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The unveiling of the notion of situation in school mathematical modeling

O descortinar da noção de situação em modelagem matemática escolar

El develamiento de la noción de situación en la modelación matemática escolar

Le dévoilement de la notion de situation dans la modélisation mathématique scolaire

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Abstract

This article addresses the problem concerning reverse mathematical modeling interpreted by the type of task that consists of finding the situation with mathematics that can be associated with a mathematical model. The objective was to create conditions in the sense of the anthropological theory of the didactic that allow us to highlight this problem. For this, theoretical-methodological resources of the anthropological theory were used, more specifically, from the investigative cycle of mathematical modeling to delimit a possible trajectory of formation with high school students from basic school. The results found, through empirical experiments based on a problem in the context of school financial mathematics, made it possible to highlight the indispensable role of habitus as a durable and transposable perception system mobilized by students. This investigative path made it possible to delimit or (re)know the situation with mathematics associated with the type of problem considered. The results make it possible to respond, not exhaustively, to the problem of interest of the anthropological theory of the didactic, as well as stimulate future research on mathematical modeling conditioned to textual genres.

Keywords: Reverse mathematical modeling, Anthropological theory of the didactic, *Habitus*, Situation with mathematics.

Resumo

Este artigo aborda a problemática concernente à modelagem matemática reversa interpretada pelo tipo de tarefa que consiste em encontrar a situação com matemática que pode estar

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associada a um modelo matemático. Objetivou-se criar condições no sentido da teoria antropológica do didático que permita evidenciar essa problemática. Para isso, recorreu-se a recursos teórico-metodológicos da teoria antropológica, mais especificamente, a partir do ciclo investigativo de modelagem matemática para delimitação de uma trajetória possível de formação com alunos do ensino médio da escola básica. Os resultados encontrados, mediante as experimentações empíricas a partir de um problema em contexto da matemática financeira escolar, permitiram evidenciar o papel indispensável dos *habitus* como sistema de percepção durável e transponível mobilizado pelos alunos. Esse percurso investigativo tornou possível a delimitação ou o (re)conhecimento da situação com matemática associada ao tipo de problema considerado. Os resultados possibilitam responder, não exaustivamente, à problemática de interesse da teoria antropológica do didático, bem como estimulam futuras pesquisas sobre a modelagem matemática condicionada aos gêneros textuais.

Palavras-chave: Modelagem matemática reversa, Teoria antropológica do didático, *Habitus*, Situação com matemática.

Resumen

Este artículo aborda el problema del modelado matemático inverso interpretado por el tipo de tarea que consiste en encontrar la situación con las matemáticas que se puede asociar a un modelo matemático. El objetivo fue crear condiciones en el sentido de la teoría antropológica de la didáctica que permitan evidenciar este problema. Para ello, se utilizaron recursos teórico-metodológicos de la teoría antropológica, más específicamente, del ciclo investigativo de modelación matemática para delimitar una posible trayectoria de formación con estudiantes de bachillerato desde la escuela básica. Los resultados encontrados, a través de experimentos empíricos basados en un problema en el contexto de las matemáticas financieras escolares, permitieron resaltar el papel indispensable del habitus como sistema de percepción duradero y transponible movilizado por los estudiantes. Este camino investigativo permitió delimitar o (re)conocer la situación con las matemáticas asociadas al tipo de problema considerado. Los resultados permiten dar respuesta, no exhaustiva, al problema de interés de la teoría antropológica de la didáctica, así como estimular futuras investigaciones sobre modelización matemática condicionada a géneros textuales.

Palabras clave: Modelización matemática inversa, Teoría antropológica de la didáctica, *Habitus*, Situación con las matemáticas.

Résumé

Cet article aborde le problème de la modélisation mathématique inverse interprétée par le type de tâche qui consiste à trouver la situation avec les mathématiques pouvant être associée à un modèle mathématique. L'objectif était de créer des conditions au sens de la théorie anthropologique de la didactique qui permettent de mettre en évidence ce problème. Pour cela, les ressources théoriques et méthodologiques de la théorie anthropologique ont été utilisées, plus spécifiquement, à partir du cycle d'investigation de la modélisation mathématique pour délimiter une trajectoire possible de formation avec des lycéens de l'école fondamentale. Les résultats trouvés, à travers des expérimentations empiriques basées sur un problème dans le cadre des mathématiques financières scolaires, ont permis de mettre en évidence le rôle indispensable de l'habitus comme système de perception durable et transposable mobilisé par les élèves. Ce parcours d'investigation a permis de délimiter ou (re)connaître la situation avec les mathématiques associée au type de problème considéré. Les résultats permettent de répondre, de manière non exhaustive, à la problématique d'intérêt de la théorie anthropologique du didactique, ainsi que de stimuler les recherches futures sur la modélisation mathématique conditionnée aux genres textuels.

Mots clés : Modélisation mathématique inversée, Théorie anthropologique du didactique, Habitus, Situation avec les mathématiques.

Introduction and justifications of the research problem

One of the primary goals of Mathematical Education pointed out by Fukushima (2021, p. 1) is "to develop the competency to solve, mathematically, problems in real-life situations", which, according to the author, it has taken a lot of researchers to work on the study of Modelling Mathematical skills (MM). In this regard, it is no coincidence that:

> Modeling and applications are essential components of mathematics, and applying mathematical knowledge in the real world is a core competence of mathematical literacy; thus, fostering students' competence in solving real-world problems is a widely accepted goal of mathematics education, and mathematical modeling is included in many curricula across the world. (Cevikbas, Kaiser & Schukajlow, 2021, p. 206).

It seems that MM is predominantly conceived as "a world-renowned field of research in mathematics education" (Greefrath & Vorhölter, 2016, p. 1) and, as such, in the last decades, it became an important topic of research focused on the teaching and learning of mathematical objects, as noted by Barquero (2020) e Florensa, Garcia e Sala (2020).

Although in a mathematical education perspective, different Modeling Mathematical research points out the difficulties revealed by the students and professors in the classroom, in particular, in the study of real context problems, according to Sodré e Guerra (2018), Sodré (2019) e Fukushima (2021), the literature (Borromeo Ferri, 2006; Blum & Borromeo Ferri, 2009, Perrenet & Zwaneveld, 2012, Blum, 2015, Greefrath & Vorhölter, 2016, Vorhölter et al., 2019, Barquero & Jessen, 2020) suggests the use of MM cycle as a didactic tool to minimize part of the complexity that provides some difficulties for students and professors.

