

## **Research and study paths: Indicators of the development of the dialectics**

### **Recorrido de estudio y de investigación: indicadores del desarrollo de la dialéctica**

Verónica Parra<sup>1</sup>

Núcleo de Investigación en Educación en Ciencia y Tecnología (NIECyT). Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN), Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)

<https://orcid.org/0000-0002-6956-0052>

María Rita Otero<sup>2</sup>

Núcleo de Investigación en Educación en Ciencia y Tecnología (NIECyT). Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN), Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)

<https://orcid.org/0000-0002-1682-9142>

### **Abstract**

In this work, we describe a set of didactic - mathematical indicators to determine the development of the dialectics. These indicators are not definitive and their first formulations are the result of the analysis of the data obtained when implementing a research and study paths (RSP) at the last year of the Argentine secondary level. From these indicators, we conclude that the most frequent dialectics are: individual and collective, subject and out-of-subject, research and study, praxeological analysis-synthesis/didactic synthesis-analysis and black boxes and clear boxes.

**Keywords:** Research and study paths, praxeological analysis-synthesis, ATD.

### **Resumen**

En este trabajo describimos un conjunto de indicadores didáctico-matemáticos para determinar el desarrollo de las dialécticas. Estos indicadores no son definitivos y sus primeras formulaciones son producto del análisis de los datos obtenidos al implementar un recorrido de estudio e investigación (REI) en el último año del nivel secundario argentino. A partir de estos

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<sup>1</sup> [vparra@exa.unicen.edu.ar](mailto:vparra@exa.unicen.edu.ar)

<sup>2</sup> [rotero@exa.unicen.edu.ar](mailto:rotero@exa.unicen.edu.ar)

indicadores, concluimos que las dialécticas más frecuentes son: individuo y del colectivo, tema y fuera-de-tema, estudio e investigación, análisis-síntesis praxeológica/análisis-síntesis didáctica y *cajas negras y cajas claras*.

**Palabras-clave:** Rutas de investigación y estudio, análisis-síntesis praxeológico, TAD.

### **Résumé**

Cet article décrit un ensemble d'indicateurs didactique-mathématiques pour décider le déroulement des dialectiques. Ces indicateurs ne sont pas définitifs et ses premières formulations ont été produit à partir de l'analyse de données obtenus en la mise en place d'un parcours d'étude et recherche (PER) à l'école secondaire argentin. A partir de ces indicateurs, on conclut que les dialectiques les plus fréquentes sont : de l'individu et du collectif ; du sujet et du hors sujet ; de l'étude et de la recherche ; de l'analyse (et la synthèse) praxéologique et de l'analyse (et la synthèse) didactique et des boîtes noires et des boîtes claires.

**Mots clés:** Chemins de recherche et d'étude, analyse-synthèse praxéologique, TAD.

### **Research and study paths: Indicators of the development of the dialectics**

Dialectics are key gestures to achieve a teaching by the research and study paths (SRP). According to Chevallard (2008) they allow “to pilot” a research teaching. But how do we determine that these gestures have been done during an implementation of a SRP? This work is aimed to show the point mentioned before. We address the problem of the “monuments of knowledge” with that specific objective: introduce a set of didactic-mathematical indicators of each of the dialectics. These indicators are not definitive and their first formulations are due to analysis of data obtained while designing, implementing and evaluating a SRP in the last level of secondary school from Argentina. This SRP go through hundreds of study themes of the official curricular: straight lines, limits, derivate and use of software.

Several authors have pointed out the importance of the dialectics (Barquero, Bosch & Gascón, 2011; Costa, Arlego & Otero, 2015; Hausberger, 2016), referring them as key gestures of a research teaching. In this work we will not center ourselves in describing in detail the dynamics of the process of SRC already implemented (Parra, Otero & Fanaro, 2015) but we will introduce a posterior phase of analysis: the construction and description of a group of didactic-mathematical indicators that will be used to establish the development of each of the dialectics. So, the research question of this work can be formulated by the following: How to determine from the results of the implementation of a SRC a group of didactic-mathematical indicators for each of the dialectics?

