

# Promote teachers' pedagogical content knowledge

ISABELLE BLOCH\*

## Abstract

Students often get a very formal conception of mathematics during their mathematics studies. When they become teachers, this formal conception makes them both confident in a direct transmission of the knowledge and unable to tolerate approximate formulations from their own students. How can we make a teacher aware on the way mathematical knowledge appears in teaching? How can we help them understand the articulation between advanced mathematical notions and the content they will be teaching themselves? Their PCK – Pedagogical Content Knowledge, referring to Shulman – includes the ability to react in a pertinent mathematical way in their classroom. We study this PCK and we use Brousseau's Theory of Didactical Situations to build situations to be experimented with student teachers. We give an account of the way student-teachers play situations and implement them in their classroom. An example of a situation on vectors is developed.

**Keywords:** teachers' training; PCK; proof; theory of didactical situations; vectors; algebra.

## Resumo

*Na universidade, os estudantes têm uma concepção formal da matemática. Quando se tornam professores, essa concepção leva-os a pensar que é natural a transmissão direta de saberes, e são pouco aptos para tolerar as formulações aproximativas e os erros de seus alunos. Como fazer para que esses professores tenham consciência dos modos de manifestação do saber matemático em sala de aula, e da articulação entre o saber científico e os saberes que eles ensinam? O que Shulman chama PCK (Pedagogical Content Knowledge) e que na França chamaríamos conhecimentos didáticos, inclui a capacidade de reagir de modo matematicamente pertinente em sala de aula. Estudamos essa PCK e utilizamos a teoria das situações didática de Brousseau para construir situações propostas aos professores em formação. Estudamos, através da memória profissional desses professores, como procedem para implementar essas situações na sua sala de aula. Uma situação sobre vetores foi analisada.*

**Palavras-chave:** formação de professores; conhecimentos matemáticos didáticos; teoria das situações didáticas; prova; vetores; álgebra.

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\* IUFM D'Aquitaine et Université de Bordeaux. E-mail: isabelle.bloch@uni-pau.fr.  
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## **Introduction**

This paper presents elements of some training methods for mathematics teachers, and an example of teaching and learning situations we offer to future teachers. As an educator in training Institute with student-teachers we frequently meet the following questions: Which conceptions do novice teachers hold on the mathematics to be taught? On teaching practice? Are young teachers able to make their students do mathematics involving validation and proof? What competency lies in this ability? Is it possible to develop it? What are the learning situations that can be introduced with that aim, in the preparation of young teachers? Are these situations similar to those that could be offered to students in the classes? What pedagogic or didactical knowledge is necessary to help teachers manage learning situations?

A professional ability is noticeable in some expert teachers' classrooms: they succeed in reacting in a very pertinent way to mathematics questions of their students. The present study has a double focus: first, try to observe this ability (or observe how it works when the teacher lacks this aptitude); and try to develop this ability by playing 'open' (but rigorously built) situations with young teachers – what Theory of Didactical Situations calls 'adidactical situations'.

This paper consists of four parts:

- 1) Conceptions of young teachers on mathematics and mathematics to be taught; the theoretical framework for building situations; some questions about situations in the student-teachers' training;
- 2) Methodology to analyse the ability of teachers to provide pertinent mathematical answers in a situation; observation of a student-teacher and definition of the criteria we keep for our research;
- 3) A paradigmatic situation introducing linear combinations of vectors;
- 4) The professional reports of two student-teachers who tried this situation with their students; A look back on the first questions and a conclusion.

A short presentation of the organisation of the academic year in the French IUFMs (Institut Universitaire de Formation des Maîtres) is to be found in annexe 1.

## 1. Teachers' training: situations for student-teachers

### 1. 1. Conceptions of young teachers about mathematics to be taught and ways of teaching

#### 1.1.1 Mathematics

Students often get a very formal conception of mathematics during their University studies. For them, a theorem has to get a proof, but no justification in terms of problem solving; its justification does not stem from a problem solving context but from its belonging to a mathematical theory. They have a rather poor culture of problems to be solved with the mathematical tools they have studied at University; and, as many authors have pointed out, their own mathematical knowledge is often inefficient and compartmentalised (Robert, 2001; Henry and Cornu, 2001, p. 483). Moreover, even for a good student-mathematician, it would not be evident to control the *didactical transposition* of the knowledge at secondary school: one can master a mathematical knowledge in well-known conditions and make errors in another environment.

#### 1.1.2 Ways of teaching

What are the novice teachers' notions of the mathematics to be taught and of teaching practice? At the very beginning of their practice, lot of them still keep the illusion that 'a good course' on mathematics is done by a teacher in front of the students, and that the teacher 'tells the law', that is, the mathematical law. They have no idea that this law could be contested, nor that the mathematical law could not be understood, overall, considering that only elementary mathematics is in question at secondary school. The mathematical formalism seems transparent to them, it is as if it were self-explaining. Indeed they themselves hardly ever question secondary mathematics: at University they are accustomed to take what the mathematics teacher says for granted and cannot imagine any other behaviour from the students in their own classes (Henry and Cornu, 2001, p. 483).

When students become teachers, they have succeeded in their studies, so they think that their mathematical training is achieved, but they know very little about how the mathematics they have learned can be applied in the secondary school (and even in mathematics themselves). It is therefore difficult for them to get a critical and reflexive point of view

on this mathematics. So they accept without many questions the didactical transposition of the mathematical knowledge at the secondary level, even if they show great willingness to adapt themselves to their students.

At secondary school didactical transposition emphasises the pragmatic point of view: mathematics are sometimes even reduced to manipulation of semiotic tools, these ones being the signs and symbols with which the mathematical work is performed (Bosch & Chevallard, 1999; Schwarz & Dreyfus, 1995). In its present state, secondary teaching is organised through conventional tasks using semiotic tools, such as developing an algebraic expression, or studying the sense of variations of a function. Teachers teach how to handle algebraic expressions, or derivatives, or link geometric assertions... but what is frequently missing – because it is not included in the curriculum – is the *reason* why such tasks must be done, and, rather often, the dimension of necessary truth relative to the consistency of the work (Sackur, 2000). Moreover, a number of young teachers are convinced that telling the mathematical truth in a formal way is sufficient.

Hence the conception of young teachers is characterized by:

- an illusion that the manipulation of ostensive objects – symbols, graphs, geometrical figures – is sufficient to give a meaning to mathematical notions;
- a lack of knowledge about pertinent problems related to the concepts taught at secondary level;
- an absence of means to take the responsibility for the organisation of a long course.

Then the didactical contract the novice teachers set in their class must evolve to enable them to:

- Organise the didactical time, on a short or long term, to define their objectives;
- Define the corpus of learning situations and exercises to offer to students;
- Give students instructions to do a task or solve a problem;
- Link how to teach and how pupils can learn by organising learning situations, and give themselves means for assessment;
- Organise students' mathematical activity and interact with them using pertinent arguments;
- Bring pragmatic proofs into play (and not only formal ones);

- Give students a real mathematical responsibility and be able to tolerate temporary formulations and procedures.

In their class teachers are responsible for the mathematical exactitude, in terms of what is right and what is false and how their students will understand mathematics and use new techniques; and they are responsible for what their students will be able to do (and learn) with the tasks they organize for them. In short, they are responsible for what the mathematics students really *do*.

