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Uses of argumentation in mathematics education: a systematic literature review in higher education

Usos de la argumentación en la educación matemática: una revisión sistemática de la literatura en la educación superior

Les usages de l'argumentation dans l'enseignement des mathématiques : une revue systématique de la littérature dans l'enseignement supérieur

Usos da argumentação na educação matemática: uma revisão sistemática da literatura no ensino superior

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Abstract

This article aims to carry out a systematic review of selected literature on empirical studies into the use of argumentation in the field of Mathematics Education in higher education. To do so, we searched for articles on the Education Resources Information Center (ERIC) database, more specifically in Mathematics Education journals published between 2012 and 2021. This period is justified for encompassing a decade of research published until the year immediately before the beginning of this study. After selecting and reading the articles, the following categories of analysis were identified: argumentation as a tool to analyze students' arguments, argumentation as a tool to analyze professors' arguments and argumentation as a teaching approach. The analysis was based on literature studies, mostly from the theoretical perspective of Toulmin's and Perelman's argumentation theories. The results showed three ways of using argumentation

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in relation to the teaching approach, investigations into the teaching of proof, assessment of the relationship between argumentation and proof and investigation into students' and professors' argumentation quality. However, we highlight that more investigations need to be carried out to assess the potential of courses focused on argumentation for higher education mathematics subjects, especially research into Perelman's approach, about which studies have been reasonably scarce.

Keywords: Argumentation, Systematic review, Mathematics education, Higher education.

Resumen

El artículo tiene como objetivo realizar una revisión sistemática de la literatura que aborda estudios empíricos sobre los usos de la argumentación en la Educación Matemática en la Educación Superior. La búsqueda de artículos se realizó en la base de datos *Education Resources Information Center* (ERIC) y se completó en revistas específicas del área de Educación Matemática entre 2012 y 2021. El período se justifica por ser una década de investigaciones publicadas antes del inicio de este estudio. Después de la selección y lectura de estos artículos, las siguientes categorías de análisis fueron identificadas: la argumentación como instrumento de análisis de los argumentos de los estudiantes; la argumentación como instrumento de análisis de los argumentos de los docentes; la argumentación como método de enseñanza. El análisis se realizó a partir de estudios de la literatura, adoptando, sobre todo, la perspectiva teórica de los argumentos de Toulmin y Perelman. Los resultados apuntan a tres formas de utilizar la argumentación en relación con: enfoque de enseñanza, investigación de la enseñanza de pruebas, evaluación de la relación entre argumentación y evidencia e investigación de la calidad de la argumentación de estudiantes y profesores. Sin embargo, destacamos la necesidad de realizar más investigaciones para evaluar el potencial de los cursos centrados en la argumentación para las asignaturas de matemáticas de la educación superior, principalmente utilizando el enfoque de Perelman, en el que los estudios han sido razonablemente escasos.

Palabras clave: Argumentación, Revisión Sistemática, Educación Matemática, Educación Superior.

Résumé

L'article vise à effectuer une revue systématique de la littérature qui traite des études empiriques sur les usages de l'argumentation dans l'enseignement des mathématiques dans l'enseignement supérieur. La recherche d'articles a été effectuée dans la base de données du Education Resources Information Center (ERIC) et a été complétée dans des revues spécifiques du domaine de l'enseignement des mathématiques entre 2012 et 2021. La période est justifiée par le fait qu'il s'agit d'une décennie de recherches publiées avant le début de cette étude. Après la sélection et la lecture de ces articles, les catégories d'analyse suivantes ont été identifié : l'argumentation comme instrument d'analyse des arguments des élèves ; l'argumentation comme instrument d'analyse des arguments des enseignants ; l'argumentation comme approche pédagogique. L'analyse a été menée à partir d'études de la littérature, en adoptant surtout la perspective théorique des arguments de Toulmin et de Perelman. Les résultats indiquent trois façons d'utiliser l'argumentation en relation avec : l'approche pédagogique, l'investigation de l'enseignement des tests, l'évaluation de la relation entre l'argumentation et la preuve et l'investigation de la qualité de l'argumentation des élèves et des enseignants. Cependant, nous soulignons la nécessité de mener des investigations supplémentaires pour évaluer le potentiel des cours axés sur l'argumentation pour les matières mathématiques de l'enseignement supérieur, en utilisant principalement l'approche de Perelman, dans laquelle les études ont été raisonnablement rares.

Mots-clés : Argumentation, Revue systématique, Enseignement des mathématiques, Enseignement supérieur.

Resumo

O artigo tem como objetivo realizar uma revisão sistemática da literatura de estudos empíricos sobre usos da argumentação na Educação Matemática no Ensino Superior. Para tanto, foi realizada uma busca por artigos na base de dados Education Resources Information Center (ERIC), em periódicos específicos da área de Educação Matemática, entre 2012 e 2021. O período justifica-se por ser uma década de pesquisas publicadas anteriormente ao início do presente estudo. Após a seleção e leitura desses artigos, foram identificadas as categorias de análise: argumentação como instrumento de análise de argumentos de estudantes; argumentação como instrumento de análise de argumentos de professores; argumentação como abordagem de ensino. A análise foi conduzida a partir de estudos da literatura, sobretudo adotando-se a perspectiva teórica da argumentação de Toulmin e de Perelman. Os resultados apontam três formas de usos da argumentação com relação à: abordagem de ensino, investigação sobre o ensino de provas, avaliação da relação entre argumentação e prova e investigação da qualidade da argumentação de estudantes e professores. No entanto, evidenciou-se a necessidade de que mais investigações sejam realizadas para avaliar o potencial de cursos focados em argumentação para disciplinas de matemática do Ensino Superior, principalmente, utilizando a abordagem de Perelman, cujos estudos realizados têm sido razoavelmente escassos.

Palavras-chave: Argumentação, Revisão sistemática, Educação matemática, Ensino superior.

Uses of Argumentation in Mathematics Education: a systematic literature review in Higher Education

Research on argumentation in the field of education arouses Mathematics Education researchers' interest in several teaching forms (Can & Isleyen, 2020; Mariotti & Pedemonte, 2019; Metaxas et al., 2016[4]; Nunes & Almouloud, 2013). More specifically, argumentation and proof[5] have been the themes of many studies (Antonini, 2018; Laamena et al., 2018; Gabel & Dreyfus, 2017. We understand argumentation and proof as processes that employ rational justification. Proof does not necessarily make use of logical deduction. On the other hand, demonstration[6] refers to proof in the mathematical scope and adopts a sequence of articulated assertions which follow a pre-established logic (Balacheff, 1988).

Studies on argumentation in Mathematics Education started to gain strength in the 1990s with investigations into students' arguments in different teaching forms (Simpson, 2015). Toulmin's argumentative model has stood out in those analyses (Simpson, 2015). According to Almeida and Malheiro (2018), although the model does not specifically focus on education in its initial proposal, it has been frequently used by researchers in Science Teaching and Mathematics Education to analyze and understand argumentation in the classroom.

