ISSN 1983-3156
Qualis A1
http://dx.doi.org/10.23925/1983-3156.2023v25i3p183-205

Propriedades comutativa e distributiva na proposição davydoviana para o ensino de matemática

Commutative and distributive properties in the proposition by Davýdov for mathematics teaching

Propiedades conmutativas y distributivas em la proposición de Davýdov para enseñanza de las matemáticas

Propriétés commutatives et distributives dans la proposition de Davýdov pour l'enseignement des mathématiques

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Resumo

A investigação, de natureza bibliográfica, foi desenvolvida no contexto da educação matemática, especificamente sobre a organização do ensino das propriedades comutativa e distributiva em relação à multiplicação, proposto por Davýdov e colaboradores. Doutor em Psicologia e seguidor de Vigotski, Davýdov coordenou o processo de elaboração de uma proposta para o ensino de matemática fundamentada na Teoria Histórico-Cultural. A proposta pedagógica foi publicada, na Rússia, por meio de livros didáticos e manuais de orientação ao professor. O objetivo é analisar o modo de organização do ensino de matemática proposto por Davýdov e colaboradores na especificidade das propriedades comutativa e distributiva da multiplicação. A análise possibilitou revelar que essas propriedades são essenciais para a constituição do sistema conceitual da tabuada, o que implica a reorganização do seu ensino, cujo ponto de partida seja a relação entre grandezas discretas e contínuas na inter-relação da aritmética, álgebra e geometria.

Palavras-chave: Teoria histórico-cultural, Educação matemática, Multiplicação, Davý.

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Abstract

Bibliographical research was developed within the mathematics education context, specifically on the organization of teaching commutative and distributive properties in relation to multiplication, proposed by Davýdov and coworkers. PhD in Psychology and follower of Vygotsky, Davýdov coordinated the elaborating process of a proposal for mathematics teaching based on the cultural-historical theory. The pedagogical proposal was published in Russia through textbooks and teacher guidance manuals. The aim is to analyze the organizing way of mathematics teaching proposed by Davýdov and coworkers in the specificity of commutative and distributive properties of multiplication. The analysis made it possible to reveal that these properties are essential for the constitution of the multiplication table conceptual system, which implies its teaching reorganization, whose starting point is the relationship between discrete and continuous magnitudes in the interrelationship of arithmetic, algebra, and geometry.

Keywords: Cultural-historical theory, Mathematics education, Multiplication.

Resumen

La investigación, de naturaleza bibliográfica, fue desarrollada en el contexto de la Educación Matemática, específicamente sobre la organización de la enseñanza de las propiedades conmutativa y distributiva, con relación a la multiplicación, propuesta por Davýdov y colaboradores. Doctor en Psicología y seguidor de Vigotski, Davýdov coordinó el proceso de elaboración de una propuesta para enseñanza de las Matemáticas basada en la Teoría Histórico-Cultural. La propuesta pedagógica fue publicada en Rusia por medio de libros didácticos y manuales de orientación para maestros. El objetivo es analizar el modo de organización de la enseñanza de Matemáticas propuesto por Davýdov y colaboradores en la especificidad de las propiedades conmutativa y distributiva de la multiplicación. El análisis hizo posible revelar que esas propiedades son esenciales para la constitución del sistema conceptual de la tabla de multiplicación, que implica la reorganización de su enseñanza, cuyo punto de partida sea la relación entre grandezas discretas y continuas en la interrelación de aritmética, álgebra y geometría.

Palabras clave: Teoría histórico-cultural, Educación matemática, Multiplicación.

Résumé

La recherche bibliographique a été développée dans le contexte de l'enseignement des mathématiques, spécifiquement sur l'organisation de l'enseignement des propriétés commutatives et distributives en relation avec la multiplication, proposé par Davýdov et ses collègues. Docteur en psychologie et disciple de Vygotsky, Davýdov a coordonné le processus d'élaboration d'une proposition d'enseignement des mathématiques basée sur la théorie historico-culturelle. La proposition pédagogique a été publiée en Russie à travers des manuels scolaires et des manuels d'orientation des enseignants. L'objectif est d'analyser le mode d'organisation de l'enseignement des mathématiques proposé par Davýdov et ses collaborateurs dans la spécificité des propriétés commutatives et distributives de la multiplication. L'analyse a permis de révéler que ces propriétés sont essentielles pour la constitution du système conceptuel des tables de multiplication, ce qui implique sa réorganisation pédagogique, dont le point de départ est la relation entre les grandeurs discrètes et continues dans l'interrelation de l'arithmétique, de l'algèbre et de la géométrie.

Mots-clés: Théorie historico-culturelle, Enseignement des mathématiques, Multiplication.

Commutative and distributive properties in the proposition by Davýdov for mathematics teaching

Generally speaking, education fulfils the task of concretizing the world's conception at a given historical moment and transforming and modifying it. Based on the principles of cultural-historical theory, its purpose is the subject's integral development from social, cultural, ethical, aesthetic, and political perspectives, which require the appropriation of scientific concepts and the development of theoretical thinking.

Theoretical thinking does not develop in everyday life but in institutions organized with this intent. In this sense, the school is one of the institutions created by humanity to transmit scientific knowledge systematized by previous generations to new generations and, consequently, provides thinking development.

In psychological terms, appropriating scientific concepts "constitutes a mental activity of analysis, synthesis, abstraction and generalization" (Rubinstein, 1979, p. 47). It is worth reflecting on what type of abstraction and generalization has predominated in the teaching-learning process of concepts in Brazilian schools and whether the teaching mode of organization has contributed to developing theoretical and empirical thinking.

