

Reference epistemological model: what form and function in school institutions?

Modelo epistemológico de referencia: ¿qué forma y función en las instituciones escolares?

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Resumen

Muchos problemas docentes están relacionados con la ausencia, en las instituciones escolares, de herramientas epistemológicas para diseñar, gestionar y evaluar procesos de estudio. Proponemos incluir entre dichas herramientas los mapas de cuestiones y respuestas que constituyen una representación parcial del modelo epistemológico de referencia. En este artículo resumimos las conclusiones de cuatro estudios experimentales que incluyen formación del profesorado y recorridos de estudio e investigación. Los resultados muestran las potencialidades de los mapas de cuestiones y respuestas y las nuevas cuestiones que se abren.

Palabras clave: Teoría antropológica de la didáctica, Modelo epistemológica de referencia.

Abstract

Many teaching problems are related with the lack in the school institutions of epistemological tools to design, manage and evaluate study processes. We propose to

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include within these tools the question-answer maps as a partial representation of the reference epistemological models. In this paper we summarize the conclusions of four experiences including teacher education courses and study and research paths. The results show the potentialities of question – answer maps and new open questions

Keywords: Anthropological Theory of Didactics, Epistemological Reference Model.

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The representation for reference epistemological models

The need for establishing explicit models of school mathematical activity is at the foundations of the constitution of the didactics of mathematics as a field of research (Brousseau, 1972). The specific form of this model in the Theory of Didactic Situations is *the fundamental situation*, defined as the minimal set of adidactical situations that allow to generate, by manipulation of the values of its didactic variables, a big enough field of problems to provide a good representation of the targeted knowledge.

The Anthropological Theory of the Didactic (ATD) and its “didactic anthropology of mathematics” (Chevallard, 1990) entails not only an *anthropologization* of mathematics but a *didactization* of the epistemology of mathematics (Gascon, 1998). This mutual enrichment is stated by Chevallard (1985, p. 59, our translation) as follows: “the genesis and the development of mathematical knowledge cannot be detached from the communication, the use and the institutional transposition of such knowledge”. Gascón (2001, p. 155, our translation) describes this new paradigm as a twofold challenge. On the one hand, a challenge for didactics “assuming the responsibility to propose new epistemological models of mathematical knowledge” and, on the other hand, for epistemology “having the need to use as an essential part of its empirical basis, the facts produced in didactic systems”.

Since the decade of the 1990, ATD has been enriched with notions and methodologies such as *praxeology*, *praxeological analysis institutional relations to knowledge and levels of codetermination*. Among them, the notion of Reference Epistemological Model (REM) or Reference Praxeological Model (RPM) was proposed to assume in a certain way the role of fundamental situation in TDS. The use of this model has many examples: García (2005) developed a REM about proportionality and Sierra (2007) about numeration systems. These examples led research within the ATD to

develop whole research projects about elementary algebra and functional modelling of discrete and continuous systems (Barquero, 2009; Lucas, 2015; Ruiz-Munzón, 2010).

The way REMs are presented in the cited works is diverse. For example, Sierra (2007) presents a REM as a sequence of praxeologies of increasing complexity, the limitations of one praxeology leading to the emergence of a more complete praxeology (Figure 1). Ruíz-Munzón (2010) uses a similar structure when presenting a REM in the field of elementary algebra: in her work three levels of algebrisation are defined and, again, the limitations of the lower levels lead to the following levels (Figure 2). In contrast to this representation, Barquero (2009) and Lucas (2015) present the REM as an arborescence of questions and answers initiated by a generating question (Q_0), taking the notion of Herbartian schema as standing point.

Figure 1

Reference Praxeological Model for numerical systems. Starting with additive systems followed by additive-multiplicative systems and finishing with positional systems (Sierra, 2007)