### **Methodology of Research**

The implementation of SRC took place during the last year of secondary school in Argentina. (Students from 16-17 years old) from the first school day, during 36 sessions of classes and the researcher was responsible for the group. On the first day of class the first question of SRC was asked. This decision was taken during the stage of design of SRC, so

there wasn't a previous training in the praxeology that would allow answers to the questions, so, the students would not know which mathematical notions would allow to provide answers

The researcher was also the teacher observing the participants, taking notes, registering the development of the class in audio and collecting the productions from the students. The generative questions are referred to microeconomics, specifically to the laws of the offer and demand of a market formed by the price of only one product. The students were distributed in groups. The teacher introduced initially the questions, each group of students had to give answers, communicate it to the rest of the members and defend it. The derivative questions formulated by the different groups of students were considered by the study community.

The path performed was designed considering certain relative hypothesis to the microeconomics, specifically the models of offer and demand:

H<sub>0</sub>: The market balance exists and it is possible to obtain.

H<sub>1</sub>: The market balance is produced when the offered quantity is equal to the demanded quantity, for a certain price.

H<sub>2</sub>: The function offers and the function demands are linear and both depend on the price of the unique product.

The questions given to the students under this hypothesis were the following:

Q<sub>1</sub>: Let's suppose that we will make a product and that our aim is to sell and collect money. The following information corresponds to information obtained in a previous test of sales (Table 1)

What model would allow to study the behavior of the offer and demand on this market?  
How to determine the price and the quantity so that the demand could satisfy at the same time that the offer doesn't have excess?

Q<sub>2</sub>: How to study the behavior of the law of offer and demand for any couple of linear functions? How to determine the point of balance in this case?

Q<sub>3</sub>: If the parameter “ordinate” of the model is modified: How to describe the variation of the point of balance?

Q<sub>4</sub>: If the parameter “gradient of a line” of the model is modified: How to describe the variation of the point of balance?

Table 1  
*Information obtained in a test on previous sales*

Price per unit (in \$ARG)	Amount of demand	Amount offered
10	300	
11		174
13	270	
14		231
23		402
24	160	
25		440
26	140	

Q<sub>5</sub>: How much does the point of balance change exactly in each case?

To answer these questions the group has to study microeconomics and mathematics. Frequently during the RSC, it was necessary to get out of the question to investigate and study different knowledge and then return to the question to finally elaborate an answer. The questions Q<sub>3</sub>, Q<sub>4</sub> and Q<sub>5</sub> correspond to the study of the variations in the point of balance after modifying the parameters of the model. The parameters were modified one by one because the official curriculum of the Secondary School Argentina prescribes only study of functions of an independent variable. The curriculum does not propose the study of functions of two or more independent variables. The students answered the questions to the variations of a parameter and at the same time build different modes calculating the point of balance (analytical way or using the software GeoGebra®) in each case. They described the variations and answered questions of the type: if it increases or diminishes one of the parameters “How does it change the balance?” The response to this question was qualitative (increase or decrease). Then, the

teacher proposed the following question: “Q<sub>5</sub>: How much does the point of balance change exactly in each case?”

Some derivative questions were the following one: What is a model of market? What is the point of balance? What is “the demand and the offer”? “How does the offer and demand behave?” These questions were answered by the students through internet search, books and asking the teacher of economy of the institution. Several groups of students not only worked on the characterization of the economic model, also researched about the factors that cause an increase or decrease of the demand; factors that cause an increase or decrease of the offer, among others. And of course, the question: “How to build a model? Here, there is a way out to the scope of microeconomics, but also to the area of mathematics. In order to build this model, it is necessary to study how to build the equation of a line, that contains two points or more, how to solve a system of two linear equations with two unknown quantities and to represent that model or situation un a system of these components.

