Some sensitive issues in the use and development of the anthropological theory of the didactic

Quelques questions sensibles dans l'utilisation et le développement de la théorie anthropologique du didactique

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Abstract

This presentation repeats the opening lecture that the author gave on January 23, 2018 in Autrans (France) during the 6th International Congress on the ATD. Something of the oral form has been deliberately preserved in the text proposed here to the reader. As the title indicates, the aim was to draw attention to certain difficulties observed in the reception and use of the ATD. It is these difficulties that the structure of the text seeks to highlight and help overcome. After having underlined the question of the vocabulary specific to the ATD, we successively examine its theory of cognition, which is the basis of the ATD, then the cardinal notion of the possibly didactic, before moving on to the theory of praxeologies, which has undoubtedly received the greatest diffusion so far, and to progress by deepening the dialectic of persons and institutional positions, before specifying the concept of inquiry, which is at the heart of the current and future developments of the ATD.

Keyword: Anthropological Theory of Didactics, Theory of cognition, Possibly didactic

Résumé

Cet exposé reprend la conférence inaugurale que l'auteur a prononcée le 23 janvier 2018 à Autrans (France) dans le cadre du 6ᵉ congrès international sur la TAD. Quelque chose de la forme orale a été volontairement conservé dans le texte proposé ici au lecteur.

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Comme l'indique le titre, il s'agissait d'attirer l'attention sur certaines difficultés observées dans la réception et l'usage de la TAD. Ce sont ces difficultés que la division du texte s'efforce de mettre en évidence et d'aider à dépasser. Après avoir souligné la question du vocabulaire propre à la TAD, on examine successivement la théorie de la cognition, qui constitue la base de la théorie, puis la notion cardinale du possiblement didactique, avant d'aborder la théorie des praxéologies, qui a reçu sans doute la plus grande diffusion jusqu'ici, et de progresser en approfondissant la dialectique des personnes et des positions institutionnelles, avant de préciser le concept d'enquête, qui est au cœur des développements en cours et à venir de la TAD.

Mots-clés : Théorie Anthropologique du Didactique, Théorie de la cognition, Possiblement didactique.

Some sensitive issues in the use and development of the anthropological theory of the didactic

Dear friends and colleagues, it is an honour and a pleasure to stand before you this morning. I want to first tell you my deep gratitude for your being here. Our shared ambition is to contribute in a myriad of ways to advancing the theorisation and practice of *the anthropological theory of the didactic*, the ATD. My own contribution today will focus on some sensitive issues that, to my way of thinking, are worthy of attention. So let me begin without further ado.

A word about words

At the outset, I would like to repeat a caveat already stated in my lecture at the 5th international congress on the ATD. When trying to describe and explain some kind of phenomena, all scientific workers are sure to be confronted with a major challenge when it comes to words and wording. How can I express that? What is the right word, if any? Or is it unutterable? Should we create some new phrase or term? How can we do that? In a word, we are often at a loss for words. As a result, it is true that, to outsiders, any ATD-oriented paper seems to teem with new words and, what may be worse, new usages of existing words. Faced with this uninviting—though necessary and fruitful—situation, one must first decline to ignore it as if no misunderstanding could ensue. But there is worse than shunning "difficult" words: whenever a "new" word pops up before your eyes, please don't go innocently to some general dictionary! In an ATD paper, words either are "well known" or deserve to be defined, explained, and exemplified. At least this is the rule that, I believe, we should follow. On this occasion of the 5th ICATD, I reminded you of the key principle as far as words are concerned—a principle I dubbed the "Humpty Dumpty principle", because of this vivid dialogue in chapter VI of Lewis Carroll's *Through the looking-glass, and What Alice Found There* (1871):

'I don't know what you mean by "glory,"' Alice said.

Humpty Dumpty smiled contemptuously. 'Of course you don't – till I tell you. I meant "there's a nice knock-down argument for you!"'

'But "glory" doesn't mean "a nice knock-down argument,"' Alice objected.

'When I use a word,' Humpty Dumpty said in rather a scornful tone, 'it means just what I choose it to mean – neither more nor less.'

'The question is,' said Alice, 'whether you can make words mean so many different things.

'The question is,' said Humpty Dumpty, 'which is to be master – that's all.'

In the case in point, we are collectively the "master". A praxeology [*praxéologie, praxeología*] is exactly what the ATD says it is—neither more nor less. Dictionary definitions according to which praxeology—not *a* praxeology—is "the (deductive) study of human action"—are of no avail. Much to the contrary, it takes a good knowledge of the ATD notion of *a* praxeology to perceive possible links between that notion and the longed-for white whale which, it seems, was first named "praxéologie" (in French) by the philosopher and sociologist Louis Bourdeau (1824-1900) in his two-volume book titled *Théorie des sciences. Plan de science intégrale* (1882), of which Book VII is called "Praxéologie. Science des functions" ("Praxeology: Science of functions"). In this book, Bourdeau writes (vol. 2, p. 463):

> On account of their dual natures of specialty and generality, these functions should be the subject of a separate science. Some of its parts have been studied for a long time, because this kind of research, in which man could be the main subject, has always presented the greatest interest. Physiology, hygiene, medicine, psychology, animal history, human history, political economy, morality, etc. represent fragments of a science that we would like to establish, but as fragments scattered and uncoordinated have remained until now only parts of particular sciences. They should be joined together and made whole in order to highlight the order of the whole and its unity. Now you have a science, so far unnamed, which we propose to call "Praxeology" (from πρᾶξις, action), or by referring to the influence of the environment, "Mesology" (from μέσος, environment).

Didactics as we try to develop it is only a *partial* science, the object of which, let me remind you, is the *didactic*, which is not the whole of social "functions". In point of fact, as you will discover in a little while, the object of didactics is the *possibly* didactic—an expression to be defined shortly.

One more point should be touched upon, which is a point known to all: most words central to the ATD have been coined in French, which de facto was (and, up to a point, still is) the original touchstone for the validity of our linguistic choices. So it should be kept in mind that the words I will use in what follows are often (though not always) renderings of French "equivalents". As a consequence, when using an "ATD word" *for the first time in this presentation*, I shall add in square brackets the original French term, followed by the usual Spanish rendering, as for example in "It proved to be a very useful praxeology [*praxéologie, praxeología*]".

A theory of cognition

It is too often overlooked that the ATD comprises a *theory of cognition* which is foundational to the rest of the ATD. This theory of cognition sets forth two kinds of characters, *persons x* and *institutions I* or, more precisely, *institutional positions* (*I, p*). To take a shortcut here, I only give two easy examples. A *class C*(*X, y*), i.e. a group of students *X* together with their teacher *y*, is an institution with two main positions, the *student position* [*position d'élève, posición de alumno*] and the *teacher position* [*position d'enseignant, posición de docente*]; a *family* is an institution with at least three main positions—father, mother, child—of which one or another may be temporarily or permanently unoccupied.

In order to present a unitary view of persons *x* and institutional positions (*I, p*), I now introduce the noun *instance* in order to designate either a *person* or a *position*[2]. An instance *u* is thus either a *personal instance*, i.e. a person *x*, or an *institutional instance*, i.e. a position (*I, p*), also called a *positional instance*.

Before we go any further, I still have to introduce the general concept of an *object*. An object is any entity that exists for at least one instance. Of course, instances are themselves objects. Objects are the stuff that the world we describe is made of. Now we

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[2] The word *instance*, from Latin *instantia* and Greek ἔνστασις (*enstesis*), has been chosen to point discreetly to the singularity of any instance—which in particular can serve as an example or a counterexample.

can reformulate the basic notions of our theory of cognition, taking for granted the twin notions of *personal relation* [*rapport personnel, relación personal*] and *institutional* (or *positional*) *relation* [*rapport institutionnel* (or *positionnel*), *relación institucional* (or *posicional*)] to an object o, classically denoted by $R(x, o)$ and $R_I(p, o)$, respectively. Given an instance u and an object o, let us set $R(u, o) \overset{\text{def}}{=} R(x, o)$ if $u = x$ and $R(u, o) \overset{\text{def}}{=} R_I(p, o)$ if $u = (I, p)$.

We say that an instance u knows the object o if and only if we have $R(u, o) \neq \emptyset$. The instance u may have a poor knowledge of o or, to the contrary, may hold a rich mastery of it: the content of u's knowledge of o is, by definition, consubstantial with $R(u, o)$, which encompasses all what u knows, thinks, imagines, dreams, fantasises about o.

With all this said, who exactly is the "we" who "has" $R(u, o) \neq \emptyset$? It may be supposed that it is the one who speaks or write—the author—, so that the statement "we have $R(u, o) \neq \emptyset$" is to be held equivalent with "the author judges that $R(u, o)$ is not empty". We can generalise this statement by introducing a person ξ supposed to be a didactician (by which I mean a researcher in didactics, especially in the framework of the ATD) and by writing $\xi \vdash R(u, o) \neq \emptyset$, to be read "$\xi$ judges that $R(u, o)$ is not empty". Of course, if we denote the didactician position by, say, $\check{r}$, we can write as well $\check{r} \vdash R(u, o) \neq \emptyset$, to be read "the instance $\check{r}$ judges that $R(u, o)$ is not empty".

All this can be—and must be—further generalised by introducing an "indeterminate" instance, which we denote by $\hat{w}$, who can be any instance, and by considering the sentence $\hat{w} \vdash R(u, o) \neq \emptyset$. If ϑ is any statement, we can more generally consider the sentence $\hat{w} \vdash \vartheta$. Obviously, if $\hat{w}'$ and $\hat{w}''$ are instances, we can write $\hat{w}' \vdash (\hat{w} \vdash \vartheta)$ and $\hat{w}'' \vdash (\hat{w}' \vdash (\hat{w} \vdash \vartheta))$; and so on. Remember that to say that $\hat{w} \vdash R(u, o) \neq \emptyset$ means that $\hat{w}$ judges that *u knows o*. It can happen at the same time that another instance $\hat{w}'$ judges that $R(u, o) = \emptyset$, in other words that $\hat{w}'$ judges that *u does not* know the object *o at*

all. Each instance is therefore at variance with the other on this issue. This is a basic example of the divergence in the way instances—including researchers in didactics ξ—generally "see" the world of cognition. *This precludes the existence of, and reference to, a naturally privileged point of view*. All instances can claim recognition of the judgements they issue; at the very least, all instances must be taken into account by the unbiased didactician doing research in the framework of the ATD.