Metaxas et al. (2016) stated that argumentation has been used as a way of learning or as an objective of instruction in the field of education. In Mathematics Education, argumentation has purposes which are different but complementary in relation to argumentation for the development of concepts and argumentation for the development of mathematical practice (Staples & Newton, 2016). We agree with Staples and Newton (2016) that both purposes are valuable: the former because it is teaching and learning practice, and the latter because it contributes to the construction and validation of a mathematical argument.

Recently, other lines of research have been conducted to investigate the effects of argumentation as a Mathematics Education teaching approach in higher education (Can & Isleyen, 2020; Gabel & Dreyfus, 2017; Metaxas et al., 2016). When such approach is harnessed in teaching, there is room for students to discuss themes. They make inferences through reasoning and from an argumentative perspective (Uygun-Eryurt, 2020). Thus, teaching is guided by actions characterized as pro-argumentative because they stimulate students' interactions and the justification processes behind reasoning. These lines of research have

[4] The translations/adaptations of the foreign language literature references that were studied and contained in the direct and indirect citations in this article were performed by the authors.
[5] Proof consists of a set of explanations that acquired a social status in its respective community (Balacheff, 1988).
[6] We will consider *demonstration* as a synonym for mathematical *proof.*

highlighted the need for more studies to be performed in order to better assess the potential of this teaching approach.

In this article, our attention is focused on argumentation in higher education. Research has intensified in this field in recent years (Can & Isleyen, 2020; Gabel & Dreyfus, 2017), but there is a limited number of empirical studies on argumentation in Mathematics Education in higher education in Brazil. Therefore, there are few updated reviews of didactical interventions in this form of teaching. Due to such lack of studies, we have considered that conducting a study to perform a systematic literature review of the aforementioned theme can be highly relevant so that we can have a better comprehension of its current status. Furthermore, a review could point out if there is shortage or problems, which in turn could generate further studies.

Based on the arguments we presented, the objective of this article is to perform a systematic literature review of empirical studies on the uses of argumentation in Mathematics Education in higher education. Thus, we will analyze how argumentation has been used in this form of teaching and what results have been produced by the studies.

In the following sections, we will describe the theoretical perspectives that supported corpus analysis, the procedures used to find the articles that compose our corpus, analysis categories, our final considerations and some implications for future studies.

Theoretical perspectives of argumentation

Different theoretical perspectives characterized argumentation in Mathematics Educations (Pedemonte, 2012). Pedemonte highlights theories proposed by Plantin (2008), Toulmin (1958), Perelman (1993) and Anscombre & Ducrot (1983) to support her research. Some of the authors characterized argumentation with regard to discourse, such as Perelman's theoretical perspective, whereas others, like Toulmin's, characterized argumentation in Mathematics Education in terms of its argumentative structures. In our first dive into the literature, we noticed the presence of the aforementioned theories. Nevertheless, as these perspectives are not so popular in Mathematics Education, particularly in Brazil, we felt the need to make a brief introduction to some aspects of Perelman's and Toulmin's perspectives and of the theory of Cognitive Unity so that readers could have a better understanding of the discussion about the results of this review.

The purpose of Perelman's argumentation theory "is the study of discursive techniques that allow us to provoke or increase people's acceptance of the theses that are presented for assent" (Perelman & Olbrechts-Tyteca, 2005, p.10). This theory aims at the audience's acceptance both in intellectual and emotional aspects. According to Perelman (1993, p. 33), the audience is “the gathering of those whom the speaker wants to influence by his or her arguments”.

To adopt Perelman’s perspective (1993), it is necessary to pay special attention to the audience’s adherence to the premises of the discourse, including argumentation and audience techniques. Perelman (1993) stated that the starting point for argumentation is composed by agreement, choice of the data and their adaptation aiming at argumentation and data presentation, and the form of the discourse. It is important for the speaker to be careful with the choice of the data and with how the presence of arguments is created because every audience has a set of accepted premises.

Thus, argumentation will produce different effects in different situations, and it is necessary to make use of appropriate methods regarding both the object of the discourse and the type of audience to which it is delivered to. That is to say that it is possible to complete – in case one finds it useful – a general study of argumentation with specialized methodologies according to the type of audience and academic subject by taking into consideration the theses and methods that are accepted in a given subject. In this sense, Gabel and Dreyfus (2016; 2017) argue that Perelman’s argumentative perspective is a reference that can be used in investigations that analyze the flow[7] of mathematical proof and in interventions in mathematical classes with regard to the teaching of proof. During the process of proof, besides formal logic, there are examples, justifications, representations and other means that are explored through argumentation (Gabel & Dreyfus, 2016; 2017).

Toulmin (2006) takes argumentation to the context of professional situations. Such discussions have been frequently used by researchers in the fields of Science Teaching and Mathematics Education as a methodological tool to analyze the arguments built in classes (Antonini, 2018). The author qualifies arguments as “justificatory” because justifications and their backings are presented to support assertions. Therefore, there are arguments in many fields like physics, mathematics, biology and so on. Moreover, Toulmin (2006) establishes a difference between field-invariant (forms and merits that do not vary with the field) and field-dependent (forms and merits that vary from field of study to field of study). According to Toulmin (2006), an argument can be understood as a train of thought or a logical sequence and,

[7] “The flow is the result of the choices made by the speaker with regard to” (i) the presentation of the logical structure of the proof, (ii) the informal way of the characteristics and considerations in the process of proof and demonstration (examples, diagrams) and (iii) instructional and mathematical contextual factors” (Gabel & Dreyfus, 2017, p. 3).

more rigorously, as interconnected sequences of reasons and claims that establish the content and the strength of the position a speaker argues for.

The elements that structure Toulmin's argumentation model are identified in Figure 1 and they allow us to assess justificatory arguments. In the layout, there are three main elements: the data (the evidence that supports a statement), the claim (the statement made based on the data) and the warrant (the statement that authorizes the connections between the data and the claim). The other complementary elements are the backing (the knowledge that is shared in the field and that warrants the justification), the modal qualifier (expresses the level of trust in a claim) and the rebuttal (the statement that refutes the justification and, thus, invalidates the claim).