In order to answer the derivative questions of the mathematics, the teacher acted in some cases as a source of information as well. For example, we reminded the students on this model because it is linear, and therefore, this one might behave as the linear functions that they had studied before. Here it was needed to “exit” in order to study linear functions and the resolution of an equation system with two unknown quantities. Once researched and studied they had to get back to the initial question and build an acceptable answer, at least for the study community (students and teachers).

The study of the questions referred to the variations that were developed during several classes, until the concept of derivative functions as a useful tool to describe reason of change between two variables. This required the study of the limits of the functions to define the derivative of a function, a new exit or way out of the theme. The questions were the following: Which is the “intuitive idea” of limit? Does the limit of a function exist always? Can function

have two different limits? Which are the properties of the limit? Which are the infinite limits? Which are the limits in the infinite? How many indeterminations can we find? How can they be “saved”?

For the analysis of the data, table 2 was created. In the first column the class number is placed (from class 1 to 36), the second column has the studied questions; the third one divided in 9 sub columns contains each dialectic. Number 1 is written to indicate that that dialectics was identified and 0 when it has not been identified. Finally, the fourth column contains the gestures indicative of the corresponding dialectic.

Table 2

*Table generated by 0 and 1*

Session N°	Questions	Dialectics (D)									Indicator
		E-I	I-C	ASP-ASD	T-FDT	P-T	CN-CC	M-M	L-E	D-R	

The initials of Table 2 belong to each of the dialectics:  $D_{E-I}$ : research and study;  $D_{I-C}$ : individual and collective;  $D_{ASP-ASD}$ : praxeological analysis-synthesis/didactic synthesis-analysis;  $D_{T-FDT}$ : subject and out-of-subject;  $D_{P-T}$ : skydiver and truffles;  $D_{CN-CC}$ : black boxes and clear boxes;  $D_{M-M}$ : media-milieus;  $D_{L-E}$ : reading and writing; and  $D_{D-R}$ : diffusion and reception.

### **Main results: Didactic-Mathematical indicators of the dialectics**

**$D_{E-I}$ . Dialectic of research and study:** we identified this dialectic when it appears at any moment during the class:

**$I_{DE-I}$ :** A search on the internet, books of different disciplines, consultation to teachers of different disciplines, consultation to different professionals and any other search in different medias who are not the teacher. For example, in this case the search on internet or in math books and microeconomy.

**I<sub>2DE-1</sub>**: A study of response  $R_i^\diamond$ , such as, the study of available answers, the works  $O_j$  that are useful in the building of the answer to the general question or its derivatives. For example:

- $R_1^\diamond$ : OMat on lineal function.
- $R_2^\diamond$ : OMat on parallel straight lines and perpendicular straight lines.
- $R_3^\diamond$ : OMat on two lineal equation systems with two unknown quantity.
- $R_4^\diamond$ : OMicr on the models of offer and demand.
- $O_5$ : OMicr on the displacement of the offer and demand curves.

**I<sub>3DE-1</sub>**: The formulation of the derivatives questions in the different groups and search for answers. For example:

- $Q_{ME1}$ : What is a model of offer and demand?
- $Q_{ME2}$ : What is the function of the offer?
- $Q_{ME3}$ : How does the function of demand behave?
- $Q_{ME4}$ : What is the point of balance in microeconomy?
- $Q_{M5}$ : How do we represent a group of data in a cartesian coordinate system?

**D<sub>I-C</sub>**. **Dialectic of the individual and collective**: We identify this dialectic when the following actions are identified.

**I<sub>1DI-C</sub>**: A group decision taken by the students, for example: to agree in a model (if the amounts offered and demands depends on the price or if the price depends of the amount offered and demanded)

**I<sub>2DI-C</sub>**: A member mentions that the production made is not his but from the group and vice versa.

**I<sub>3DI-C</sub>**: Each group pact how to expose it and defend it the answer knowing that is it a production of the group collective, not individual, assigning tasks and individual responsibilities in this spread out of information.