However, in the Anthropological Didactic Theory perspective (ATD), here assumed as theoretical-methodological resources for the construction of the investigation problem, the notion of the MM cycle, presented in the specific literature, is transposed to a solid theoretical framework (Garcia et al., 2006) including, as highlighted by Sodré and Guerra (2018) and Sodré (2019), by proposing the Investigative Cycle of Mathematical Modeling (ICMM)

ICMM (Sodré & Guerra, 2018, Sodré, 2019) can be interpreted as a praxeological organization and, with that, assumes "a dual functional role as a didactic-methodological device: for the teaching-learning of MM and teacher formation" (Sodré, 2021b, p. 17), by drawing upon itself a praxeological infrastructure of mathematical and non-mathematical knowledge.

In another way, ICMM integrates itself in a dynamic of an epistemology functional in whose knowledges appear as 'machines' producers of useful skills to the creation of Responses

R to questions Q (Bosch & Gascón, 2010, p. 82), whose knowledge pointed out by Bosch e Gascón (2010) are comprehended using Mathematical models.

The praxeological organization of the ICMM is endowed with six sequential tasks to do, namely:

- Task T_0 – Building an Initial Reference Situation for the problem in context;
- Task T_1 –To Investigate the mathematical models that live in the school institution related to the problem in context;
- Task T_2 – Finding situations with the mathematics that can be associated with a mathematical model;
- Task T_3 – To Evaluate the mathematical models;
- Task T_4 – Developing a mathematical model;
- Task T_5 – to Disseminate and defend the mathematical model.

Although the study of a problem under conditions and restrictions imposed or created by the use of ICMM does not accomplish all the tasks, it can direct the subject (student or teacher) to possible changes in the quality of relations with different knowledge, and with that, allowing the construction of responses to issues raised by Chevallard (2005) about a given knowledge, here interpreted by the use, study, production and didactic-institutional transposition of mathematical models.

In this regard, we put our attention to the problem of ICMM task $\mathbf{T_2}$, which consists of finding mathematical situations that can be associated with a mathematical model, but without disregarding dynamics and the functional role of the other ICMM tasks in relation to the $\mathbf{T_2}$ task, whose problem raised by Sodré (2021b) is called *reverse mathematical modeling*, or simply RMM, in which the dynamic of the investigation starts from the mathematical model for the (re)cognition of the situation with its associated mathematics.

Thus, delimiting a type of *situation with mathematics*, here paraphrased from the notion of *social practices with mathematics*, created by Chevallard (2005, p. 174) “it remarkably exceeds the ground originally granted: the teaching of scholar mathematics; it penetrates the set of uses of mathematics; it infiltrates an infinity of space which mathematical knowledge is relevant and its manipulation is observed”.

Social practices with mathematics (Chevallard, 2005) can be observed in diverse institutional spaces such as engineering, biological sciences, and/or/correlated areas, and, even so “at companies, repair shops, laboratories and in the manipulation of no-mathematical knowledges, but whose work like it was (Ibidem, p. 175), as the study of problems in a real context at scholar MM presupposes.

Looking from this angle, the notion of the situation with mathematics, in the effort of social practices with mathematic (Chevallard, 2005) looks like way more extensive, then, it includes diverse manipulations of mathematical knowledge in different institutional contexts, normative or not, in view that it is not always that a kind of problem in real context is passive by the delimitation of a type of situation with mathematics, from which a possible mathematical model associated with it can emerge. In other words, it is not always that a kind of problem is equipped with a mathematical situation or something mathematical.

In this regard, “To consider a situation is a first process of abstraction: considering a situation, we keep only from reality the features that we think relevant to our action”. (Revuz, 1971). Furthermore, the author highlights that “A situation is a portion of reality which is separated from its environment and which we want to consider in itself” (Revuz, 1971).

According to extracts from texts, the kind of problem to be faced by a subject or an academic community must not be confused with the notion of the situation with the mathematics that can emerge from it, thus, the product from the subject’s abstraction in face of a kind of the considered problem, and, with that, it draws on its accumulated experiences in its personal and institutional life history. Our affirmations about this discussion can be explained using Chevallard’s observations below:

> When a tile falls from a roof onto your head, that is a fact, just a fact, even if it is a very unpleasant one. But science is not interested in this particular event. Physics, to giving an example of this, studies the phenomena related to the fall of heavy bodies; and medicine studies other relevant phenomena, such as the consequences of a tile falling on your head... (Chevallard, 2013a, p. 2)

The episode above, about the fall of a tile, reveals, for different people, doctors and physicians, distinct delimitations of a situation, that are completely subjective, about the same fact. For a physician, his point of view seems to be in direction of the falling bodies phenomenon, seeking to establish the laws which apparently rule it, whose relationships indicate are broader, based on their experiences with theoretical and/or practical knowledge.

In any case, theoretical and/or practical knowledge will allow us to make assumptions and reveal “more relationships than our direct familiarity with the world of facts would allow us to recognize” (Chevallard, 2013a, p. 2). From the doctor's point of view, this phenomenon allows the delimitation of the situation regarding the consequences of a tile falling on a person's head.

Two of the dangers highlighted by Revuz (1971) that can threaten mathematics teaching must be listed: confusing the situation and the mathematical model and studying the model as if it were the situation, which prevents them, the situation and mathematical model, from being studied separately.

Another danger, according to Revuz (1971), is to separate the situation from the model, and, with that, completely to ignore one of them without never being references to the other, as it looks to be dominant in some primary schools' cases, when the study of mathematical models in diverse disciplines is concentrated, without making connections with situations associated to them. Paraphrasing Wittgenstein (1999, p. 129, "every single sign seems dead. What does give it life? – In use, it lives", that is, a mathematical model, like a sign, only makes sense in its use in relation to a situation.

Using a mathematical model in front of the study of a situation with mathematics includes, in a determinant way, "knowing and knowing, in ATD case, is using in a suitable mode (Sodré, 209, p. 96), namely, according to the associated mathematical situation, we presuppose the necessity of subject to manipulate the mathematical model to make constructions of high-quality links, in TAD's point of view, (Chevallard, 2005), between non-mathematical knowledges and mathematical ones in social practices, in general, including scholar context.

Our precepts are based on the indispensable role of non-mathematical knowledge, such as cultural knowledge or, more precisely, "practical knowledge, which functions in the operation, is learned, enriched, without, however, being used, taught, produced" (Chevalard, 2005, p. 154,) and, as such, they are instituted in the culture of social practices, here interpreted in the sense of Bourdieu's habitus (2004, 2013), as "systems of durable and transposable provisions, structured structures predispose to function as structuring structures, that is, as generating and organizing principles of practices and representations [...]" (Bourdieu, 2013, p. 87)

From this point of view, the kind of tasks $\mathbf{T_2}$ (ICMM tasks), that is finding situations with the mathematics that can be associated with a mathematical model, would become problematic, as highlighted by Sodre (2021b), more precisely, because we assume that the construction of the situation with mathematics can gradually reveals itself in consonance with the habitus of the practical field, that refers to the kind of problem in considered context, because a practice that includes the action of finding the situation with mathematics "is the product of a dialectical relationship between the situation and the *habitus*", according to Wacquant (2007, p. 66), based on Bourdieu's observations.