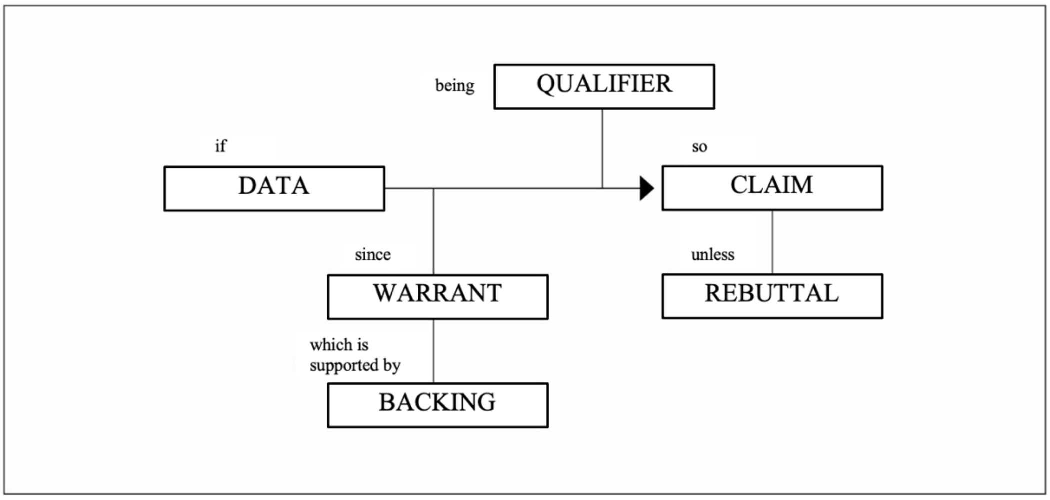

Figure 1.

Toulmin's argumentation model (adapted from Toulmin, 2006)

Mathematics Education research on argumentation intensified with the publication of Krummheuer's study (1995). Since then, this model has been used to analyze arguments in middle school (Almeida & Malheiro, 2018), high school (Mariotti & Pedemonte, 2019) and higher education, both in undergraduate courses (Fukawa-Connelly, 2014; Wawro, 2015) and in post-graduation (Kwon et al., 2015; Metaxas et al., 2016).

Some researchers use a reduced form of Toulmin's model, that is, they only observe the main elements in the structure of the argument (Nunes & Almouloud, 2013; Uygun-Eryurt, 2020), whereas others use the full model (Laamena et al., 2018; Metaxas et al., 2016) depending on the context and on the analysis proposed by each researcher. When performing argument analysis through this model, some researchers link it to another argumentative scheme (Metaxas et al., 2016) or adapt it according to their investigation objectives (Inglis et al., 2007).

Metaxas et al. (2016) stated that Toulmin's model does not show the quality of arguments, so they decided to link it to Walton's (2012) argumentative scheme. These schemes are inference structures that are linked to types of reasoning. Walton (2012) identified twenty-five schemes. Here are some examples of schemes: argument from cause and effect, argument from analogy, argument from authority, argument from classification.

The structure of Cognitive Unity[8] provides us with tools to investigate the relationship between argumentation and proof (Antonini, 2018; Kaplan et al., 2019; Mariotti & Pedemonte, 2019). According to the aforementioned authors, it is possible to analyze the differences and analogies between argumentation and mathematical proof with this structure. In addition, it allows us to analyze the whole process: the emergence of conjecture, the argumentation to support it and the final mathematical proof (Mariotti & Pedemonte, 2019). Pedemonte (2007) used Toulmin's model to investigate the relationships between arguments connected to a conjecture and the content of mathematical proof.

Antonini (2018) states that Cognitive Unity is important for the processes of conjecture generation and argumentative production, which provide support to hypotheses in the production of mathematical proof. Moreover, Pedemonte (2007, p. 25) states that the cognitive unity framework is "an effective tool that can both predict and analyze the difficulties that students might have while performing the process of mathematical proof".

Methodological procedures

This article is a systematic review which, according to the parameters of the *Preferred Reporting Items for Systematic Reviews and Meta-Analyses* protocol (PRISMA) (Galvão et al., 2015), is a type of review whose question is devised in a specific way. In this sense, this type of review uses systematic and explicit methods to identify, select and evaluate relevant studies on a given theme in a critical way, and also to produce and analyze the data contained in these studies. Thus, inspired by these parameters and by the review performed by Teixeira et al. (2012), we will use the following organization: (i) identifying the research subject; (ii) identifying the articles; (iii) selecting the articles; (iv) applying exclusion criteria; (v)

[8] "(...) while developing some conjecture, students build their explanations progressively through intense argumentative activity, which is interconnected by the justifications for their choices; in the following stage, students coherently link to this process the organization of some of the arguments produced previously, in a logical sequence" (Boero et al., 1996, p. 96).

systematizing the selected articles; and (vi) analyzing the results. We crafted the following questions to specify the proposal of this review:

1) How has argumentation been used in Mathematics Education in higher education?

2) What results have been produced?

After the establishment of the review questions, we defined the objective of mapping out empirical studies in the literature on argumentation in Mathematics in higher education. To form the corpus for this review – regarding the identification and selection of articles -, we searched for published articles on the Education Resources Information Center (ERIC) database. ERIC is considered an important international database of research publications in the field of education. In addition, we included the following journals due to their relevance in the field of Mathematics Education: Mathematics Education, Journal of Mathematics Teacher Education (ZDM) and Educational Studies in Mathematics.

Furthermore, we searched across websites of Brazilian journals that are not part of ERIC, although they are qualified in the Qualis CAPES system as B2-level or higher, such as Revista de Educação em Ciência e Tecnologia (ALEXANDRIA), Boletim de Educação Matemática (BOLEMA), Educação Matemática Pesquisa (EMP), Grupo de Estudos e Pesquisas em Educação Matemática (BOLETIM GEPEM), Revista de Educação Matemática (ZETETIKÉ) and other international journals like Revista de Investigação em Educação Matemática (QUADRANTE), Revista Latinoamericana de Investigación en Matemática Educativa (RELIME) and Revista Iberoamericana de Educación Matemática (UNIÓN). As listed above, these were our databases and the first criterion we followed to perform our search.

The second criterion was the use of keywords and the definition of a time period. We used the keywords argumentation and mathematics and argumentation and proof for the *ERIC* database and for the international journals that are not part of it. For the Brazilian journals, we adopted the same system; however, we used the Portuguese writing of the keywords when they were not available in English. The search was limited to the period between 2012 and 2021, which encompasses a decade of studies published until the year before the beginning of this study. The end of the search took place on March 24th, 2021. Moreover, for the purpose of justifying our choice, in a systematic review-type study, the decade immediately before the beginning of the review is a sufficient time window in order to understand what the status of recent knowledge of the investigated field is. We identified 227 articles on ERIC, 34 of which were duplicated in the system. Across the journals that are not part of ERIC, we found 30 more articles. Therefore, we found a total of 223 articles.

The third selection criterion was the exploratory reading of article titles and abstracts in order to find articles that matched the theme of our research, and we identified 48 articles that were related to argumentation and mathematics teaching. Still, as our interest is the use of argumentation in Mathematics Education in higher education through didactical interventions, we applied the following exclusion criteria: (i) theoretical articles; (ii) articles with didactical interventions in basic education and (iii) articles with didactical interventions related to software. As a result, we excluded 10 theoretical articles, 21 articles with didactical interventions in basic education and 4 articles with didactical interventions related to software.