This being said, can we determine how much "good knowledge" of the object o the relation $R(u, o)$ incorporates? Once again, there is no privileged viewpoint. In other words, there are no absolute criteria of judgement. In order to realistically model the kinds of situations of interest to us, the first step we have to consider consists in supposing that a (more or less explicit) basis for comparison—a touchstone—exists, which takes the form of a given positional relation to the object o, $R_I(p, o)$. The chosen institutional relation is the standard by which an instance v will judge $R(u, o)$: v will say that, by definition, "u has a 'good' knowledge of o" if, in v's view, $R(u, o)$ "resembles" $R_I(p, o)$. In such a case, one will say that, according to v, $R(u, o)$ *conforms with $R_I(p, o)$* or *is in conformity with $R_I(p, o)$*, a statement written in symbols like this: $v \vdash R(u, o) \cong R_I(p, o)$. Likewise, if, in v's view, $R(u, o)$ fails to conform to $R_I(p, o)$, one shall write*: $v \vdash R(u, o) \ncong R_I(p, o)$.

Up to now, our model comprises four parameters, to wit u, o, I and p; or, in a condensed form, u, o, and (I, p). However this model is still incomplete in one important respect. The social reality we are trying to model and study always hypothesises a more or less hidden or implicit instance, v, which we will call an *evaluating instance*. This instance is supposed not only to judge whether $R(u, o) \cong R_I(p, o)$ or, to the contrary, $R(u, o) \ncong R_I(p, o)$, but more generally to assess the conformity of $R(u, o)$ with $RI(p, o)$. In particular, v is supposed to be able to judge whether the relation of u to o at time t_1,

denoted by $R(u, o, t_1)$, is more, or less, or (almost) equally conforming to the standard $R_I(p, o)$ than $R(u, o, t_0)$, where $t_0 < t_1$. Let us denote by $\varphi(R, \bar{R})$ the (supposed) degree of conformity of $R = R(u, o)$ with $\bar{R} = R_I(p, o)$. It is assumed, specifically, that v will be able to compare the degrees of conformity with $\bar{R} = R_I(p, o)$ of $R_0 = R(u, o, t_0)$ and $R_1 = R(u, o, t_1)$, and let us know whether $v \vdash \varphi(R_0, \bar{R}) < \varphi(R_1, \bar{R})$, or $v \vdash \varphi(R_0, \bar{R}) > \varphi(R_1, \bar{R})$, or $v \vdash \varphi(R_0, \bar{R}) = \varphi(R_1, \bar{R})$, or, less precisely, $v \vdash \varphi(R_0, \bar{R}) \approx \varphi(R_1, \bar{R})$. With this fifth parameter, we arrive at a quintuple—a 5-tuple—which we call a *cognitive nucleus* (or *kernel*) and denote by $\tilde{n} = (u, o, I, p, v)$.

The expression $\tilde{n} = (u, o, I, p, v)$ calls for a number of comments, a few of which we shall review in what follows. A cognitive kernel $\tilde{n}$ exists for an instance $\hat{w}$ provided both the instance u and the object o exist for $\hat{w}$. Moreover, the object o has to be seen by $\hat{w}$ as "knowable" par u. Also o has to be seen by $\hat{w}$ as "knowable" by at least one institutional position (I, p). In point of fact, empirical observation shows that such an institutional position, as well as the existence of an evaluating instance v, are more often than not merely hypothesised by $\hat{w}$. By modelling $\tilde{n}$ as a quintuple, we therefore bring to the fore a part of it which, for a number of instances $\hat{w}$, is an "immersed part" hidden from clear consciousness—the "emerged part" being the couple (u, o). A major reason for the forgetting of $R_I(p, o)$ seems to be the ingrained, uncriticised premise that there would exist an absolute, universal position $(*I, *p)$, depending on o, whose relation to o, i.e. $*\bar{R} = R_{*I}(*p, o)$, is held to be the knowledge of o incarnate, regarded as unsurpassable[3]. The same can be said of the forgetting of v: it is as if $\hat{w}$ held an entrenched (but equally erroneous) assumption relating to the existence of an absolute, universal evaluating instance $*v$, automatically available without request, issuing the one "true" verdict about

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[3] The use of a prefixed asterisk to denote a supposed entity, as in $*I$, takes up the convention adopted in the study of Proto-Indo-European (PIE), as can be seen in the *Online Etymology Dictionary*'s entry "Didactic (adj.)": "1650s, from French *didactique*, from Greek *didaktikos* 'apt at teaching,' from *didaktos* 'taught,' past participle of *didaskein* 'teach,' from PIE root *dens- 'wisdom, to teach, learn.'

$R(u, o)$. In many cases, therefore, a cognitive nucleus is thus reduced to a mere *cognitive base* $\bar{n} = (u, o)$, surreptitiously associated with a fictitious and illusory cognitive kernel $*\tilde{n} = (u, o, *I, *p, *v)$. In any case, we must remember that, quite often, a cognitive nucleus $\tilde{n} = (u, o, I, p, v)$ evoked by an instance $\hat{w}$ lies grossly *underdefined*—and the more so as $\hat{w}$ is foreign to the didacticians' community. The instance $\hat{w}$ may, for example, simply hold that "somewhere" there exists a "touchstone" position (I, p) and an evaluating instance v, without being able to say more about them. Naturally, when $\hat{w}$ is a positional instance, it can happen that $\hat{w} = (I, p) = v$. Similarly, when $\hat{w}$ is a personal instance x, it can be that $v = x$, the touchstone being in this case the personal relation $R(x, o)$ itself— the person x being unwittingly turned into an institution in which, so to speak, x's dealings with o give tangible form to a supposed specific position within this institution.

What may be termed the "missing half" of a cognitive nucleus $\tilde{n} = (u, o, I, p, v)$ when reduced to the cognitive base $\bar{n} = (u, o)$, i.e. the triple (I, p, v), is, however, no less important. Indeed, the life of institutions is replete with this kind of 3-tuples. We will call this a *cognitive frame of reference* and denote it by $\underline{n} = (I, p, v)$. Formally, we can write $\tilde{n} = (u, o, I, p, v) = (u, o) \frown (I, p, v) = \bar{n} \frown \underline{n}$. To borrow an easy metaphor, we shall speak of the process of "nuclear fission" as expressed by the equation $\tilde{n} = \bar{n} \frown \underline{n}$ and of the reverse process of "nuclear fusion" as expressed by $\bar{n} \frown \underline{n} = \tilde{n}$. What has been mentioned earlier is a state in which "fissioned" cognitive nucleuses proliferate in the form of cognitive bases $\bar{n} = (u, o)$. But isolated cognitive frames of reference $\underline{n} = (I, p, v)$ are no less numerous and no less present. Any instance v who happens to believe that a certain institutional position (I, p) knows the object o, i.e. such that $v \vdash R_I(p, o) \neq \emptyset$, can purport to issue judgments on the conformity of the relation $R(u, o)$ to $R_I(p, o)$ and ends up creating out of the blue a cognitive frame of reference $\underline{n} = (I, p, v)$ with respect to cognitive bases of the form $\bar{n} = (u, o)$. To put it in plain language, the relation of many instances u

(and in particular of many persons x) to a given object o may thus come to fall under the jurisdiction of unheralded and possibly unwished-for cognitive frames of reference $\underline{n} = (I, p, v)$. Cognitive nuclear fusion is more often than not a spontaneous phenomenon.

There is, however, a case of particular relevance to the didactician, in which some research instance $\hat{r}$ builds up, within the institution $\hat{\Delta}$ of research in didactics, a "dedicated" position $\hat{p}$ whose relation to o, $R_{\hat{\Delta}}(\hat{p}, o)$, will be taken as the standard by which to judge the relation $R(u, o)$, with u belonging to some set U of instances. The position $(\hat{\Delta}, \hat{p})$ or, more precisely, the institutional relation $R_{\hat{\Delta}}(\hat{p}, o)$, is conceived of as a comparison basis. This evaluating basis is used, in such a case, as an analytical tool. Evaluation, here, is first and foremost a means to an end, which is the analysis of $R(u, o)$. Conversely, we can state that any analysis of the relation $R(u, o)$ by some instance v follows on from comparisons made with relations $R_I(p, o)$, some of which may have been designed on purpose by v as analytical evaluation tools. We are here getting closer to the notion of *reference praxeological model* [*modèle praxéologique de référence*, *modelo praxeológico de referencia*] of o, to which we shall come back later.