Studies such as those by Moraes (2008), Rosa and Hobold (2016), and Rosa and Marcelo (2022) point out that teaching in various Brazilian propositions and teaching practices has contributed to developing empirical thinking, corresponding to traditional (or classical) formal logic, which developed from the perception of the external characteristics of specific groups of objects and/or phenomena. As a result of this starting point, the abstraction and generalization of the essential, fundamental characteristic common to all objects occur. The formal concept is reached by designating the common indications through words (Davýdov, 1982). Such concepts, developed through abstractions and generalizations based on the perception of the external characteristics of objects and phenomena, he nominates empirical. The author warns us that intuitive thinking is linked to this type of generalization, as suggested or understood by the traditional school and psychology, as he put it. For thinking turns only to the external appearance of objects and phenomena, lacking analysis in internal relations, since the empirical concept results from the perception, abstraction and generalization of characteristics externally and directly exposed to the sensory organs.

The formation of concepts based on traditional formal logic limits attention to the external aspects of objects and phenomena, which has implications for students' learning in each school context that adopts such a conception, among which we can highlight the students' memorization of some empty notions only to get grades in school assessments. In this type of memorization, there is an emptying of the content in one's thinking, which is limited to the external characteristics of the objects analyzed in the conceptual elaboration process instead of the essence, the internal relationship, as dialectical logic predicts. As a result, the content memorized for the evaluation now has an expiration date: the assessment day.

Would changing logic as a method of knowledge in teaching practice and textbooks solve the problems related to the appropriation of theoretical concepts? We believe it is not enough, but its solution involves changing the logic supporting the method of knowledge adopted in school education.

From the context mentioned above, the need to rethink the contents and methods developed in school education emerges. It is essential to break, in a concrete and lasting way, with the logic of internalizing education processes. It is not enough just knowledge and awareness but to experience processes that contribute to disrupting the current logic.

For students to be creators of themselves and their environment, it is necessary to appropriate scientific concepts and, consequently, the development of theoretical thinking (Davýdov, 1982), given that, with the theoretical thinking developed, the subject seeks the essence of things, including what is apparently camouflaged by contradictions.

In this context, this study aims to present the results of bibliographic research on the mode of organization of mathematics teaching presented in the Davydovian proposition. In the specificity of this article, we analyze Davýdov's approach to teaching the commutative and distributive properties of multiplication.

Vasily Vasilyevich Davýdov, born in Moscow, Russia (1930-1988), dedicated more than 25 years of his life to research to formulate a teaching theory focused on the development of children's and young people's thinking (Libâneo & Freitas, 2013). They believed the school's function is to teach students to "guide themselves with autonomy in scientific information and any other sphere of knowledge" (Libâneo & Freitas, 2013, p. 315) and develop students' theoretical thinking.

Davýdov (Давыдов), together with collaborators such as Gorbov (Горбов), Mikulina (Микулина), and Savieliev (Савельева), elaborated on and developed, on an investigative basis in Russia, a proposition for the teaching of mathematics based on the cultural-historical theory and published it in textbooks, teacher guidance manuals, among others. This material constitutes the data source of the investigation that gave rise to this article. These are the tasks corresponding to the exercises of Brazilian textbooks for teaching multiplication. These tasks were extracted from the original version of the textbook (in the Russian language) of the third year of elementary school (Давыдов et al., 2009). We present the resolution of the tasks in accordance with the teacher guidance manual, written by Gorbov and Mikulina (Горбов; Микулина, 2003), collaborators of Davýdov, in a separate book in the form of an experience report.

The reflections on the experience carried out in the classroom by Davýdov's group of collaborators enabled a broad process of elaboration and re-elaboration of the proposal. This movement is reflected in teacher guidance manuals. Without these reflections, it is not possible to conceive the tasks presented in the textbook as an expression of the assumptions arising from the cultural-historical theory since, apparently, the Davydovian tasks are similar to the exercises presented in Brazilian textbooks (Rosa, 2012).

In the first contact with the Davydovian didactic material, we detected some tasks that indicated the existence of systematization of the commutative and distributive properties of multiplication in the third year of elementary school.

The Davydovian proposition is not fragmented into concepts, such as addition first, then subtraction, multiplication, multiplication table, and division (Davýdov, 1988). On the contrary, concepts are presented in a system of internal connections, such as multiplicity/divisibility relations, whole/parts. In the connection system, one concept contributes to the development of the other. We take as an example the whole/parts relationship, one of the internal connections of the concepts of addition, subtraction, multiplication, and division. If the whole under analysis consists of parts, and each part with the same measure, this internal connection is related to the concept of multiplication, interconnected with its inverse operation, division. In addition, this connection is present in other concepts such as multiplication tables, equations, arithmetic progression, and function, among others, thus forming an integral system based on the essential relationship: the relationship between quantities.

During the investigation, we adopted the following methodological procedures: initially, we identified the methodological guidance presented in the teacher guidance manual, corresponding to the tasks presented in the textbook. Subsequently, we proceeded to the concomitant study of the tasks with the guidelines, the basis for identifying that Davýdov and collaborators develop the commutative and distributive properties from a conceptual system (concept of number, multiplicity relationship, divisibility, part-whole relationship, addition, subtraction, among others). However, for this article, we selected only three tasks related to mathematical properties and reproduced them with the resolution proposed in the teacher guidance manual (Hobold, 2014). The analysis made it possible to reveal that the teaching of multiplication proposed by Davýdov and team, through the essential relationship between the commutative and distributive properties of multiplication, is developed based on the relationship between discrete and continuous quantities in the interrelation of arithmetic, algebraic, and geometric meanings that make up the system.

The Davydovian Tasks

Davýdov (1982) advocates the possibility of students appropriating theoretical knowledge. To this end, it is necessary to organize the teaching-learning process in accordance with activities that boost student development. Davýdov (1982) and other researchers, such as Elkonin, identified that the child, from birth, goes through stages of development marked by main activities, that is, activities that boost the development and appropriation of human objectivations. Thus, around six to ten years of age, the child's main activity is study. During this period, students have the capacity for reflection, analysis, planning, needs and reasons for study (Davýdov, 1988). The "study activity should be understood as an activity for the subject's self-transformation" (Repkin, 2014, p. 88).