Finally, after applying the exclusion criteria, there were 13 articles left for an in-depth analysis. Among these articles, 12 were found on the ERIC database and 1 in QUADRANTE, an international publication. Therefore, all of the articles are from international journals. The selection and exclusion criteria were based on the parameters established by the PRISMA protocol (2015) and on the systematic review performed by Teixeira et al. (2012). Besides, our corpus analysis was guided by the argumentative theory perspectives proposed by Toulmin (2006) and Perelman & Olbrechts-Tyteca (2005).

Based on the data obtained from the selected articles (Table 1), we will perform an analysis of the results of the review study in the next section.

Table 1.

Aspects identified in the study of the corpus (produced by the authors)

Id.	**Authors**	**Objective**	**Participants**	**Method**	**Reference**	**Main results**
A	Fukawa-Connelly (2014)	To propose a model to investigate undergraduate teaching with a focus on mathematical proof.	Abstract Algebra Professor	Qualitative	Toulmin	Students had an inconsistent proof model.
B	Kwon et al. (2015)	To understand the characteristics of students' argumentation and to identify design principles.	Specialization in Mathematics Education Students	Qualitative	Toulmin linked to van Eemeren	Students' argumentation schemes were improved, resulting in more quality as the teaching strategy progressed.
C	Wawro (2015)	To investigate how students argue about linear algebra solutions.	Students from several undergraduate courses	Qualitative	Toulmin	Thorough comprehension and complex justifications are possible when students are engaged in the argumentative process.

Id.	Authors	Objective	Participants	Method	Reference	Main results
D	Metaxas et al. (2016)	To examine a shift in pedagogical argumentation when the participant of the study is involved in a teaching proposal with hypothetical settings about calculus teaching in the classroom	Didactics of Calculus post-graduation students	Qualitative	Toulmin linked to Walton's schemes	The combination of methodologies contributed to identifying several aspects of argumentative activity.
E	Tristanti et al. (2016)	To describe the construction of deductive justification in mathematical argumentation.	Mathematics Education undergraduate students	Qualitative	Toulmin	When students developed deductive justification, they initially made use of inductive justification.
F	Can and Isleyen (2016)	To explore the effects of a teaching approach with an argumentative focus related to the teaching of probability.	Mathematics Education undergraduate students	Quantitative	Toulmin	The experimental group was more successful than the control group, who was taught through the traditional approach.
G	Zazkis et al. (2016)	To describe the activities that mathematicians get involved in to successfully record proof of verbal and symbolic analyses supported by graphical arguments.	Mathematics undergraduate students	Qualitative	Toulmin	Devising, synthesizing and restructuring are tasks that work as a bridge between graphical arguments and verbal-symbolic proof.
H	Gabel and Dreyfus (2016)	To design methods to investigate aspects of the flow of mathematical proof.	Number theory professor	Qualitative	Perelman and Toulmin	The intervention generated global and local changes in the flow of proof.
I	Antonini (2018)	To investigate proof by contradiction in geometry with a focus on the processes related to the treatment of geometric figures.	Basic education and undergraduate students	Qualitative	Cognitive Unity	Indirect argumentation supports proof by contradiction.
J	Laamena et al. (2018)	To investigate the use of examples in mathematical proof activities and to obtain relationships with argumentation.	Mathematics Education undergraduate students	Qualitative	Toulmin	Examples have various functions according to students' needs in the act of mathematical proof.
K	Kaplan et al. (2019)	To analyze the relationship between argumentation and proof regarding verbal and visual	Mathematics Education undergraduate students	Qualitative	Cognitive Unity and Toulmin	Students have a more convincing process of proof as long as they are involved in the exploration and argumentation of the

Id.	Authors	Objective	Participants	Method	Reference	Main results
		representations and algebraic-mathematical concepts.				corresponding conjecture.
L	Uygun-Eryurt (2020)	To investigate how the conception of mathematical induction through written argumentation is developed.	Mathematics Education undergraduate students	Qualitative	Toulmin	The more organized and structured the production of written argumentation is, the better the performance in the process of mathematical induction is.
M	Can and Isleyen (2020)	To investigate the effect of probability teaching with an argumentative focus in relation to academic performance and knowledge permanence.	Mathematics Education undergraduate students	Quantitative	Toulmin	Teaching based on an argumentative approach improved academic performance, but it did not have any effect on the permanence of knowledge.

Based on the study of this corpus (Table 1) and guided by the review questions, we observed intersections between the aspects approached in these investigations and identified evidence that constituted the analysis categories described in the following section.

Corpus analysis

We identified the following categories: **(i)** argumentation as a tool to analyze students' arguments; **(ii)** argumentation as a tool to analyze professors' arguments and **(iii)** argumentation as a teaching approach.

To address the review questions, in each category we will discuss objectives, participants, uses of argumentative references, methodological aspects and main results in order to perform an analysis of the studies.

Argumentation as a tool to analyze students' arguments.

This category encompasses the largest number of studies (**B**, **C**, **D**, **E**, **G**, **I**, **J**, **K** and **L**) that form the corpus selected for this review. Regarding the objectives of these studies, all of them focus on students' arguments in order to describe, explore or analyze them. Three of the objectives investigate the understanding of concepts, theorems and conceptions by studying students' argumentations (**C**, **D** and **L**). All of the other studies analyze or describe the arguments produced when students are engaged in tasks to build mathematical proof (**B**, **E**, **G**, **I**, **J** and **K**).

The participants involved in these studies are undergraduate (**C**, **E**, **G**, **I**, **J**, **K** and **L**) and post-graduation students (**B** and **D**). They are mostly Mathematics Education undergraduate students, except for studies **C** and **I**. The exceptions were focused on mixed groups formed by Mathematics undergraduate students and students from other courses. In Brazil, the undergraduate course in Mathematics Education corresponds to the *Licenciatura em Matemática* course (Mathematics Teaching, in free translation) and *graduação em Matemática* (Bachelor's degree in mathematics, in free translation).

For the studies of this category, argument analysis was performed based on different theories. Toulmin's perspective was used in all cases. For studies **B** and **D**, Toulmin's full model was linked to other argumentative schemes. In study **B**, Toulmin's model was combined with schemes by van Eemeren et al. (2012).[9] The researchers analyzed students' arguments during the construction of mathematical proof and their respective justifications while they were engaged in discursive tasks. The aspects of such discussions were codified by the researchers based on the elements in Toulmin's model. Then, they observed what structure they were linked to: single argumentation, multiple argumentation and compound argumentation. According to the researchers, a better-quality argumentation structure shows that students take part in various discursive tasks, such as argument construction, counterargument proposals, additional arguments or rebuttal of counterarguments (Kwon et al., 2015).