The possibly didactic

Why should researchers in didactics feel concerned with the analysis of personal and institutional relations $R(u, o)$? In order to arrive at an answer to this question, we have one important remark to make. Until this point, we have considered only *the cognitive*. Although we have hinted at the phenomenon of cognitive *change* (in dealing with the evaluating instance v), there has been until now no hint whatever of *the didactic*. Before we can do that, we'll have to introduce an umbrella notion, that of the *possibly* didactic. To do so, let us start with a cognitive kernel $\tilde{n} = (u, o, I, p, v)$. A *possibly didactic situation*

is defined to be a quadruple $\varsigma = (\tilde{n}, \mathcal{C}, w, \delta)$ of a certain kind[4]. In this quadruple, $\mathcal{C}$ is the set of all conditions of any kind—i.e. of any level in the *scale of levels of didactic codeterminacy* [*échelle des niveaux de codétermination didactique*, *escala de niveles de codeterminación didáctica*]—that are satisfied at some given time t. Let me remind you of what this scale looks like:

<div align="center">

Humanity

⇕

Civilisation

⇕

Society

⇕

Schools

⇕

Pedagogy

⇕

Didactic system

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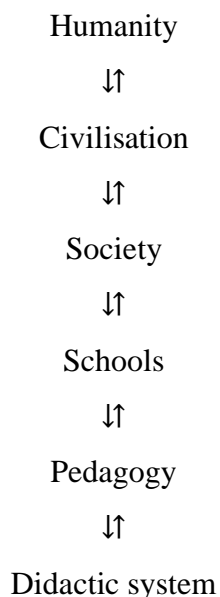
A *didactic system* is a structured set $s = S(X, Y, o)$ where X is the set of *students*, Y the set of "didactic helpers" (teachers, tutors, etc.), and the object o is the *didactic stake*, that students $x \in X$ have to "study" and "learn" with the help of $y \in Y$. A didactic system s comes into existence, lives and dies in the framework of an institution that enables its existence and functioning, which we call a *school*. Indeed, what is meant by "school" is twofold: it is either a "school system" Σ or a local establishment σ of the school system Σ. Among the set of conditions that Σ and σ can create are those that determine the school's *pedagogy*, which is essentially the manner through which the $x \in X$, together with the $y \in Y$, have access in σ to the object o as an object to study. In many pedagogies, students and teachers are grouped together on a permanent basis, which is a *class*,

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[4] The Greek letter ς is the lower-case letter sigma (σ) when used in final position, as in the words πρᾶξις (*prâxis*), λόγος (*lógos*), or μέσος (*mésos*). Paradoxically, although without malice aforethought, we knowingly use this letter in initial position in the expression $\varsigma = (\tilde{n}, \mathcal{C}, w, \delta)$.

generically denoted by $c = C(X, Y)$. In "class" pedagogies, didactic systems $s = S(X, Y, o)$ normally form within the class $c = C(X, Y)$. A lot more rarely, a didactic system $s = S(X, Y, o)$ in σ can be formed from distinct classes $c_1 = C(X_1, Y_1)$, $c_2 = C(X_2, Y_2)$, ..., with $X \subset X_1 \cup X_2 \cup ...$ and $Y \subset Y_1 \cup Y_2 \cup ...$

The upper levels of the scale (society, civilisation, and humanity) provide the opportunity to point out the following: although the name of each level is in the singular, there is a multitude of didactic systems, of pedagogies, and a large diversity of schools. There exists also a great many societies and civilisations. A special comment is in order with respect to the notion of civilisation. Consider two societies S and S' and two institutions I in S and I' in S' that, in the eyes of some instance $\hat{w}$, appear to be "homologous"—it can be marriage, school, leisure, etc. Then we shall say that, from the point of view of $\hat{w}$, the societies S and S' belong to the same civilisation as concerns institutions I and I' with respect to a set $\mathcal{O}$ of objects, if, for all $o \in \mathcal{O}$ and for all positions p and p' in I and I' held to be homologous by $\hat{w}$, we have: $\hat{w} \vdash R_I(p, o) \approx R_{I'}(p', o)$. Of course this definition admits variants: one can specify it to the case of a single pair of positions (I, p) and (I', p'), for example. With this said, let me stress an especially riveting class of phenomena, to wit, the *civilisational changes* that from time to time occur within a given civilisation. These changes share three characteristics: they are *local*, they come about historically in several stages spanning decades or even centuries, and they are hotly disputed until some day, having reached a point of no return, they are accepted and even taken for granted by a large majority of people, thereby wrongfooting the rearguard of the old civilisation. At the level of *society*, one can refer to the rise (or not) of secondary education for girls for example. At the *school* level, one can mention the current decline of manual calculation methods, among many other limited but sweeping changes in the set of possibly didactic conditions. As for the level of *humanity*, the only aspect I shall

put forward here is that this level is the only exception to the rule stated above: there is definitely only *one* humanity since the extinction of the Neanderthals some 30 000 years ago. Each level in the scale of didactic codeterminacy is the seat of conditions specific to that level which may be effective in the entire social space: a condition grown out of a society's specificities, for example, may bear upon any other level.

Of course, no instance whatever can be aware of *all* the conditions that the scale of didactic codeterminacy would allow us to situate at their specific level, particularly when these conditions have not yet been revealed by scientific research. To be more precise, given an instance $\hat{w}$, we can, if need be, denote by $\mathcal{C}_{\hat{w}}(t)$ the subset of conditions that exist for $\hat{w}$ at time t. Among the conditions $c \in \mathcal{C}$ are those pertaining to u, and in particular to u's relations $R(u, \bar{o})$, for a host of objects $\bar{o}$, including o. The instance w appearing in the expression of $\varsigma = (\tilde{n}, \mathcal{C}, w, \delta)$ is any instance who can "do something", "perform an act", "take action" in one way or another. An act which w does in such a context will be generically called a *gesture* [*geste*, *gesto*], where we take the word "gesture" to mean any act of *any* kind whatsoever. A gesture will be generically denoted by the Greek letter δ, which completes the quadruple $\varsigma = (\tilde{n}, \mathcal{C}, w, \delta)$. Such a quadruple is exactly what we shall call a *possibly didactic situation* [*situation possiblement didactique*, *situación posiblemente didáctica*]. The action δ performed by w may result from a *declared intention* on the part of w to "do something" so that the degree of conformity of $R(u, o)$ with $R_I(p, o)$, as appraised by v, increases. In that case, we shall say that w makes an *intendedly didactic* [*à visée didactique*, *con intención didáctica*] gesture δ with respect to $\tilde{n}$ and $\mathcal{C}$. In other cases, however, such an intention is not declared by w and may even not exist at all. In all cases, let us denote by R_0 the relation $R(u, o)$ before δ occurs, by R_1 the relation $R(u, o)$ after δ has occurred. In the case of an intendedly didactic gesture on the part of w, we can say that the instance w prognosticates, however implicitly, that the

following statement will become true: $v \vdash \varphi(R_1, \bar{R}) > \varphi(R_0, \bar{R})$. To make a short story longer, let us say that w anticipates that v will declare the degree of conformity of $R(u, o)$ with $R_I(p, o)$ to be higher once δ has occurred.

A crucial aspect of w's situation must be emphasised, which functionally distinguishes the role played by w from that of v: while v judges the *effects* of δ on $R(u, o)$ once they have occurred, w is forced to "prophesy" v's judgement about these effects *before they occur*. In other words, w makes an *a priori* judgement on v's judgement, which, on the contrary, is based on an *a posteriori* analysis. Of course w's prediction is generally based on a great many elements, including the observed effects on $R(u, o)$ of seemingly nearby situations $\varsigma' = (\tilde{n}', \mathcal{C}', w', \delta')$ previously observed. A posteriori observations available to w are usually taken into account in the formulation of w's conjecture with respect to δ—a posteriori knowledge is almost always a key ingredient of an a priori decision.

The notion of possibly didactic situation can be easily and fruitfully generalised. Instead of an instance w, let us consider a set W of instances w, and, instead of a gesture δ, a set Δ of gestures δ, with members of W making the gestures $\delta \in \Delta$. This results in a (generalised) possibly didactic situation denoted by $\varsigma = (\tilde{n}, \mathcal{C}, W, \Delta)$. The notion of an intendedly didactic situation can as well be generalised. But we shall now reformulate the symbolic expression of a (possibly didactic) situation $\varsigma = (\tilde{n}, \mathcal{C}, W, \Delta)$. A gesture δ or a set of gestures Δ alters the set $\mathcal{C}$ of prevailing conditions, $\mathcal{C}$ becoming the set $\mathcal{C}'$ of *newly* prevailing conditions. We call $\mathcal{C}'$ the *derangement* [*dérangement*, *desarreglo*] of $\mathcal{C}$ by δ (or Δ) and denote it by $\mathcal{C}' = \mathcal{C}^{\wedge\delta}$ (or $\mathcal{C}' = \mathcal{C}^{\wedge\Delta}$), to be read "$\mathcal{C}$ deranged by δ (or Δ)", where the (typographical) symbol $\wedge$, used here metaphorically, is the "caret insertion point" (the Latin *caret* means "it lacks"). We can write $\mathcal{C}' = \mathcal{C}_0 \cup \mathfrak{D}_\delta$ (or $\mathcal{C}_0 \cup \mathfrak{D}_\Delta$), where $\mathcal{C}_0 \subset \mathcal{C}$ and where $\mathfrak{D}_\delta$ (or $\mathfrak{D}_\Delta$) are the *new* conditions generated by δ (or Δ). This leads to consider the

following alternative expression of a possibly didactic situation: $\hat{\varsigma} = (\tilde{n}, \mathcal{C}, W, \mathfrak{D})$, where $\mathfrak{D}$ is the set of "*deranging conditions*" [*conditions dérangeantes*, *condiciones desarreglandas*] engendered by the instances $w \in W$.

Let us now return to our main point: what criteria should we use to say that a *possibly* didactic situation ς is a (plainly) *didactic* situation—or not? To answer this cardinal question, we have to return to the indeterminate instance $\hat{w}$. We shall say that the possibly didactic situation $\varsigma = (\tilde{n}, \mathcal{C}, W, \Delta)$ is a *didactic situation for* $\hat{w}$, or is $\hat{w}$-*didactic*, with respect to $\tilde{n}$ and $\mathcal{C}$, if $\hat{w}$ anticipates that v will declare the degree of conformity of $R(u, o)$ with $R_I(p, o)$ to be *higher* once the gestures $\delta \in \Delta$ have been performed by the instances $w \in W$. In case $\hat{w}$ forecasts that this degree will be found *lower* by v, we shall say that ς is *antididactic* for $\hat{w}$ or $\hat{w}$-*antididactic* with respect to $\tilde{n}$ and $\mathcal{C}$. Finally, if $\hat{w}$ foretells that v will declare that the degree of conformity of $R(u, o)$ with $R_I(p, o)$ is approximately unchanged, we shall say that ς is *isodidactic* for $\hat{w}$ or $\hat{w}$-*isodidactic* with respect to $\tilde{n}$ and $\mathcal{C}$. To put it in a nutshell, in the first case $\hat{w}$ believes that "it will work"— according to v, u will learn with respect to $R_I(p, o)$, i.e. $v \vdash \varphi(R_1, \bar{R}) > \varphi(R_0, \bar{R})$; in the second case, $\hat{w}$ thinks that "it will make things worse"—according to v, u will lose relevant knowledge about o, i.e. $v \vdash \varphi(R_1, \bar{R}) < \varphi(R_0, \bar{R})$; in the third case, $\hat{w}$ concludes that "it won't really make any difference"—according to v, u will neither learn nor unlearn with respect to $R_I(p, o)$, i.e. $v \vdash \varphi(R_1, \bar{R}) \approx \varphi(R_0, \bar{R})$.