In this context, Davýdov and collaborators structured a teaching proposition following the study activity, consisting of particular tasks. The methodological guidelines in this proposition are that the students develop tasks under the teacher's guidance through six study actions:

> 1) Transforming the data of the study task to reveal the universal relationship of the study object;
> 2) Modeling of the universal relationship in object, graphic and literal forms;
> 3) Transforming the universal relationship model for the study of its properties in 'pure form';
> 4) Constructing a system of particular tasks that a general procedure can solve;
> 5) Controlling the performance of the previous actions;
> 6) Evaluating the appropriation of the general procedure as a result of the solution of the given study task (Davýdov, 1988, p. 181).

The starting point for the study of concepts, in the Davydovian proposition, is the relations between magnitudes. Quantities are the properties of objects and phenomena of reality that allow determining their size: larger, smaller, equal; in short, their measure. Each concept has a different relationship between the magnitudes that give rise to it. Thus, at the beginning of the teaching and learning process of each concept, Davýdov proposes that the elements that make up such a relationship be revealed and how they interconnect (1st study action). Subsequently, this essential relationship is abstracted from the sensorially given material and modeled in object, graphic and literal form (2nd study action). In this process, the 3rd study action is transforming the model to study the concept in its general form and, finally, the 4th study action is the one in which several particular tasks are proposed to be developed by the general procedure, in other words, reproducing the concrete. Controlling and evaluating (5th and 6th study actions) are developed throughout the teaching and learning process.

As mentioned in the previous paragraph, the starting point for developing mathematical concepts is the measurement of quantities, which can be discrete or continuous (length, width, volume, mass, among others). However, when the unit of measurement is minimal compared to the object to be measured, it is necessary to create a larger unit of measurement, called intermediate measurement, composed of basic units, which makes it more effective to control the variation of quantities.

This movement also occurs in the study of commutative and distributive properties regarding multiplication in the Davydovian proposition. The intermediate unit of measure is not given directly to students; they need to analyze the object to determine it, as we will present in task 1 below.

Task 1 consists of *three* distinct steps (*a*, *b* and *c*). In the first part (item *a*), the generic measures of the areas (*A* and *B*) are presented in Figure 1: Choose an intermediate unit of measurement and complete the scheme (Давыдов et al., 2009).

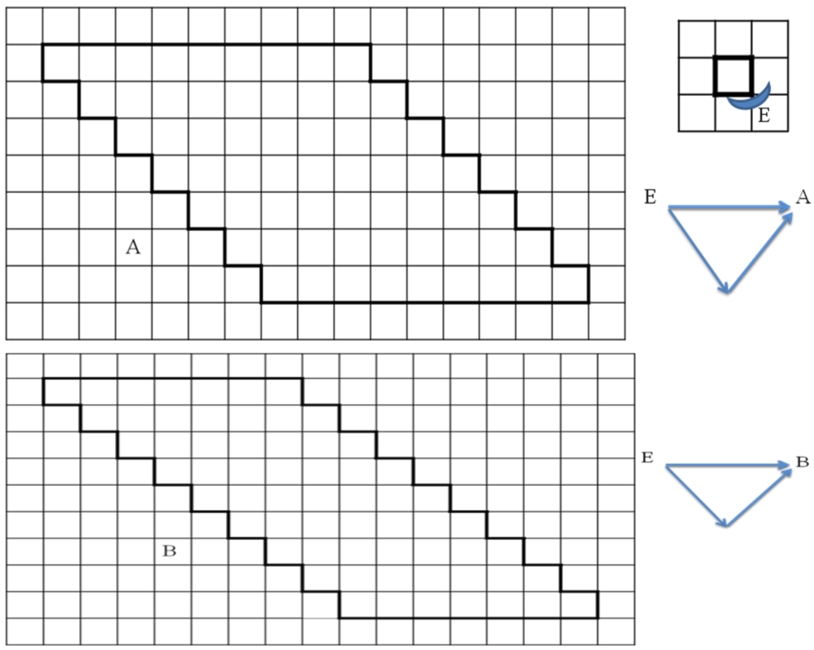

Figure 1.

Task 1) Commutative property of multiplication. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

The task consists of measuring the surface areas and recording the measurements in the scheme (Figure 2). This suggests the construction of an intermediate unit of measurement since the basic unit of measurement (*E*), formed by a square of the mesh, is *small* compared to the total area to be measured. Children will conclude, with teacher guidance, that the line (units arranged horizontally) can be considered as an intermediate measure in both figures (Горбов & Микулина, 2003).

In the first figure (measure *A*), children will represent the intermediate unit of measurement (*K*), composed of *nine* (9) basic units, which repeat *seven* (7) times (Figure 1). In the second figure (measure *B*), the intermediate measurement (*L*) is composed of *seven* (7) basic units of measurement, taken *nine* (9) times. Students should record the numbers in the schematics corresponding to each area. The unknown values are indicated by a question mark in the arrow scheme (Горбов & Микулина, 2003).