In study **D**, to investigate a student's shift in argument, the researchers employed Toulmin's full model and Walton's (2012) argumentation schemes. They applied a combination of these schemes because they were searching for an analytical framework that was able to analyze the quality of arguments through critical questions within the context. Thus, the researchers observed the structure of the argument in relation to all of the elements in Toulmin's model. The more elements the argument had, the more solid it was considered. Furthermore, they examined if the corresponding critical questions were answered. According to the researchers, a well- founded discourse makes use of a larger number of backings and answers more critical questions [10]. The absence of rebuttals and answers to critical questions characterized an argument as weak or unjustified.

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[9] van Eemeren and his collaborators suggest the following types of argumentation structure: "single argumentation (it includes only a claim and its justification), multiple argumentation (a claim supported by several justifications) and compound argumentation (it includes several justifications to support a claim that induces a new assertion)". (Kwon, Younggon & Hwan, 2015, p. 999).

[10] From Walton's perspective (2012), an argument is appropriately evaluated through critical questions within the context of the dialogue in which it occurs. The critical questions provide a list of categories to assess each argumentative scheme. There are considerations within the context of the dialogue that allow us to evaluate an argument as weak or strong. Thus, an argument will be considered weak, erroneous or fallacious in case it does

In studies **G, K** and **L**, the tool used to analyze the arguments was a reduced or simplified form of Toulmin's model (without its complementary elements). In study **G**, the researchers focused on the construction of arguments based on graphical representations that students used to justify the veracity of their assertions while building proof. Therefore, the observation of each inference by the students was focused on the data-warrant-claim axis. In some cases, however, the justification was not explicitly declared by the student. According to Toulmin (2006), justification plays an essential part because it allows the connection between the evidence and the assertion which the student aims to validate.

Yet, in study **K,** the analysis of students' arguments happened in two stages: in the first stage, the researchers focused on the observation of data-justifications-claims while students solved problems. Even though the researchers acknowledged the importance of these elements to mathematical arguments, they were not interested in observing rebuttals or qualifiers. Nevertheless, for the objective of their research, which was to understand the relationship between argumentation and proof, the use of Toulmin's model in its simplified form was sufficient.

In the second stage of the analysis, they observed the justifications and backings presented by the students while they answered questions related to the use of representations in the process of argumentation and proof, in terms of the referential system. According to Toulmin (2006), the backing is an important element to the argument because it warrants the justification.

The analysis of study **L**, on the other hand, encompassed two processes. The first one was related to students' written argumentation through Toulmin's model based on the observation of the data, justifications and claims. The second process was focused on the data collected in interviews, and the following categories were used: nature, function, meaning, importance and need for mathematical induction.

In studies **C, E** and **J**, argument analysis was performed through the full form of Toulmin's model, albeit with different focuses. Study **C** made use of microgenetic analysis (forms of reasoning in an isolated way) and ontogenetic analysis (over time) combined with Toulmin's model to investigate the structure of arguments related to solutions to linear algebra equations while establishing connections between the proposals and a theorem's concepts. The model was used to analyze or rebut students' arguments when they provided a solid foundation

not present any substantial justification to the reasonability of a given criticism, that is, in case it does not answer critical questions that are appropriate to the context of the argument in question (Walton, 2012).

to understand conceptual frameworks in algebra. Rebuttals play an important part in the establishment of content consistence conditions and of a claim's limits (Toulmin, 2006).

The analysis in study **E** is focused on the element of justification while students are engaged in problem-solving tasks. Toulmin (2006) highlights the importance of acknowledgement and comprehension – in mathematical argumentation, especially – when it comes to the reconstruction of justification elements. In addition, the researchers were interested in the type of justification (inductive, structural-inductive and deductive). The objective of the analysis was to describe the construction of deductive justification based on non-deductive justification.

In study **J**, the analysis was centered around the use of examples, aiming to explain the use of examples when students are engaged in the process of mathematical proof. As a result, the use of the model enabled the researchers to explain the structure of the argument which provides a path for using the examples in the claim. Each example was categorized according to three different functions: exploration tool, investigative tool for justification and persuasive tool. The uses of examples are useful to understand students' arguments (Laamena et al., 2018; Yopp & Ely, 2015).

For study **I**, the theoretical tool Cognitive Unity was used to analyze students' answers. The analysis compared the process of argumentation and the mathematical proof with focus on the expressions used, the structure, the content and the geometric figures. Students' proof construction process was investigated so as to describe differences and relationships between argumentation and proof by contradiction. Thus, argumentation plays a fundamental part in the construction of mathematical proof. In this sense, Pedemonte (2007) emphasizes that we need to search for similarities in the process of argumentation and proof in order to encounter cognitive unity.

With regard to the methodological procedures employed in the studies analyzed in this category, all studies adopted a qualitative approach, whose purpose is to describe and interpret the research subject because it is more focused on the comprehension than on the explanation of the phenomena being studied (Lichtman, 2010). This aspect is in line with the objectives of this category, which include description, exploration and analysis. During the tasks developed with the students, we could observe individual activities, activities in small groups, in pairs or involving the whole class as a group.

About the procedures used for data production in the analyzed studies, the researchers usually combined more than one technique: observation, interviews, images (drawings, sketches) and written documents. The employed techniques are, in fact, coherent considering that the qualitative studies were carried out in an educational context. As for the form that the procedures were recorded, we identified notes, field diaries, reflexive notes, audio recordings and video recordings.

When it comes to the main results, we identified three subcategories stemming from the convergences that were encountered: **(i) analysis tool linked to two argumentative schemes; (ii) analysis tool for specific elements of the argumentation; (iii) tool to analyze the relationship between the processes of argumentation and proof.**

Studies **B** and **D** form the first subcategory. In these studies, Toulmin's model was linked to another argumentation scheme to analyze students' arguments. This combination contributed to the identification of several aspects of the possible construction of knowledge in argumentative activities, mainly to the observation of an evolution in terms of argumentation quality. The use of tools in an integrated way enabled a better assessment of argumentative discourse, which paved the way for the evaluation of discourse and knowledge.

The second subcategory encompassed studies **C, E, J** and **L.** As for study **C,** Toulmin's analysis tool allowed the identification of specific mechanisms through which students were able to understand the meaning in representations of linear algebra concepts. In study **E**, it was possible to make an inference about the use of justification by students during the argumentative process. When students built a deductive justification, they initially used inductive justification, which is employed when one wants to reduce the uncertainty of the result, whereas deductive justification removes the possibility of a rebuttal (Tristanti et al., 2016).

Study **J** pointed out that examples served several functions depending on students' needs. Examples were used as an exploratory tool, as an investigative tool for justification and as a persuasion tool. About study **L**, it was possible to conclude that the more organized and structured the production of written argumentation was, the more skills students had to build and use mathematical induction.

