

Editorial

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This special issue of the *Educação Matemática Pesquisa* Journal focuses on the philosophy of mathematics education. It includes twenty-four articles with different topics, assuming diverse ways of thinking, which involve a broad scope and a plurality of conceptions and questions suitable to the discussion among researchers and teachers present and active in the community of mathematics educators.

At core of the way of being of the philosophy of mathematics education is the search to transcend the explanations provided by different theories that deal with education, learning, teaching, philosophy, and mathematics, just to mention the disciplines that, more strongly, are called to answer the questions raised, and to be faced in this sphere of action. It also seeks to go beyond the practices carried out by those who teach, investigate, and form educators and researchers, always asking what they say about the world, society, man, and education. Its driving force lies not only on understanding, but also on indicating paths and possibilities so that philosophical thinking arises among students, teachers and researchers who are learning, teaching, and researching. It points to the current pressing urgency of establishing collaborative

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environments in which analyses, criticism, and reflection can be produced, considering self-understanding and self-care and, in an intertwined and concurrent way, the understanding and care of the other and the world, which is understood in its fluid and dynamic historicity. That requires strength and courage. It is necessary to tear down the walls of generalized massification, disseminating a *pseudo* acceptance of what is ‘different’ based on a proclaimed equality, as the features are dissipated and erased so that there is no longer a need to accept what is different, as there is no one who sees and assumes themselves as singular. It takes courage to proclaim what is equal and what is different in each of us, considered from a micro perspective of the world, and from equality and difference considered from the macro perspective, that is, of cultures with their values, beliefs, and social practices.

The authors of the twenty-four articles committed themselves to contemplating reality and pointing out possibilities. Thinking is an activity performed in the corporality of the living-body that each of us is, nourished by worldly reality, a soil of existence in which the other, different from us, but also equal, is always thinking, saying, and doing, expressing thoughts. Therefore, a thinking that is not conceived as an activity that can be performed in an abstract way, which hovers above individuals, separate, and insensitive to the clashes of everyday life reality.

The philosophy of mathematics education performs this thinking full of certainty, shaken by doubt, making room for uncertainties, advancing through analyses, criticism, and reflection. Investigations made within this context often cause dissatisfaction to professionals and ordinary people, because they do not provide answers to support definitions or determine how to *do it*. It is easier to work and live with the certainties that can be given by theories, political and religious ideologies, or massive repetitions uttered by the media. It takes strength to operate with certainties that are necessary for specific times and activities, and with uncertainties, when questions are posed about the value of the same operations.

The attitude of researchers and teachers who work within the realm of philosophy of mathematics education is characterized by assuming and updating thinking. Philosophy is always *'thinking about'*..., which means that thinking is nourished by the reality of what is focused by the attentive eye of the researcher. As stated above in this *Editorial*, the authors of the twenty-four papers were nourished by different realities which they have read and interpreted from different philosophical perspectives. Focused realities: training mathematics educators, the teaching practice; understanding mathematics teachers regarding mathematics; ways of understanding epistemology, articulating it with ontology and ethics; the movement of knowledge taking place while learning of geometry; algebraic thought; debates present in classical philosophy and their articulation with mathematics education; concepts present in recent history; works by renowned authors and, based on their ideas, point out possibilities for mathematics education to be effected; language, representation and games; the movement of research; interdisciplinary articulations and possibilities that can be foreseen in the pedagogical horizon; the beauty of mathematical demonstrations and the enjoyment of that beauty.

Below there is a brief presentation of the aspects dealt with in the articles that are part of this edition of the *Educação Matemática Pesquisa* Journal.

The first article, “Mathematics education in the context of certain classical debates in philosophy and mathematics” by Michael Otte and Mircea Radu, highlights the links between philosophy, logic, mathematics and mathematics education since the 16th century, focusing on the reflections of classical philosophers (Aristotle, Leibniz, Spinoza, Euclid, and Descartes) and dialoguing with contemporary thinkers in various fields of knowledge.

In the second text, reflecting on the “Epistemology of research in mathematics education”, Maria Aparecida Viggiani Bicudo wonders if it makes sense to affirm that research epistemology exists, indicating that this term is understood in daily life without reflection about

what it means. Throughout the text, the author explains meanings and analyzes the epistemology of research in mathematics education.

In the third article, “The contributions of the model of semantic fields to early years and primary school teachers’ education”, João Pedro Antunes de Paulo and Rejane Siqueira Julio analyze the strangeness present in interactions with undergraduate students of a degree in pedagogy course. Peano axioms were the object of discussion in the course and the authors' analysis shows the understanding of mathematics of that mathematician, which allowed an expansion of the mathematical understanding of future pedagogues.