We can observe student-teachers (and even more experienced teachers) in their classrooms, and some of them are quite able to foster students' mathematical activity; as others are unable to provide pertinent answers to students' actions or questions and to stimulate their interest. We want to question this professional knowledge: how does it appear in the teacher's behaviour? Are there means to develop it? Could it be linked with the teachers' conceptions of mathematics?

## **1.2 Taking the epistemological dimension into account**

Mathematical pertinence of the teacher seems to be linked with an epistemological component in the teacher's role that is emphasized by a number of authors (Arsac & al., 1992; Lenfant, 2001; Jaworski, 2003; Malara & Zan, 2002). Those last two authors consider the model of the teacher as being him/herself a decision maker, which leads to pay attention not only to the teacher's epistemological knowledge but also to the teacher's knowledge on children's thinking, and to his/her ability to question mathematical knowledge and to 'put it on stage' in a pertinent situation. This last ability we link with the one Shulman calls PCK (Pedagogical Content Knowledge).

By the fact Shulman (1986) classifies the teachers' knowledge in (at least) three components:

- Content knowledge: mathematics student-teachers have worked during their University studies and more specialised mathematics they study the year they prepare the CAPES (see Annex 1);
- Pedagogical knowledge: ability to make the class run, regulating students' work and behaviour;
- Pedagogical Content Knowledge (PCK): ability to interact with the students on mathematical subjects, to interpret the students'

formulations and productions, and to build pertinent learning situations.

During the year of professional training, Content Knowledge is no more the matter, at least in the way it has been taught at University; Pedagogical Knowledge is going to be improved by practicing and the training can organise specific development about it, such as looking at classrooms' videotapes, talking about students' needs and behaviour and receive advices from older teachers.

*Pedagogical Content Knowledge* is related to *specialised mathematics for teaching*. To improve PCK, teachers usually try in their class scenarios about a specific knowledge, and maybe they can regulate for the next time, basing themselves on the way it worked. It is the usual but empirical and hazardous way teachers' knowledge has always been left to private practice. Teachers' trainers can also *tell* young teachers the 'best way to do', but then, experience proves that they hardly believe it, especially if this best way is not the way they have been accustomed when they were students themselves.

We think it can be more efficient if young teachers have to try out teaching situations related to a concept, and to convince themselves that it is possible to build that kind of situations, where the knowledge is not transparent but will appear as a tool to solve the problem. Their formal conception of mathematics makes it difficult for them to imagine a kind of game – a problem – whose (hidden) solution could be a mathematical notion.

The theory of didactical situations offers valuable means to build such situations (see Bloch, 2003).

### 1.3 Principles for building situations

The dimension of *problem solving* is not part of the mathematics as taught at University. It is then very difficult for a young teacher to imagine problems relative to a mathematical concept, and all the more problems that can be attainable by students at the secondary level. Yet society now considers problem-solving as an unavoidable reference in teaching; since 1981 at least, a number of authors insist on the necessity of constructing situations where students are led to: take responsibilities about the knowledge (Douady 1994), debate with other students on the necessity and the truth of mathematical assertions (Legrand 1993),

understand that a technique is relative to a type of problems and have to be generalised (Chevallard 1998). For Vergnaud (1981)

La résolution de problème est la source et le critère de savoir.  
(Problem-solving is both source and criterion for knowledge).

Besides, we notice that for a few years, the French syllabus in mathematics has emphasised the importance of problem-solving; this it is settled since the earlier Primary school and continues at Secondary school level:

“... ce qu'est une véritable activité mathématique : identifier un problème, conjecturer un résultat, expérimenter sur des exemples, bâtir une argumentation, contrôler les résultats obtenus et évaluer leur pertinence en fonction du problème étudié” (Programme de Sixième, –12 year-old pupils – p. 15)

(What a real mathematics activity is : identify a problem, conjecture a result, experiment on examples, build an argumentation, control results and assess their pertinence).

But a situation – in the sense of the Theory of Didactical Situations (TDS) – is not only a ‘problem’ as in ‘problem-solving’. Sackur (2000) notices that only pertinent kind of problems, by way of their special structure, allow students to understand the necessity and coherence of mathematical statements. Constructing situations, Brousseau (1997) points out the necessity of organising a milieu in which students can work assuming a maximum of responsibilities about mathematical knowledge. We have illustrated in a previous work how to organize such a milieu – a graphic one – to teach the notion of function (Bloch, 2003); this milieu allows focusing on questions related to functions’ *properties*.

In each case, the situation provides first a ‘material’ milieu that allows experiments for pupils; and the milieu gives feedbacks (success or not). The material milieu is made of “material things” to act with (when we say “things” we mean that for the students they do not necessarily represent mathematical objects, or at least coherent ones); this milieu gets a heuristic dimension. A second phase, creating a validation milieu, includes procedures of verification which must lead students to formulate mathematical properties. Then the didactical situation allows the teacher

to declare the aimed knowledge (see Brousseau 1997, p. 8-17, p. 248; Bloch 2003, p. 12).

There are not so many 'good' situations for secondary school and high school. A main difficulty at this level is that the 'material' milieu is already constituted of mathematical representatives; but while the teacher sees them as mathematics, the students sometimes see them only as sorts of conventions that the teacher introduces.

To build situations that are relative to a notion we apply principles from the theory of situations:

- identify a 'game' where the concept is pertinent (epistemological component); Brousseau refers to the game theory, with two 'players': a proponent and an opponent, and a milieu that returns feedbacks on the game;
- make the main didactical variables appear and choose their value, so as to build an heuristic milieu followed by a milieu of reference (the milieu of reference is a milieu for validation);
- after the validation phase, the didactical situation allows to introduce the knowledge through institutionalisation and mathematical proofs.

What is the difference between a 'direct' and an 'inverse' situation? A knowledge being previously introduced by the teacher in an ostensive way, a direct situation is a task where the drill is to use this knowledge to obtain a result: for instance, students are told what an orthogonal basis is, and they must use this basis to construct the point M corresponding to  $\overrightarrow{OM} = 2\vec{i} + 3\vec{j}$  (this is a very usual task to 16-17 years-old students). We notice that in this case the knowledge is a contingent fact in the sense that this knowledge is used because the teacher says so, but the necessity does not come from the situation. If we ask students to place the point defined by  $\overrightarrow{OM} = 2\vec{i} + 3\vec{j}$  in a frame, they can as well proceed the right way as they cannot: it depends on their own understanding of the rule; if they do not use the good procedure they can answer the question, only their answer will be false.

But if we ask them to find coordinates, or to find points or vectors under any conditions, they cannot succeed without having at least an idea about how to do it: in that phase the aimed knowledge is necessary. Maybe they could find one point or some coordinates at random, but they cannot win the whole game without an idea on 'how to do'. Although in



this case it is not quite didactical – because the milieu gives a feedback on the success but not about the pertinence of their method – they cannot succeed in any case without putting the aimed knowledge in their way of doing. We call it an inverse game because students have no longer to find a result deduced from given elements: they have to find an object on which conditions are given, and they must find by themselves the good knowledge and the actions that will lead to the aimed mathematical object.

