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"When we take 1, we have to put 1...": actions that support mathematical reasoning performed by a teacher when discussing an addition task

"Quando nós tiramos 1, temos que pôr 1...": ações que apoiam o raciocínio matemático desempenhadas por uma professora ao discutir uma tarefa de adição

"Quando tomamos 1, tenemos que poner 1 ...": acciones que apoyan el razonamiento matemático realizadas por un profesor cuando se habla de una tarea de suma

"Quand on prend 1, il faut mettre 1...": actions qui appuient le raisonnement mathématique effectuées par un enseignant lors de la discussion d'une tâche d'addition

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Abstract

This paper presents the results of a qualitative and interpretive research and has as its theme the mathematical reasoning and the actions of the teacher when supporting their development. The purpose was to analyze the actions developed by a teacher in the elementary school when conducting a collective discussion of an exploratory math task with 1st grade class. There was a theoretical discussion about mathematical reasoning and its processes and about teacher actions that support the development of mathematical reasoning. The data were collected by

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audio and video recordings, during the collective discussion of the task and they were analyzed for four categories of actions: invite; guide/support; inform/suggest and challenge. The results indicate that the actions performed by the teacher involved the four categories and that these led the students in the processes of conjecturing, identifying patterns, validating, justifying and generalizing.

Keywords: Mathematical reasoning, Mathematical reasoning processes, Teacher actions

Resumo

Este artigo apresenta os resultados de uma pesquisa de característica qualitativa e interpretativa a qual tem como tema o raciocínio matemático e as ações do professor quando apoia o seu desenvolvimento. O objetivo foi analisar ações desenvolvidas por uma professora dos primeiros anos de escolaridade ao conduzir a discussão coletiva de uma tarefa exploratória de matemática com uma turma de 1º ano do Primeiro Ciclo. Uma discussão teórica a respeito do raciocínio matemático e seus processos, bem como sobre ações do professor que apoiam o desenvolvimento do raciocínio matemático foi realizada. Os dados que compõem o *corpus* de análise dessa pesquisa foram coletados durante a discussão coletiva da tarefa exploratória na respectiva turma, através de registros de áudio e vídeo. Os dados foram analisados considerando quatro categorias de ações: convidar; guiar/apoiar; informar/sugerir e desafiar. Os resultados indicam que as ações desempenhadas pela professora envolveram as quatro categorias previstas na literatursa, e que tais ações conduziram os alunos nos processos de conjecturar, identificar padrões, validar, justificar e generalizar.

Palavras-chave: Raciocínio matemático, Processos de raciocínio matemático, Ações do professor.

Resumem

Este artículo presenta los resultados de una investigación cualitativa y tiene como tema el razonamiento matemático y las acciones del docente a la hora de apoyar su desarrollo. El objetivo fue analizar las acciones desarrolladas por un docente en los primeros años al realizar una discusión colectiva de una tarea exploratoria de matemáticas con una clase de 1er año. Realizamos una discusión teórica sobre el razonamiento matemático y sus procesos y sobre las acciones docentes que apoyan el razonamiento matemático. Los datos fueron recolectados mediante grabaciones de audio y video, durante la discusión colectiva de la tarea y los datos se analizaron considerando cuatro categorías de acciones: invitar; guía/apoyo; informar/sugerir y desafiar. Los resultados indican que las acciones realizadas por el docente involucraron las cuatro categorías previstas y que estas condujeron a los estudiantes en los procesos de conjeturar, identificar patrones, validar, justificar y generalizar.

Palabras clave: Razonamiento matemático, Procesos de razonamiento matemático, Acciones del maestro.