The above conclusions may sound disappointing to some readers. They simply remind us that there is no such thing as an absolute, "privileged", "preferred" instance $\hat{w}$. In fact, any instance $\hat{w}$ may happen to be of interest to a researcher ξ—we can have, for example, $\hat{w} = u$, or $\hat{w} = w$, for some $w \in W$. Note that the situation $\varsigma = (\tilde{n}, \mathcal{C}, W, \Delta)$ can be regarded as didactic by $w_1 \in W$, as antididactic by some $w_2 \in W$, and as isodidactic by $w_3 \in W$. This generalises to instances which may happen to get connected in one way or

another with $\varsigma = (\tilde{n}, \mathcal{C}, W, \Delta)$—for example, if u is a student position, the students subjected to this position, their parents, their teachers, etc. One case in particular may trouble some readers more than any other: the case of researchers in didactics $\xi_{\mathcal{T}}$ subjected to a research position $\check{r}_{\mathcal{T}}$, where $\mathcal{T}$ is a (broadly acknowledged) theory of the didactic and $\check{r}_{\mathcal{T}}$ the position occupied by the researchers working within this theory. In that case, $\check{r}_{\mathcal{T}}$ may have developed a priori criteria according to which a situation $\varsigma = (\tilde{n}, \mathcal{C}, W, \Delta)$ ought to be labelled didactic, to the exclusion of any other viewpoint. More generally, any instance $\hat{w}$ may have worked out some "didactic" orthodoxy or orthopraxy (in a lay and non-medical sense of the word) which proves no less effective for being unscholarly. By contrast, the scientific aim of the ATD is to explore, analyse and take stock of the variation across space and time of the multifarious personal and institutional relations to the didactic. In other words, our aim should be to eventually understand and master the *economy* (what gestures are made, by whom, and for what reasons?) and the *ecology* (which gestures can and cannot be made, and why?) of the possibly didactic. In studying the ecology and economy of the possibly didactic, *all* gestures must be taken into account, even if w does not intend them to be didactic. More broadly, all conditions, regardless of their level in the scale of didactic codeterminacy, must be regarded as *possibly* didactic and studied accordingly. To make headway now, we have to delve deeper into what the ATD offers.

The theory of praxeologies

The theory of praxeologies is certainly the most well-known tool proffered by the ATD. So I shall skip a lengthy introduction and simply remind you that a praxeology is a quadruple traditionally denoted by $\wp = [T / \tau / \theta / \Theta]$. The first two components compose the *praxis* block [*le bloc de la praxis*, *el bloque de la praxis*] or "know-how" part [*le savoir-faire*, *el saber hacer*], denoted by $\Pi = [T / \tau]$, while the second two form the *logos*

block [*le bloc du logos*, *el bloque del logos*] or "knowledge" part [*le savoir*, *el saber*], denoted by $\Lambda = [\theta / \Theta]$. We shall write: $\wp = [T / \tau / \theta / \Theta] = [T / \tau] \oplus [\theta / \Theta] = \Pi \oplus \Lambda$. The letter T designates a *type* of tasks [*type de tâches*, *tipo de tareas*]. An element t of T is a (particular) task, i.e. a specimen [*spécimen*, *espécimen*] of type T. The Greek letter τ denotes a *technique* [*technique*, *técnica*] relative to the type of tasks T. The tasks $t \in T$ that τ allows one to successfully perform constitute *the scope* [*la portée*, *el alcance*] of τ, which is usually a *strict* subset of T. As for the Greek letters θ and Θ, they refer respectively to the *technology* [*technologie*, *tecnología*] of τ and the *theory* [*théorie*, *teoría*] of this technology. Note that the usage of calling (by synecdoche) a whole praxeological system "a theory" has become common: the "theory of rational numbers" (in a given institution), for example, is usually understood to include at the same time theoretical parts, technological parts, types of tasks, and techniques.

A cardinal point in the theory of praxeologies is that *any* personal or institutional "doing" is regarded as a task t of a certain type T, or a concatenation of tasks $t_1, t_2, ..., t_n$ of types $T_1, T_2, ..., T_n$, respectively, which implies a technique τ together with a *logos* block $[\theta / \Theta]$, or techniques $\tau_1, \tau_2, ..., \tau_n$ together with *logos* blocks $[\theta_1 / \Theta_1], [\theta_2 / \Theta_2], ..., [\theta_n / \Theta_n]$. If, for example, an instance $\hat{w}$ regards a possibly didactic situation $\varsigma = (\tilde{n}, \mathcal{C}, W, \Delta)$ as *anti*didactic, such a judgement is based on a (more or less hidden) praxeology $\wp$ that it behooves the researcher ξ to study. In particular, the study of $\wp$ must reveal, beyond the technique τ, the technological and theoretical constituents that play a part in $\hat{w}$'s judgement—praxeological analysis is the key lever of cognitive analysis. Here a word of caution is in order. There is no such thing as an isolated *praxis* block, unrelated to any corresponding *logos* block: whatever the instance u using a technique τ, there exists for u, at least in an embryonic form, a *logos* block with more or less predictable content,

which will often prove to be "*u*-made" rather than the by-product of a larger institutional world.

How does the notion of praxeology connect with the cognitive and didactic theories outlined so far? Firstly, let us define the *object universe* [*univers objectal*, *universo objetal*] $\Omega(u)$ of an instance *u* by $\Omega(u)) \overset{\text{def}}{=} \{o \,/\, R(u, o) \neq \emptyset\}$. We then define the *cognitive equipment* [*équipement*, *equipamiento*] $\Gamma(u)$ of *u* by: $\Gamma(u) \overset{\text{def}}{=} \{(o, R(u, o) \,/\, o \in \Omega(u)\}$. A praxeology $\mathscr{p}$ is simply an object of a special kind, to which an instance *u* may therefore have a nonempty relation $R(u, \mathscr{p})$. Consequently, we define the *praxeological universe* of *u*, $\Omega^{\blacklozenge}(u)$, by $\Omega^{\blacklozenge}(u) \overset{\text{def}}{=} \{\mathscr{p} \,/\, R(u, \mathscr{p}) \neq \emptyset\}$ and the *praxeological equipment* of *u*, $\Gamma^{\blacklozenge}(u)$, by $\Gamma^{\blacklozenge}(u) \overset{\text{def}}{=} \{(\mathscr{p}, R(u, \mathscr{p})) \,/\, \mathscr{p} \in \Omega^{\blacklozenge}(u)\}$. It is obvious that $\Omega^{\blacklozenge}(u) \subset \Omega(u)$ and $\Gamma^{\blacklozenge}(u), \subset \Gamma(u)$. Conversely, we posit that $\Gamma^{\blacklozenge}(u)$ is a "generating" subset of $\Gamma(u)$, which is to say that the relation $R(u, o)$ emerges from the relations $R(u, \mathscr{p})$, for all those praxeologies $\mathscr{p} \in \Omega^{\blacklozenge}(u)$ that appear to *u* to bring into play the object *o*, either technically, technologically, or theoretically. This definition calls for some comments. Firstly, it applies to *all* possible objects *o*. As a consequence, the relation that an instance *u* has to a person *x*—to one's mother, for example, when *u* is a person—comes out of the wealth— or the dearth—of praxeologies belonging to $\Gamma^{\blacklozenge}(u)$ in which the person *x* plays a part. Although it may not be an obvious fact at first glance, because we are generally imbued with an idealistic view of the human world, this is the view advocated by the ATD; and the same applies to institutional instances, not persons, as well. Secondly, it may happen that, given the relation $R(u, \mathscr{p})$, the instance *u* will *not* enter in contact with the object *o* through $\mathscr{p}$, whereas another instance *u'* will typically know the object *o because* of its being a "key" ingredient of $\mathscr{p}$. A nearby but distinct situation occurs when two different praxeologies $\mathscr{p}$ and $\mathscr{p}'$ exist, both relative to the type of tasks *T*. Here is an elementary,

mathematical example. The type of tasks consists in removing the radical from the denominator of a fraction like, say, $\dfrac{5}{3-\sqrt{2}}$. The classical technique τ consists in multiplying the numerator and denominator by the so-called "conjugate" of the denominator, in that case $3+\sqrt{2}$, which gives: $\dfrac{5}{3-\sqrt{2}} = \dfrac{5}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{15+5\sqrt{2}}{9-2} =$

$\dfrac{15}{7} + \dfrac{5}{7}\sqrt{2}$. In such a case, even a limited knowledge of τ implies some knowledge by the user of τ of the object o called the "conjugate". However, another, more elementary, technique τ' exists that *does not* require such a notion. It is based on the following technological theorem: Given positive numbers a, b, c, and d, if $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{\lambda a + \mu c}{\lambda b + \mu d}$ for all λ and μ such that $\lambda b + \mu d \neq 0$. (When a, b, c, d, λ, and μ are positive integers, this result can be readily interpreted in terms of proportions of balls in urns.) The first step in the technique τ' consists in creating the "new" fraction $\dfrac{c}{d}$ by multiplying by the radical in the denominator the numerator and denominator of the fraction $\dfrac{a}{b}$. Here we have: $\dfrac{5}{3-\sqrt{2}} =$

$\dfrac{5\sqrt{2}}{3\sqrt{2}-2}$. The second step will be to linearly combine b and d so as to get rid of the radical,

as follows: $\dfrac{5}{3-\sqrt{2}} = \dfrac{5\sqrt{2}}{3\sqrt{2}-2} = \dfrac{5 \times 3 + 5\sqrt{2} \times 1}{(3-\sqrt{2}) \times 3 + (3\sqrt{2}-2) \times 1} = \dfrac{15+5\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-2} =$

$\dfrac{15+5\sqrt{2}}{7} = \dfrac{15}{7} + \dfrac{5}{7}\sqrt{2}$. It should be clear that there is no reason for the "user" u of τ' to come across the object "conjugate"—which may therefore remain absent from $\Omega(u)$. Let us note also that, even if $\tau' = \tau$, $\wp$ and $\wp'$ may differ because of distinct technologico-theoretical blocks $\Lambda = [\theta / \Theta]$ and $\Lambda' = [\theta' / \Theta']$, in which case the user will come across distinct technological or theoretical elements.