**I<sub>4DI-C</sub>**: The teacher and the students decide what subject to study.



**I5DI-C:** The teacher prepares the common settings in regard to the need to move forward in the study process.

**I6DI-C:** The students incorporate questions during the common settings to re direct the study process according to the production of each group.

**D<sub>ASP-ASD</sub>. Dialectics of the praxeological analysis-synthesis/didactic synthesis-analysis:** we identify this dialectic when we observe an action of the following type:

**I1DASP-ASD:** An analysis of the different answers  $R_i^\diamond$  that requires to decide what to study of this work to build the answer  $R^\heartsuit$ . For example: what and how to study the system of two lineal equations with two unknown quantity? what and how to study the displacement of the functions? How to study the models of offer and demand? What and how to study the relations between variables?

**I2DASP-ASD:** An analysis of the information obtained by different information systems: internet, books, micro economy books, teachers, economists, merchants, etc.

**I3DASP-ASD:** An analysis of the questions asked in each study group.

**I4DASP-ASD:** A synthesis of techniques, technology and theories that make up the different  $R_i^\diamond$ .

**I5DASP-ASD:** A synthesis of the information obtained by the different media prioritizing what is necessary and adequate to give answers to the different questions.

**I6DASP-ASD:** A synthesis of the answers to the derivatives questions.

**D<sub>T-FDT</sub>. Dialectic of subject and out-of-subject:** The separation between mathematics and microeconomy it is done in terms of exploring different environments that apparently don't have any direct relation with the issue considered. For example, the study of limits of the functions was produced when there were questions asked about the variations of price and amount of balance. This exploration was not obvious when considering question Q4. That is how we identified this dialectics for the following actions:

**I<sub>1DT-FDT</sub>**: Students go to a different discipline of mathematics. For example, to microeconomy. The decision over the domain of validity of the parameters of the model implies to study laws of offer and demand and adjust them.

**I<sub>2DT-FDT</sub>**: In mathematics, a solution to the same discipline. For example:

- The study of limits of the functions in order to enter the study of the derivative of functions as limits of the incremental quotient.
- The study of equation systems to enter the calculation of point of balance.

**D<sub>P-T</sub>. Dialectics of skydiver and truffles**: This dialectic starts working when it is introduced for the first time to a new question, a derivative question, a  $R^\diamond$  and or any other work, that when doing a search in different media and without a strict analysis, it seems to be useful to the construction of answer  $R^\heartsuit$ . We identify the functioning of the dialectics when at some point of the class we observe:

**I<sub>1DP-T</sub>**: the group of students cannot determine how to start answering the question and the productions delivered do not give a partial answer to the questions.

**I<sub>2DP-T</sub>**: the search on the internet is wide and it starts to focus on what can be useful.

**I<sub>3DP-T</sub>**: The search in books lead us to rule out different chapters that weren't useful for answering the questions.

**D<sub>CN-CC</sub>. Dialectics of black boxes and clear boxes**: We identify this dialectic when at some point in the class there is a partial study of fragments or parts of some work. So, when a study is produced in a *grey level*. For example: actions belonging to this level of *grey* are the following:

**I<sub>1DCN-CC</sub>**: To study only one way to solve a system of equations.

**I<sub>2DCN-CC</sub>**: To build the equation of the line that goes through two points without doing the mechanical study of the formula.

**I<sub>3DCN-CC</sub>**: To study straight lines without studying perpendiculars.

**I<sub>4DCN-CC</sub>**: To study the derivatives of functions as a limit of the incremental quotient.

**D<sub>M-M</sub>: Dialectics of media-milieu**: We identify this dialectic when at some point in the class:

**I<sub>1DM-M</sub>**: Questions are asked in terms of “why?” and the results obtained or proposal of a media (source of information) are questioned. For example: which of the two models obtained are correct? Why both models of offer and demand are suitable?