Our assumptions are based on observations by Sodré and Guerra (2018) and Sodré (2019), when they highlight that the study of a kind of problem in a concrete context requires considering a domain of reality, that is, a situation with mathematics, because, according to Revuz (1971, p. 50, our translation), “as in all cases, the most difficult thing is to know what the situations are”. Knowing the situation with mathematics includes the ability to recognize it and, with that, to question the naturalness illusion of the necessary knowledge, if not indispensable, for the study of a problem in the context of MM. Therefore:

> It should be stressed that amalgamation processes are in no way “natural”: they are the product of human work. Like other theoretical approaches in the social sciences, ATD endeavors to “deconstruct” the illusion of the naturalness of human works, which is maybe the central illusion that we have to come to grips with in the scientific approach to knowledge and its diffusion. (Chevallard, 2019b, p. 95).

According to Chevallard (2005), the ratification (or not) of our hypothesis about MM’s problems, pointed out by Sodré (2021b), delivers us to the research problem below, following ATD premises here assumed, in particular, the Basic Problem highlighted by Chevallard (2009b):

> Given certain restrictions, what set of conditions present in the study of a problem in context can an institution or a person integrate into their practices to forward the delimitation of the situation with mathematics, which can be associated with a mathematical model?

Following this concept, it is necessary to observe that “In ATD, one studies the conditions that favor or preclude the diffusion of knowledge”. (Chevallard, 2019b, p. 95). In principle, according to Chevallard (2009b), everything is condition, and as one of didactic’s objects of interest, this are not enumerate, in priori, so: “leur découverte progressive et la compréhension de leur rôle dans la diffusion de telle ou telle entité praxéologique $\wp$ sont l’objectif permanent de la recherche en didactique” (Chevallard, 2009b, p. 12).

Thus, in the ATD’s sense, we aim to create conditions that allow highlighting the MM’s issue, based on the study of a problem in the context of financial mathematics learned at school with primary students. Students would be faced with a series of appropriate situations, which enable the subject to build his knowledge (Chevallard, 2005, p. 102), because “At all levels, among the conditions of interest to didactics research, a great number are created intentionally by persons and institutions” (Chevallard, 2019b, p. 96).

The dialectic between the notion of the situation with mathematics and the habitus

According to Chevallard (2005)

> To try to advantageously delimit (particularly thanks to certain retrospective economies) the socio-historical genesis of the knowledge designated to be taught. Taking current achievements into account, it would be possible to constitute an artificial epistemology as an improved summary – that is, leaving aside the dead ends, and the failures, but reimplanting all the wealth of fertile developments and sometimes forgotten in the historical construction of knowledge (Chevallard, 2005, p. 54-55).

From this standpoint, it is possible to point out some historical-epistemological fragments of works like Maclaurin's (1753) and Euler's (1795), which clippings based on trajectory, in our understanding, can generate didactic transpositions (Chevallard, 1999, 2019b) in the sense of creating conditions that make the teaching of mathematical objects possible, including under the eyes of the analysis, as Maclaurin (1753, p. 64) highlights: "analysis is the art of solving problems. Problems are solved utilizing equations".

This fragment reveals that problems can be solved using mathematical equations, without possibly considering the role of another knowledges, like practical knowledges that eventually can limit, or even prevent the alleged "translation" of a problem into an equation, as desired by the MM.

This way of thinking and acting seems us also dominant in Euler's works when he stresses:

> The main object of Algebra, as of all parts of Mathematics, is to determine the value of previously unknown quantities. This is achieved by carefully weighing the prescribed conditions, which are always expressed in known quantities (Euler, 1795, p. 451)

However, Fukushima (2021) points out, following Frejd and Ärlebäck (2011), Galbraith and Stillman (2006), Miwa, (1986), and Treilibs et al., (1980), that "the process called 'mathematization,' which connects extra-mathematical domains with mathematical domains is the most difficult for students". (Fukushima, 2021, p. 1) and not just that, because this situation has been shown problematic to teachers when they are inserted in no general contexts, following Grandsard (2005). Thus, "the transformation of a problem from the "non-mathematical world" to the "mathematical world" and, vice versa, does not seem to show the role of other knowledge, including practical non-mathematical knowledge that is considered by ATD" (Sodré, 2021b, p. 3).

"The transformation" of a problem into a mathematical problem has been shown problematic. It is not a coincidence that Bum (2015, p. 79) ratifies that "any students get stuck already here. This is not only or even not primarily a cognitive deficiency". In any case, although Euler (1795) anticipates some prescribed conditions, in the sense of thinking about the kind of the considered problem. As well as Maclaurin (1753) presupposes, making it seem like every problem can be solved, Blum (2015) points out students' strategy for problems in contexts: "Ignore the context, just extract all data from the text and calculate something according to a familiar schema" (Blum, 2015, p.79). In face of this, it is urgent considering Brady and Lesh's (2021) appointments:

> Simplifying real-life situations to the point that they focus on a single mathematical construct or topic more often than not to remove these aspects of the system: while this may be desirable from an applications perspective, it is unacceptable from a modeling and modeling perspective. (Brady and Lesh, 2021, p. 20

Beyond Brad and Lesh's observations (2021), it is necessary to consider that Maclaurin (1753) doesn't explain how the subject can make associations between equations and the kind of problem, or even how to recognize the proper equation or type of task to be faced using the mathematic equation, here interpreted as mathematical models. Those practices don't seem simple, because, "it is not easy to talk about practice in a not negative way and, mainly, about practice when it is more mechanical than appearance, more opposed to the logic of thought and discourse (Bourdieu, 2013, p. 133).

In another way, Maclaurin (1751) and Euler (1795) seem to assume, based on presumptions, that problems can be solved using equations, although it is not possible.

> For example, there may be a sacred site for an indigenous population that is known also to be rich in minerals. It may well be possible to analyze the economic costs and benefits of mining that site through a detailed mathematical description; however, it is both inappropriate and impossible to "mathematize" the cultural significance of the site. (Christensen, Skovsmose & Yasukawa, 2008, p. 78).

To demarcate a situation with the mathematics that can be associated with a mathematical model doesn't seem to be a simple task, even possible and contextualized problems in primary schools that uses the rule of three and proportion, as so below:

Prints of scholarly books are made in a printing house. In 2 hours, 40 impressions are made. In 3 hours, the same machine produces 60 more prints, in 4 hours, 80 prints, and, in 5 hours, 100 prints.

Frame 1.