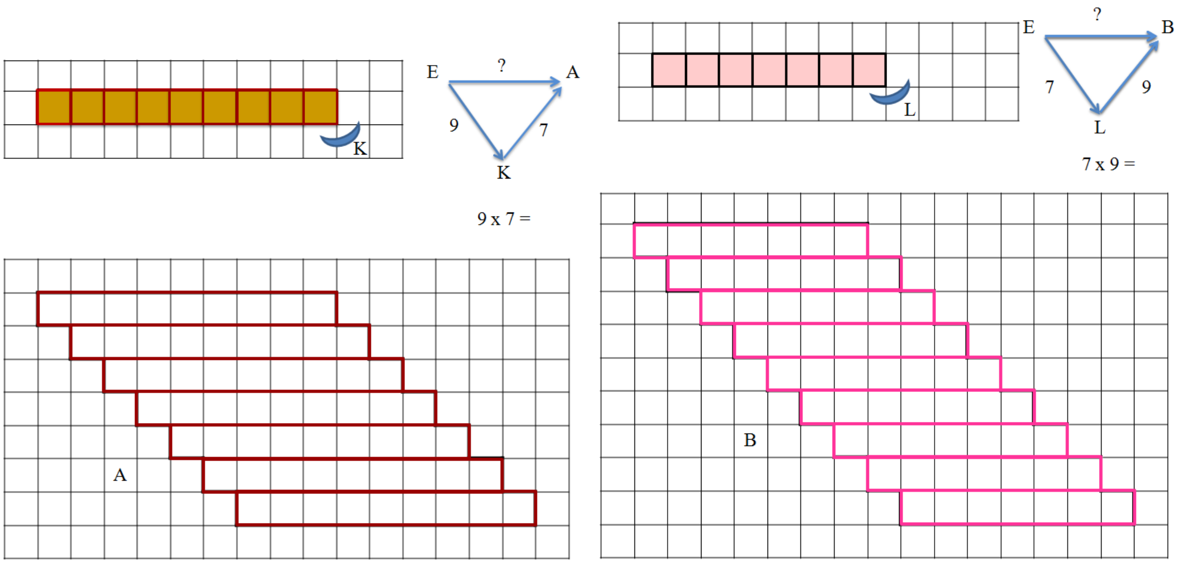

Figure 2.

Task 1 a) Construction of the intermediate unit of measurement. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

According to the registration proposed in the schemes, it is still necessary to determine the number of basic units that make up each figure. This unknown value was represented in the schema with a question mark. The proper operation to determine the total of basic units, when you have the value of the intermediate unit and how many times it is repeated, is multiplication (9 x 7=__ and 7 x 9=__). The results are recorded in the schematics (Figure 3).

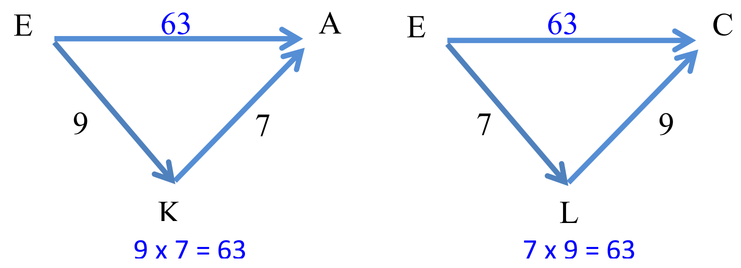

Figura 3.

Tarefa 1 a) Registro dos valores nos esquemas. Elaboração das autoras com base na proposição davydoviana (Давыдов et al., 2009)

Task 1 b) Calculate the area of the figure shown in Figure 3 with different intermediate units of measurement and represent the result of the measurement process in the schemes (Давыдов et al., 2009).

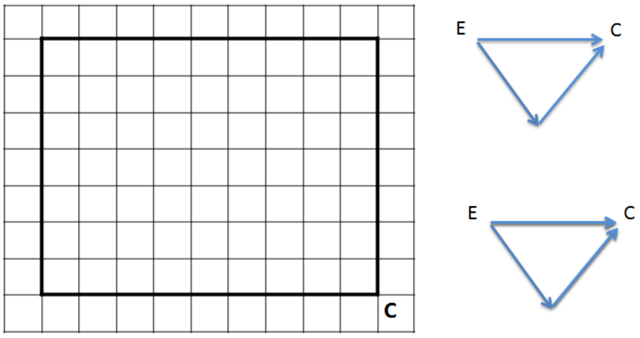

Figure 4.

Task 1) Commutative property of multiplication. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

The task consists of determining the surface area in two different ways, that is, of two distinct intermediate units of measurement: row and column (Figure 4). The teacher suggests dividing the class into two groups to perform the task. One group uses the line as an intermediate unit of measurement, and the other, the column (Горбов & Микулина, 2003).

Children who adopt the line as an intermediate unit of measurement make the proper representation in the mesh (*L*), which is composed of *nine* (9) basic units and is repeated *seven* (7) times in the total area. The numbers are recorded in the scheme, followed by the operation (9 x 7 =__), which allows the calculation of the total area, according to Figure 5 (Горбов & Микулина, 2003).

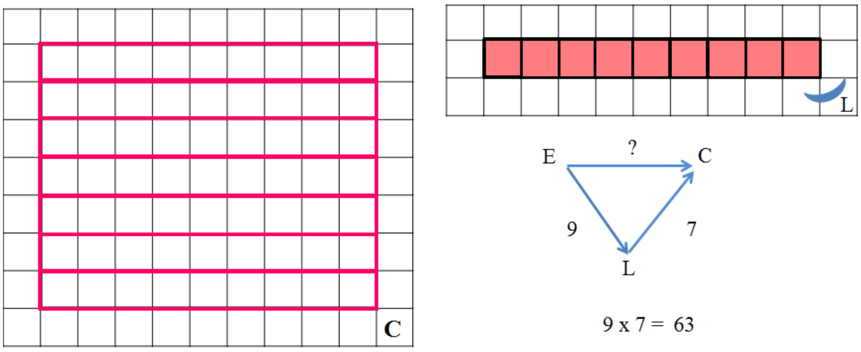

Figure 5.

Task 1 b) Construction of the intermediate unit of measurement – line. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

Children who establish the spine as an intermediate unit of measurement proceed to the measurement similarly to the one we have just shown. However, the column is composed of *seven* (7) basic units of measurement and is repeated *nine* (9) times in the total area (Figure 6). The numbers are recorded in the scheme, followed by the operation that allows the calculation of the area in question (Горбов & Микулина, 2003).

