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Asymptote in prospective mathematics teachers' graphing praxeologies

Asymptote dans les praxéologies graphiques des futurs professeurs de mathématiques

Ana Katalenić[1]
Faculty of Education, University of Osijek, Croatia
http://orcid.org/0000-0001-9857-5758
Aleksandra Čižmešija[2]
Faculty of Science, University of Zagreb, Croatia
https://orcid.org/0000-0002-5017-4143
Željka Milin Šipuš[3]
Faculty of Science, University of Zagreb, Croatia
https://orcid.org/0000-0002-0726-3335

Abstract

Graphical representation is one of the fundamental and widely spread representations. We performed a comprehensive research of the didactic transposition of asymptote and asymptotic behaviour in the upper secondary education in Croatia, within the framework of the anthropological theory of the didactics. Our study included textbook analysis, questionnaires with university students and interviews with two mathematicians. In this poster, we present a part of our research with an emphasis on university students' graphing praxeologies. Results showed that students' graphing praxeologies differ from scholarly praxeologies. Further, students grounded their praxeologies mostly on their high-school graphing knowledge, even after being exposed to advanced mathematics that could foster their autonomous thinking.

Keywords: Prospective mathematics teachers, Function graph, Asymptotes and asymptotic behaviour.

[1] akatalenic@foozos.hr
[2] cizmesij@math.hr
[3] milin@math.hr

Résumé

La représentation graphique est l'une des représentations fondamentales et largement répandues. Nous avons effectué une recherche approfondie de la transposition didactique de l'asymptote et du comportement asymptotique dans l'enseignement secondaire supérieur en Croatie, dans le cadre de la théorie anthropologique du didactique. Notre étude comprenait une analyse de livres scolaires, des questionnaires avec des étudiants de l'universités et un entretien avec deux mathématiciens. Dans cette affiche, nous présenterons une partie de notre recherche en mettant l'accent sur les praxéologies graphiques des étudiants de l'université. Les résultats montrent que les praxéologies graphiques des élèves diffèrent des praxéologies académiques. En outre, les élèves ont fondé leurs praxéologies principalement sur leur connaissance graphique de lycée, même après avoir été exposés à des mathématiques avancées qui pourraient favoriser leur réflexion autonome.

Mots-clés: Futurs professeurs de mathématiques, Graphe fonctionnel, Asymptotes et comportement asymptotique.

Asymptote in prospective mathematics teachers' graphing praxeologies

Graphical representation is one of the fundamental and widely spread representations, in mathematics and elsewhere. Mathematics curriculum for general secondary mathematics education in Croatia includes standard topics related to graphing such as graphing elementary functions, conics and functions within calculus.

We performed a comprehensive study within ATD on asymptote and asymptotic behaviour as a body of knowledge in the Croatian upper secondary mathematics education. Here, we present a part of our research emphasizing university students' graphing praxeologies.

Literature Review

The existing studies related to asymptotes in graphing do not focus on appearance and the role of the asymptote in graphing. A certain deal of research was conducted on the use of various software to enhance mathematics teaching and learning (Yerushalmy, 1997). Dahl (2016) investigated students understanding and concept image of asymptotes and Kidron (2011) students' construction of knowledge related to the horizontal asymptote. Zarhouti, Mouradi and Maroufi (2014) described how obstacles of different nature affected students' performance in graphing function within calculus.

The graphing activity is the *praxis* that should promote or apply the l*ogos* of point-wise, local and global properties of a function or a curve, as suggested by Vandebrouck (2011). We investigated what mathematical knowledge prospective mathematics teachers draw on when graphing functions and curves especially with respect to appearance of asymptotes.

Methodology

In the comprehensive study, we examined the didactic transposition of asymptote and asymptotic behaviour in the upper secondary education in Croatia, following the ATD (Chevallard & Bosch, 2014; Chevallard & Sensevy, 2014).