Relation between time and number of impressions s

Time (h)	2	3	4	5
Impressions	40	60	80	100

The proportionality constant between the magnitudes is found by the ratio between the machine's working time and the number of copies made: $\frac{2}{40} = \frac{3}{60} = \frac{4}{80} = \frac{5}{100} = \frac{x}{y} = \frac{1}{20}$. (Source:https://www.todamateria.com.br/grandezas-proporcionais-grandezas-diretamente-inversamente-proporcionais).

In this specific fragment, which is interesting to us, the magnitudes of time (x) and the number of impressions (y) are immediately revealed, anbeing directly proportional. Then, from the relationship: $\boldsymbol{\frac{x}{y} = \frac{1}{20}}$, is possible to it is possible to designate the number of impressions (y) by the mathematical model given by: **y = 20.x**, as one of the possible models associated with the situation that uses a simple rule of three. This way of doing and thinking can reveal that the subject assumes quantities as directly proportional, even if types of problems do not show enough elements to ensure proportionality or not.

Namely, the mobilization of the subject's practices, in face of the type of problem, directs the dialectic between the situation with mathematics and the habitus, the latter understood here as a producer of practices, that is, "the operating presence of the entire past of which it is a product" (Bourdieu, 2013, p. 93). On the other hand, habitus, "like every art of inventing, is what allows the production of practices in an infinite number, and relatively unpredictable ones (such as the corresponding situations)" (Ibidem, p. 92).

In this point of view, the dominant culture of scholar practices seems not to evince what Barquero, Bosch, and Gascón describe, in the fragment below:

> No se plantean, por tanto, preguntas sobre la "comparación" del grado de adecuación de dos o más modelos de un mismo sistema, ni sobre la necesidad de modificar progresivamente un modelo determinado para dar respuesta a las nuevas cuestiones problemáticas porque el sistema se supone construido de una vez por todas (no aparecen cuestiones "nuevas" no previstas de antemano), ni sobre la necesidad de elaborar modelos de los modelos (la recursividad de la modelización matemática es completamente ignorada en la práctica escolar). (Barquero, Bosch & Gascón, 2007, p. 7).

It is necessary to consider that numerical data of rule of three's and proportionality' topics can construct another mathematical model, as such as:

$y_1 = 22{,}87. e^{0{,}303.x_1}$, with an error of $R^2 = 0{,}982$ (model I);

$y_2 = 64{,}67. \ln(x_2) - 7{,}411$, with an error of $R^2 = 0{,}982$ (model II);

$y_3 = 20. x_3 - 1 + Cos[(x_3 - 2).(x_3 - 3).(x_3 - 4).(x_3 - 5)]$ (model III);

$y_4 = 20. x_4 + Sen[(x_4 - 2).(x_4 - 3).(x_4 - 4).(x_4 - 5)]$ (model IV).

It seems to us that a great part of teaching objects that involves proportional quantities is defined by a subject's decision in face of the kind of problem, although other models can assure the relation between evolved quantities and, with that, even produce acceptable answers to the considered problem Thus, "many models can be imagined for one situation, and many different situations may be represented by the same model. A difficult task is to choose, if possible, the best model". (Revuz, 1971, p. 49).

The designated mathematical models I, II, III, and IV help us to understand that a situation with mathematics can produce different mathematical models associated with it. Along this path, a question may emerge: what does determine the choice of a mathematical model? Now, the choice of a model, beyond the experiences and habitus incorporated by the subject who models it, in general, is conditioned by the conformity of the social practice that is at stake, that is, it depends on the institutional environment where the subject who models is an integral part, since, in this dynamic, practical knowledge, in habitus' role, is crucial for decision-making, including the choice or not of a mathematical model. Otherwise, "right and false is what men say; and in language, men are agreed. It is not an agreement on opinions, but on the way of life" (Wittgenstein, 1999, p. 98).

Under this bias, the MM perspective contradicts the notion of conversion proposed by Duval (2011), although he recognizes the existence of difficulties for students to "find the equation of a straight line starting from its graphic representation, even for the most elementary cases" (Duval, 2011, p. 97).

For Duval (2011, p. 97), "we should not find, in the mathematical concepts linked to the affine function, the profound reason for these difficulties, but in the lack of knowledge of the semiotic correspondence rules between the registration of the graphic representation and the registration of the algebraic expression", what it seems to evince a straight relationship between graphic and equation registration. The author does not seem to reveal possibilities of using other mathematical equations from the numerical data of a graph or even a table, as mentioned above.

Ultimately, the strategy used by students to ignore the context, and from which a situation with mathematics can emerge, as Blum (2015) points out, can prove to be problematic, if we consider the classic problems of practical arithmetic, the rule of three's case, described by Silva (2017), in the following example: "If a sixteen-year-old girl, with her sweet voice like a crane, dancing and chirping like a peacock, receives 600 coins, how much would a 25-year-old receive? (Brahmasphutasiddhanta: 769, apud Sarma, 2002, p. 150) (apud Silva, 2017, p. 61).

This kind of problem, in the initial teacher's formation course, reveals answers in which two professors' voice register' transcription (**P_1** e **P_2**) are clarified:

- ***P_1*** *– I would use direct proportionality, but for the information about the voice and the dance, for me, there must be some "gotcha" that I am not seeing... using proportionality being one of sixteen and the other of twenty-five... if the sixteen's person get six hundred coins, so I would have it... that if from sixteen to twenty-five is nine years then I would use that same proportionality... in this case, I multiply six hundred by nine then I would find the answer of five thousand four hundred coins, then I would do it this way. However, I'll stick with this idea if there's no other information behind it. [Sic]*
- *P_1 - But it is necessary to observe if this proportionality were linear... it is necessary to observe if this linearity really exists, if this proportionality is according to the years, or if there are other factors that make the variation really changes. [sic]*
- *P_2 - This text is curious, I don't know if there's a catch in there, the first question I would ask myself... to earn coins it's enough just to be a certain age, because, from the question's report, it seems that the girl who won six hundred coins, in addition to having this age, I could simply think of a problem of proportion and directly proportional... Initially , I could think like this, how much older, more coins, but I start to realize through reading that there were factors that I don't know if they were important for that , like a cache that voice, the way of dancing, because if that all matters and not just age, how could I pay this twenty-five-year-old girl? since the only information I know about her, the only report on the matter for her to try to aim for something is that she would have to have a similar voice? Would she have*

to dance like a peacock? And another thing, I think it's already a bit of a journey, not that the other placements haven't been... is how much would a twenty-five years old receive? With a slightly remote doubt, these twenty-five years would it be referring to a new person? Or suddenly the age of another factor like coins? It is interesting to note that the first age of sixteen is written in full, while the other has a number written before it... these are some statements like that, with sincerity. [sic]

These fragments, pointed out by teachers $\mathbf{P_1}$ and $\mathbf{P_2}$, help us to understand part of the complexity that involves the issue of delimiting a situation with the mathematics that may be associated with the type of considered problem, as they presuppose, in our understanding, the existence of cultural elements that condition the way of doing and thinking about the type of problem, that is, the specific habitus of the culture of practice. Thus, the habitus, as a system of durable and transposable dispositions, can be interpreted in an approximate view as the "way of acting common to all men as a reference system, through which we interpret an unknown language" (Wittgenstein, 1999, p. 93).