Let us consider again a cognitive nucleus $\tilde{n} = (u, o, I, p, v)$ and a possibly didactic situation $\varsigma = (\tilde{n}, \mathcal{C}, w, \delta)$. The gesture δ performed by the instance w may be intendedly didactic or not. In any case, δ is a task (or a sequence of tasks) whose performance draws on a praxeology $\wp$ (or a sequence of praxeologies $\wp_1, \wp_2, ..., \wp_n$). As a result, the gesture δ is dependent on w's praxeological equipment $\Gamma^{\bigstar}(w)$, notably when w is an institutional position. When δ is intendedly didactic, we shall speak of an *intendedly didactic praxeology*. All this remains true when $w = u$, in which case δ may therefore pertain to an intendedly *auto*didactic praxeology. Let us add that the evaluating instance v issues judgements that proceed from *evaluation praxeologies*, which may be intendedly *un*didactic.

The dialectic of persons and positions

So far, we have not distinguished between the cases where the instance u is a person x and those where u is an institutional position (I, p). In other words, we have deliberately treated unlike cases alike. A hurried reader, skimming through the above developments, may unthinkingly take u in $\tilde{n} = (u, o, I, p, v)$ to represent a personal instance, not an institutional position—and the same goes for the instance v. This state of things should be regarded as one of the most critical issues for the future of didactics. What is the problem exactly? It seems that, up to now, didacticians have focused almost exclusively on the making of *persons*. Let us limit our attention momentarily to the case of a classroom. Here a person, the teacher y, makes gestures to help persons x better conform to some positional relation to some object o. Of course the teacher y is not a "pure" person, but first and foremost a *subject* of the teacher position p_t. In like manner, the student x is not an artless person: x is essentially subjected to the student position p_s. In standard, "classical" didactics, the positions p_s and p_t are taken for granted: they are "givens", and we usually don't bother about them. In other words, we are concerned by

the process through which a person x adapts to the position p_s with the help of y, or, for those of us concerned with the training of teachers, we focus on the process by which a person y becomes a teacher, in full conformance with the requirements of the institutional position p_t. Where, then, does the problem lie? In the scale of didactic codeterminacy, teachers usually restrict themselves to considering the two *lowest* levels, that of pedagogies and of didactic systems. Worse still, many didacticians haughtily ignore even the level of pedagogies and confine themselves to the "interior" of didactic systems— which boils down to largely ignoring the set $\mathcal{C}$ in $\varsigma = (\tilde{n}, \mathcal{C}, w, \delta)$. For any didactic system $S(X, Y, o)$ to exist, it is necessary that (at least) two institutional positions, p_s and p_t, come into being—their general existence is a cardinal condition for the blossoming of didactic systems. More generally, in a given society, it is necessary for the possibly didactic to exist that a whole array of institutional positions be available. Consequently, it is the task of the researcher ξ to study not only sets of conditions that can make a student adapt to a given position p_s but to investigate also the creation and shaping of this very position p_s— the conditions of the birth of an institutional position involved in some possibly didactic situations are themselves possibly didactic conditions, that needs to be analysed as such.

Once again, let me illustrate this with some easy examples. One possible scenario is when some instance w decides to replicate in a given institution I' a system of positions (p_s, p_t) already existing in some institution I known to w. A replica of a position p according to an instance $\hat{w}$ is defined to be a position p' such that $\hat{w} \vdash \Gamma(p') \approx \Gamma(p)$; or more basically such that $\hat{w} \vdash \Gamma^{\blacklozenge}(p') \approx \Gamma^{\blacklozenge}(p)$. In the case in question, we must therefore have $\hat{w} \vdash \Gamma^{\blacklozenge}(p_{s'}) \approx \Gamma^{\blacklozenge}(p_s)$ and $\hat{w} \vdash \Gamma^{\blacklozenge}(p_{t'}) \approx \Gamma^{\blacklozenge}(p_t)$. The first example we can look at happened to be a total failure, if we are to believe the following account given by the French mathematician André Weil (1906-1998) in the preface to his book *Number Theory*

for Beginners (1979), in a passage where he happens to mention the mathematician Maxwell Alexander Rosenlicht (1924-1999) and where $\hat{w}$ = André Weil:

> In the summer quarter of 1949, I taught a ten-weeks [*sic*] introductory course on number theory at the University of Chicago; it was announced in the catalogue "Algebra 251". What made it possible, in the form which I had planned for it, was the fact that Max Rosenlicht, now of the University of California at Berkeley, was then my assistant. According to his recollection, "this was the first and last time, in the history of the Chicago department of mathematics, that an assistant worked for his salary". The course consisted of two lectures a week, supplemented by a weekly "laboratory period" where students were given exercises which they were asked to solve under Max's supervision and (when necessary) with his help. This idea was borrowed from the "Praktikum" of German universities. Being alien to the local tradition, it did not work out as well as I had hoped, and student attendance at the problem sessions soon became desultory. (p. v)

Didactic systems are rarely in total isolation. Much to the contrary, they usually live in *associations* comprising generally at least a core system that we call the *principal* didactic system (PDS) and a number of peripheral systems called *auxiliary* didactic system (ADS). The most widespread, though often debated, of these associations is, in all likelihood, that of a class $C(X, y)$ (which is the place of the succession of PDSs) together with the auxiliary system $S(x, y^*, o)$ devoted to *homework*—in that case, the "helper" y^* may be x's mother or father or some older sibling of x, or may not exist at all. In the second place, we find the ADS dedicated to "(supervised) practical work". As defined by the online Collins Dictionary, a *practicum*—a word written the German way by Weil— is "a course in which theory is put into practice, a practical training or research session"; it is, according to the *Wiktionary*, "a college course designed to give a student supervised practical knowledge of a subject previously studied theoretically". The introduction of a practicum component in the teaching format substantially altered the *pedagogy* of the "school"—the Department of Mathematics of the University of Chicago—where Weil's lectures took place. The online Merriam-Webster Dictionary asserts that the "first known use" of *practicum* in the United States dates back to 1874, in the restricted sense of "a course of study for teachers, doctors, nurses, etc., that involves actually working in the

area of study and using the knowledge and skills that have been learned in a school". The insuperable strangeness to students that Weil reports on seems to be due to using the notion of *practicum* outside the training of, say, teachers or nurses (and a few other sorts of professionals), and most particularly in that "purest part of pure mathematics" which is number theory. The result was that, whereas the position p_t (held by Max Rosenlicht) came into existence (which Rosenlicht's mocking comment on his earning his salary for once bears out), although it didn't come to fruition, the position p_s remained in limbo all the while. Weil's account is therefore the story of a didactic flop. This fiasco suggests that the successful creation of a new kind of didactic systems in a given teaching format requires not only students and teachers deemed intrinsically "good" but also students and teachers prepared to take part in the construction of the institutional positions to which they will eventually conform. It is not unreasonable to consider that in the early stages of creating an institution, the shaping of appropriate institutional positions must take precedence over the actors' accomplishments, for the right functioning of the institution almost always lags behind its structural tailoring.

We shall now go through an example somewhat different. Andrew Hacker is currently an emeritus professor of political science in the Department of Political Science of Queens College of the City University of New York (CUNY). In 2016 was released his book *The Math Myth and other STEM delusions* (the acronym STEM means "science, technology, engineering, and mathematics"), a book that generated no little excitement and controversy among math and nonmath people alike! In chapter 12 of his book, Hacker reports on a teaching experiment he carried out. Here is the introduction to his account of it:

> In the fall of 2013, I visited my college's Department of Mathematics with a proposal. Most of our students are required to take an introductory mathematics course. I offered to teach an experimental section, which would focus on

quantitative reasoning. I made it clear that my assignments would rely almost entirely on arithmetic, but at a rigorous level and often in ways not ordinarily employed. Its aim would be to make students agile with numbers, including the use and analysis of statistics. So along with being a professor of political science, I added being a professor of mathematics to my résumé. It's been said that at New York's Bellevue Hospital, the interns practice on their patients. In that spirit, I want to thank my students in Mathematics 110 [*sic*] for the trials and errors they amiably endured. What follows is a sampling of what we covered. (p. 181)

In the case at hand, w = Andrew Hacker makes a gesture δ in the form of a proposal presented to Queens college's Department of Mathematics—Hacker being himself a member of the Department of Political Science of that college. The proposal aimed to set up a new course, to be called "Numeracy 101", which came down to creating a (seemingly) "brand-new" SDP, $S(X, y, o)$, in which $y = w$ = Andrew Hacker, o = "quantitative reasoning", X being a (potential) group of students of political science. The field called "quantitative reasoning" by w consists in an apparently unlimited list of questions, of which Hacker gives a small sample in his book: When and how does a number become a statistic? Is a ten-month year possible? How large is West Virginia? Surveys show that most Americans—even if fewer than in the past—view that country as being the best in the world. But can this sentiment be backed by facts?... It seems that, contrary to the *praktikum* advocated by Weil, the "experiment" of Numeracy 101 was a success, at least from the point of view of Andrew Hacker = $\hat{w} = w = y = v$. The weakest point, however, is that Hacker's attempt at changing the mathematics education of "nonmath" students proved to be a once-only, impermanent feat—in point of fact, it seems to have lasted no more than three years. Can we think of conditions that would make it into a really sustainable and generalisable achievement? Or is Hacker's attainment a one-off operation? Certainly we can suspect some critical conditions to be nonrenewable or little renewable. In particular there was possibly the fact that it was looked upon by many stakeholders and observers as a first-time experiment, "just to see". And there was the fact that the proposer was a well-known scholar and, at the start, an

appreciated personality in academe and elsewhere. Also, Hacker's teaching project supposedly addressed mathematical needs of *nonmath* students, a fact that may have surreptitiously tipped the scale in favour of its endorsement by the Department of *Mathematic*s. More generally, what appears problematic is the sustainability of a set of conditions which would ensure success according to a variety of criteria. In truth, this very problem undermines the credibility of *most* overpersonalised "one-shot" innovations.