Résumé

Cet article présente les résultats d'une recherche qualitative et interprétative et a pour thème le raisonnement mathématique et les actions de l'enseignant pour accompagner leur développement. L'objectif était d'analyser les actions développées par un enseignant des premières années de scolarité lors de la conduite de la discussion collective d'une tâche mathématique exploratoire avec une classe de 1ère année du premier cycle. Il y a eu une discussion théorique sur le raisonnement mathématique et ses processus et sur les actions de l'enseignant qui soutiennent le développement du raisonnement mathématique. Les données qui composent le corpus d'analyse de cette recherche ont été recueillies par le biais d'enregistrements audio et vidéo, lors de la discussion collective de la tâche exploratoire dans la classe respective. Les données ont été analysées en considérant quatre catégories d'actions: inviter; guider/soutenir; informer/suggérer et interpeller. Les résultats indiquent que les actions réalisées par l'enseignant impliquaient les quatre catégories prévues dans la littérature et que celles-ci conduisaient les élèves dans les processus de conjecture, d'identification de modèles, de validation, de justification et de généralisation.

Mots-clés: Raisonnement mathématique, Processus de raisonnement mathématique, Gestes de l'enseignant.

“When we take 1, we have to put 1...”: actions that support mathematical reasoning performed by a teacher when discussing addition

A great deal of research has shown that the development of mathematical reasoning should be one of the main objectives of mathematics education at different levels, from the early years of primary school to higher education (Araman & Serrazina, 2020a; Jeannotte & Kieran, 2017; Mata-Pereira & Ponte, 2018; Stylianides, 2009). National curriculum documents such as the National Curricular Parameters (PCN, 2002), and the National Core Curriculum (BNCC, 2018), present mathematical reasoning as relevant in the process of learning mathematics.

Despite the different theoretical perspectives assumed regarding mathematical reasoning (Jeannotte & Kieran, 2017; Lannin et al. 2011, 2012; Mata-Pereira & Ponte, 2018; Ponte et al. 2012) there is consensus among scholars that mathematical reasoning is a vital skill that should be cultivated for learning mathematics.

Thus, it is essential to create teaching opportunities that contribute to the development of mathematical reasoning and, therefore are conducive to learning. However, implementing teaching practices which contribute to the development of mathematical reasoning in regular classrooms is still a dauting task for teachers.

It is necessary to bear in mind that students do not develop mathematical reasoning only by memorizing concepts and procedures; a stimulating learning environment, in which students can use different strategies to solve tasks, communicate mathematical ideas, and listen to the ideas of others, must be established. In light of this, the work with exploratory and investigative tasks, combined with the actions of teachers while conducting mathematical discussions, are promising alternatives (Araman et al. 2020). Teachers must select good exploratory tasks, and question students about what they did and how they did it, as suggested by Wood (1997), eliciting justifications for their choices, thus promoting learning.

In this context, this article aims to highlight and analyze the actions of a primary education teacher, when discussing an exploratory task involving addition with her students, in the 1st year of the First Cycle, in a school on the outskirts of Lisbon, Portugal. Data were collected through audio and video recordings and analyzed under the perspective of the theoretical framework studied.

Actions that support mathematical reasoning in mathematics classes

According to Mata-Pereira and Ponte (2018), for teachers to be able to promote mathematical reasoning in the classroom, it is especially necessary that they have knowledge about mathematical reasoning itself and the reasoning processes of students.