With this view:

> It is to the extent and only to the extent that habitus is the embodiment of the same history – or, more exactly, of the same history objectified in habitus and structures – that the practices they engender are mutually comprehensible and immediately to the structures [...] (Bourdieu, 2013, p. 95)

Following this way, the notion of the situation with mathematics is in close relation to habitus from the practical field, which it is inserted and, perhaps for this reason, not only, it is a Theory of Didactic Situations interests (Brousseau, 1995), in addition to ATD, for constituting the strong hypothesis of the very definition of mathematical knowledge. In turn, knowledge is described in terms of situations, that is, "knowledge is a situation" (Bosch & Chevallard, 1999, p. 3) and, with that, the "reason for being or the rationality that gives meaning to mathematical activity" (Bosch, Chevallard & Gascón, 2006, p. 3).

In the same way, the close dialectical relationship between the habitus and the situation with mathematics can be interpreted by the notion of personal and institutional relations theory, announced by Chevallard (2019b). Like this:

> Given an object o and a person x, we posit that there exists an entity—regarded, in what follows, as a set—called the personal relation of x to o. This set is denoted by R(x, o) and comprises all the "ways" in which x relates to o—through pondering over o, speaking or writing about o, using o, handling o, dreaming or daydreaming or

fantasizing about o, spiffing it up, etc. In an expanded sense of the verb to know, the relation of x to o, R(x, o), encapsulates all that x "knows" about o. (Chevallard, 2019b, p. 77).

Under our assumptions about the complexity that the accomplish of the CIMM Task $\mathbf{T_2}$ can reveal (Sodré & Guerra, 2018, Sodré, 2019), we believe that facing it may even demand the creation of conditions about the didactic transposition, in addition to making it possible for the subject to encounter the situation with mathematics, that perhaps can be associated with a mathematical model, based on the theoretical-methodological notion of a Study and Research Path, (SRP), in line with Chevallard (2013b), calling for the development of disciplinary and non-disciplinary knowledge, including the habitus of the practical field, in dialectics with situations with the mathematics that is necessary, if not indispensable, for the study of a problem in scholar MM context.

To confirm or not our hypothesis, we forwarded an empirical study carried out with elementary school students from a public school.

Empirical cut and analysis of founded results

The study process was thought in a course accomplished to twenty (20) high school students from the Application School of the Federal University of Pará (AS/FUPA), which we symbolize here by $X = (x_1, x_2, x_3, .. ., x_{18}, x_{19}, x_{20})$. Four groups of five students were formed. In each group, didactic systems described by $S_i = (S_1, S_2, S_3, S_4)$ were installed, based on the problems faced, under the guidance of a director (d), as highlighted by the dynamics of the PEP (Chevallard, 2013b):

> In general, a PEP is forwarded from undetermined questions Q_i that is answered by certain questions Q_{ij} during the investigation (Chevallard, 2009), which "may lead a class to rediscover a complex of works that may vary depending on the route taken (what depends on the activity of X, on the decisions of Y, but also the praxeological resources $R_i^{\lozenge}$ and O_j currently accessible)" (Chevallard, 2009, p. 28), which can be modeled by the notion of main didactic system S (X, Y, Q), capable of producing or not auxiliary didactic systems to build strong answers (Sodré, 2021a, p. 105).

The study process was carried out through a type of capital investment problem of scholarly financial mathematics, described in the following terms:

- **Kind of specific question – Q_0:** How to determine the return value on an investment in a saving account of R$ 500.00, at the end of the three months?

Based on this type of problem, didactic systems $\mathbf{S_i}$ expressed the following question: Q_1 – What is the saving rate?

After consulting websites, didactical systems consensually elected an average value of approximately 0.5% per month. In this sense, S_i didactical systems seem to reveal actions that can be translated by ICMM tasks $\mathbf{T_0}$ and $\mathbf{T_1}$, by forwarding ways of doing and thinking about an initial situation with mathematics, designed by using the mathematical models available in scholar literature, as guided by the figure 1.

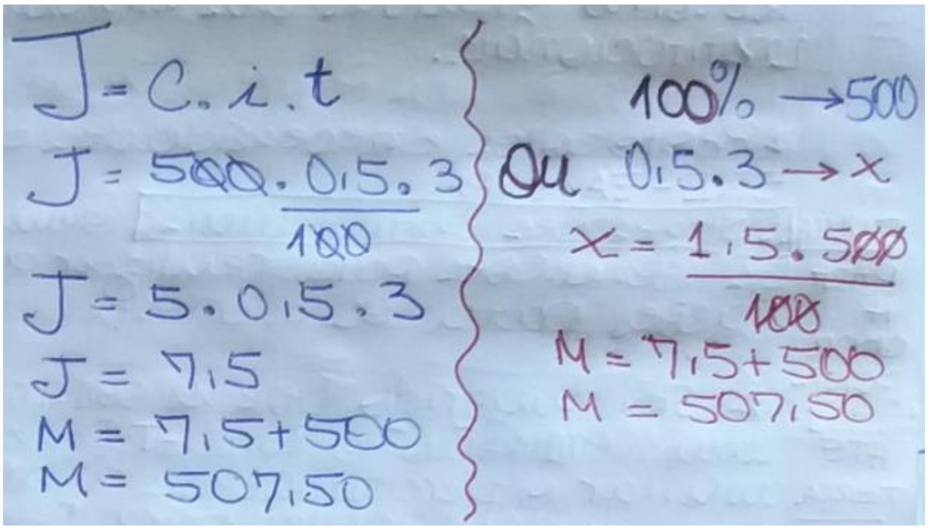


Figure 1.