In the case just examined, we can surmise that both the students $x \in X$ and the teacher y (= Andrew Hacker) were "good". But the positions p_s and p_t they occupied were undeniably fragile, as if they were passing dreams with no future. These remarks can be generalised. Whenever students study and teachers teach, they willy-nilly embark on a collective process *of creation and development* of institutional positions p_s and p_t. This is true in the cases of incipient didactic systems that have to be started from scratch, which was the case, for instance, when some of us, in the early 1990s, set to work to create the late IUFMs ("instituts universitaires de formation des maîtres"): we were then creating a position p_t at the same time that we tried to validly occupy it; and we contributed as much as we could to creating, against, mostly extraneous, backward-looking and reactionary forces, a position p_s that "our" student teachers would, even if unwillingly, eventually come to occupy. But this is true as well in *every classroom* where students and teachers routinely meet to work together. All teachers can be aware that *they don't just teach*. Jointly with their students and many other stakeholders, they shape and reshape these students' position as much as their own position. Students, whatever their age, can be made aware, through appropriate means, of the creative process in which they are, by necessity, engaged.

The dialectic between institutional positions and persons is central to the life and history of institutions and peoples. It is critical to the understanding of the way the didactic works. Up to now, however, we have told only part of the story. Two reciprocally connected questions remain to be considered. The first one is: What does it mean to make some gesture δ so that $R(u, o)$ achieves greater conformity with $R_I(p, o)$ in the case when u is not a person but an institutional instance $(\bar{I}, \bar{p})$? As for the second question, it is: Does a didactic system have a role in shaping institutional positions, apart from p_s and p_t? Without oversimplifying things, let us suppose that $\bar{I}$ is an occupation and $\bar{p}$ a given position in that occupation. In all cases, the position (I, p) that serves as criterion is known through a description of its praxeological equipment $\Gamma^{\bigstar}(I, p)$, which is quintessentially $(\bar{I}, \bar{p})$'s *desired* equipment $\Gamma^{\bigstar}(\bar{I}, \bar{p})$. The position $(\bar{I}, \bar{p})$ becomes reality when *persons* subject themselves to it, be they veteran professionals or novices in that position. In such a case, particularly when $\bar{p}$ is regarded as a high-skilled occupational position, there usually exists a vocational school σ, endowed with specific positions p_s and p_t, which trains and produces novice professionals $\bar{x}$ to occupy the position $\bar{p}$. Such a school σ, which has to continuously create and re-create within itself appropriate positions p_s and p_t, simultaneously contribute to reshaping the budding position $(\bar{I}, \bar{p})$ by dispatching to it new but like-minded professionals, whose coming into office and involvement will at least partially alter the existing cognitive and praxeological equipments of $(\bar{I}, \bar{p})$. This is typically how (professional) institutions change. In this regard, the name "normal school", used formerly to designate a teacher-training institute, should be understood in two different ways. It refers first to the idea that aspiring teachers should be instructed in the current "norms" of the profession they want to take up. But a "normal school" can also be regarded dually as an institution that, essentially through the intervention of its former students who have become teachers, reshapes the teaching trade. This remark can be

generalised to all vocational schools and all trades: a vocational school at the same time *enforces* the (current) norms of the trade *and* more or less covertly (though not unwillingly) *alters* them.

We can now go back to the first question. What can an instance w do to help a positional instance $(\bar{I}, \bar{p})$ to better conform with the criterial position (I, p)? Whether (I, p) is only a paper entity or a functional reality, an expert on (I, p) may be appointed to help develop $(\bar{I}, \bar{p})$ in conformity with (I, p). Also, short in-service training courses can be organised for would-be subjects of $(\bar{I}, \bar{p})$, for example. Above all, a specialised school offering a complete initial training course can be created (or updated), along with its pedagogy and its didactic systems. We are thus brought back to our central point of interest. But, now, we should know better. It should be obvious that the ecology and economy of the didactic cannot be conceived of, and studied, in ignorance of the dialectic between persons and institutional positions. Much too often, working didacticians content themselves with questions of the following kind: *Given the established system of positions*, what can y or x, respectively, do to help x learn o? (The opening phrase in italics is usually unstated.) Generally speaking, the prevalent system of positions is taken for granted by ξ (in the wake of y, whose focus is understandably narrower), so that the didactic effects of possible variations in this system are never clearly explored, even though they can significantly change cognitive dynamics. By contrast, it should be a burning ambition of the ATD to take into account the aforesaid dialectic, in order to come to grips with the didactic as a "total social fact", to use a notion borrowed ("Social fact," n.d.) from the French anthropologist Marcel Mauss (1872-1950).

The ATD and the concept of inquiry

Any researcher ξ is obviously concerned with the cognitive and praxeological dynamic of *persons*. The foregoing demonstrates that ξ has no choice but to also study

the dynamic (and, for that reason, the static too) of *institutional positions*, notably that of u, v, w, and $\hat{w}$ as they appear in the above formalisation, where $\tilde{n} = (u,\, o,\, I,\, p,\, v)$ is a cognitive nucleus and $\varsigma = (\tilde{n},\, \mathcal{C},\, w,\, \delta)$ a possibly didactic situation that some $\hat{w}$ may hold to be didactic, or antididactic, or isodidactic with respect to v's (future) judgement. To do so, ξ may have to play different roles—that of $\hat{w}$ (indeed, ξ is a $\hat{w}$ of a sort), of w, of v, or even, naturally, of u—without ever ceasing to be a researcher, and notably without ever stopping to identify and analyse the interplay between u, v, w, $\hat{w}$, o, $(I,\, p)$, δ, and $\mathcal{C}$.

In this part of my talk, I shall focus on some aspects of the question *What about o?* It has been said that o is whatever object considered by at least some instance $\hat{w}$ as worthy of being studied by at least some instance u with the help of instances $w \in W$ making gestures $\delta \in \Delta$. Concretely, this will lead to the setting up of a didactic system $S(X,\, Y,\, o)$, where each $y \in Y$ is a w of a certain kind. (Beware! A minister of education or a town mayor, for instance, can be a w too.) Now, as it appears on the scale of didactic codeterminacy, a didactic system has to be "accepted" by some school σ and prove dynamically compatible with σ's pedagogy. The personal instances $u = x \in X$ and $w = y \in Y$ have to be "inducted" into the school σ. Possibly still more important, σ must accept the creation and existence of the positions p_s and p_t that the $x \in X$ and $y \in Y$ will respectively occupy. For lack of space, I shall hereafter confine myself to the case of the object o—that must likewise be "endorsed" by σ.

The accreditation of o by a school σ commonly requires that o be subsumable under some "academic" discipline that has or could have its authorised place in σ. Broadly speaking, this process of validation is a complicated matter, which has to do with the concept of *didactic transposition*. In the episode related by André Weil, where σ was the Department of Mathematics of the University of Chicago, it was certainly straightforward to reach approval for teaching the course "Algebra 251"—in fact, Weil stayed at Chicago

for over a decade (1947-1958) at the invitation of Marshall Stone (1903-1989). In the case of Hacker's teaching at the Department of Mathematics of Queens College, it seems that things went not so smoothly. The course number, "Numeracy 101", provides some information in this respect. According to Wikipedia's article entitled "Course (education)", "the course number 101 is often used for an introductory course at a beginner's level in a department's subject area". In other words, this course was very modestly ambitious. Moreover, the course's name was not *mathematic*s but *numeracy*, a term which most dictionaries define plainly as "the ability to use numbers, especially in arithmetical operations". As for Hacker himself, he simply advocated "adult arithmetic", and "citizen statistics" at the very most, an unthreatening aspiration, which may have defused the situation. As a general rule, given an object o, and provided o is not a "classic", it is not easy for an instance w to track down a school σ that has o in its collection of objects of study. (Naturally, most instances w do just the opposite: they come across some school σ and let the instances u they care about study whatever σ has to offer.) Let me add that, in actual fact, it is even more difficult to find a school σ that includes in its curricular repertoire not only the object o but also a *given* institutional relation $R_I(p, o)$ that a normally achieving student x attending that school will eventually conform with, at least from the point of view of some evaluating instance v_σ accredited by σ—which translates as: $v_\sigma \vdash R(x, o) \cong R_I(p, o)$. It is even more unlikely to encounter a school σ that fully handles a given cognitive nucleus $\tilde{n} = (u, o, I, p, v)$, with $v = v_\sigma$, where u, whether a person or an institutional position, is designated beforehand.