The understandings about reasoning mathematically presented in the literature point to consensus among scholars. According to Morais et al. (2018), mathematical reasoning can be described as "a set of complex mental processes from which new propositions (new knowledge) are derived, based on propositions which are known or assumed to be true (prior knowledge)" (p. 782). Stylianides (2009, citado em Araman & Serrazina, 2020a), describes this process as "an inference process through which previously known mathematical information is used to obtain new knowledge or draw new conclusions". Jeannotte and Kieran (2017) understand mathematical reasoning as "a communication process, with others or with oneself, which allows one to infer mathematical statements from other mathematical statements" (p. 7). Even though the above-mentioned authors phrase their definitions differently, all agree on the need to develop new mathematical knowledge based on existing knowledge. However, in order to comprehend that, it is necessary to understand both aspects of mathematical reasoning: structural and processing. For Jeannotte and Kieran (2017), such aspects are related, albeit studied separately. Regarding the structural aspect, these authors highlight deduction, induction, and abduction (Jeannotte & Kieran, 2017). With regard to processing, Jeannotte and Kieran (2017) identified eight mathematical reasoning processes divided into two categories – (i) search for similarities and differences (which comprises the processes of conjecturing; generalizing; identifying patterns; comparing; and classifying), and (ii) validation (comprising justification; proof; and formal proof), and a ninth process, which consists in exemplifying, which supports the processes described in the two previous categories. As a result of such understanding, in order to develop mathematical reasoning in classrooms, the teacher must consider some aspects, among which, the need to present Challenging tasks, which give rise to rich collective discussions and promote mathematical reasoning, as well as revising "certain expectations, creating an environment in which children express their thoughts" (Wood, 1997, p. 37).

In addition to proposing tasks with exploratory characteristics, the teacher must enable interactions in the classroom, encouraging students to explore, present and discuss their solutions, mobilizing different reasoning processes. The questioning of the teacher is fundamental for the development of students' mathematical reasoning, and dialogue must be promoted to foster the interaction among students and teachers.

Ponte et al. (2013) studied teacher actions that promote mathematical reasoning and organized such actions into four categories: Inviting actions, when teachers ask questions in order to introduce students to the discussion context; Guiding/Supporting actions, which include those through which teachers lead students to provide explanations about how they think; Informing/Suggesting actions, when teachers provide information, suggestions, explanations that support students' reasoning; and Challenging actions, through which teachers challenge students' to deepen their mathematical reasoning, putting them in a situation through which they advance into new terrain, "whether by means of representation, interpretation of statements, establishment of connections, or reasoning, arguing or evaluating" (Ponte et al. 2013, p. 59).

In more recent studies, Araman et al. (2019) advanced the research on teacher actions based on the model by Ponte et al. (2013). In addition to the categories, those authors prepared a summary table that describes the actions envisaged in each category, as shown in Table 1 below:

Table 1.

Analysis table of teacher actions that support mathematical reasoning (Araman, et al., 2019, p. 476)

<table>
<tr><td rowspan="4">CATEGORIES</td><td>Inviting</td><td>- Asks for answers to specific questions.
- Asks for reports on how they reach the solution.</td><td rowspan="4">ACTIONS</td></tr>
<tr><td>Guiding/Supporting</td><td>- Provides clues to students.
- Encourages explanation.
- Guides students’ thinking.
- Focuses students’ thinking on important facts.
- Encourages students to repeat their answers.
- Encourages students to re-elaborate their answers.</td></tr>
<tr><td>Informing/Suggesting</td><td>- Validates correct answers provided by students.
- Corrects incorrect answers provided by students.
- Re-elaborates answers provided by students.
- Provides information and explanations.
- Encourages and provides multiple resolution strategies.</td></tr>
<tr><td>Challenging</td><td>- Asks students to give reasons (justifications).
- Proposes challenges.
- Encourages evaluation.
- Encourages reflection.
- Pushes for precision.
- Pushes for generalization.</td></tr>
</table>

The Inviting category includes the actions through which the teacher "elicits information from students, either through direct questions, or through explanations of what they did, aiming to observe how students are thinking and what their understanding is about the theme" (Araman et al. 2019, p. 476).

In turn, the Guiding/Supporting category comprises actions in which the teacher, through questioning or explanations, "steers students' thoughts towards a certain situation, or focuses on important facts, or even gives students clues and encourages them to (re)consider their answers" (Araman et al. 2019, p. 476).