“Relations between mathematics and philosophy in the emergence of pure mathematics: mathematics as the foundation of thinkability”, by Vinicius Linder and Gert Schubring, focuses on the epistemological turn that occurred during the 19th century in Prussia. For this, the authors use studies by Jakob Fries and Hermann Grassmann, mathematicians whose works would influence new conceptions, and are now of greatly regarded in mathematics and physics.

Paulo Vilhena da Silva, Valdomiro Pinheiro Teixeira Júnior, Daniana de Costa and Alyne Maria Rosa de Araújo Dias present a way of seeing philosophy of mathematics education under a Wittgensteinian perspective. In the article “A philosophy of mathematics education from the perspective of Wittgenstein”, the authors show the therapeutic character of Wittgenstein's philosophy of language and the nature of mathematical knowledge in that perspective, pointing out the epistemology of its use and its consequences for the pedagogy.

The article “Almost the same thing: thinking about a topology of translation and/in mathematics education research from a Wittgensteinian perspective”, aims to discuss aspects of the act of translating, when contemplated in line with the particularities of the field of mathematics education under a philosophical perspective. The authors, Rafael Montoito and

Andreia Dalcin, analyze two excerpts of the works by Lewis Carroll, original and translated, based on Wittgenstein's theories about language-games and family similarities.

The analysis of an activity performed with students of an undergraduate course in mathematics, following the therapeutic attitude according to Wittgenstein, is exposed in the article “Labyrinths of language: language games as a means of action in mathematical modelling activities”. For Lourdes Maria Werle de Almeida and Emerson Tortola, modeling activities enable language games that function as a way to organize our experiences with the world through mathematics.

Carolina Cordeiro Batista and Rosa Monteiro Paulo present discussions on situations experienced during a class study conducted with a group of mathematics teachers within a phenomenological posture. In the text “I didn’t see this in class!”: realizing oneself in the lesson study discussion”, the authors explain understandings about how *turning to* teaching practice using technology favored *awareness of how to be* a teacher.

In the article “Prepredicative knowledge: understandings about the idea of angle”, Marli Regina dos Santos uses part of a research conducted with first-year students in a undergraduate course in mathematics in which she promoted a discussion regarding the idea of angle. The author focuses on the close relationship between the spatiality experienced by the individuals in daily life and geometry as a deductive science. She demonstrates the pre-predicative knowledge, indicated by Merleau-Ponty, as a way of understanding geometric doing.

“The philosophical problem of representation and its ramifications for mathematical modeling in mathematics education”, by Tiago Emanuel Klüber, Carla Melli Tambarussi and Gabriele de Sousa Lins Mutti, approaches the philosophical problem of representation, both from the point of view of the classical theory of knowledge and phenomenology. The authors point out differences between such perspectives, perform a hermeneutics of the concepts

involved and explain understandings of work with modeling in mathematics education which focused on the theme of reality and correlations.

The study by Juliano Cavalcante Bortolete, Vanessa de Oliveira and Manoel Francisco Guaranha points out, in BNCC (National Common Core Curriculum), a technical, pragmatic, and computational view for teaching and learning practices of algebra, with a view more centered on language than on the meanings of the thought this language explains. In the paper entitled “Algebraic thinking within BNCC: reflections and alternatives”, the authors reflect on the act of thinking subsidized by readings by Husserl and, with Freudenthal, discuss the importance of understanding algebra beyond its operative rules.

The key objective of the article “Philosophical concepts of creative insubordination in ethnomodeling research” is to discuss the philosophy of ethnomodeling as creative insubordination within mathematics education and as a globalization process. The authors, Milton Rosa and Daniel Clark Orey, explain that ethnomodeling is not an attempt to replace globalized school/academic mathematics. They emphasize the need for the school curriculum to recognize the existence of local mathematical knowledge and propose a dialogue between local and global approaches.

Reflections made by philosophers and historians of science on mythology, astronomy and pre-Socratic principles are exposed in Tania Baier's article “Do we inhabit flying arks linked with the Earth-ark?” The text invites the reader to contemplate the fragmentation and hierarchy of values present in scientific knowledge and in the teaching of curriculum contents. It focuses on the phenomenological sense of the universe, as explained by Husserl, through the metaphor of flying arks: the home-places along with the Earth-ark.

In the article “Aesthetic experiences in the education of science and mathematics teachers: influences of Gadamerian hermeneutics”, Ana Paula Carvalho do Carmo, Robson

Simplicio de Sousa and Maria do Carmo Galiuzzi show the results of a project involving the study, creation, and planning of aesthetic experiences in science and mathematics. The authors explain the concept of aesthetic experiences described in the text based on Gadamer's philosophical hermeneutics.