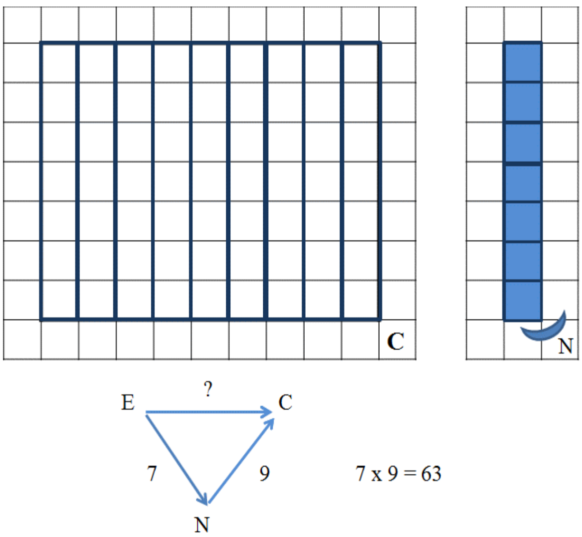

Figure 6.

Task 1 b) Construction of the intermediate unit of measurement – line. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

The teacher proposes that the groups present their records and the operation (Figures 5 and 6). He remembers that they both measured the same surface, so the basic units are the same, a square of the mesh. However, the intermediate measures were composed differently. The teacher guides the children to verify that the result is the same (Figures 5 and 6) and ends with the analysis of the composition of the records, both of the first part of the task and of the second (9 x 7 = 63 and 7 x 9 = 63) (Горбов & Микулина, 2003).

Task 1 c) Use the calculator to determine the total area of the figures presented in the previous items (areas with generic measures *A*, *B* and *C*) and compare the results (Давыдов et al., 2009).

The reflection that will support the resolution of the task is as follows: The three surfaces, *A*, *B* and *C*, were built with the same basic unit (a grid square). However, the intermediate units of measurement were composed differently. On the surface with measure *A*, the intermediate unit was composed of *nine* (9) squares and repeated *seven* (7) times: 9 x 7 = 63. In *B*, the inverse, *seven* (7) taken *nine* (9) times, i.e., 7 x 9 = 63.

On the surface with measure *C*, its area was calculated in two different ways. First, the line was considered an intermediate measurement unit consisting of *nine* basic units. The figure consists of seven lines; then, the operation consists of multiplying 9 x 7 = 63. In the second, the column was taken as an intermediate unit of measurement, with *seven* basic units, which translates into 7 x 9 = 63, as there were *nine* columns. Thus, the three surfaces have the same area: A = B = C = 63 (Горбов & Микулина, 2003).

Are products composed of the same factors but placed in a different order always the same? Yes. The teacher suggests statement analysis by solving new tasks that are presented in the textbook (Горбов & Микулина, 2003).

The resolution of the tasks suggested above leads to the generalization of the commutative property of multiplication and can be expressed in algebraic form: given any two numbers *m* and *p*, taken in a different order, the products will be the same, which is synthesized by the equality $m \times p = p \times m$: the displacement of the factors does not change the product (Горбов & Микулина, 2003); however, it changes the meaning for the multiplicand and the multiplier.

Algebraic resources pave the way for the expression of any arithmetic operation without it being necessary to operate with the quantities. In the task under analysis, the model ($m \times p = p \times m$) that represents the commutative property of multiplication stems from the generalization of the relationship between the quantities (areas, in its specificity).

However, reflection with the use of symbols provides work "with the relationship between quantities, without it being associated with numerical, geometric, or any other kind of entities, which stands out as essential to algebraic knowledge, as well as the generalization of these relationships between quantities" (Panossian, 2014, p. 104). Symbolic representation synthesizes and enables the work with the relationship between quantities, in general, in particular tasks, as presented below (2).

Task 2: Calculate the products 7 x 2, 5 x 2, 9 x 2 and 6 x 2 from the commutative property of multiplication (Давыдов et al., 2009).

According to Hobold (2014), at this stage of development of the Davydovian proposal, children have not yet studied the multiplication tables of the numbers *five*, *six*, *seven* and *nine*. However, through the commutative property of multiplication, it is possible to determine the products based on the knowledge referring to the multiplication table of the number *two*:

7 x 2 = 2 x 7 = 14

5 x 2 = 2 x 5 = 10

9 x 2 = 2 x 9 = 18

6 x 2 = 2 x 6 = 12

The task consists of mentally relating the multiplication table of the number *two* to other tables through the commutative property. It becomes another procedure for memorizing the multiplication table, which only occurs after studying its essence. On the mental plane, this relationship does not arise out of nothing, but is preceded by a movement involving several real relationships between magnitudes. As stated by Núñez and Oliveira (2013, p. 295), the formation of mental actions and concepts is carried out "with the support of external objects and, as they are manipulated, passing through a series of stages, they are later carried out on the mental plane and become the property of the psyche".

Davýdov proposes developing mental actions through mathematical properties in the context of the object of study. In the previous tasks, the starting point was the analysis of the relationships between continuous quantities, specifically between areas. In the following task, reflections occur based on discrete magnitude.

Task 3 consists of three steps (*a*, *b*, and *c*). In the first item (*a*), the four-pointed stars are arranged in 38 columns. Each column consists of three stars (Figure 7). Make up the operation to calculate the total number of stars (Давыдов et al., 2009).

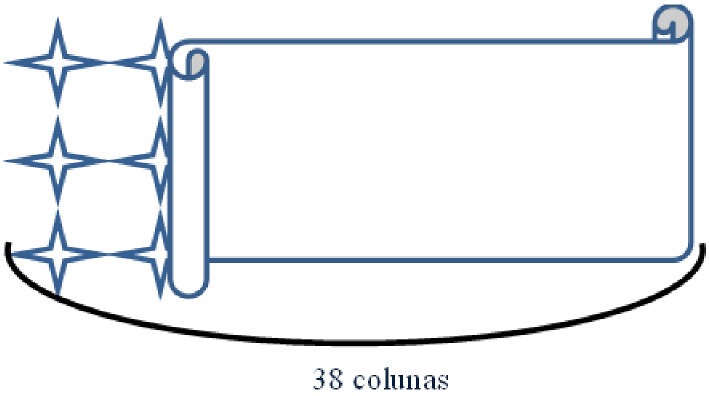

Figure 7.