The work we have to do in the Theory of Didactical Situations is to find a game that students could play to succeed with the rule of vectors' decomposition; playing this game students must have a criterion of success at their disposal, as a material verification, before they question to find why the result could not be else. What is interesting is that the knowledge of the first part of the situation is absolutely necessary to solve the second problem: the first part represents a heuristic milieu, and the second game a milieu for validation where the proofs appear. The necessity of some mathematical statements can then be debated, in terms of: 'Why – with which mathematical arguments – are we sure that we found the right coordinates?' This game about decomposition of vectors is The Grid Game we expose in Part 3.

#### **1.4 Professional knowledge to manage situations**

On another hand, it is now well established that complex situations are not easy to manage in a class; we can think that knowing a 'good' situation is not sufficient for a teacher – even an expert one – to enable him/her to have students performing mathematical reasoning. Arsac & al. (1992) analyse the reproducibility of situations: they indicate that, because of his/her own epistemology, it is difficult for a teacher to act:

only as a chairperson and as the collective memory of the class during the debate period.

It can also be problematic to make real practice of teachers compatible with the aims of the situation, as their implicit epistemology and beliefs can block the satisfying process of the situation: Arsac & al. give an example of a teacher who does not allow herself to write a false statement on the blackboard.

This uncertainty about reproducibility also makes it necessary that student-teachers gain the experience of such situations by playing them by themselves. A research-team of the INRP (Institut National de la Recherche Pédagogique) has experimented with young teachers the last phase of a situation, the most problematic one, when students tell the class what they have found and the teacher must coordinate the interventions of the different groups; all novice teachers say that they must have had an experience of that work, or have seen a more expert teacher do it, to dare to try it in their class. (Douaire & al. in Colomb, Douaire, Noirfalise, INRP 2003, p. 53).

In Bloch 1999 we performed a very detailed study of the teacher's milieu in didactical situations (while Margolinas (2002) rather studies the teacher in 'ordinary' situations); we notice that the teacher has the responsibility of organising the material milieu, and anticipating the actions students can perform on the objects at their disposal. During the process of the situation, the teacher has to guide the students, interact with their knowledge and productions, be aware of his/her own knowledge and the constraints of the teaching situation (particularly time). We observe that in the heuristic milieu for students, the teacher must keep her/his own knowledge in reserve to let the responsibility of the mathematical research to his/her students. In the phase of validation, the teacher has to discuss students' productions from their point of view and not from his/her own knowledge.

Since the heuristic milieu is a milieu for experiment, at this level there is no need to debate and prove: students have to make up their mind about the task, try and re-try and conclude about the actions they can perform in the milieu and how the milieu 'reacts'. Concerning the second level of milieu that includes proofs, we can see that in didactical situations, there are generally at least two levels of proofs in the work of the students.

These levels of proof are what Hana & Jahnke call: 1) *pragmatic proof* (in the phase of validation) and 2) *purely deductive notion of proof* (proofs that prevail in the didactical milieu – for teaching and institutionalising – or in academic mathematics as well). For Hana & Jahnke the last level is specific of scholarly mathematics; but the pragmatic one belongs to school teaching, and more widely is part of the work of any practitioner

of mathematics: in order to understand mathematics it is necessary to connect concepts, to see how they interact in a pragmatic way.

As Hana & Jahnke say,

In school mathematics as well most of the concepts are first defined by formal properties, while their meaning becomes clear only when they are applied. ... A proof may be in the position, in fact, of deducing a new theorem from a proposition which itself may have little credibility, and which will acquire credibility and meaning only in the future, when the system of which it is a part becomes more fully developed and capable of wider applications. (Hana & Jahnke, 1993, p. 427)

And page 432 they add:

These considerations imply a fundamental difference between scholarly mathematics and school teaching. (...) Teachers must take into consideration the contribution which a given proof makes to our understanding of reality. (...) This implies a high level of epistemological complexity for the relevant processes of teaching and learning. The teacher cannot avoid these perplexing issues by simply communicating to the students Hilbert's view of modern axiomatics.

In school teaching, and particularly when the teacher organizes an didactical situation, the meaning is at stake before the definition and formal proof of the concept occur; then in a phase of validation mostly pragmatic proofs are discussed, debated, justified. We can see that this fact actually emphasises the complexity of the teacher's task from an epistemological point of view. Taking into account the pragmatic proofs produced by students in the milieu of reference is not an easy task for the teacher, considering that his/her culture is essentially related to formal proofs as said above. We already noticed this epistemological complexity in the teacher's work and we can imagine that all these difficulties are all the more important when teachers are novice ones. But this epistemological 'jump' is necessary: when in a too evident situation – telling students the mathematical truth – young teachers react by routine. Complexity will compel them to take into account more open students' activity.

### 1.5 What outcomes do we expect from playing situations with student-teachers?

Situations we offer to teachers must be carefully chosen and built; they must relate to sensitive subjects of the secondary school's mathematics *and* be organized so as to make them aware of new dimensions of the knowledge. We make the hypothesis that it is possible to play with such situations during the training time, and the objectives are:

- to provide them with situations to apply in their classes; to make them discover new kinds of interactions between teacher and students;
- to enable them to make their own mathematical knowledge evolve, thanks to the interactions with situations. In that way it is quite essential that they play the situations themselves; it is the only way they can realise the cognitive amazement produced when a well known knowledge is unrecognisable because it is 'hidden' behind a situation and you see that this knowledge will come out (even if it is not under its definitive mathematical form) through the actions students can perform.

The introduction of situations must achieve another aim: allow teachers to improve their reactions to students' actions. We can logically suppose that among students, an open problem will make more differentiated reactions occur, so that teachers have to accustom themselves to unusual questions; we expect that they become able to tolerate more unusual mathematical formulations. To achieve this aim there is a fundamental condition: *to play themselves the situation* is the only way that could make them confident that the situation will reach the right aim. Under this condition only they could become able to tolerate the uncertainty of the first phase and the 'incorrect' formulations of the students.

## 2. Methodology and observations

### 2.1 Methodology of the research

Considering the research methodology, Malara & Zan (2002, p. 559) recall the need for social and anthropological approaches to study teachers' beliefs: these two authors note that the choice of following a few teachers in an individual approach is more and more frequent; it provides detailed analysis on the cognitive process at work.

As an introduction to our study we want to better shape our main subject (ability to mathematical pertinence in a classroom): we present an observation of a teacher who – at the time we saw him – lacked this ability to organize a confrontation with a real mathematical task and to recognise pertinent formulations, not only in the students' work but even in his own interventions.

Many studies point the great interest of narratives to allow an access to 'tacit knowledge' underlying practice (Arsac & al. 1992; Bishop 1998 quoted by Malara & Zan). Similarly, to link narratives and situations, we chose to follow two young teachers in the process of their professional report (this report is an academic requirement, see annexe 1); one part of this report is the account of a situation they implemented in their class. To resume our intentions and ways of investigation:

- 1) We want to analyse the mathematical ability of the teachers, that is, the way they can foster students' activity by pertinent interventions; this ability we call  $C_1$ ; it includes the fact that the teacher leaves his/her students a real responsibility in the situation.  $C_1$  is an important part of PCK.
- 2) We make student-teachers play themselves the chosen situations; first we ask teachers to do the students' task; we observe them at work (attempts, procedures, calculations); then we ask them questions about the aims of the situations, the didactical variables they can manage and fix, the alternatives they dispose to teach a given theme.
- 3) We practice direct observation in the classrooms of some student-teachers.
- 4) We study the professional reports of student-teachers who try to implement at least one of these situations in their class; these reports containing also students' drafts.