**I<sub>2DM-M</sub>**: A different answer is studied in any media (that is not the teacher).

**I<sub>3DM-M</sub>**: Questions in terms of “how?”, that is, questioning how to prove that the model chosen is the correct one? How do we prove that the point of balance varies? How do we prove that the point of balance and the parameters are related? Etc. This implies the need to look for new information

**D<sub>L-E</sub>: Dialectics of reading and writing**: We identify this dialectic when at some point of the class the students:

**I<sub>1DL-E</sub>**: The student underlines or highlight what they consider important from the internet researches or when they copy on their folders what can be useful in this search and the use of books or asking for information from economy and math teachers.

**I<sub>2DL-E</sub>**: They prepare the synthesis of their own work or from the information obtained in a media.

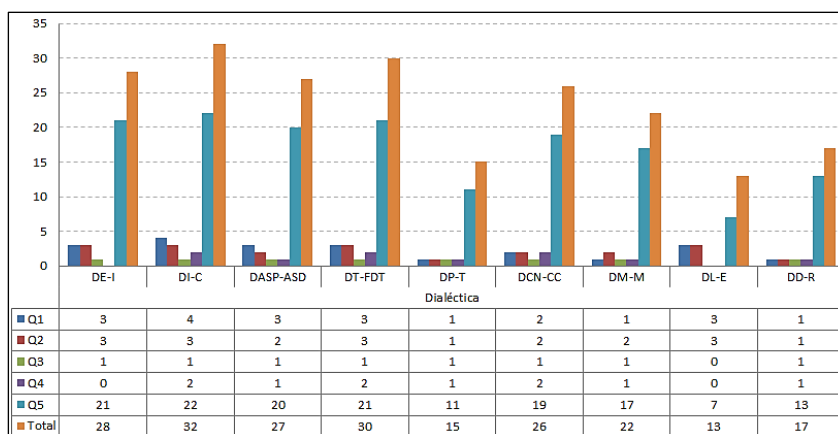
**D<sub>D-R</sub>: Dialectics of diffusion and reception**: We identify this dialectic when at some point in the class study groups communicate and defend their answers. When they share the productions in each common setting.

### **Discussion and final reflection**

Regrouping the numbers 0 y 1 – obtained according to table 2 – in regards to each question and each dialectic, we summarize this information in the following diagram.

Diagram 1

*Frequency of each dialectic according to each question.*



Question number 5 presents higher occurrence of indicators of all of the dialectics, specially the dialectic of research and study, subject and out-of-subject and individual and collective. This indicates a higher occurrence in the search of information and development of the investigations, more agreements from each group and higher entering and coming out of different subject (such as mathematics and microeconomy). There is a significant difference with the remaining questions, possibly because Q<sub>5</sub> has allowed to generate more derivate questions (characteristics not determined beforehand) and a study sustained in time. More sessions of the class were destined for the construction of answers to Q<sub>5</sub> and its derivate questions than for the rest of the issues. This particularity can be influenced in the apparition of the dialectics, as well as the fact that Q<sub>5</sub> allowed to address aspect of the curricular that had not been studied before by the class.

The search for answers to Q<sub>4</sub> did not present indicators of the dialectics of research and study nor the dialectic of reading and writing, possibly because Q<sub>4</sub> did not generate derivate questions and in consequence it was not necessary to look or research in different sources of information and so there were not any lectures with subsequent re writing and interpretations form the students. In Q<sub>3</sub> there were no indicators of the dialectics of reading and writing but

there was of the dialectic of research and study, possibly because  $Q_3$  generates some questions related to straight parallels, to the use of GeoGebra®, and the intersection of two straight lines: mathematics that the students knew and used. Generally, the occurrence of indicators was similar for  $Q_2$  and  $Q_1$  detecting an inferior number in the corresponding to  $Q_5$  but higher to  $Q_3$  and  $Q_4$ .

