiple of 6	**T_5**: Prove that b divides N(n) by distinguishing all possible rests in the division of n by b.
T_{5bis}: … by using operations on remainders. Ex: Prove that for every n, $3^{n+6}\text{-}3^n$ is divisible by 7	**T_{5bis}**: … by using operations on rests.

If we focus only on the types of tasks given by the two teachers, the table shows very strong similarities. In fact the only significant difference is that T_2 is missing for P2.

The description of teacher's activity in the classroom through the MO that he proposes is interesting in order to characterise the nature of the mathematics presented to students. But such a description is not sufficient to understand the complexity of each teacher's activity. Indeed,

7 $MO=[T/\tau/\theta/\Theta]$ where T is a mathematical type of tasks, τ the technique to solve T, θ the technology that justifies the use of τ and Θ the theory (the justification of θ).

to comprehend teacher's didactical choices to teach a MO, it is necessary to analyse the way the MO is actually presented: the detail of actual tasks, when tasks are given, etc. If we distinguish the exercises from the tests used in the evaluations, we obtain a different table:

Exercises	
P1	P2
T_1	T_1
T_2	
T_3	
T_4	T_4
T_5	

Tests	
P1	P2
T_1	
T_2	
	T_3
T_4	
T_5	T_5
$T_{5\ \mathrm{bis}}$	$T_{5\ \mathrm{bis}}$

It illustrates clearly that even if P1 and P2 made similar mathematical choices, their didactical choices are radically different: P1's evaluation is based exclusively on types of tasks that have already been seen in exercises while P2's is only based on new type of tasks. These choices are very likely to have an influence on students' learning.

Analysing teacher's activity in this way is a particular means in order to point out the importance of the mathematical and didactical choices made by teachers.

Teacher's discourse

In this part, we have used Hache's methodology. Hache (1999) analyses teacher's discourse according to three axes: the **object** of the discourse[8] (what is the teacher talking about), the **tenor** of the discourse (in which terms does s/he express him/herself) and the **function** of the discourse (what is the aim of the discourse).

To analyse P1 and P2's object of the discourse, we used the same three dimensions as defined by Hache that we adapted according to the specificity of our study which concerns lectures while Hache analysed more interactive sessions. We first coded P1 and P2's discourses distinguishing the three following dimensions: **contextual** discourse (when the discourse

8 In this paper, we will only present our work on the object of the discourse.

evokes mathematics that are linked with a situation, an exercise or when an example is given as a help to understand a proof), **non-contextual** discourse (when the mathematics at stake are very general or theoretical discourse), **link** discourse (discourse that evokes contextual mathematics to illustrate a non-contextual discourse or vice-versa). The results are summarised in the following table:

	P1	P2
Contextual	35 %	11 %
Non-contextual	50 %	83 %
Link	15 %	6 %

It shows that P2's discourse is predominantly non-contextual and that P1's one has a more significant contextual/non-contextual dynamic. The explanation of this phenomenon is that P2, makes only small digressions from the formal proof of Euclidean division whereas P1, at the beginning of the lesson, clearly expresses her willingness to use numerical examples when working on the Euclidean division's proof.

As we can see, P1's contextual/non-contextual dynamic is due to a particular choice:

> So we'll always have a real time work [numerical example] beside in order to help you understand what we are doing [the proof].
> (link discourse, extract of P1's discourse, our translation)

Because of this choice, she makes explicit connections between the numerical examples and the theoretical proof.

As for P2, she changes her non-contextual discourse for a contextual one when students do not manage to answer her questions. To make them find the answer, she lowers her demands and ask less theoretical questions:

> P2: In other words, q is the greatest whole number which is not more than a/b. P2: Yes or no?
> Student: Yes
> P2: Oh, you know it, what's its name?
> Students: ...
> Student (after a time): The integral function...
> P2: Nearly... The...
> Student: f(x)

Students: ...
P2 (after a time): What is the greatest whole number which is not more than 3.5?
Student: 4
P2: Less?
Student: euh, 3
P2: To 2.7?
Student: 2
P2: So, what is 2 for 2.7? What is 3 for 3.5?
Student: a lower approximate value
P2: Yes, a lower approximate value to the unit. What else?
Students: the integral part
P2: It's the integral part. So q is what we call the integral part of a/b. Do you know the notation?
(*link discourse*, contextual discourse, non-contextual discourse, extract of P2's discourse, our translation)

In the previous example, P2 thought that most of the students knew what the integral part of a number is. In the event this was obviously not the case. The change in her discourse can be seen as an adaptation to a disturbance which was not anticipated in her course's planning. So, during the lesson (the example that we have chosen is representative of all lessons), P2's discourse becomes contextual when the expected elements of knowledge are not produced by the students.

Consequently, this analysis of teachers' discourse emphasises the fact that the variability of teacher's activity during a lecture does not only depend on the mathematical choices that teachers make for the proof of a theorem.

Conclusion

In this paper, the research work that we have exposed studies teacher's activity according to a double point of view: an institutional analysis and an analysis of classroom's practices.

We hope that this brief account of our work show **the interest of analysing teacher's activity without separating the "mathematical" and the "didactical" aspects**. Indeed, at every level (institutional constraints and space of freedom, course's project, practice in the classroom), they are influencing one another. Despite this insight, teacher's activity remains extremely complex to analyse and understand. Hence, it is necessary to carry on studying this activity from every angle in order to understand it in order to improve strategies for teacher's education.

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