For an assessment of our tentative to make student-teachers more expert at fostering students' mathematical activity, we retain three main criteria:

$C_1$  is the mathematical ability of the teachers, that is, the way they can foster students' activity by pertinent interventions;  
 $C_2$ : the ability to manage the situation to its end with a phase of debate and validation; this ability includes the aptitude to guide a debate, select some formulations and leave some others, while keeping the situation's fundamental aims and features.

C<sub>3</sub>: the tolerance to approximate formulations from the students, which is a part of PCK too.

## 2.2 The case of Jeremy: driving students' activity outside of mathematical criteria

We made the hypothesis that the professional ability C<sub>1</sub> in teaching consists in driving students' activity with adequate mathematical interventions. To identify pertinent signs – or not – of this ability we observe teachers in their classrooms. The case we present here is the one of a 40 years-old teacher, Jeremy; some years ago he has been teaching to students of a technical high school to which he could explain mathematics in a rather direct way. In the present observation he teaches mathematics to 14 years-old children. He wants to introduce the resolution of linear equations at the beginning of the academic year (November).

First he gives them very plain equations, as:  $x + 5 = 6$  or  $x + 300 = 500$ , with a schema – the real straight line with adequate values – to help them. All the students solve the equations, and then he announces:

J: Yes but equations could have been more difficult and you would not have known how to solve them; so I give you a general method to transform and solve equations.

Then he gives his students a paper with four schemas of a weighing machine being balanced; the four balances are:

$$a = b \quad a + c = b + c \quad x + x = 6 \quad 3x + 5 = x + 11$$

The numbers 6, 5, 11 are symbolised by the same number of drawn bricks; the letters a, b, x, are just written on the pans of the weighing machine. Students must write the equations in front of the schemas.

The task is not at all evident for his students; so he tells them that they just have to cross the same elements (the bricks) on each side of the weighing machine; he does not seem to see that this task is impossible at least on the first and the third schema: they cannot cross out letters if they are not the same. Then he adds: 'for the third and the fourth equality, you have to divide the weighing machine in two equal parts'. Since his schema is ambiguous for the fourth equality – a brick of 5 versus 11 bricks of 1 – the teacher himself fails in dividing the rest of the bricks on the blackboard; the students seem to be lost.

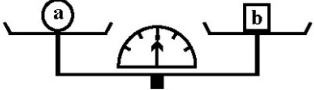
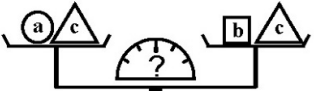
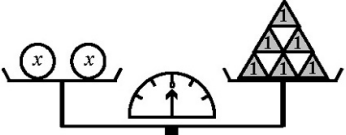
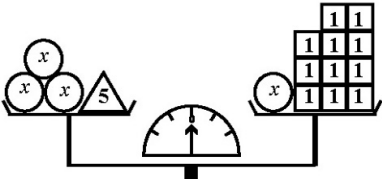
After that phase he expresses and writes the rules for solving equations. He then proposes an exercise to apply the rules; students have an equation and, instead of solving it, they must calculate the value of other algebraic expressions using the equality. The expressions are:

$$x + 7 \quad 1/3(x + 4) \quad x - 12 \quad 3(x - 5)$$

The equation is:  $x + 5 = 6$ . We can observe that the students do not follow the instructions, but solve the equation and substitute  $x$  by 1.

We analyse this introduction to equations and find that the two first equalities have not the same status than the others (as they are illustrations of the rules, while the third and fourth ones give an application), and this has not been clear for the students.

Considering the third equality:  $x + x = 6$ , in the teacher's mind it is a stage of the resolution of the last equation, but actually this equation becomes:  $3x - x = 6$ , which is not the same. The students' real work is not related to the foreseen resolution: they cross out bricks on the schemas when they can do it; the teacher has to express almost all sentences to say what should be done; most students stay waiting at their desk, without showing any noticeable mathematical activity.

	La balance est équilibrée, donc
	1) .....
	2) .....
	3) .....
	4) .....

During the discussion with the trainer, the teacher says that students must see the rules and solve the last equation using the hints he gives them. But what is the meaning of: 'divide a weighing machine in two equal parts'? The teacher does not seem to be aware of the mathematical non-pertinence of his situation and of his language. He is in the illusion that what he sees is also seen by the students, and *he* sees mathematical solutions of equations in this task – because equalities are written in regard of the schemas, and balances are often an illustration of equation solving in text books. He does not see either, that the last calculations he proposed are done by another way than the one he intended. When the trainer makes him notice that students may have learnt nothing since they could solve the last problem without the aimed knowledge, he answers that this is a good thing: even if they have understood nothing, they can do the job; for him, this is an indication of the good technique he holds to make the class run anyway.

Jeremy simply cannot see that his language during this occasion is not a mathematical one, does not allow students to take a responsibility in mathematics (write mathematics, express mathematics, decide truth or not in mathematics), and that he proposes something like a metaphor for equations, but no *mathematical* construction of how to solve linear equations. Jeremy is an example of what we call non-pertinence of mathematical intervention. He is not a 'bad' teacher: he is concerned with his students, and he really thinks that this metaphor of a balance helps students to understand equations. Anyway he is not the only teacher to do that, and this legitimates him in his mind: actually equations are rather often introduced this way in secondary school. Analyse the way mathematics are talked and written in a class is a way to improve PCK in this case; it is efficient but it does not give an alternative route.

### 2.3 The study of students' professional reports

We have chosen to follow two trainee-teachers, Justine and Séverine, as we intended to work in depth with a few teachers through the process of writing their professional report. We taped the interviews of the teachers and disposed of some transcriptions of their classes, and written productions of students. Their final report is also a witness of how they solved the difficulties and contradictions they met while trying situations in their classrooms.



We point out especially the ways they become aware of *the situation* they introduced and *the model of situations* (the theory), because it is awareness of the principles that they carry out to organize their action. As noticed by Malara & Zan, and present in the Italian model of teacher-researcher, theory modifies the teaching, as it modifies the teacher's knowledge of the situation, her/his meta-cognitive skills, beliefs on what students are able to do, and emotions, making her/him more confident. So it modifies the teacher's decision processes and consequently her/his practice. (Malara & Zan, 2002, p. 564). We can say that the two teachers became aware of the theory they used and of the way they were using this theory. We analyse the decisions they took to answer students' productions and their mathematical pertinence in section 4.

### 3. A paradigmatic situation about vectors: the grid game

#### 3.1. The grid-game: Principles of the situation

A situation (from a first idea of A. Berté) to introduce the multiplication of vectors by real numbers has been tested with both novice teachers and students. The aim is to build lessons on vectors that permit to make the functionality of this notion appear in different kinds of problems. It consists of a game, whose support is a grid (see Annexe 2). There are two phases; a direct one and an inverse one.