Within the third subcategory, we identified studies **G, I, K** and **L,** which employed Toulmin's argumentative tool or Cognitive Unity, or a combination of both for argument analysis. In study **G**, it was possible to describe the three activities that contributed so that students could establish a connection between graphical arguments and verbal-symbolic proof. The identified activities were elaborating, synthesizing and restructuring. Moreover, through this connection, it was possible to detect mistakes in the proof students devised.

Study **I** showed that in order for students to restore the rupture between figural and conceptual elements, they needed to search for geometric sense and support to their indirect argumentation. As contained in the discussion promoted by Antonini (2018), an indirect argumentation has a structure that is analogous to the structure of a proof by contradiction, but it is presented more spontaneously in terms of reasoning. Thus, it stems from the denial of what should be sustained for a more articulate definition.

The results of studies **K** and **L** pointed out that students can experience a more convincing process of proof when they are previously involved in the exploration and argumentation of the conjecture, especially in written argumentation. Other studies in this field (Antonini, 2018; Mariotti & Pedemonte, 2019; Pedemonte, 2007) strengthen these results and emphasize the positive impacts of argumentation on the construction of proof.

In the studies contained in this category, there is predominance of Toulmin's model as an argumentative analysis tool because it offers the possibility to analyze mathematical argumentation in the justification processes, especially in demonstrations or investigations into the relationships between the processes of argumentation and demonstration.

Argumentation as a tool to analyze professors' arguments

In this category, there are studies **A** and **H**. In the former, the argumentative tool was used to analyze an Abstract Algebra professor's argument, whereas in the latter, the analysis was based on a Number Theory professor's arguments. The objective of these studies was to provide models to assess the teaching of mathematical proof. However, in addition to assessing teaching, study **H** also proposes changing it.

As for the type of argumentative tool employed to analyze the professors' arguments, study **A** adopted Toulmin's model. The study focused on observing if there was a detail pattern in the professor's teaching of proof regarding the elements of the model both in writing and speaking, as well as the opportunities students had during that teaching moment to learn how to word their proofs in writing.

The analysis focused on the justifications contained in the arguments employed to verify the mathematical properties that were being studied. In addition to the justifications, the researchers observed if the professor used the backing to give credence to the justification. In advanced mathematics, in general, there is a need for complete arguments. Therefore, we emphasize the importance of the backing to support the justifications. The backing is a categorical assertion or a solid foundation for the justification (Toulmin, 2006).

Study **H** was supported by Perelman's argumentative theoretical perspective to analyze the flow of proof in relation to the following aspects: scope, organization and presence. About scope and organization, the analysis of the proof was carried out based on the choices made by the professor in terms of the organization of proof modules, the scope of each module and the effort that was expended in each module. As for presence, the analysis was focused on the observation of rhetorical figures, like repetition, or the use of several justifications and backings to reinforce their presence. According to Perelman and Olbrechts-Tyteca (2005), the language, the form of insistence and the presentation techniques that are employed aim to strengthen the presence of the object of study among the audience.

Moreover, regarding the analysis of the professor's arguments, the researchers paid attention to several characteristics that could qualify a proof as efficient (coherence, clarity). As for the logical presentation of the proof, it is interesting to notice that the researchers also made use of Toulmin's full model to analyze the elements that enabled the presence of the flow of proof. When observing the structure of the argument, they focused on the number and type of justifications and backings that were presented to support a claim and the direction of the argumentation (abductive[11]/deductive).

The researchers also analyzed the rhetorical elements that impose presence in the act of proving, as follows: emphasizing some aspects, repetition of the same ideas through different words, uses of examples and the time allocated for each stage of the proof process. Thus, two argumentative instruments were used for the analysis of the presence of each proof module. While Toulmin's model allows the presentation and analysis of the argumentation structure, Perelman's argumentative framework enables us to complete the analysis because it promotes the investigation of other aspects of argumentation, for example, its adaptability to the audience (Gabel & Dreyfus, 2017).

When it comes to the methodological procedures of the studies analyzed in this category, the researchers adopted a qualitative approach focused on case study, which is line with the objects of the investigation, because researchers use deep strategies and detailed descriptions in case studies.

In terms of the techniques harnessed for data production, observation was employed in studies **A** and **H**. In the latter, the professor used after-class questionnaires about affective and cognitive aspects related to the proof presented by her. Next, students were interviewed based on the answers contained in the questionnaires. Then, the professor was interviewed and invited to reflect on her presentation of proof. A field diary (**A** and **H**), an audio recording (**H**) and a video recording (**A**) were adopted as recording procedures.

[11] According to Pedemonte (2002), abductive argumentation involves finding the best or more likely explanations stemming from a set of facts or information. "The search for the solution to a problem is frequently developed based on a claim" (Pedemonte, 2002, p. 67).

The main results of the studies in this category were directed to the teaching of mathematical proof. In study **A**, through Toulmin's argumentative framework, it was shown that the professor presented different levels of detail during the act of proof, and the researcher inferred that the use of justifications and backings for the teaching of proof contained inconsistencies. Thus, she concluded that the professor presented an inconsistent model to carry out her proof procedures, which could reduce the pedagogical value of the proof. The professor's difficulty in justifying and backing her arguments is in line with other studies involving students in the field of Science in higher education (Teixeira et al., 2015).

In study **H**, the researchers emphasized the analytical potential of Perelman's theoretical-argumentative perspective to evaluate the teaching of proof. They state that rhetorical figures could produce change in the visibility of proof by the students. Besides **H**, other studies aimed to expand the visibility and potential of this analytical framework for research on science teaching and Mathematics Education (Silva Júnior, 2019).

Argumentation as a teaching approach

In this category, we identified two subcategories: **(i)** argumentation as an explicit teaching approach (**F, L** and **M)** and **(ii)** argumentation as an implicit teaching approach (**B, D** and **H**). Studies **F** and **M** are exclusive to this category. Therefore, we will address the objectives, the participants, the use of the argumentative framework, the methodological aspects and the main results related only to these studies, as we did in the previous categories. For the other studies in this category, we will discuss their teaching approaches and their respective results, since the discussion of the other aforementioned aspects was performed in the previous sections.

We identify an explicit approach when there is a direct form of argumentation teaching. The participants of the study receive information on the argumentation concept, the definition of what an argument is, the elements that constitute an argument, the construction of arguments, types of argument, the process of justification of ideas and of rebuttal, the identification of evidence, and other information depending on the theoretical framework adopted by the researcher. On the other hand, in the implicit approach, the professor or researcher is guided by strategies that encourage argumentation even though they are not teaching any topics related to an argumentation theory.

Unlike in a traditional approach, to approach teaching from an argumentative perspective, the classroom environment must be carefully considered in a way that allows students to share their ideas, assess and analyze other students' ideas (Can & Isleyen, 2020).