Simple interest formula (J=c.i.t) and the practice of rule of three (Research file, 2019)

Figure 1 enhances the objectivation of two praxeological organizations by the didactical system $\boldsymbol{S_4}$, that is: $\wp_{\mathbf{1}}$ can appoint the simple interest formula and $\wp_{\mathbf{2}}$ can represent the classical practice of rule of three under the structural graphic discourse of proportional fourth. Thereby, didactical system $\boldsymbol{S_4}$ took place as directly proportional quantities, however, these described praxeological organizations ($\wp_{\mathbf{1}}$ e $\wp_{\mathbf{2}}$) interpreted here as mathematical models, produce the same mathematical response, they both can not be confused, because even they have similarities in praxis, the discourse which guides them are distinct. Otherwise:

> A nearby but distinct situation occurs when two different praxeologies $\wp$ and $\wp'$ exist, both relative to the type of tasks T. [...] Let us note also that, even if $\tau' = \tau$, $\wp$ and $\wp'$ may differ because of distinct technologic theoretical blocks $\Lambda = [\theta / \Theta]$ and $\Lambda' = [\theta' /$

Θ′], in which case the user will come across distinct technological or theoretical elements. (Chevallard, 2020, p. 30-31).

It is necessary to consider that in ATD "A praxeology can be considered as a model or a system depending on the kind of questions put; being a model of a system is a function of a praxeology, it is not in its nature" (Garcia et al., 2006, p. 233).

The $\boldsymbol{S_4}$ s'way of thinking seems ratified by the didactic system's objective manifestation $\boldsymbol{S_2}$ when it highlights the praxeological organization $\wp_3$, from the mathematical model of calculation of reduction to the unit, guided by the logic of practical arithmetic's discourse, as shown in figure 2.

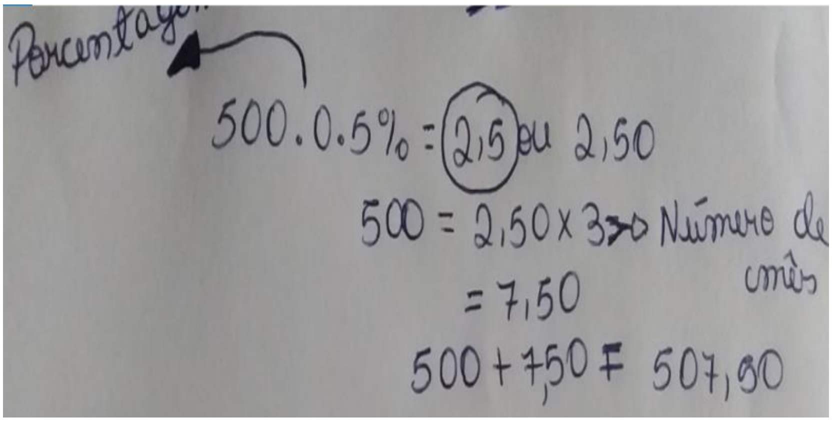


Figure 2.

Mathematical model of the calculation of unit reduction according to practical arithmetic (Research file, 2019)

Until that, the investigation path revealed that delimiting the situation with mathematical, which can be associated with mathematical models of capital investment problems, according to the $\mathbf{T_2}$ ICMM Task's objective, is problematic, perhaps because these two didactical systems don't have a perception filter (Chevallard, 2005) or don't be used to specific habitus from practical field of banks and related financial institutes, because habitus can be understood as:

> a system of durable and transposable dispositions that, integrating all past experiences, functions at every moment as a matrix of perceptions, appreciations and actions, and enables the fulfillment of infinitely different tasks thanks to the analogical transfer of schemes acquired in a previous practice (Bourdieu, 2002 [1972], p. 261).

The lack of habitus from the practical field of banks and related financial institutes to didactical systems can be interpreted, in ATD's view, as the existence of low-quality relation between the interdisciplinary knowledges (Chevallard, 2005), in particular, non-mathematical knowledges that make sense and meaning to financial institutions' mathematical situations

In other words, "the habitus is that kind of practical sense of what we should do in a given situation – what we call, in sport, the sense of the game, the art of anticipating the future of the inscribed game, in outline, in the current state of the game." (Bourdieu, 1996, p. 42).

The study of the capital investment problem by didactic systems produced "much more than a single solution. It produces new knowledge (new problems, new techniques, new technologies, and theories) and new arrangements of previous knowledge (Bosch, Chevallard & Gascón, 2006, p. 5), including, in the construction of situation of simple interest's mathematical model, based on the study of other situations in context, that they were socialized in the class, as highlighted in figure 3.

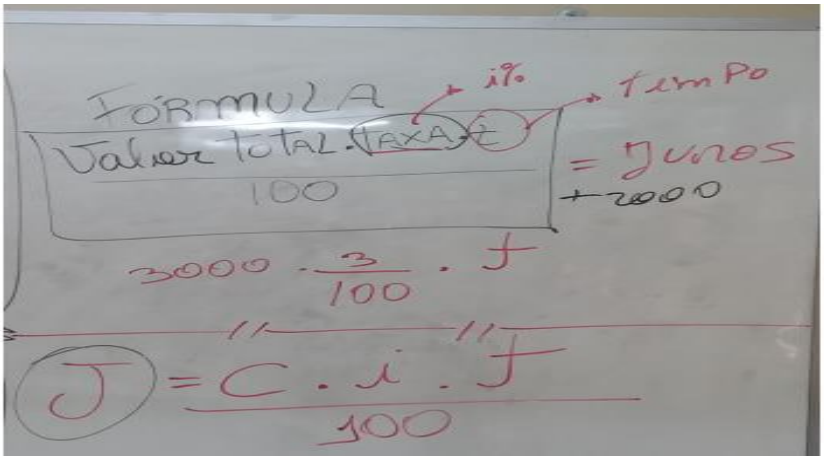


Figure 3.

Construction of the mathematical model of the simple interest situation (Research file, 2019)

Situations with mathematic in financial contexts like "création de la culture, sont explicitement réglés, de manière parfois fort précise, par convention sociale. C'est le cas des transactions financières, du prêt à intérêt, etc., pratiques sociales qui sont en fait définies a priori par un modèle mathématique (Chevallard, 1989, p. 27).

The knowledge of the situation with mathematics demands that the subject enables of recognize it, as Wittgenstein highlights (1976), because depends on the quality of relations between knowledges (Chevallard, 2005) whose compose the practical field of financial institutes, there is, the habitus as analogous practices' generators to incorporated practices by subject. In another way, the knowledges of situations with mathematic by didactical systems objectively revealed the encounter with mathematical models, described by praxeological organizations $\wp_1$, $\wp_2$ e $\wp_3$, without, however, reaching out to the encounter with the situation with mathematic associated to the kind of capital investment problem, as guided by ICMM's $\mathbf{T_2}$ task.