Let us return to the problem of the object o and the didactic system $\mathscr{s} = S(X, Y, o)$. To take things further, we need one more notion, that of a *work* [*œuvre*, *obra*]. Just like we talk about a work *of art*, or a work *of literature*, we shall refer freely to a work *of mathematics*, or of algebra, physics, etc. Generically, a work is anything *purposely*

human-made. Algebra is a work, the notion of an equation is a work, any technique is a work, etc. Despite the immemorial prejudice in favour of the "natural" and against the "artificial", we shall posit that any object *o* whatsoever is an "artefact", although it is never created out of nothing. In truth, we understand an object *o* to be anything identified by some person or institution as an ingredient of a praxeology, which makes it into a work. Even a small stone, when propelled by a slingshot, is such an object *o* and is therefore a specific work in itself—as such it is not "given" by nature but devised by humans. In spite of its possible sheer materiality, an object *o* that appears in the didactic system $s = S(X, Y, o)$ should therefore be regarded as a work. But here we will introduce yet another distinction. A work *o* can be a notion, a word or phrase, a written symbol like $\sqrt{}$, a theory, an answer to a question, a process, a tool, a machine, a show, a book, a poem, a symphony, a button, a shoelace, a building, a spiral staircase, an atom of cobalt, etc. All these objects are *works of the first kind*. They contrast with the works of *the second kind*, which are nothing else than *questions*. A question is undoubtedly human-created and purposive. It is therefore a work. Works of the second kind, i.e. questions, are precious resources, integral to our praxeological life. This is where the idea of *inquiry* [*enquête*, *investigación*] comes in. In this respect, it is worth quoting almost in full the *Online Etymology Dictionary*'s entry on "inquire":

> c. 1300, *enqueren*, *anqueren*, "to ask (a question), ask about, ask for (specific information); learn or find out by asking, seek information or knowledge; to conduct a legal or official investigation (into an alleged offense)," from Old French *enquerre* "ask, inquire about" (Modern French *enquérir*) and directly from Medieval Latin *inquerere*, from *in-* "into" (...) + Latin *quaerere* "ask, seek" (...), in place of classical Latin *inquirere* "seek after, search for, examine, scrutinize."

The "related entries" listed by this dictionary include *enquire*, *inquest*, *inquirer*, *inquiring*, *inquiry*, *inquisition*, *inquisitive*, *inquisitor*, and *query*. Let us begin with the case of a "didactic stake" *o* which is a question *q*, so that we shall write the didactic system

$s = S(X, Y, q)$, i.e. as a didactic system "of the second kind". The study of q in s aims to formulate an answer a that will resist at least some currently possible attempts to disprove it. It has become usual to write it as follows: $S(X, Y, q) \to a$. This is the *Herbartian schema in reduced form.* At this point, a divergence occurs between ways of producing the answer a in s. The didactic scenario through which the answer a is arrived at in s may be one of a series that can take place on a continuum. At one end, we will locate the "magisterial" scenarios, in which the students' *topos*, that is to say the system of (possibly) didactic gestures students are allowed to make and for which they have full responsibility, is the *smallest* possible one, while the teacher's *topos* is at its largest. In this scenario, the teacher y—it is supposed here that the set Y is a singleton—is fully a "teacher": y inquires into the question q on one's own—or rather *has* inquired in advance, sometimes a long time before—and has in store an answer a_y that this very y will "teach"—i.e. show, point out—to the students. For these students, y's answer a_y will be the class's answer to q, traditionally denoted by $a^\heartsuit$, which the students $x \in X$ will have, from this point forward, to retain and use. At the opposite end of the continuum are the "collegial" scenarios, in which the inquiry into q is entrusted to the students X, with y to oversee them as the "chief of inquiry". In this case, we rewrite the Herbartian schema in *a semi-developed form*, as follows: $(S(X, Y, q) \to M) \to a^\heartsuit$, where the set M, progressively created by X under the supervision of y and called the *didactic milieu*, brings together entities of potentially all nature regarded as possibly didactic tools for studying q. (In purely magisterial scenarios, the milieu M exists as well but its makeup belongs entirely in the teacher's *topos*.) The theory of inquiry developed within the framework of the ATD is concerned with studying formats *across the entire continuum* between the two poles of the "purely magisterial" formats and the "purely collegial" formats.

What happens in the case of a didactic system of the first kind $s = S(X, Y, o)$, where o is not a question? In general terms, the study of a work o amounts to raising questions q_i about o (what is the structure of o? what is it made for? how does it works? can we dispense with it, and should we? etc.) and studying them in themselves. We are thus led back to the case of a didactic system of the second kind, $s_i = S(X, Y, q_i)$. Conversely, the study of a question q leads to study other works. In this perspective, the didactic milieu M can be written as follows: $M = \{ a_1^\lozenge, a_2^\lozenge, \dots, a_m^\lozenge, o_{m+1}, o_{m+2}, \dots, o_n, q_{n+1}, q_{n+2}, \dots, q_p, d_{p+1}, d_{p+2}, \dots, d_q \}$. Here, the $a_i^\lozenge$ are answers to q that can be found in a variety of institutions (in which they have been "hallmarked"—whence the lozenge $\lozenge$ in superscript); the o_j are "other works" (for example those works that answers $a_i^\lozenge$ rest upon), the q_k are questions raised about the question q, the answers $a_i^\lozenge$, the other works o_j, and the data sets d_l—which, naturally, are works to be questioned in the course of the inquiry. The interplay between works of the first kind and works of the second kind can continue almost endlessly. At every step of the way, the respective *topoi* of the students and the teacher may vary between the "almost all magisterial" and the "almost all collegial". Of course, one can also consider the case of an (intermittently) *auto*didactic system, in which $Y = \emptyset$.

One important question currently under investigation in the framework of the ATD is that of the conditions that would allow defining and implementing a student position ι_s and a teacher position ι_t appropriate to a teaching format based on a given *collegial* scenario. I shall not dwell on this point, but three easy remarks are in order here. First, the Greek letter used, ι, is the letter iota (ἰῶτα), the first letter of the word ἱστορίᾱ (*historíā*), which produced English *history* and is defined as follows by the *Wiktionary*: 1. inquiry, examination, systematic observation, science; 2. body of knowledge obtained by systematic inquiry; 3. written account of such inquiries, narrative, history. The aim of

my second remark is to avoid confusion between *creating positions* ι_s and ι_t and preparing *teachers* (respectively, *students*) to adequately occupy these positions. The third caveat is that there exist today a plethora of learning and teaching formats that use the *umbrella* term *inquiry* as an emblem, so that we must be careful not to loosely identify the concept of inquiry as it keeps developing in the framework of the ATD with any of the many pedagogic formats which (not illegitimately) show an interest in the mere idea of inquiry. In fact, a permanent research focus of the ATD is nothing less than to make sense of *all* possible learning and teaching formats by modelling and analysing them appropriately *in terms of inquiry*.

Such a modelling approach responds to two major requirements. In the first place, it aims to develop a scientific approach to the didactic in which institutional (or personal) *normativities*—i.e. the continuing ability to propose and disseminate one's own norms—are first of all *objects of research*, which, if need be, can give rise to (norms-abiding) implementations that are first and foremost *tools* in an experimental or quasi-experimental perspective reminiscent of Gaston Bachelard's concept of "phenomenotechnique". Secondly, it is meant to enable the researcher to take into account, and account for, *all* forms of the didactic, without giving scientific prominence to any one of them, but with a will to make good scientific use of their anthropological diversity, both synchronically and diachronically. One can note for example that, due to the presence of the institutional answers $a^\diamond$ in the milieu M—regarded as a modelling tool—, the concept of inquiry set forth by the ATD allows us to model "inquiry-based" pedagogic formats plainly at variance with those pedagogic formats which, in keeping with an old school tradition, purport to restrict students' access to information in the classroom and beyond, for example by banning Wikipedia or, more generally, online sources.

Inquiry-based modelling and the disciplines

What may be the biggest obstacle we have to face as we try to develop inquiry-based modelling in the framework of the ATD is the problem of the works o that will appear in didactic systems $s = S(X, Y, o)$. What are they, what can they be, what instances "choose" or impose them? The problem that arises is bound to be complex. To clarify it, let us distinguish two broad study paradigms. The first is the *paradigm of visiting works*—understood to be works of the first kind. The didactic trajectory of a class $C(X, Y)$ can then be described as a finite sequence of works $o_1, o_2, ..., o_n$. Roughly speaking, we can differentiate three cases. Some of these works may have been "politically" imposed at the level of the *society S*, others at the level of the *school* σ, while yet others are decided upon at the level of the class $C(X, Y)$ itself, either by the teaching staff Y of their own accord (in the case of a "magisterial" scenario), or by the students X under the guidance of Y (in the case of a more "collegial" scenario). Given a work of the first kind o, the study of o unfolds a list of questions q that may be decided on by Y or, collegially, by X supervised by Y. In all these cases, a didactic system $s = S(X, Y, o)$, and specifically Y and X, enjoy a varying degree of freedom. An essential problem indefinitely ahead of us is the elucidation of the ecology and economy of such curricular organisations.

In this perspective, I would like to conclude my presentation by addressing what I regard as a *vital* question of our discipline—the controversial issue of "disciplines" or "subjects". A word first about the history of the word *discipline* itself. According to the *Online Etymology Dictionary*, *discipline* entered the English language in the early 13th century as meaning "penitential chastisement", "punishment". In Latin *disciplina* meant "instruction given, teaching, learning, knowledge" and also "object of instruction, knowledge, science". The word came from *discipulus*, "learner", "disciple", deriving itself from the verb *discere*, "learn", related to *docēre*, "teach" (the source of *doctor*,

doctrine, *document*). The sense of "treatment that corrects or punishes" *derives* from the notion of "order necessary for instruction". In English, the meaning "branch of instruction or education" is first recorded in the late 14th century. The meanings "military training" and "orderly conduct as a result of training" appear to date back to the late 15th century.

A discipline $\mathcal{D}$ will be defined to be a system of praxeologies $\wp_{\mathcal{O}}$ that bring into play a field of objects $\mathcal{O}$, this system being, as much as can be (in the institution that hosts it), "well-defined", "well-constructed", and "well-controlled". Among disciplines are the "academic" disciplines[5] as well as the primary and secondary school subjects, such as arithmetic, mathematics, physics and chemistry, physical education, languages (English, Spanish, French, etc.), history and geography, Latin, Health and hygiene, or art[6]. In accord with our definition, we can extend these lists to include systems $\wp_{\mathcal{O}}$ that are part of the praxeological equipments of many "non-scholastic" institutions.

As everyone knows, many academic disciplines claim to be "scientific", thus using a term which is, admittedly, a "difficult" word, with a long and winding history. The *Online Etymology Dictionary* reveals, in the entry "scientific (adj.)", that "*sciential* (mid-15c., 'based on knowledge,' from Latin *scientialis*) is the classical purists' choice for an adjective based on science", while "*scientic* (1540s) and *scient* (late 15c.) also have been used." According to the same source, the first record of *scientific revolution* is from 1803, that of *scientific method* from 1854 and *scientific notation* from 1961. John Ayto's *Dictionary of word origins* (2005) says of the word *science*: "its modern connotations of technical, mathematical, or broadly 'non-arts' studies did not begin to emerge until the 18th century."