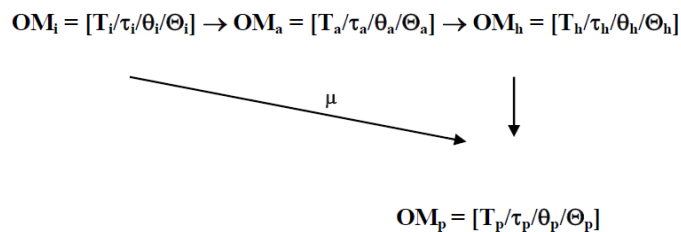
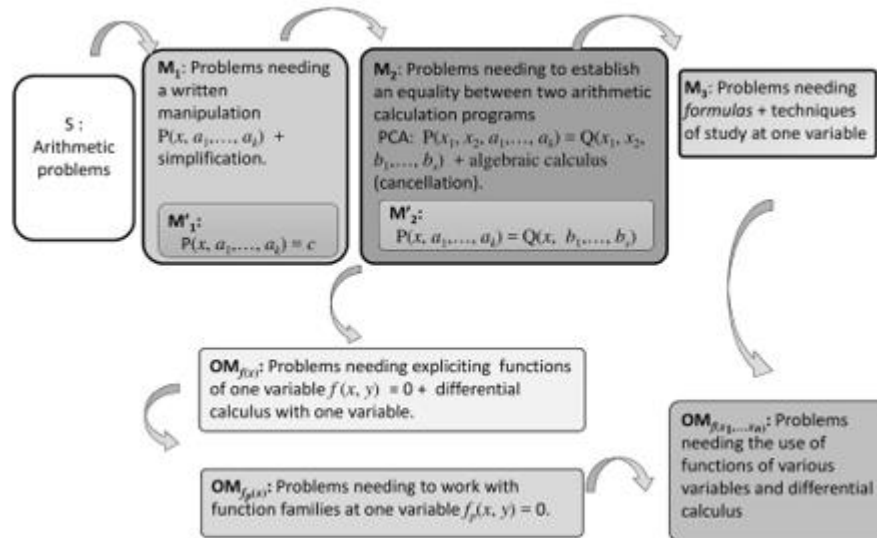


Figure 2

Reference Epistemological Model proposed in Ruiz-Munzon (2010) about elementary algebra (Ruiz-Munzon, 2010)



The use of these models is indispensable in didactic research within the ATD framework. The model acts as a necessary emancipating tool (Gascón, 2014) enabling the researcher to detach from the school and the scholar institutions and to propose explicit alternative models for the knowledge to be taught. In consequence, the REM plays a crucial role in the analysis of didactic transpositive processes, the study of didactic phenomena and the design of new processes of study.

Epistemological tools at the school institution

These new study processes (that in the ATD framework usually take the form of a Study and Research Path, SRP) will be experienced by a specific set of teachers and students of a specific institution. This fact leads to a problematic situation, already stated by Florensa, Bosch and Gascón (2015, p. 2640):

The most remarkable feature is the shortage and inadequacy of tools available in the teaching institution to describe, manage, and evaluate the dynamics of mathematical activity. This lack of tools could in the first place be attributed to the scarcity of spontaneous epistemological models (...).

Consequently, the following research questions are formulated:

Which new notions or tools are needed to describe and manage the dynamics of the mathematical activity that will take place in study processes? How to describe these tools depending on the role addressed (didactic researcher, teacher and students)? How to make them available in the teaching institution and to the participants of the didactic process?” (Ibid.)

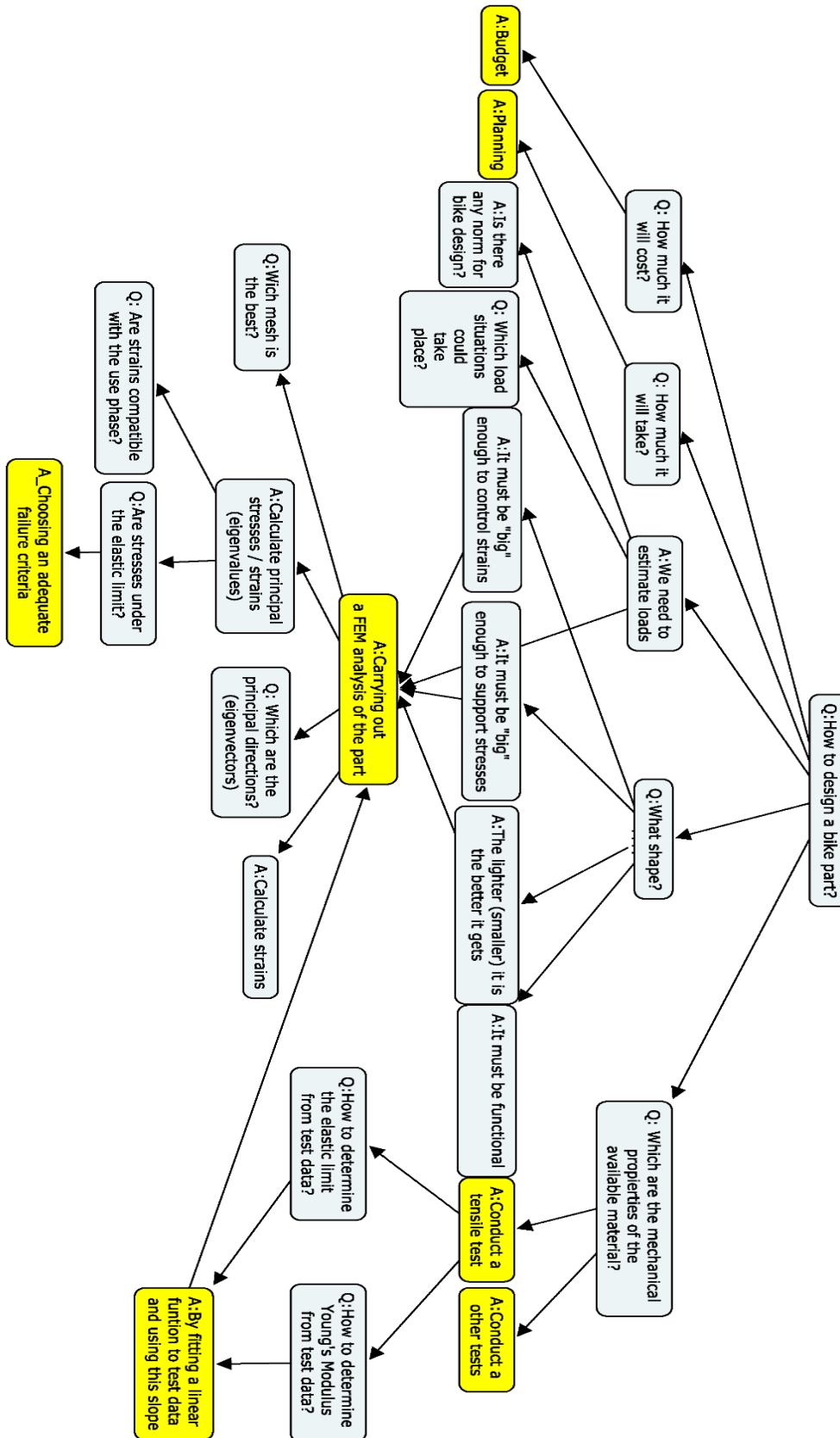
Winsløw, Matheron and Mercier (2013) present the question-answer maps (Q-A maps) as a tool to model “mathematical knowledge from a didactical perspective”. In addition, they hypothesize that “such a representation is sufficiently close to teachers’ concerns, and also captures such essential parts of a didactic design, that one could use it as a tool for collaboration and communication with and among teachers, regarding concrete teaching designs” (p. 281). As said before, the Q-A maps have already played the role to materialise a REM (Barquero, 2009; Lucas, 2015). We hypothesize that Q-A maps will enrich the epistemological tools available at scholar institutions.