However, during the course of studies, the didactic system $\boldsymbol{S_1}$ emphasized "confrontations" of practices in face of the objective manifestations of the other didactic systems, as shown in figure 4:

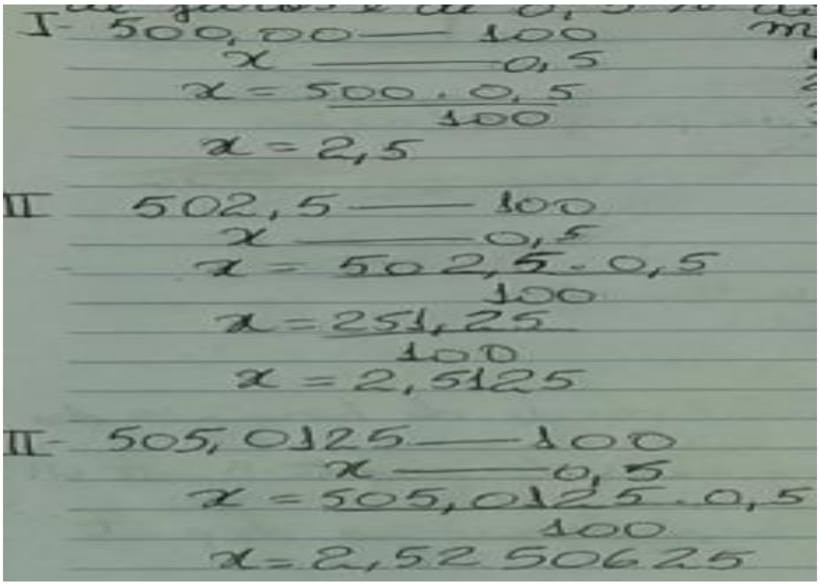
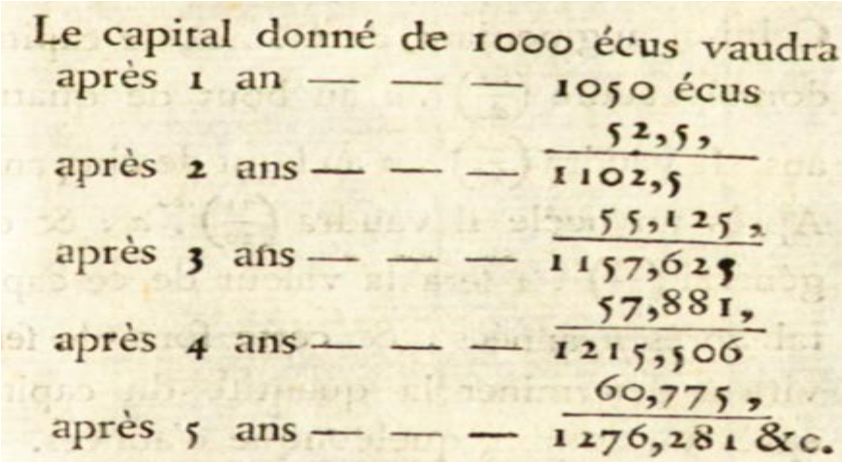


Figure 4.

The variability of the rule of three discourse as a modeling praxeology (Research file, 2019)

In figure 4, didactic system $\boldsymbol{S_1}$ shows a situation with mathematics that puts into use a mathematical model described by the praxeological organization $\wp_4$. Although it bears similarities with the praxeological rule of three's organization $\wp_2$, perhaps because it presents similarities in *praxis*, but the *logos*, that is, the way of thinking or, more precisely, the discourse that guides praxis, is different for the same type of considered problem.

The praxeological organization $\wp_4$, cited by $\boldsymbol{S_1}$, goes back to, in your point of view, historical-epistemological fragments of a didactic creation (Chevallard, 2005) for teaching, as postulated by Euler (1795), based on a specific situation, as shown in Figure 5.



Le capital donné de 1000 écus vaudra
après 1 an — — — 1050 écus
52,5,
après 2 ans — — — 1102,5
55,125,
après 3 ans — — — 1157,625
57,881,
après 4 ans — — — 1215,506
60,775,
après 5 ans — — — 1276,281 &c.

Figure 5.

Traces of a possible didactic creation for the calculation of interest (Euler, 1795, p. 433)

The specific situation (Figure 5), considered as a capital of 1000 écus, applied to 5% per year is given by:

1 year - - - 1050 écus

2 years - - - (1050 + 52,5) = 1102,5

3 years - - - (1102,5 + 55,125) = 1157,625...

In the meantime, a condition was "imposed" by S_1: the use of a scientific calculator, leading to the installation of a specific didactic system: $\mathbf{S_{1_{\mathbb{C}}}}$ $(\boldsymbol{S_1}, \wp_{\mathbb{C}})$, which $\wp_{\mathbb{C}}$ designates "the praxeologies related to calculation tasks on the scientific calculator, including, with respect to floating-point arithmetic, such as rounding and truncation" (Sodré & Guerra, 2018, p. 258).

The "confrontations" of practices provoked by $\mathbf{S_1}$, by contrasting the situation with mathematics revealed by $\boldsymbol{S_2}$, $\boldsymbol{S_3}$, and S_4, led S_1 to encounter other non-mathematical knowledge, such as the use of the scientific calculator, and, not least, knowledges of specifics situation with mathematics that can be associated with the mathematical model. In any case, "all these answers are, in fact, draft answers, which remain for you to work on" (Chevallard, 2009b, p. 26).

The response produced by $\mathbf{S_1}$ allows the contrast between didactical systems and, with that, cognitive and praxeological dynamics were highlighted, specially by construction and reconstruction knowledges' process. That is:

> de même qu'il y a une histoire de la personne comme sujet, il y a une dynamique cognitive, qui fait que certains objets disparaissent de UC(x) tandis que d'autres y apparaissent, et il y a une dynamique praxéologique par laquelle l'équipement

praxéologique de x, qu'on peut noter EP(x), change – une partie de cet équipement perdant une partie de son caractère opérationnel tandis que d'autres de ses parties sont rénovées et que des éléments nouveaux viennent s'ajouter au fil du temps. (Chevallard, 2009a, p. 6-7).

Beyond that, the use of calculation techniques carried out, period by period, seems to be recognized by Euler (1795) as a limited technique, since " you can continue, in the same way, for many years if you want, but when the number of years is too big, the calculation became long and tedious; here is how we can short it. (Euler, 1795, p. 433), referring to the use of an algebraic mathematical model given by $\left[\left(\frac{\mathbf{21}}{\mathbf{20}}\right)^{\boldsymbol{n}} .\ \boldsymbol{a}\right]$, to calculate the capital's amount (a) in a (n) years, as a period, using 5% as a rate.

In ICMM's $\mathbf{T_2}$ Task realization, although this proves to be a type of problematic task for primary school students, the route that was taken produced an encounter with different knowledge through the deconstruction and reconstruction of mathematical models used by the didactical systems, as one of the conditions, in the sense of ATD, which made it possible the gradual introduction of other conditions, for example, the use of scientific calculators to generate answers, in close relationship with mathematical situations. Under these conditions, but not only, the didactic system S_3 forwarded the structuring of the situation with broader mathematics and, with that, but it also endowed it with a "new" style of rationality (Chevallard, 1999), motivated by the technique – $\boldsymbol{\tau_1}$ – of the rule of three, as it is shown in Figure 6.