We performed mathematical and scientific literature review, praxeological analysis of curriculum and textbooks as representatives of knowledge to be taught (see Čižmešija, Katalenić and Milin Šipuš (2017)), questionnaires with university students and focused interview with two scientists. The interview was designed to gain insight into mathematicians' beliefs and ideas as members of institution of scholarly mathematics. Students' available knowledge is product of both secondary and university level mathematics education. It is also potentially taught knowledge since participating students were prospective mathematics teachers. Winsløw and Grønbæk (2014) emphasized that prospective teachers should advert to university mathematics when teaching.

Research setting and questionnaire items

Participants in our study were 51 students in their final, fifth year (age 21-23) at the largest mathematics department in Croatia. Majority of students attended general upper secondary school and graduated as bachelor's in mathematics education that also included formal mathematical education.

We administered three questionnaires with mainly open-ended items characterized as routine, non-routine, and problem mathematical tasks. In this poster, we present results of three items, two from the first and one from the last questionnaire. In the first two items (item 1.1. and item 1.3.), we expected students to use some of the following praxeologies:

– P: plotting corresponding points (x, $f(x)$) and connecting them by a smooth curve;

– T: transforming a prototype graph of an elementary function to get a graph of a composite elementary function; this evokes praxis of algebraic manipulation and discursive knowledge about function composition with linear function;

– F: drawing a curve using discursive knowledge about graph shape and point-wise, local and global properties of the function.

Item 1.1. Graph a function f given with $f(x) = (2x - 1)/(x - 1)$. Describe the function.

Item 1.3. It is expected that percentage (expressed as a decimal) of viewers, who will respond to the commercial message for a new product after t days, behaves according to the formula $o(t) = 0.7 - 2^{-t}$.

(a) Represent graphically the given relationship $o(t)$.

(b) What is the expected percentage of viewers who will respond to the commercial message after 7 days?

(c) Describe behaviour of the expected percentage of viewers who will respond to the commercial message as days pass.

Item 3.3. A hyperbola is given by its equation $x^2 - 4y^2 = 4$.

(a) Find the equations of asymptotes of the given hyperbola.

(b) Find the equation of the tangent line of the given hyperbola from the point P (-1, –1/2).

(c) Draw the hyperbola and the lines from the tasks (a) and (b). What do you notice? Explain the meaning of the obtained results.

Results

Praxeologies in the item 1.1.

In this item, 46 students answered as given in table 1.

Table 1:

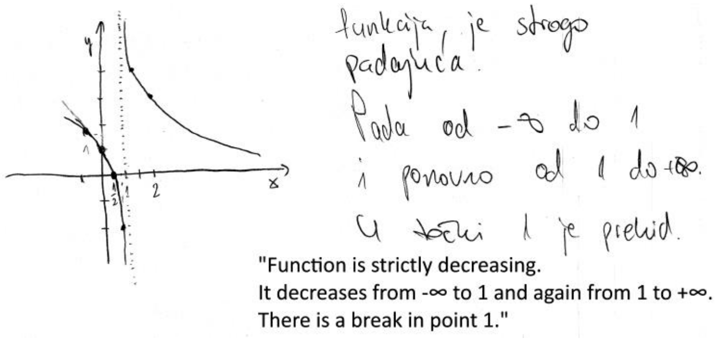
he number of students with regard to the praxeology used when graphing rational function[4]

Praxeology	P	C	C and P	Partial C	T	Total
Incorrect or no graph	4	6	2	2		14
Incorrect graph in hyperbola shape	5	6	3		1	15
Correct graph with vertical or no asymptotes	4		1			5
Correct graph with horizontal or both asymptotes	1	5	4	2		12
Total	14	17	10	4	1	46

Among students who plotted points, some drew a hyperbola-shaped curve, that would intersect the line $y = 2$, thus making a graph with incorrect function range (see figure 1), and some students drew a curve through appropriate points without displaying function behaviour at infinity. Only one student used transformation but made a blunder in the algebraic manipulation of the given expression.

Figure 1

An example of student's incorrect graph of the rational function



[4] We used C to denote drawing a curve using analysis with calculus. Figure 1: An example of student's incorrect graph of the rational function

Majority of the participants analysed the function with calculus. Those students explored first derivative, function domain, function limit at infinity, and some of them the second derivative.