The Informing/Suggesting category encompasses teachers' reactions to the information provided by students. These are the actions that "validate or correct answers given by students, re-elaborating information given which might be incomplete or needs improvement, providing explanations and information, and requesting, or presenting other resolution strategies (Araman & Serrazina, 2020b, p 152). The Challenging category includes actions in which the teacher "attempts to expose students to a Challenging situation so that they can advance their mathematical reasoning, looking for new representation forms, establishing new connections, reflecting and evaluating the situation, generalizing and justifying" (Araman et al. 2019, p. 476).

It is worth pointing out that there is no strict sequence or a hierarchy among the categories. The actions performed by teachers do not necessarily need to occur in all instances, they can be presented in cyclic movements; advancing and retracting; nonetheless, some have greater potential to support mathematical reasoning.

Methodology

This article analyzes the actions performed by a primary education public-school teacher, working in the 1st Cycle, on the outskirts of Lisbon, when conducting a collective discussion while conducting an exploratory mathematical task in the classroom. This investigation uses a qualitative approach with an interpretative character and is part of a broader project that uses design-based research methodology (Cobb et al. 2016; Ponte et al., 2016). Data were collected through audio and video recordings of the class, which were then transcribed, with the free and informed consent of all participants, and the agreement of the school. The names of students and the teacher were changed in order to guarantee confidentiality.

The resolution of the task was conducted with 26 students in a 1st year class. The objective of the task was to develop calculation flexibility in addition problems (Figure 1).

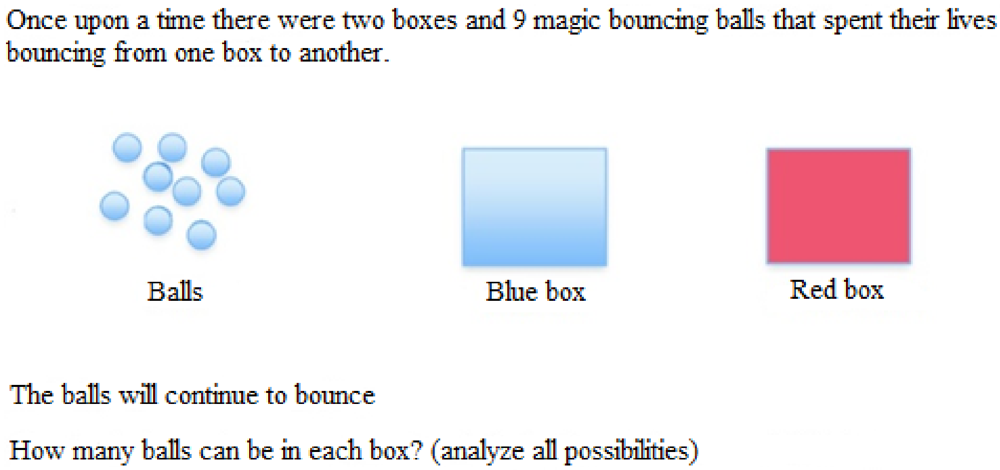

Figure 1.

Task "Box of Balls" (Research data)

Students formed pairs who first worked autonomously on the task, making individual written records on an answer sheet. After the autonomous exploration stage, some students were invited to present their resolutions on the board which sparked discussions, led by the teacher. The focus of the analysis is on the actions of the teacher during the collective discussion. For the analysis, we considered four action categories organized according to the synthesis of models regarding the actions of teachers that support mathematical reasoning (Araman et al., 2019), as shown in Table 1.

Results

Students sat in pairs and solved the task called “Box of Balls” (Figure 1) which consists of nine bouncing balls and two boxes, a blue and a red one. The balls bounce from one box to the other several times, and the students' task is to verify how many and which are the possible conditions of the balls while alternating between the boxes.

To do so, the teacher distributed a printout of the task and manipulable material, similar to buttons, to represent the balls. The students performed the task by drawing boxes with different numbers of balls in each, and making notes regarding the possibilities they found, with the condition that only nine balls were used.

After the class finished the proposed task, the teacher started a collective discussion of the task, which according to Wood (1997), presents potential for the development of mathematical reasoning.