The direct game simply consists in calculating sums of vectors, and associating them to the correct points, as usually done at upper secondary school. This first direct game institutes a heuristic milieu, the milieu where students can get the technique and the basic strategy: they discover that if they multiply a vector by a number they can start from a point and reach another point. The type of instruction at this phase is: let  $A$  be a point of the plane, a given vector  $\vec{u}$ ; place the point  $B$  such as:

$$\overrightarrow{AB} = \vec{u}, \text{ or } C \text{ such as: } \overrightarrow{AC} = -3 \cdot \vec{u}.$$

The inverse game – that we present here – has got two phases itself:

- In *Phase 1* the game aims to find points by doing the multiplication of *one* given vector by numbers. What is at stake in this *Phase 1* is the way how students relate real numbers and lines in the plane, and research in Mathematics Education shows that this is a serious problem. In *Phase 1* the students have to understand that: given a non zero vector and

a point, one can reach any point of the one dimensional space defined by the point and the vector by multiplying the vector by an adequate real number. Phase 1 is a communication game and students work in groups in which there are two emitters and two recipients. (See annexe 2 for instructions and schemas).

- The second phase works with the functionality of a *two vectors basis* in the plane. It is a communication game too, but in a two dimensional system (a basis). In *Phase 2* students have to find that: given two non collinear vectors and a point, one can reach, by sum and multiplication, any point of the plane. If reaching *every* point is not actually possible, restraining to integer coefficients is not enough to understand (through action) the generality of the rule. In some way, the situation makes the students meet constructive mathematics: the task allows students to become convinced that every point can be reached, a pragmatic proof being found in some non trivial cases.

In this inverse game students cannot succeed if they do not use – in action – the knowledge aimed at this level of milieu: a basis of two vectors permits to reach every point – at least, every point with plain coordinates and some non trivial points – of the affine plane. The main objective is that the students understand *the rule of how a vector basis operates, before* they are told – in the teaching situation, by institutionalisation – the formal expression of this rule:  $\forall M \in P, \exists ! (x,y) \in \mathbb{R}^2 / \vec{OM} = x \cdot \vec{u} + y \cdot \vec{v}$

### 3.2 The grid-game with student-teachers

This situation has got objectives of different level for teachers:

- making understand *by action* that with a basis of two vectors they can reach every point of the affine plane; according to the sense of Hana & Jahnke it is a *pragmatic proof* of the functionality of the concept of basis; it makes student-teachers discover that pragmatic proofs are *not* evident even when a formal proof is well-known;
- for that purpose, it is necessary to let young teachers actually reach some points with real coefficients as  $\sqrt{2}$  or rational numbers (constructible numbers). Before they discover this game, they usually think that they can only draw on the grid, points as  $2\vec{u} + 3\vec{v}$  (points with entire or simple rational coordinates);
- related to the theory of didactical situations, this experience can clarify the principles we use to build such a situation: the knowledge should be

used first as a tool in an action phase; it makes them also encounter the notion of didactical variable, as they can see that coordinates, number of vectors, and position of the points play this role in the situation.

During the training, the instructions are: do yourself the task, and then, explain the mathematical knowledge 'hidden' behind this task; find the didactical variables. They have to discover that it is not trivial to find the good coefficients for the first game (although this game appears to them as evident), and even more for *Phase 2* in Game 1 where:  $\overrightarrow{OM} = \sqrt{2} (\vec{u} - \vec{v})$  or Game 2 where:  $\overrightarrow{OM} = 15/4 (\vec{u} + \vec{v})$

This situation is particularly interesting to try with young teachers because they have overall a very formal view of linear algebra. The idea that it is possible to 'realise' the concept of basis of a vector space in such a way – without telling students the definition – is very amazing for them. Moreover, they experience difficulties just to find the good coefficients and the didactical variables:

- supports of the vectors following the lines of the grid or not;
- nature of numbers – integers, rational or irrational numbers; place of the points to be found;
- number of vectors of the system: 1) one vector – in which case only some points can be reached from given points with the multiplication of a vector by a real number, like in *phase 1* of the inverse game; 2) two vectors – in which case all the points can be reached in one way, 3) or three vectors – in which case the points could be reached by different ways.

After a while they propose the use of theorems like Thales's theorem about parallels or the construction of the diagonal of a square. Very few of them think that they can use the definition of a basis just with the decomposition of the vectors; and actually this would be a theoretical, well-tried mean but would not lead definitely to pragmatic success (be able to calculate the coefficients). So they are led to undertake themselves a heuristic research by drawing rotations, new vectors, doing calculations... The remarkable fact is that they produce themselves temporary 'incorrect' writings on their drafts; as they see these writings are meaningful for them and do not jam the process of the knowledge in the situation, they realise that even on a subject they thought they were mastering, they are led to produce pragmatic proofs without a great respect of the mathematical formalism.

This is the main event in the process of their experience: at the same time they get convinced that the situation of the grid game really allows to discover the functionality of a basis. This will make the teachers confident that, in spite of the uncertainty of the heuristic phase and of the impropriety of the formulations, students are led to the aimed knowledge. We identify this confidence as a fundamental gain for student-teachers when they play the situation.

#### **4. Teachers' experiment with their students and conclusion**

We first give a brief description of how the two teachers whose report we followed succeeded in driving the situations in their class; then we examine the results our device allows to state, with respect to the questions we quoted in the first parts of our paper.

##### **4.1 The description of the situation 'The grid game' in professional reports**

###### **4.1.1 The situation in the class of Séverine**

Séverine teaches in a technological class of 16 years-old students. She tries to convince the students of the usefulness of mathematics. She explores a few directions, as the visit to a heating factory and tries to make students write the equations of the blowing heat; this leads her to analyse the importance that her students are able to attribute to mathematics in their studies and in their professional project.

The title of her professional report is:

“Comment donner du sens aux mathématiques pour intéresser les élèves d'une classe de Seconde technologique à cette discipline d'enseignement général?” (How to give sense to mathematics to make students of a technological class interested?)

In the introduction she says:

Teaching mathematics in a technological class is a problem because of the pertinence of mathematics with respect to the needs and vocational aims of the students. We want to organise situations that could be both adequate to the knowledge and pleasant for the students. But in such situations, the teacher has

to manage a number of elements in her class, as communication or mathematical research; her role will be more complex. These situations 'put in stage' a mathematical knowledge; we want to study how the teacher think about them, how she builds them, how she manages them in the class, and finally, what these situations bring to the students but also what difficulties they can generate. (p. 5)

She identifies two types of motivation: *external* and *internal* one. External motivation is obtained when the teachers calls upon history of mathematics, or a concrete problem, to make them work mathematics; internal motivation is just due to the fact that students may happen to feel self-confident in understanding mathematics, making progresses, and that they take some intellectual pleasure in the mathematical problem they have to solve.

Séverine analyses very well the principles that are used for building an adidactical situation:

First, a situation is built with respect to an aimed knowledge. A mathematical notion is built with respect to a need (inside a mathematical theory); it is functional when it is used as a tool to do a work. In a didactical situation, the knowledge is used first as a tool to solve a problem; the teacher does not intervene to tell the students what they are to find, the students must deduce the solution from the given conditions.