Task 3 a) Four-pointed stars. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

By way of example, we take the column as an intermediate unit of measurement. Thus, *three* (3) are taken *thirty-eight times* (38). The teacher records, on the board, the scheme and the corresponding operation to determine the total of basic units (3 x 38 =__), as shown in Figure 8 (Горбов & Микулина, 2003).

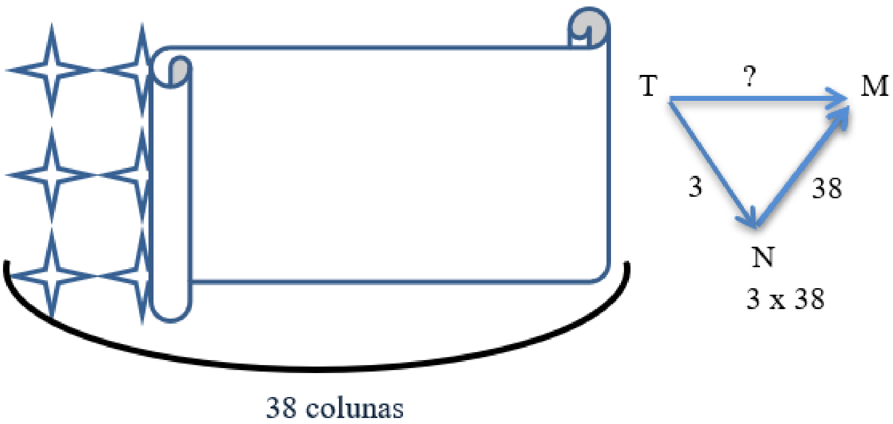

Figure 8.

Task 3 a) Schema and operation record. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

Task 3 b) Five-pointed stars are distributed in 56 columns of three stars each (Figure 9). Compose the operation to calculate the total number of stars (Давыдов et al., 2009).

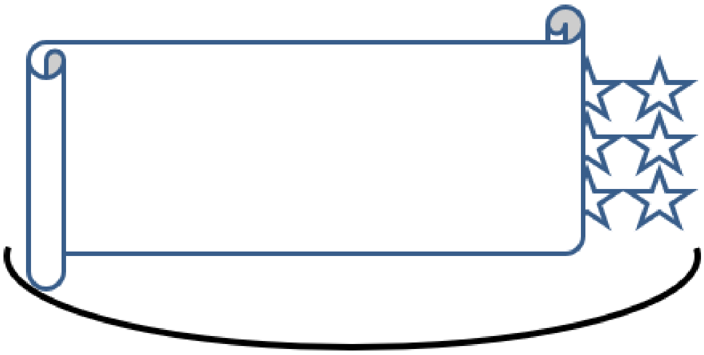

Figure 9.

Task 3b) Five-pointed stars. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

To determine the total value of *five-pointed* stars (Figure 9), the teacher suggests that the column be considered an intermediate measurement unit. The conclusion is that *three* (3) are taken *fifty-six* (56) times. The teacher and children construct the scheme on the board and notebook, respectively, and record the operation: 3 x 56 =__, as shown in figure 10 (Горбов & Микулина, 2003).

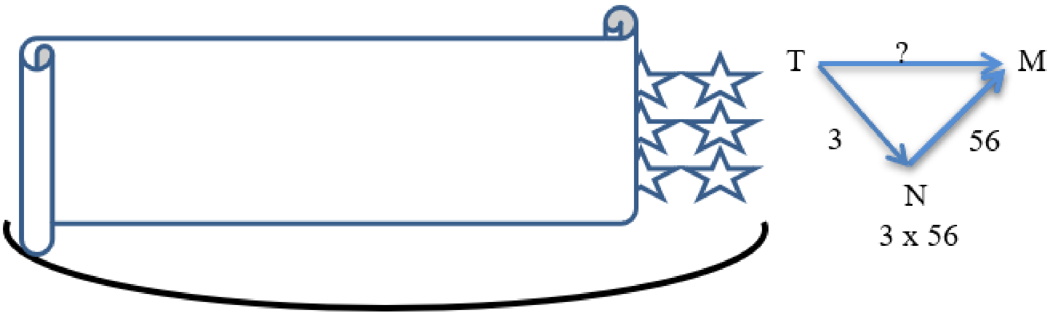

Figure 10.

Task 3 b) Construction of the second scheme. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

Task 3 c) Determine how many of four- and five-pointed stars (Figure 11) are hidden behind the screen. Represent through arrow schemes (ДАВДОВ et al., 2009).

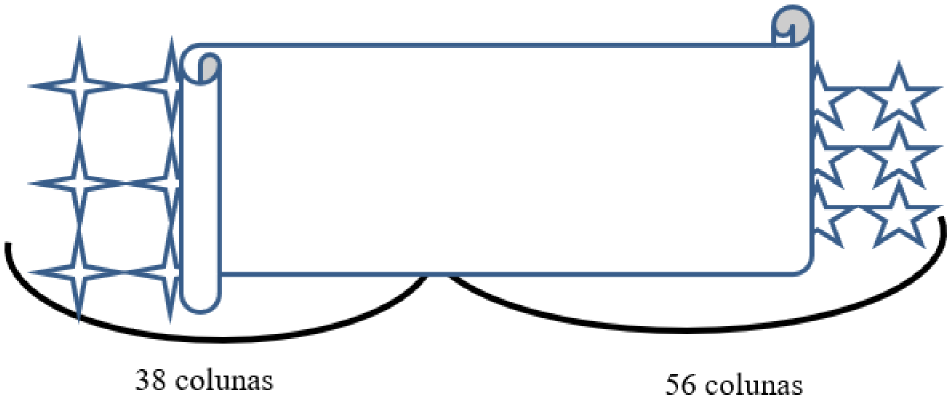

Figure 11.