In summary the most frequent dialectics was from the individual and collective, this is due to the way of working of the group in class. The dialectic of subject and out-of-subject is another frequent one as well as praxeological analysis-synthesis/didactic synthesis-analysis and black boxes and clear boxes. The dialectic of the praxeological analysis-synthesis/didactic synthesis-analysis and the black boxes and clear boxes were used in class immediately. Both of them are strongly linked since the realization of the analysis to a synthesis requires to determine a level of grey useful to the study of the works.

The dialectics of media-milieu is at a lower level than the ones mentioned before. There is a stunning result since the dialectic of media-means dialectics is a key gesture to SRC (Bosch, Gascón, 2007, Barquero, Bosch, Gascón, 2011). Besides, if we got out of a subject to study a work probably some element should have been incorporated to the media when we returned to the subject. This result can be due to the disjoint classification of the dialectics since the indicators of one of the dialectics also could be an indicator of the other. There can be a possible interrelation between the different dialectics since for example all searches of external answers or creation to new answers are elements that constantly are incorporated in the milieu.

However, the indicators introduced here correspond to this particular implementation and have been developed under conditions and limitations of SRC. We conclude that as more generative a question is, it is possible to build more indicators of dialectics. This work expects to move forward in the construction, amplitude and generativist of the set of indicators

extending them to future implementations and other researches. The work of Salgado, Otero and Parra (2017) expect to contribute to this sense.

### References

- Barquero, B., Bosch, M. & Gascón, J. (2011). Ecología de la modelización matemática: los recorridos de estudio e investigación. In M. Bosch, J. Gascón, A. Ruiz Olarría, M. Artaud, A. Bronner, Y. Chevallard, G. Cirade, C. Ladage & M. Larguier (Eds.), *Un panorama de la TAD* (pp. 553-577). Bellaterra: Centre de Recerca Matemàtica.
- Bosch, M. y Gascón, J. (2007). Fundamentación antropológica de las organizaciones didácticas: de los “Talleres de Prácticas Matemáticas” a los “Recorridos de Estudio e Investigación”. In A. Bronner, M. Larguier, M. Artaud, M. Bosch, Y. Chevallard, G. Cirade, G. y C. Ladage (eds.), *Diffuser les mathématiques (et les autres savoirs) comme outils de connaissance et d'action* (pp. 55-91). Uzès: IUFM de l'Académie de Montpellier.
- Chevallard, Y. (2008). Didactique de l'enquête codisciplinaire et des parcours d'étude et de recherche. *Colloque international « Efficacité et Équité en Éducation »*, Rennes. <https://esup.espe-bretagne.fr/efficacite-et-equite-en-education/programme/symposium-chevallard.pdf>
- Costa, V. A., Arlego, M. & Otero, M. R. (2015). Las dialécticas en un Recorrido de Estudio e Investigación para la enseñanza del Cálculo Vectorial en la Universidad. *Revista de Formación e Innovación Educativa Universitaria*. 8(3), 146-161.
- Hausberger, T. (2016). Dimensions collaboratives et dialectique médias-milieux : un questionnement didactique autour d'une retranscription d'échanges sur un forum de mathématiques. *Enjeux et débats en didactique des mathématiques : 18ème Ecole d'été de Didactique des Mathématiques* (pp.613-622). Brest : La Pensée Sauvage.
- Parra, V., Otero, M. R. & Fanaro, M. (2015). Recorrido de Estudio e Investigación codisciplinar a la microeconomía en el último año del nivel secundario. Preguntas generatrices y derivadas. *Uno. Revista de Didáctica de las Matemáticas*, 69, 1-10.
- Salgado, D.; Otero, M. R. & Parra, V. (2017). Gestos didácticos en el desarrollo de un recorrido de estudio e investigación en el nivel universitario relativo al cálculo: el funcionamiento de las dialécticas. *Perspectiva Educativa*, 56(1), 84-108.