This teaching proposal can provide an environment that offers a space for discussion with interactions among students, which will foster the argumentation process and, consequently, allow the process of justification to bloom. An example of this kind of approach is inquiry-based learning. With this teaching proposal, students learn through discursive activities (Kwon et al., 2015).

In the first subcategory, studies **F** and **M** were conducted by the same researchers. They aimed to investigate the effect of probability teaching through an argumentative approach on Mathematics Education undergraduate students. In addition to focusing on students' knowledge performance, like study **F**, study **M** also focused on knowledge permanence. In these studies, after students had taken a pre-test, they were split into two groups: an experimental one and a control one. Then, they were submitted to an 18-hour-long probability course. The experimental group learned probability through an explicit argumentative teaching approach whereas the control group learned by a traditional approach, centered on the professor.

In the experimental group, students were taught with an argumentative focus from Toulmin's perspective. During the course, there was a process of discussion of questions about probability, and students searched for solutions in different ways. They had to defend their claims by using backings, which allowed them to build arguments. The professor conducted the process by using pro-argumentative actions through questions like "why do you think so?", "how would you convince your classmate that your thought is true?", "does anybody oppose the presented solution?", and the solution to the questions was guided by an argumentative process. The students were encouraged to discuss and justify their ideas, so the method enabled students to participate actively (Can & Isleyen, 2016).

On the other hand, in the control group, the solution to the problems was answered by the researcher. Students did not have any opportunities to participate actively. After the course, students from both groups answered a questionnaire devised by the researchers (a post-test questionnaire without prior notice). In study **M**, besides the post-test, there was a retention test three months after the post-test. The researchers decided to administer this test because not only did they want to investigate performance, but also knowledge permanence.

The research approach that was in line with the methodology adopted in these studies was a quantitative one with focus on the investigation into experimental design. The tool used to collect the data was a probability performance test with open-ended questions. For data analysis, the researchers used statistical analysis software. In these studies, argumentation was used just as a teaching approach, not as an argument analysis tool.

Based on the sampling results of these studies, the researchers presented some considerations. In study **F**, they noticed that there was performance improvement after the course, both in the experimental and control groups. Nevertheless, the students from the experimental groups were more successful in terms of performance. According to the researchers responsible for the study, the significant statistical difference between the experimental group and the control group occurred as a result of the teaching of probability with as argumentative focus. There was more effective contribution to mathematical knowledge because in this type of teaching proposal, students participate, aim to persuade other students, express their own ideas and listen to their peers' positions until they reach a consensus about the solution to the questions or about the assertions made in the discussion.

A similar result was obtained in study **M**, in which the explicit argumentative teaching approach increased students' performance. However, there was no noticeable difference in students' knowledge permanence between the experimental and control groups. The researchers argued that it could have occurred because it was the first time that the students were taught from an argumentative perspective. In this sense, the researchers pointed out the need to increase teaching time through an argumentative approach.

Still with regard to the first subcategory, we have study **L**. In the first stage of the intervention, students were exposed to explicit argumentative teaching from Toulmin's theoretical perspective. They received information from this perspective and constructed arguments according to its model about mathematical induction during group tasks to build proof. They had the opportunity to write their argumentations regarding the practices that were conducted in the intervention. The discussions focused on the necessary steps to solve the questions related to Mathematical Induction. Students aimed to justify if each step was true besides discussing the relationships between concepts and presenting their evidence according to the related data. The argumentations were written in smaller groups and then exposed to the whole group.

After that, the researcher gave feedback on their constructions, and, in a procedural way, students continued to produce written argumentations about proof construction activities related to mathematical induction during the intervention, which lasted for 12 hours. After the intervention, there was improvement in their written argumentation, in their conception of mathematical induction and in students' proof construction development. Moreover, the students were able to convert mathematical statements and prove them by using Mathematical Induction procedures (Uygun-Eryurt, 2020). The adopted teaching approach contributed significantly because it generated motivation and logic in every step of the Mathematical Induction proof construction process.

In the second subcategory, there are studies in which the intervention strategies generated pro-argumentative results although there was no explicit argumentative teaching. In study **B**, the didactical intervention happened through a teaching approach centered on the students and based on an investigation conducted over a whole Multivariable Calculus course. A large part of the content was taught through online videos so that students could have more time for mathematical discussions during their in-person teaching period, which was done in small groups. Time is an important element because, in this teaching approach, students need to justify their answers, build hypotheses and participate in the proposed discussion. Moreover, it is sometimes necessary to restructure the problems that are being studied.

The results of the study considered that the intervention was effective. By the employment of the didactical strategy, students' argumentation schemes were improved and evolved in terms of complexity as the strategy progressed. According to van Eemeren et al. (2008), as students get engaged in discursive activities, there is growth in the complexity of their argumentative structure. Such growth occurs because students have a chance to justify their answers, question their peers' answers, offer counterarguments, ask for additional justifications and other elements that are present in the process of mathematical investigation through discussions.

With regard to studies **D** and **H**, both interventions aimed to study a professor's argumentative discourse. In the former, the professor was a Didactics of Calculus postgraduate student. The course followed a hypothetical scenario analysis approach in the classroom about the teaching of Calculus. The course was 26 hours long and encompassed 7 tasks, and each one of the tasks described a hypothetical scenario based on a student's understanding or misunderstanding about mathematical concepts related to the teaching of Calculus.

In the intervention, after the professor answered the course tasks, the mathematical and pedagogical discussions on the tasks were conducted through an argumentative perspective between the professor and the researcher. The researcher conducted the intervention with questions based on Walton's (2012) argumentation schemes and Toulmin's (2006) model. These discussions comprised uses of examples, uses of counterexamples, situations that led to a fallacy in the teaching of Calculus and specific questions about the objectives of each task. The professor under analysis presented justifications and also backings to support his arguments during the explanations of the tasks. As the intervention progressed, in addition to the minimum elements to a consistent argument (data-warrant-claim), the professor started to present the backing, the qualifier and rebuttals in his arguments, and there was an increase in answers to critical questions.

In this sense, the result presented by the researchers is justified, that is to say, the structure of the course encouraged argumentation, and the professor under analysis developed the articulation of multiple pedagogical interpretations. In study **D**, as in study **B**, there was an increase in argument quality after the intervention with an argumentative focus, so the participants' argumentative schemes became better structured.

On the other hand, in study **H**, the intervention took place through a three-hour meeting between the researcher and a Number Theory professor. The intervention occurred after an analysis of the flow of proof presented by the professor and before the professor conducted the same proof with another group of students the following year. After the analysis of the flow (the choices made by the professor about the presentation of the logical structure of the proof, about the characteristics and informal considerations related to the process of proof and about contextual mathematical factors), the researcher aimed to change such choices by using Perelman and Olbrechts-Tyteca's (2005) theoretical lens.