An analysis of the restrictions of didactic transpositions proposed in Spain in the 1960s, resulting from the changes that modern mathematics provoked, is presented in the text “Didactic transposition in the teaching of geometry in Spain during the 1960s”. The investigation by Julián Roa González and Mercedes Hidalgo-Herrero was based and developed through the analysis of school texts and legal provisions seeking evidence of epistemological change in the period under analysis.

The article “Articulations between ethnomathematics & history of mathematics: conditions of possibility from pedagogical actions” exposes Foucault’s concepts of power, knowledge and counter conduct, and Wittgenstein’s language games and life forms. In this article, Juliana Batista Pereira dos Santos and Isabel Cristina Machado de Lara propose reflections about the contributions of those philosophers who articulated ethnomathematics and history of mathematics. In this articulation, the authors highlight possible implications for teaching of mathematics in elementary education.

Christian Lovis and Rita de Cássia Pistóia Mariania present the research “Relationships with mathematics: understandings of researchers in the field of mathematics education”. This article describes elements that characterize relationships with mathematics pointed out by researchers of *stricto sensu* productions. Through a qualitative approach guided by principles of content analysis, the authors systematized the data into categories: philosophical currents; scientific and school knowledge; interdisciplinary aspects; sociocultural aspects, and social practices.

Phenomenology as a possibility of research in philosophy of mathematics education is the focus of the reflections made by Paulo Wichnoski in the article “*Researching-with phenomenology in the philosophy of mathematics education*”. In the text, the author describes some characteristics of phenomenology for research in mathematics education and exposes the procedures executed for a PhD thesis conducted within phenomenological bases to exemplify what is stated in theoretical terms.

The purpose of the work of Saul Rodrigo da Costa Barreto, José Messildo Viana Nunes and Saddo Ag Almouloud, entitled “Research in mathematics education: a historical-philosophical look at teacher education” is to highlight philosophy as a framework for reflection on the contributions of the research field of mathematics education and the didactics of mathematics for the education of elementary school teachers. The text focuses on the understanding of teaching and learning mathematics in the context of teacher training using the didactic-methodological device called study and research path (SRP), constructed from the anthropological theory of the didactic (ATD).

In the article “Philosophies of difference and mathematics education: possible and invented conversations”, reflections are developed about conditions to inquire about the power of mathematical education in the midst of philosophies of difference and what spaces there are to develop education that is not mere reproduction. Virgínia Crivellaro Sanhotene, Gilberto Silva dos Santos and Samuel Edmundo Lopez Bello describe understandings about a mathematical education that becomes possible by dialoguing with Nietzsche, Foucault, and Deleuze within a philosophy that produces questions and displacements.

Rosemeire de Fatima Batistela questions the sense of the beauty mathematics. In the article “On the sense of beauty in mathematics and what seemed beautiful to us in the demonstration of Gödel's incompleteness theorem”. She argues that the sense of mathematical

beauty of a theorem is shown as enlightenment that evidences the result; a light that can be seen and provides familiarity with the theory considered for the demonstration that provides clarity to understand the axioms used, the conciseness of the evidence, its originality, the articulation of ideas, and the generality of the results. Beauty is also evident in the openings that a theory brings to new research in mathematics.

Hans-Georg Steiner systematized theories and methodologies for research in mathematics education and its establishment as a scientific discipline, giving rise to his theory of mathematics education program (TME). Steiner's texts are analyzed by Marluce Alves dos Santos and Saddo Ag Almouloud through a bibliographic review and the explanation of their understandings in the article "Theory of mathematics education (TME) program by Hans-Georg Steiner: philosophical and epistemological aspects".

In the article "The construction of vector mathematical knowledge in the light of the development of Bachelard's scientific spirit and epistemological obstacles", Rita de Cássia Florêncio Rocha Kasahara, Ivanilde Apoluceno and Pedro Franco de Sá present Gaston Bachelard's theory of epistemological obstacles. Starting from Classical Antiquity up to the early 20th century, they analyze theories that gave rise to the idea of vectors, highlighting the epistemological ruptures present in their development. The authors draw attention to the sense of "vigilance" advocated by Bachelard so that new obstacles or obstacles that have already been overcome do not prevent the development of the scientific spirit.

Finishing this dossier, Elisangela Pavenelo thematizes thought under Heidegger's conception, in the article "A work with digital technologies in training courses of mathematics teachers: possibilities for thought". The author analyzes a problem-solving situation conducted by students of the undergraduate course in mathematics regarding the fundamental theorem of

calculus and argues that the experience with *digital technologies* encourages students to elaborate, evaluate and validate hypotheses, giving them possibilities for thinking.

With these twenty-four works, the dossier “Philosophy of Mathematics Education”, published in this *Educação Matemática Pesquisa*, invites the community of educators and researchers to engage in reflection and self-understanding. We hope you enjoy reading it!