She speaks of "theoretical tools to build a situation which gives sense to Mathematics", and notices that the teacher must respect three phases in the driving of such a situation: 1) devolution of the problem where students can make tests in the milieu and errors – the utility of this phase for students is that they constitute a repertoire of results of their actions; 2) mathematical debate, this one being the phase she gets the most struggle with; 3) institutionalisation.

She chooses three situations to implement in her class, including the grid game (see 3.1 below).

In her personal questioning of building situations for the internal motivation of students, she thinks that inverse situations (Bloch, 2005) are likely to interest students because they put a real problem that can puzzle them. She points out to the main didactical variables as being constraints

at disposition of the teacher. Séverine notices that the recipient's role is easier than the emitter's, and she attempts modifications, as letting students playing both roles at the same time, or giving more simple values to coefficients, to adjust the situation. She shows her ability to let the students play their role in the responsibility of mathematical knowledge; productions of the students are quite interesting and show that Séverine was able to cope with incorrect writings, as: "length  $\overrightarrow{KV} = \overrightarrow{KL} + 45^\circ$ "; students made a numbers of new constructions on the grid to find the required coordinates, and did even measurements, which shows that they seized the means and the freedom the situation offers. It is a result of significant interest since research shows the difficulty students encounter in doing new constructions (e.g. on a figure in geometry). We also notice that the teacher was able to tolerate measurements and incorrect writing: this was our criterion  $C_3$ . It is worth noting the fact since measuring is globally disqualified at upper secondary school.

In her conclusion Séverine points at the difficulties of her role as a teacher. It is necessary to anticipate the procedures and the errors of the students without telling them what to say: ability to leave students take a mathematical responsibility without telling them the truth at once, this is  $C_1$  or a part of it. The management of groups' work, of the way students speak in the second phase, is important: the teacher must keep master of the order students speak. The difficult role of the teacher in the last phase of institutionalisation is to clarify the aimed knowledge: Séverine owns  $C_2$  – the ability to bring the situation to its term. She carries out that this situation allows a significant gain on the students' motivation and the pertinence of the mathematical work in the class. Séverine ends the report with a very pertinent question: 'What are the key concepts in mathematics, for which it would be necessary to organize such a situation, in order that students would get a correct conception of the notion?'

#### 4.1.2 The situation in the class of Justine

Justine teaches in a class of thirty-three 16 years-old students from a regular class. The title of her professional report is:

"Réflexion sur l'enseignement des mathématiques, une transmission de connaissances, un apprentissage de la vie".  
(Considerations on teaching mathematics as a transmission of knowledge and a life learning).

She merges, in a very personal way, humorous scenes in her class and reflective knowledge on how to reach her objectives. So quoting the official instructions of that level:

*Search, find partial results, ask questions... are a few aspects of the diversity of mathematical action. Any student, at his/her level, should have occurrences of doing experiences about the efficiency of mathematical concepts and simplification that allows the mastering of abstraction.*

Very altruistic but is it really conceivable when you have got 33 complex human beings, all different? (...) the teacher must think of the links that exist between: – students' groups (homogeneous or not); – the aimed objectives (knowledge, know-how, remedial, deepening); – didactical treatment (pedagogical organization, texts, didactical tools). (p. 3)

From the beginning of her narrative work she shapes very interesting features of the aims and means of mathematics teaching. She points the necessity of recognising knowledge behind students' errors, and to organize a work with little groups of students – no more than four – to make students more independent in the research ('the teacher is no longer the unique owner of the knowledge'). During these phases she wants to foster the activity of the students and to make them share the responsibility of the mathematical truth. She wants to organize pertinent situations so that:

Learning situations must be pertinent from the knowledge point of view, and they must be attractive. (p. 11)

The third part of the report is especially interesting for our development as she points out the role of the teacher: tell the students the instructions, anticipate their errors, their conceptions; keep some information and make some other explicit; regulate the students' work.

The teacher must not forget that the work is under the students' responsibility; she must be aware that she would not induce the answers, which would deprive this phase of any interest and would cause interferences on the following one. (p. 20)

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She thinks that after a phase of research, the teacher must organize a phase of debate:

Students must confront their solutions, discuss, debate, validate ... (p. 21)

It appears that she is ready for organising situations in her class; she even anticipates the theory in a way, as she is remarkably aware of the work it is, to make such an artificial construction that hides the knowledge it would be so easy just to tell students:

The organised debate is *in no way* natural. It cannot be else but the consequence of an artificial, extremely delicate and constraining construction; it creates a frame where the discussion appears fictitiously free; but the hypothesis is that this freedom area be sufficient to allow a social interaction suitable to promote learning. (p. 6)

With such a state of mind, Justine seizes the situations she experiments in the training as an occasion to verify her ideas on teaching and her high idea of mathematics. She chooses three situations: a communication game about geometrical figures, the grid-game (see 3.1), and the product of functions (Bloch 2003).

Justine identifies perfectly the objectives of the grid-game situation and the conditions of devolution: instructions to the students, stake for the emitter and for the recipient, necessity to reject measure instruments even if they are accepted in a first time (she said 'yes' in the following excerpt):

Student: I tried a measurement to get an idea    J: Yes but I do not want only the number I want to see why it could not be else  
Student: So make a computation ...

In this excerpt we can see that she is stressing the student to make her calculate and find the rule; anyway we could observe that situations with an adidactical dimension are always led quickly to their validation phase when they are played with rather old students. This is an effect of the didactical contract in the mathematics classroom; and this contract is under the responsibility of the students as well. At secondary school,



students know perfectly well that they are not here just to play a game, but to learn a mathematical rule.

Justine sets up an analysis of the errors the students are likely to make and she compares their actual work to this a priori analysis. She speaks of the validation process, and she identifies a material validation by superposition of the reconstitution of the grid to the model: she indicates that this validation does not permit to see the *cause* of the errors. Then she insists on the theoretical validation by argumentation, the one that permits to institutionalise. She observes that for most students the question is no more: 'Have we the permission of the teacher to do that?', but: 'Is the transformation I intend to do pertinent to solve the problem or not?' that is, they are no more in a position to do the task because of the didactical contract but because they question themselves on the mathematical pertinence of their action. This is an indication of  $C_1$  on the teacher's hand.

About the development of the situation in her class, she observes positive and negative aspects:

- Positive ones: the devolution does take place; earlier knowledge is remobilised.
- Negative ones: in very heterogeneous groups there is little discussion about the validity of the result; in very weak groups the teacher must help students to find the most difficult points. The role of the recipients is too easy, in relation to the role of the emitters: so she adapts the game to make students at the same time emitters and recipients, as Séverine does. She says that the validation is a process because it cannot be punctual, there are some comings and returns; she identifies different types of erroneous procedures to be analysed in the process of validation.

Productions of students show that they used various means to do the task: theorems like 'Thales', linear combinations (even if they divide vectors, which Justine was able to accept on students' drafts), counting the squares on the grid, measurements ...

In a final part, whose title is: 'How to think the teacher's role?' she summaries the phases of devolution, instructions to students, the work of the students when they are searching solutions, and the phase of synthesis; about the last one she says:

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This phase must make the students able to have a clear vision of what have been done and said during the course; it is also an institutionalisation phase that does not get against the process of devolution, as it is an inverse but complementary process.

Evidently she masters  $C_2$  ; she also gains  $C_3$ .