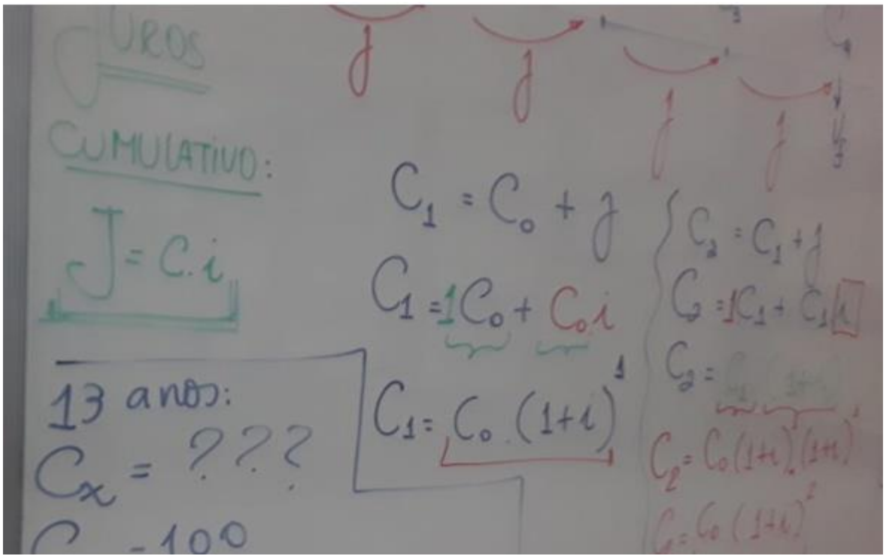


Figure 6.

Algebraic reconstruction of the mathematical model of the compound interest situation (Research file, 2019)

Under this way of thinking that guides the construction of a new praxeological organization, “usually, the praxeological absence is translated, in the first place, as the lack of techniques. How to perform T-type tasks? And also: how to carry out these tasks in a better way? These questions demand production of techniques and, therefore, of praxeologies”. (Chevallard, 1999, p. 228), here designated as $\wp_5$, which representation translates the mathematical model of compound interest situation with the functionality of a new technique described by τ_2: $\mathbf{C_x = C_0\ (1 + i)^{x}}$.

Anyway, Chevallard (2019a, p. 12) emphasizes that “the person resorts not only to a technique but also to a technology and a theory, which praxeological analysis has to make clear”. In our case, it can be interpreted by the institutional relativity of knowledge, whose τ_1 rule of three techniques doubly functionated, beyond the technique, but also as an $\wp_5$ praxeological organization’s technological-theoretical discourse. Perhaps, it is the reason Chevallard (1999, p. 224) reveals that: in elementary arithmetic, where the same small speech has a double function, technical and technological, which allows finding the requested result (technical function) as much as justifying the expected result is correct (technological function)”.

Considerations and future perspectives

The accomplishment of ICMM’s $\mathbf{T_2}$ task proved to be problematic for elementary scholar students, based on the study of a problem in a real context from the scholar’s financial mathematics perspective. This article aimed to create conditions, not all according to the ATD, that would allow highlighting the MMR problem, translated by the performance of the ICMM’s $\mathbf{T_2}$ task, without disregarding the functional role of the $\mathbf{T_0}$ and $\mathbf{T_1}$ tasks of the referred m

The chosen path validated the role of non-mathematical knowledge in articulation and integration with mathematical knowledge, whose conditions, based on mathematical models, recognized by the school institution, were decisive in the study process and, not least, for the “demagification” of the type of capital investment problem faced by elementary school students to delimit the situation with mathematics that can be associated with the mathematical model of the case in question. The models used or reconstructed, in a functional epistemology of knowledge, acted as good “machines” for the production of knowledge about the questioned reality domain (Bosch, Chevallard & Gascón, 2006).

The accomplishment of ICMM’s $\mathbf{T_2}$ task reveals deconstructions and reconstructions of situations with mathematics that were objectified by didactic systems, whose responses evidenced the use of knowledge that was incorporated by the subjects, in the sense of habitus

(Bourdieu, 2002, 2013). in their personal and institutional life history, and in which they were introduced into articulation and integration by the dynamics of the faced situations. “the habitus are spontaneously inclined to recognize all expressions which they recognize themselves” (Bourdieu, 2013, p. 181).

Habitus manifested by the didactic systems, in addition to their function as a system of perception, appreciation, and action, allowed the construction of practices with the use of the rule of three, materialized by the reconstruction of a mathematical model of compound interest, as objectively highlighted by the didactic system $\boldsymbol{S_3}$.

It is worthy to talk about the functional role of the rule of three as one of the knowledges that were mobilized the by didactic system to the construction of another knowledges, in addition to highlighting the institutional relativity of knowledge (Bosch, Chevallard & Gascón, 2006), since the rule of three sometimes fulfilled the role of technique, for facing tasks in praxeological organizations; sometimes the technological-theoretical role, for the production of other praxeological organizations that were needed to reveal the situation with mathematics that is associated with the mathematical model of the capital investment’s kind of problem of

This line of thought revealed, in our understanding, the “confrontation” of practices between the didactic systems by the different praxeological organizations put into action, by allowing the establishment of cognitive and praxeological dynamics, leading to the disappearance of some objects and the emergence of new qualities of relations, thus remodeling the embodied practices of didactic systems. In addition, the confrontation of practices led, in some sense, to the old/new dialectic, according to Chevallard (2005), through the construction of a teaching object with a balance between the past the and future.

In this dialectic, the object of knowledge, paraphrased here by the type of problem in a real context, appeared “as an object with two contradictory faces” (Chevallard, 2005, p. 77). In the first moment, as *something new*, with a certain openness to exploring the knowledge already incorporated by the didactic systems and, in the second moment, it appeared as an old object, through the students' recognition of some situation with mathematics by the cognitive domain, that is, by their habitus, a structure predisposed to be mobilized in the face of a situation.

Ultimately, we assume that scholar MM should consider a wide variety of contexts of situations that allow students to incorporate situations with mathematics, given Blum's observations (2015, p. 84) when “we cannot wait for any mystical transfer from one example or context to another”.

Thus, it is necessary to consider that every problem in a real context or not, can operate under specific linguistic conditions, whose textual genres are equally indispensable for the student's cultural background as "models that are corresponding to recognizable social forms in the communication situations in which they occur" (Coutinho, 2004, apud Marcuschi, 2008, p. 84).

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