We consider that these new tools to be provided to teachers and students should empower them to (1) describe the knowledge involved during a study process overcoming the limitations of the previous conception of knowledge, (2) contrast the new study process with the previous one specially in terms of responsibilities assumed and richness of the media and the milieu and (3) make explicit the *raison d’être* of the knowledge to be taught. In fact, these aspects are, among others, expected to be developed by the REM.

In order to validate this hypothesis we have developed four empirical studies: (1) a secondary teacher education course (Florensa, Bosch, & Gascón, 2016a), (2) the design, experimentation and analysis of an SRP in General Elasticity (Florensa, Bosch, Gascón, & Mata, 2016), (3) a lecturer’s course on didactics (Florensa, Bosch, & Gascón, 2016b), and (4) the design, experimentation and analysis on an SRP in strength of materials developed with one of the participants of the lecturers’ course. In all these experiences the role attributed to the Q-A maps is central both in the teacher education courses and in the experienced SRP with students.

Figure 3

A priori Q-A map for the SRP in Elasticity



Q-A maps: a crucial epistemological tool

Regarding the description of knowledge, in both experienced SRP, Q-A maps have been the tool that have helped teachers and students to describe the development of the SRP. One of the maps produced by the lecturers as tool for the a priori analysis of the SRP is presented in Figure 3. The maps have been used to assign tasks to parts of the community of study and to plan the weekly activity. The maps also have helped students to identify one of the possible *raison d'être* of the knowledge at stake. A very illustrating example of these phenomena is the statement of one of the students participating in an SRP in an interview after the SRP. The student stated: “Q-A maps in the weekly reports were useful: iterations appeared there, we saw where we were progressing.” Also during the interviews, one of the lecturers managing the SRP stated: “implementing the SRP and using the Q-A maps has changed how I teach the course (...) the SRP generating question has become the *raison d'être* of the taught knowledge. Now I feel that my teaching task has a rationale.”

The Q-A maps also played an important role when introducing the media-milieu dialectics to in-service teachers. The notion was presented in order to highlight the scarceness of objects in the media and the milieu in the scholar institution and to compare them to an experienced SRP. The Q-A map enabled them to describe for each question which objects were used to generate a specific answer and how and why did the community of study accepted the answer as a correct one. This capacity of the Q-A maps to analyze and highlight the individual-collective dialectic was stated by Bosch (2015).

We conclude that these four experiences reveal that Q-A maps used by teachers and students can empower them to develop tasks that usually are absent from scholar institutions but that are crucial to manage an SRP. We consider that these maps fulfil, at least partially

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