At the end of her report, Justine is very much convinced that situations are an effective means of sharing mathematical responsibility with her students; she organizes another situation that she heard during the training (product of functions, see Bloch, 2003) and she allows herself to pursue with another situation on how to solve equations graphically. She says that she enjoys teaching that way: this brings insurance that she will continue working with such situations.

#### **4.2 Situation as a tool to focus relationship between theory and practice within teachers` education**

In this device, our choice was to work explicitly on the epistemological component of the teacher's work through complex situations; a fundamental component of the training was to let the student-teachers play themselves the situations before they try them in their class, so as to raise their awareness both of the subject – mathematical knowledge – and of the role of the teacher, the whole being PCK. But we could not be positive on the hypothesis that student-teachers were able to understand so deeply the structure of situations. The second component was to raise this awareness through the privileged mean of a narrative, the professional report.

##### **4.2.1 The mathematical knowledge at stake**

About mathematical knowledge we can assess that the situations play their role: work on the pedagogical content knowledge, that is, both on the teachers' mathematical knowledge and the means at their disposal to teach that knowledge. During the training, all student-teachers get involved in the situation, try to find the solution and then to analyse the situation with its didactical variables.

At the same time we were wondering to which questions such an experience could reasonably give rise for the trainee-teachers; and which would remain inaccessible even when the student-teachers have

experienced the situation both by themselves and with their own students (Lenfant 2001). Concerning the mathematics we can say that a very deep understanding of the pedagogical content knowledge regarding every mathematical theme to be taught at secondary school is a very high level of expertise; student-teachers only begin their own learning in that way. So we did not expect very sharp questions about the mathematical aspect of the situations. They take the situation for granted – as they have been convinced of its pertinence by experience during the training time – and just express the hope that somewhere such a situation is to be found each time they need it.

About 25% of the student-teachers achieve implementing at least a situation in their classroom. It could seem that it is not a wonderful result, but it must be related to the long time it takes, to really change traditional ways of teaching. Concerning the didactical aspect – driving the situation and the students' knowledge – the student-teachers who tried it revealed a very good aptitude to analyse the development of the situation in their classroom, and to manage the situation to its end with a phase of debate and validation.

#### 4.2.2 Situations and ability to pertinent interventions

We were wondering on the ability to give pertinent answers that student-teachers could build with the situations we were offering; this work emphasises two points related to this question:

- playing oneself the situation is a fundamental component to anticipate the role of the teacher and especially his/her mathematical responsibility, and to be confident enough in the situation's outcome, in order to be able to tolerate uncertainty in the first heuristic phase;
- the two teachers we followed learned new ways of 'expressing mathematics' while adapting themselves to their students' formulations.

We notice that Justine and Séverine became aware of the epistemological dimension of teaching even if only in pragmatic terms: when and how to tell the students the mathematical knowledge, what kinds of interventions of the teacher preserve the sense of the situation or do not, how to cope with the 'false' or 'incorrect' formulations of the students and not reject them. An important dimension of this epistemological awareness is the aptitude they show for accepting these

formulations, and even more: they take the students' formulations as contingent statements, according to Durand-Guerrier (Durand-Guerrier 2003), that is, statements to be discussed and put in the mathematical debate. So it appears that they are able to cope with pragmatic truth and to organize a debate aiming to improve the students' formulations; and this ability makes them very vigilant about their own declarations in their classroom.

Anyway the ability to pertinent interactions cannot be defined in itself: interactions are relevant when they lean on the students' work and formulations about the conditions of the situation. So we can conclude that this ability is not a kind of aptitude: it depends on the significant milieu that has been built to teach a mathematical notion. We could see Jeremy, who could not succeed in introducing a method for equations solving from an inefficient metaphor, we can see that the grid game helps the student-teachers to interact with their students because it provides precise mathematical answers to pertinent questions. This emphasizes the relevance of the chosen situation.

Why is it the situation that proves to be effective to make teachers improve their mathematical pertinence? We see two main reasons:

- 1) The situation provides a substantial mathematical material (instead of inefficient metaphors) to think, to talk about, to write, to draw... so it is easier for the teacher to seize good occasions to express mathematics;
- 2) The situation makes students try much more attempts, temporary procedures and writing... this many dimensional work offers the teacher a lot of opportunities to interact with her students, and the matter to do it.

A most interesting result is the level of theoretical thinking we could observe in the two young teachers' reports. Each of them went further than we could think possible in that direction; according to our previous analyse, they did not make a sharp evaluation of the mathematical pertinence of the situation but they employed themselves to understand the way the situation operates in their class.

#### **4.3 Situations at secondary school**

From a more general point of view, we could question the way didactical situations can work at secondary school or upper secondary

school. At primary school, situations built by the TSD have sometimes the reputation of being very demanding in time and capability of the teacher; they have given rise to questions about the way of managing such situations with very heterogeneous classes.

The situations we have proposed are short enough to be implemented in a classroom even during a few hours (4 or 5 séances), and they can insert into a more classical device. The heterogeneity of the students population is not an obstacle: we are aware that not all students will achieve the goals of the situation; but, even if they start on the heuristic phase, they will learn *more* than just by doing the direct task (the only task that is done usually). During the debate, they will have an occasion of hearing the other students' solution and of learning by their formulations. In many occasions we could notice that knowledge is very well spreading in a classroom, and students learn by their own interactions as well as by the teacher's answers or assertions. So we can say that situations at secondary school are compatible with the ergonomics of teaching, subject to the conditions we state above: be rather short and fit into the usual organization of the academic year.

### **Conclusion**

In the general problematic of the relationship between theory and practice in mathematics teachers' education, the question of the means trainers can mobilise to help student-teachers to evolve is a fundamental one. We think that this work attests that complex situations are an interesting means – if not the only one – to make student-teachers' PCK evolve. They are improving their PCK in two directions: the way they think about a mathematical topic, and the way they can answer their students' questions and achieve the implementation of situations in their classrooms.

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## **ANNEX 1**

### **Organisation of the training in the French IUFMs**

Teachers' education in France changed in the 90ths, when the second year of training passed under the IUFM's responsibility; the new organisation of training has been described in Comiti & Ball (1996) and more recently in Henry & Cornu (2001).

The first year remains a recruiting competition (the CAPES – Certificat d'Aptitude Professionnel de l'Enseignement Secondaire) that the students prepare in at the French Universities. The preparation for CAPES, though oriented towards mathematics, is nevertheless the first time when students can reorganize what they have learnt in the first years of their University courses. They generally have little new knowledge to learn even if there is some, mostly geometry and probability. Theoretically, mathematical knowledge required for the CAPES is related to secondary mathematical topics, and “does not go much beyond the first two years at University” (Henry and Cornu, 2001, p. 486).

CAPES has a written theoretical part on mathematics and two oral parts. The second oral part of the examination consists in selecting and presenting a set of activities and exercises on a given theme of the secondary school: this is a reflexive task about mathematical contents of this level. As Henry and Cornu say,

...the preparation of CAPES certainly offers an opportunity for developing more synthetic and integrated views about mathematics and for developing some reflexive thinking about mathematical objects. (Henry and Cornu, 2001, p. 486)

But candidates still remain in a perspective of an examination, and the lessons and exercises they have to build are designed for a jury.