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[5] Note that the extensive list given in *Wikipedia*'s article "Outline of academic disciplines" ignores didactics and pedagogy.
[6] See *Simple English Wikipedia*'s article titled "Subject (school)".

While some disciplines (in the sense propounded above) present themselves as non-scientific, or even as antiscientific, many of them crave to be called scientific—and sometimes fear to be deemed pseudoscientific. It has to be noted that the same institution—for example the *mathematical* institution—may combine an allegedly scientific discipline and, at the organisational level for example, scientifically unpretentious disciplines, that scaffold its scientific production.

What is the relevance of disciplines to the paradigm of inquiry currently developed in the framework of the ATD? In the first place, given a discipline $\mathcal{D} = \wp_{\mathcal{O}}$, let $p_{\mathcal{D}}$ be the position of whoever submits to $\mathcal{D}$. The relation $R(p_{\mathcal{D}}, o)$, where $o \in \mathcal{O}$, can provide a precious support—together with the set $\mathcal{D}|_o$ of the praxeologies $\wp \in \mathcal{D} (= \wp_{\mathcal{O}})$ that bring o into play—to whoever has to do with o in the framework of some inquiry $\mathfrak{I}(q)$ about a question q. In other words, the existing disciplines—whether scientific or not—lay the foundation for all investigative work. In concrete terms, many disciplines may have a controlling influence on the inquiry's progress—more or less indirectly through institutional, ready-made answers $a_i^{\diamond}$ to the question q, or, more directly, through works o_j brought together into the didactic milieu M. There exists a limiting case that, too often, takes all the light—when q is a question officially studied in some discipline $\mathcal{D}$ in which it has an "official" answer $a_{\mathcal{D}}^{\diamond}$. In such a case, the institution that hosts $\mathcal{D}$ and its emissaries often exert a strong pressure on the didactic system $\mathscr{s} = S(X, Y, q)$ to adopt $a_{\mathcal{D}}^{\diamond}$ —or some paler copy of it—as its "own" answer $a^{\heartsuit}$. Such an intradisciplinary approach often does not lead to satisfactory answers $a^{\heartsuit}$ because the answer required by $\mathscr{s}$ must satisfy exogenous constraints that $\mathcal{D}$ may legitimately ignore. This is a major stumbling block. Contrary to pseudoscience, a scientific discipline $\mathcal{D} = \wp_{\mathcal{O}}$ does not pretend to know everything about an object $o \in \mathcal{O}$, but only what can be ascertained through research praxeologies meeting the criteria proper to $\mathcal{D}$. More generally, we must not be led into

believing that, given an object o, there exists an institution which can tell us "the whole truth" about o and holds absolute knowledge of it. Every discipline must be taken as offering a circumscribed but to some extent reliable access to whatever object is considered. The cognitive limitation of a discipline is the price to pay for the soundness and accuracy of its statements. A consequence of this is that no academic discipline can cater to all the needs of a researcher ξ.

However, in the real world of institutions, scholastic and otherwise, things are going a little differently. From a widespread academic perspective, for example, knowledge is regarded as divided up into exclusive territories, each jealous of its purported possessions. For a discipline $\mathcal{D} = \wp_\mathcal{O}$, any object $o \in \mathcal{O}$ tends to be looked at as if it were its inalienable property, that cannot be shared with any other institutional discipline. If $\mathcal{D}$'s knowledge of o is indeed limited, the insiders of $\mathcal{D}$ often take these limits to be intrinsic to o and not liable to be truly overcome in any other field of study. Such a solipsistic stance is a trap for would-be users, who are deterred away by the apparent epistemological insularity of $\mathcal{D}$ and end up believing that you cannot be a relevant end-user of $\mathcal{D}$ unless you are a full-fledged, legitimate insider of $\mathcal{D}$. Our present-day scholastic institutions are a main factor in this situation since they traditionally practice disciplinary seclusion. Such a regime of separate development is clearly antagonistic to the paradigm *of questioning the world*, in which didactic systems $\mathcal{S} = S(X, Y, q)$ form around questions q that supposedly raise key issues relating to X's present and future needs—around what I have called "omphalic" or "umbilical" [*omphaliques* or *ombilicales*, *onfálicas* or *umbilicales*] questions relative to X and their generation and the society they enter into. But there is more to it than that.

In the course of an inquiry into some question q, whenever we come across some object o to which a well-established discipline $\mathcal{D}$ lays claim, we should never forget that,

however "pure" its insiders may believe it to be, $\mathcal{D}$ is the ever-changing result of a never-ending historical process—a reason for which $\mathcal{D}$ should be denoted by $\mathcal{D}(t)$, with $\mathcal{D}(t) = \wp(t)_{\mathcal{O}(t)}$, where t is the time at which we consider submitting ourselves to it. This entails two main consequences. Firstly, a discipline is always in the making: the set of objects $\mathcal{O}(t)$ as well as the set of praxeologies $\wp(t)$ resting on $\mathcal{O}(t)$ change over time. More than that, whenever some instance draws on $\mathcal{D}(t)$, this may result in some change both in $\mathcal{O}(t)$ and in $\wp(t)$. Despite our tendency to view the world *sub specie aeternitatis*—from the standpoint of eternity—, we must not relinquish the notion that the substance of a discipline and the discipline itself are indefinitely unfinished. To borrow from computerese, they are perpetual betas. Purism is, in this respect, beside the point. Rather we should ponder over what contribution to the inquiry $\mathcal{I}(q)$ the discipline $\mathcal{D}(t)$ would have made in the year $t = 1920$ or $t = 2070$, etc. If we had more time, I would have allowed myself to dwell on a revealing example, that of probability "theory" through the ages—starting from Geralomo Cardano (1501-1576) and his *Liber de ludo aleae* (written circa 1564 but not published before 1663), in which the author tackled the question "How many throws of a fair die do we need in order to have a fair chance of at least one six?" and answered that the number of throws should be *three*, whereas *present-day* probabilistic praxeologies entail that it must actually be *four* (Gorroochurn, 2011). In fact, just as we have considered the *possibly* didactic, we must consider the *possibly mathematical*, the *possibly biological*, etc. What we need is, indeed, an anthropological theory of $\mathcal{D}$, for example an *anthropological theory* of the mathematical, of the biological, of the linguistic, etc. In this connection, we should not forget that the objects and praxeologies composing a given discipline $\mathcal{D}(t)$ at some time t have been brought together by an effort to solve problems almost always *foreign* to the question q investigated: a critical and cautious approach to $\mathcal{D}(t)$ is therefore advisable.

At this point, we can take up again the notion of "reference praxeological model" of an object o. Let us consider two instances u and $\hat{v}$—the latter of which can be thought of as a research position $\hat{r}$. How can $\hat{v}$ make sense of $R(u, o)$, analyse it and, if necessary, make suggestions as to what some instance w could do to help $R(u, o)$ get closer to $R_I(p, o)$? In such a situation, key elements are, among others, the relations $R(\hat{v}, R(u, o))$ and $R(\hat{v}, R_I(p, o))$. In the same line as Cardano's error, we can illustrate this situation with a famous example in which u is Jean Le Rond d'Alembert (1717-1783). In the entry "Croix ou pile" (Heads or tails) of the *Encyclopedia*, d'Alembert (1754) considers the following probability problem: "One asks what is the chance of obtaining a cross in two consecutive throws." In other words, in two tosses of a fair coin, what is the probability that heads will appear at least once? D'Alembert's solution is puzzling to the modern mathematical reader. According to d'Alembert, the actual possibilities are three in number: either the first toss gives heads (H), and *the game is over*; or it gives tails (T) and there is a second toss, which gives either heads (TH) or tails (TT). The three outcomes are "therefore" H, TH, and TT. If these three outcomes are deemed equiprobable, one concludes that there are two chances out of three of getting "heads" in two tosses, whereas present-day probability theory dictates that the probability is 3/4, not 2/3, since there are *four*, not three, equiprobable outcomes (HH, HT, TH, TT). Given that the error made by u (= d'Alembert) was not a casual error caused by carelessness but proceeded from a deep-rooted personal theory of probability which u never repudiated, how can an instance $\hat{v}$ come to make sense of $R(u, o)$, where o = "probability"? What "stuff" can $R(\hat{v}, R(u, o))$ be made on? An essential constituent is the set of all the praxeologies $\wp \in \Gamma^{\blacklozenge}(\hat{v})$ that involve the object o. In keeping with notation introduced earlier, this set can be denoted by $\Gamma^{\blacklozenge}(\hat{v})|_o$ and called $\hat{v}$'s "reference praxeological model" (RPM) of o. Such a "model" is, in truth, a *modelling basis* that will allow $\hat{v}$ to develop the relation $R(\hat{v}, R(u, o))$ and, at

the same time, will set limitations on its development. In this respect, it is crucial to consider the dynamic of $\Gamma^{\blacklozenge}(\hat{v})|_o$, that is to say of $\Gamma^{\blacklozenge}(\hat{v}, t)|_o$. Be it personal or institutional, the instance $\hat{v}$ will see $\Gamma^{\blacklozenge}(\hat{v}, t)|_o$—that is, $\hat{v}$'s own "discipline" of o—change in time, due to gestures made by instances w, including $w = \hat{v}$. In the case of d'Alembert's error, for instance, many authors have tried to enrich our modelling basis to go beyond the (pithy) interpretation (in terms of equiprobability) given above (see, for example, Vargo, 1977; or, in quite a different direction, Daston, 1979). These references should remind us that, certainly more than most instances $\hat{v}$, researchers ξ and research positions $\hat{r}$ struggle to make their reference praxeological model of o explicit to themselves and to others, in order both to use it in elucidating $R(u, o)$ and to further work on this very model. But it is now high time to put an end to this introductory talk. Thank you all!

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