Task 3 c) Determination of total four- and five-pointed stars. Prepared by the authors based on the Davydovian proposition (Давыдов et al., 2009, p. 11-12)

This part of the task asks students to determine the total number of units hidden behind the screen, consisting of four- and five-pointed stars (Figure 11). From the records obtained in items a and *b* (Figures 8 and 10), the children will conclude, with the teacher's guidance, that they obtain the solution by adding the two parts (Горбов; Микулина, 2003):

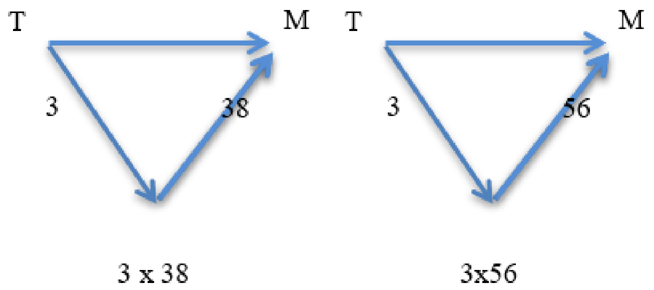

Figura 12.

Tarefa 3 c) Determinação do total de estrelas de quatro e cinco pontas. Elaboração das autoras com base na proposição davydoviana (ДАВЫДОВ et al., 2009)

The solution, therefore, is obtained by summing the two multiplicative expressions: 3 x 38 + 3 x 56. Is it possible to think of a new scheme that represents the actions that make it possible to determine the amount of four- and five-pointed stars? The reflections are oriented towards the elaboration of the scheme presented in Figure 13 (Горбов & Микулина, 2003).

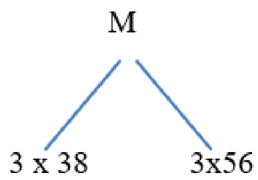

Figura 13.

Tarefa 3 c) Esquema que representa a soma das partes. Elaboração das autoras com base na proposição davydoviana (Давыдов et al., 2009)

The teacher asks: Does this scheme (Figure 13) offer the conditions to determine the amount of four- and five-pointed stars? Reflections must be oriented to conclude that it is possible to determine the integer when the value of the parts is known. In this case, the parts are (3 x 38) + (3 x 56). It is the scheme that represents the sum of the parts[3]. In other words, the scheme refers to the whole divided into parts. In each part, there are the same number of rows. Thus, to determine the whole, the parts are added.

Task 3 d) Determine another way to calculate the four- and five-pointed star total. Represent your actions through an arrow scheme (Давыдовет al., 2009).

The teacher resumes the analysis of the last star screen (Figure 10): Is it possible to calculate the number of four- and five-pointed stars using another method? He directs the analysis so that the children can see that it is possible to determine the value of the whole through another procedure. But, for this, it is necessary to determine the number of columns (38 + 56). There is in common, in the sentences (3 x 38 and 3 x 56), the factor *three* (3) - the intermediate unit of measurement is the same in both. The teacher builds the arrow scheme on the board (Figure 14) and records the numbers that represent the actions of items *a* and *b* (Горбов & Микулина, 2003).

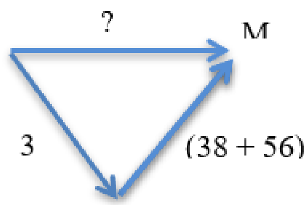

Figura 14.

Tarefa 3 d): Esquema com setas para o cálculo do total de estrelas. Elaboração das autoras com base na proposição davydoviana (Давыдов et al., 2009)

3 This scheme, in the Davydovian proposition, is used in the addition operation and indicates the number of parts that constitute the whole. Gradually, this scheme will be associated with the multiplication operation.

After recording the numbers in the scheme, the teacher reminds students of the two procedures for calculating the total of the two types of four- and five-pointed stars (Figure 15).

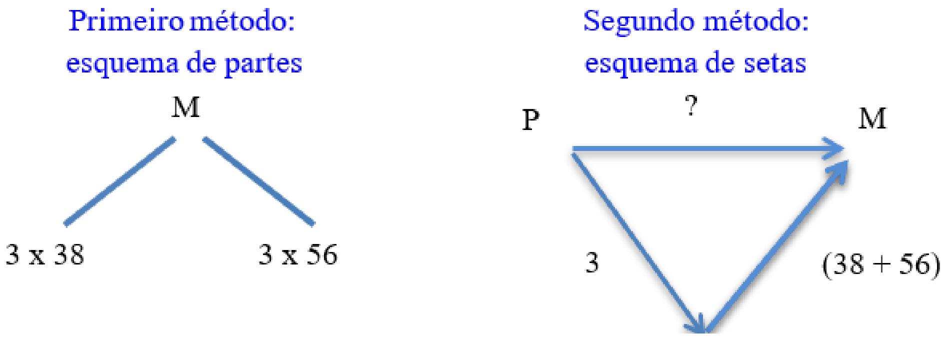

Figura 15.

Tarefa 3 d) Métodos de cálculo. Elaboração das autoras com base na proposição davydoviana (Давыдов et al., 2009)

Then, the teacher asks: What result will we obtain when doing the calculation by the first method? And for the second? Which result is greater? (Горбов & Микулина, 2003).

In the resolution process by the first method (3 x 38) + (3 x 56), the children will initially determine the products within the parentheses and then proceed to the sum (114 + 168), as shown in Figure 16:

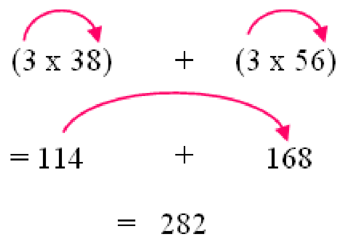

Figura 16.