Once they have passed the theoretical examination, French student-teachers are made responsible for the teaching of mathematics in one secondary school class, even if they have no experience in teaching, which is frequent. An older and more “expert” teacher has the responsibility to help the trainee-teacher. During nine months, the novice teachers must also follow about 15-18 days of training in one of the twenty four national training Institutes (IUFMs). The role of the training in the Institute is

*Isabelle Bloch*

first to help new teachers to conceive and perform lessons of mathematics in front of their students. But beyond that, the role of the trainers is to let the novice teacher build reflexive tools to analyse his/her practice and to improve it.

A second part of the training has to supply teachers with didactical mathematics knowledge to help them understand the articulation between advanced mathematical notions and the contents that they have to teach. This second component of the training leads to revisit some mathematical notions, but in a different way from what they have been taught at the University. A large part of this component of the training is to study secondary teaching organizations and link them to “higher” mathematical knowledge; As Comiti says,

The second year is focused on professional issues, based on a purposive link between theory and practice intended to help students understand that it is necessary to reorganize, process, sometimes even deepen their knowledge in order to be able to teach it, develop their capacity to analyse and plan learning situations, take into account pupils' ideas and representations as regards notions to be taught, choose a pedagogical approach according to their aims, conceive, carry out and assess learning sequences, set their practice in a broader theoretical and professional frame, and vary their teaching patterns. (Comiti & Ball, 1996, p. 1141)

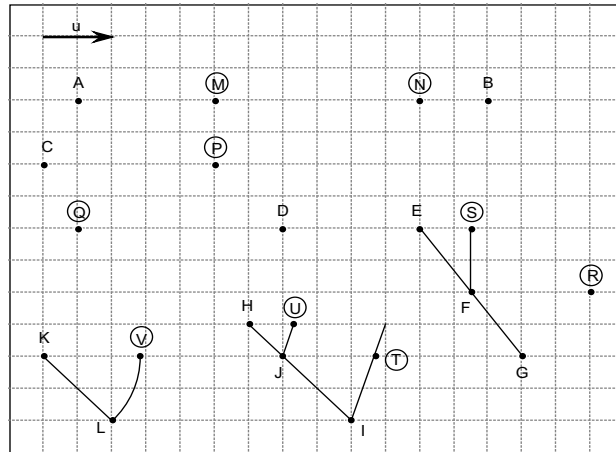
A third component of the training is very important for our project: student-teachers have to write a professional report that must be a reflective study about teaching mathematics in their class. The initial questions of that report are often vocational ones: How to get a quiet atmosphere in the class? How to rise students' interest in mathematics? But other questions can easily be turned into didactical ones: for instance and more specifically, how to interest students to vectors? Or, how to make them improve in algebra? What is the significance of their mistakes? Why do they encounter so great difficulties in understanding and using formal language and symbols?

The direction of professional reports offers the trainers an opportunity: to work with student-teachers about their personal questions, get enough time to elaborate a teaching project and follow the realisation

at least through the account of the student-teachers (time is a drastic problem during the second year of training in the IUFMs): professional reports give a fundamental occasion of linking theory and practice. For a trainer it is impossible to go into each class of the directed students to control the realisation of the project; student-teachers are incited to videotape their lessons anyway, but this is also very time demanding to see and analyse, supposing that every student-teacher should agree to take a video in his/her class, which is not the case).

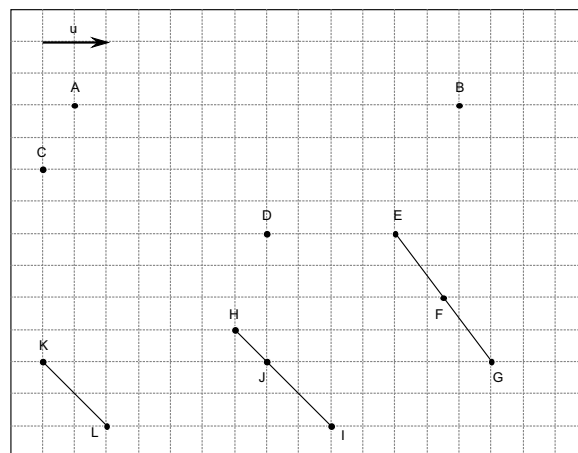
**ANNEX 2**

**Phase 1: Grid for the emitters**



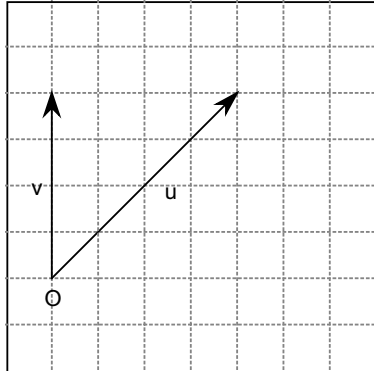
The recipients have got the same grid as you, but with only the vector  $u$  and the points A to L. You must send them a message to make them find the other points: M to V. Your message must contain only the given points,  $u$  and numbers.

**Grid for the recipients**

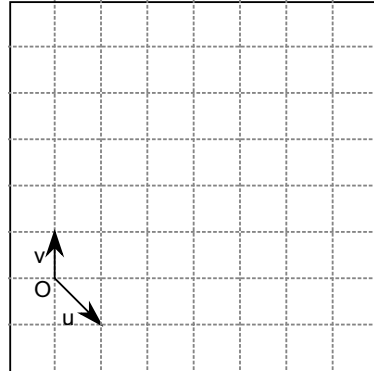


Phase 2

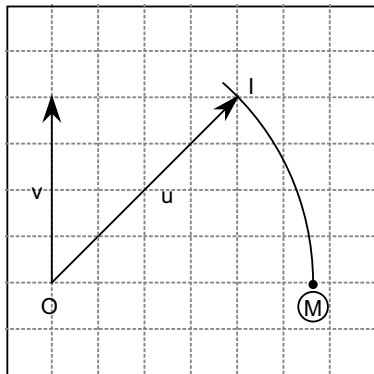
Game n°1  
Grid for the recipients



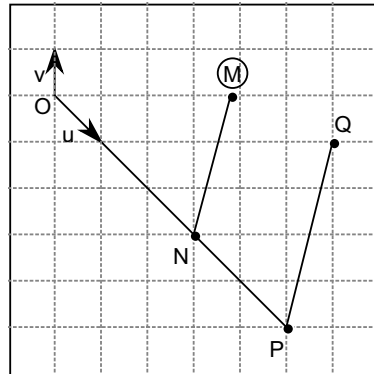
Game n°2  
Grid for the recipients



Game n°2



Game n°1  
Grid for the emitters



The other team has got the same grid as you, but with only the point O and the vectors u and v. Send them a message to place the point M. It is on the circle (O, OI), and on a straight line orthogonal to v. But you're not allowed to tell it in your message, that must contain only O, u, v and numbers.  
The answer is:  $\vec{OM} = \sqrt{2}(\vec{u} - \vec{v})$

The other team has got the same grid as you, but with only the point O and the vectors u and v. Send them a message to place the point M. It is at a place so that (MN) // (PQ) and the points N, P, Q are exactly at crosses of the grid. But you're not allowed to tell it in your message, that must contain only O, u, v and numbers.  
The answer is:  $\vec{OM} = 15/4(\vec{u} + \vec{v})$