Less than half of the students described the function, and 20 students wrote about function monotonicity. Two students elaborated asymptotic behaviour in some manner.

Praxeologies in the item 1.3.

In this item, 47 students answered as given in table 2.

Table 2

The number of students with regard to the praxeology used when graphing exponential function

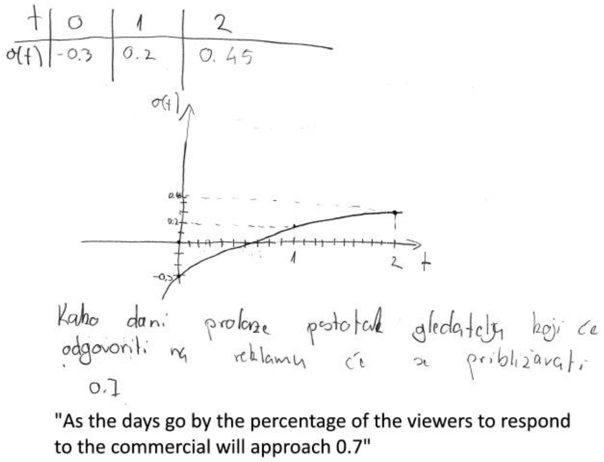
Praxeology	-	P	T	F	Total
Incorrect or no graph	4	8			12
Incorrect graph with horizontal asymptote		6			6
Correct graph		13	1		14
Correct graph with horizontal asymptote		11	1	3	15
Total	4	38	2	3	47

Students who drew incorrect graph displayed a convex curve or a line through plotted points or made errors when evaluating function or entering points. Students, who drew correct graph without horizontal asymptote by plotting points, displayed curves that increase unboundedly, or the behaviour at infinity is (graphically) unavailable (see figure 2) or depicted with stagnation.

Students wrote about monotonicity, asymptotic behaviour or both properties of the given exponential function. Students who plotted points wrote the values were increasing more often than they wrote about the asymptotic behaviour. Almost all of the students who depicted the horizontal asymptote also mentioned the asymptotic behaviour of the values; and 44% of the students, who did not depict the asymptote, mentioned the asymptotic behaviour of the values (see figure 2).

Figure 2

An example of student's correct graph of the exponential function



Praxeologies in the item 3.3.

In this item, 37 students answered and majority of them used the expected technique of drawing a curve passing through points marked as vertices and approaching lines marked as asymptotes. Students made errors in obtaining equations of the asymptotes, graphing asymptotes as lines or entering the hyperbola vertices.

The scholarly perspective of the graphing praxeology

Both interviewed scientists agreed that all discursive properties should be discussed and recognized when graphing elementary functions. They found that the graph shape – i.e. global properties of elementary functions, should be familiar already to students in their upper secondary mathematics education. Both scientists preferred graph transformations over other graphing techniques, especially over plotting points and graphing by function analysis using calculus.

Discussion

Three tasks presented to students showed that students' graphing praxeologies to a large scale differ from praxeologies provided by two mathematicians. Although students rarely provided comprehensive discursive answers, their used techniques showed vague correlation and conversion between different representations and settings which require asymptote, especially related to graphing of rational functions and a hyperbola. Students predominantly used "too strong tools" in graphing simple rational function by relying on analysis of the given functions within calculus. In contrast, they mainly relied on plotting points when graphing exponential function. When graphing a hyperbola, they mostly focused on its global shape complemented by pointing up the vertices. Overall, the asymptote was underrepresented in the praxis and logos of students' answers.

Conclusion

The three tasks in our study problematized the appearance and the role of the asymptote in graphical representation of basic functions and curves. Previous research and scholarly perspective suggest that different properties of functions and curves are helpful for graphing. With respect to the asymptote, students' strategies were found to be predominantly dependent on the settings in which the task was presented, thus fragmented and partially incoherent. The study also revealed that majority of students did not autonomously upgrade their high-school graphing praxeologies to more coherent ones, not even after being exposed to advanced university mathematics. In our further research we would explore constraints that hinder students in using their academic knowledge in the context of secondary mathematics education – e.g. by interviews with prospective teachers and institutional analysis of university education.

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