During a dialogue with the professor that was being analyzed, the researcher made suggestions adapted to Perelman's argumentative theory in order to make the proof more coherent. She encouraged the professor about the use of argumentation to support the proof and highlighted parts of the proof that could have been more emphasized, which would reinforce its presence. According to Perelman and Olbrechts-Tyteca (2005), in the construction of an argument, the speaker (in this case, the professor) needs to choose some elements around which they want to center attention. From this argumentative perspective, the creation of presence and the communion with the audience are essential elements to an argumentation.

Thus, it is necessary to be concerned with the audience's adherence to the premises of the discourse. In this approach, when we consider that teachers are speakers before their audiences, they must address their students supposing that they adhere to what is part of the acknowledged corpus of the discipline. When ideas and explanations for a mathematical proof are exposed in the classroom, for example, teachers must ensure their students' adherence to their reasoning sequence as they progress.

It is important to highlight that, in that intervention, the researcher offered suggestions in order to change and support the flow of proof, thus emphasizing the professor's discursive characteristics in terms of presentation, which could be improved by highlighting his mains ideas and suggesting rhetorical forms, and of time distribution for each flow of proof, and other aspects that could help make the proof more coherent. The main results of the intervention as highlighted by the researchers were: global and local changes in the flow of proof, Perelman's theoretical perspective was efficient at capturing the changes, and the professor's encouragement towards the use of informal arguments and combinations during the presentation of the proof.

Considerations and implications

This study aimed to perform a systematic review of empirical studies on the use of argumentation in Mathematics Education in higher education, given the importance of argumentation for research in the field of education, especially in Mathematics Education. We used two questions to specify the proposal of this review: How has argumentation been used in Mathematics Education in higher education? What results have been produced?

With regard to the first question, we identified three ways argumentation was used (Figure 2): argumentation as a tool to analyze students' arguments, argumentation as a tool to analyze professors' arguments and argumentation as a teaching approach.

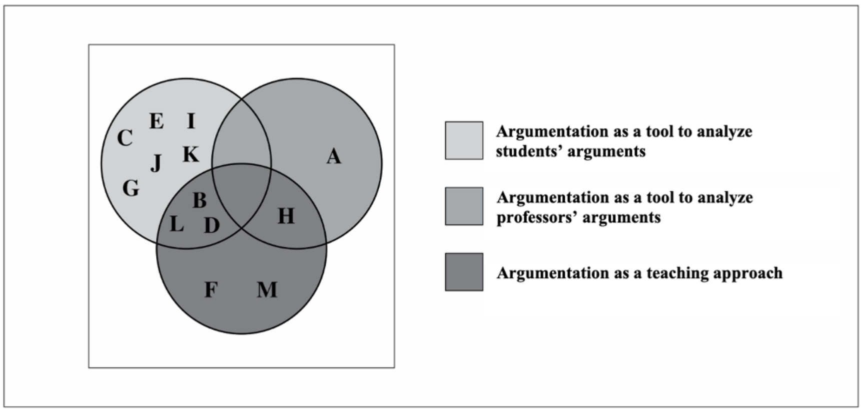

Figure 2.

A summary of argumentation uses in the studies contained in the corpus (devised by the authors)

Most of the analyzed studies used Toulmin's argumentative perspective to analyze students' arguments, professors' arguments and teaching approaches. Only two studies focused on professors' arguments, which shows that there is a need for more research to investigate professors' arguments, especially within the context of mathematical proof.

Among the studies that analyzed professors' arguments, only study **H** was based on Perelman's perspective to perform the analysis. However, the study also employed Toulmin's model to analyze the elements that enabled presence in the flow of proof. We understand that

Perelman’s perspective was less used in the studies because it is focused on rhetorical aspects in teaching contexts.

Studies **I** and **K** were the only ones which employed the theory of Cognitive Unity to investigate the relationship between argumentation and mathematical proof. In four studies (**B, D, H** and **K**), the researchers chose to combine argumentative schemes to analyze arguments. The combination of schemes enabled a more in-depth analysis of argumentative processes during the investigation of different discursive elements. Furthermore, when a scheme presented limitations as an analysis tool, it was possible to employ another scheme in order to solve the problem.

With regard to the use of argumentation as a teaching approach, we identified 6 (six) studies, which were divided into two subcategories: argumentation as an explicit teaching approach and argumentation as an implicit teaching approach. Argumentation as a teaching approach has proved to be a promising option to improve students’ argumentation quality and increased their skills in the construction of written or spoken mathematical proof. It is important to emphasizing that when proposing this type of teaching, the classroom environment must allow students to interact, share their ideas, assess and analyze other students’ ideas.

About the second review question, we inferred that three forms of argumentation use have a potential for research in the field of Mathematics Education in higher education and have proved to be efficient as contained in the results of the analyzed studies in terms of teaching approaches, investigations into proof teaching, assessment of the relationship between argumentation and proof and investigation into students’ and professors’ argumentation quality.

The studies that employed Toulmin’s model proved to be efficient at addressing issues related to the teaching of mathematics in Higher Education. It is a useful theoretical tool to study argumentation structures both in the process of justification of mathematical proposals and in mathematical proof. Despite the weak presence of Perelman’s framework in the studies selected for this review, we agree with Gabel and Dreyfus (2016; 2017) that this framework has potential for the investigation of mathematical proof teaching and for presenting alternatives with regard to making a proof clearer. On the other hand, the studies that employed the Cognitive Unity framework highlighted important results to Mathematics Education by discussing the process of generation of hypothesis and well-founded arguments for the production of mathematical proof.

Besides, we identified relevant results when argumentation was used both in an implicit and in an explicit way in the teaching approach. In general, these approaches generated argumentative situations in mathematics classes, thus enabling justifications for the claims, a better articulation of mathematical ideas and construction of written and oral arguments by students and also by professors who teach mathematics.

In this study, the 13 (thirteen) analyzed articles were obtained from international journals because we did not find any articles that met the adopted criteria for the systematic review in Brazilian journals, which emphasizes the lack of studies on didactical interventions in higher education in Brazil, although we acknowledge that there could be other Brazilian or foreign studies that could answer our review questions. However, the analyzed studies were sufficient to answer the review questions and, therefore, we were able to bring contributions to this research field so that researchers who are interested in argumentation in Mathematics Education in higher education can perform new investigations, reflect and support the results presented here.

Based on the findings presented in this systematic review, we highlight the need for research devoted to argumentation in Mathematics Education in higher education, especially studies that encompass didactical interventions with a teaching approach focused on argumentation in several academic subjects in order to assess the potential of this type of approach, mainly investigations based on Perelman's theoretical perspective, about which studies have been reasonably scarce.

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Data availability statement

Not applicable because this is a systematic review.