Tarefa 3 d) Cálculo do total de estrelas por meio do primeiro método. Elaboração das autoras com base na proposição davydoviana (Давыдов et al., 2009)

By the second procedure, the children will first determine the sum within the parentheses (38 + 56) and then the product (3 x 94), as shown in Figure 17:

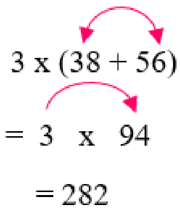

Figura 17.

Tarefa 3 d) Cálculo do total de estrelas por meio do segundo método. Elaboração das autoras com base na proposição davydoviana (Давыдов et al., 2009)

The teacher and the children compare the results (Figures 16 and 17) and conclude that the value obtained by the two procedures is the same, as it is the same amount, but they differ in the resolution method, as shown in Figure 18 (Горбов & Микулина, 2003):

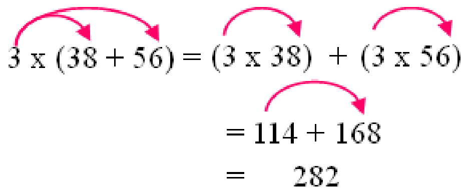

Figura 18.

Tarefa 3 d) Relação de equivalência entre os dois métodos de resolução. Elaboração das autoras com base na proposição davydoviana (Давыдов et al., 2009)

Thus, to multiply a number by the sum, one can multiply each term separately and add the result to the end. This conclusion supports the elaboration of the following rule: *a* x (*b* + *c*) = *a* x *b* + *a* x *c* (Горбов & Микулина, 2003). This consists of the distributive property of multiplication in relation to the sum (Caraça, 1959). This property is related to the initial stage of the development of the concept of multiplication as the addition of equal portions. That is, when the whole is composed of several equal parts.

It is worth noting that the scheme of the parts (Figure 15) is related to the concept of addition (sum of the parts) and can be transformed into another scheme, that of arrows, which is linked to multiplication. This movement of transformation of models contributes to the formation of theoretical thinking, as it reinforces the understanding of the essential relationship of addition and multiplication. What is more, this movement culminates in the mathematical property.

Properties, in mathematics, are a form of generalization of the arithmetic concept. This "[...] is considered as a particular case of a more general [algebraic] concept" (Vygotsky, 2000,

p. 372). Operating from the more general formula, such as the distributive property of multiplication, means becoming independent of a given arithmetic expression (Figure 18). Algebra allows generalization and has, as a product, the awareness and apprehension of arithmetic operations (Vygotsky, 2000).

Davýdov and collaborators, in their teaching proposition, contemplate a series of mathematical properties that consist of the conceptual roots of the multiplication table and that, in Brazil, are presented only at the end of elementary school (Hobold & Rosa, 2017). In our country, students' access to algebra usually begins around the seventh grade, with the introduction of equations and letters. In this sense, algebra is conceived as letter manipulation. For Panossian (2012, p. 14), this conception extends throughout the other years of schooling until high school and "is a source of difficulties for students, who do not understand the meaning attributed to the symbol and even to algebraic knowledge as a whole".

For the author in question, some difficulties related to teaching and learning mathematics are due to how teaching is organized. In the specificity of algebra teaching, for example, with the symbolic form in the most formalized stage, the movement that arises from the relations between quantities is not considered. Thus, "teachers can highlight their characteristics, make classifications, develop techniques and treat them only empirically, by their appearance, without reaching the essence of this form of knowledge" (Panossian, 2014, p. 263, our translation). In the Davydovian proposition, in turn, algebra arises from the relationship between magnitudes and advances to abstraction and generalization.

The generalization process is fundamental for appropriating systematized knowledge and developing thinking in the school environment. It is worth reflecting on what type of generalization the organization of teaching and pedagogical activity are anchored, for the modes of actions developed during the resolution of the proposed tasks can determine the type of thinking to be developed, which for Davýdov can be theoretical or empirical.

Final considerations

In the Davydovian proposition, the commutative and distributive properties of multiplication are developed through object actions, focusing on the relationships between discrete and continuous quantities. The starting point is the general, essential relationship that engenders the given conceptual system and enters the internal connections not given directly to the sense organs. Such connections originate from the relationship between the units of intermediate measurement, the number of times it fits in the quantity in measurement and the total of basic units.

During the development of tasks, the essential relationship of multiplication is modeled by different semiotic means, such as object, schema, graphic and literal. Object modeling is performed by means of magnitudes. Discrete (counting) and continuous (length, width, volume, among others) quantities are used. Another modeling consists of the arrow scheme, which indicates, in the abstract, the movement taken to prepare the intermediate unit of measurement, the total number of times it is repeated in the whole and the total number of basic units. Finally, in the literal form, by means of letters that represent the same relationship between magnitudes revealed in the object plane. This process culminates in algebraic abstraction and generalization, in other words, in substantial generalization. There is no fragmentation between arithmetic, algebra and geometry. These are part of an interconnected conceptual movement with support in the relationship between quantities.

In this mathematical context, based on the properties of multiplication, the multiplication table is systematized theoretically. The modes of actions, both of the teacher and the student, are fundamental for appropriating historically produced knowledge and developing the subjects' thinking.

We envision in the Davydovian proposition, as well as Rosa, Garcia, and Lunardi (2021), possibilities to rethink the problems related to the teaching and learning of mathematics in Brazil, as it allows reflections on the theoretical level of mathematical concepts from the first years of schooling.

Thus, for future research, some questions emerge, such as: What modes of action in the pedagogical activity, understood as the teacher's teaching activity and the student's learning activity, can strengthen the development of theoretical thinking in the school environment? What are the challenges and possibilities to develop the Davydovian proposition in the Brazilian context? How to organize teacher education to enhance the development of students' theoretical thinking? These and other questions are part of our future research intentions.

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Tradução: Maria Isabel de Castro Lima