In order to do that, students were called to the board so that they could show their solutions to their colleagues.

The first student at the board wrote: $6 + 3 = 9$. Then, the teacher asked:

Teacher: Explain that, please, Nina… **(Inviting)**
[The student immediately observed what she wrote on the board]
Nina: Well, what I did... [continued watching but did not respond].
Teacher: So what did you write, Nina? [pointing to the calculation written on the board]. **(Inviting)**
Nina: six plus three equals nine.
Teacher: Why 6+3? **(Challenging)**
[The student seemed to be reflecting on it but did not answer the question].
Teacher: the class can help. Why 6+3? **(Challenging)**
Student: Because... Because... from three to six it takes five.
Teacher: From three to reach nine it takes six... **(Informing/Suggesting)**

The first action performed by the teacher is in the category of Inviting, with the objective of initiating the discussion of the task with the students. However, when noticing the student's delay in explaining her solution, the teacher renewed her Inviting action, in order to encourage the student to provide her answer.

Faced with the answer provided by the student, the teacher asked her to explain the possibility on the board, asking her to present reasons to justify her answer, thus performing an action in the Challenging category.

The student did not present a justification to validate her conjecture, and the teacher extended the question to other students in the room, once again through the action of Challenging. Still, the teacher, in order to correct the student's train of thought, re-elaborates the wrong answer given, action listed under the Informing/Suggesting category.

Realizing the difficulty of the students in answering, she continues:

Teacher: *By the way*, in this question, we are working with boxes and with balls [reinforcing aspects of the task]. **(Informing/Suggesting)**
Teacher: Renato, why did you write 6+3? [indicating the numbers on the board] **(Challenging)**
Renato: Like 5 more... [the teacher interrupts, pointing to number six in the calculation].
Teacher: No! Number five is already here. **(Informing/Suggesting)**
[Renato continued his explanation.]
Renato: 5 plus 5 is 10, we thought that 6 plus 3 makes 9.

The teacher revisited some information regarding the task which could contribute to students' understanding, as indicated in the Informing/Suggesting category. Regarding this category, she corrected a wrong answer given. However, the teacher insisted that the students give reasons, asking Renato why he wrote $6 + 3$, an action provided for in the Challenging category (she asked students to give reasons). Renato presented an answer, but the answer provided still did not validate the conjecture.

Resuming the discussion, the teacher asked Renato:

Teacher: And where are the boxes here? [pointing to the calculation]. **(Guiding/Supporting)**
Renato: If 6 goes into the blue box, 3 goes into the red one.
Teacher: How about this 9? What is it? **(Guiding/Supporting)**
Renato: It's what it adds up to...!
Teacher: It's how much it adds up to? How come? **(Guiding/Supporting)**
Renato: The number of balls in the two boxes.
Teacher: The balls in both...? **(Guiding/Supporting)**
Students: [...] Boxes!

This excerpt shows the teacher performing several actions provided for in the Guiding/Supporting category, asking consecutive questions in order to guide students' thoughts and focus them on important facts. Through the teacher's intervention, Renato managed to elaborate a justification for his solution.

In order to continue the presentations of students' resolutions, the teacher called Gil, a student from another group, to show his resolution, once again, an Inviting action. After writing 8+1=9 on the board, Gil explained the resolution by saying:

Gil: We have eight balls in the red box, and one in the blue box. It's 9! [Gil exclaimed, pointing to the calculation].
Teacher: Does everyone agree?
Students: Yes!

Continuing with the presentations, the teacher told Gil to return to his place and invited another group to the board. Two students, Luana and Ilda, walked up. Ilda writes 9+0=9 on the board, while her colleague Luana watched. After doing so, the teacher asked Ilda to explain her solution, again performing the action of Inviting.

Ilda: Nine boxes...
Teacher: Are they boxes? [pointing to the calculation]
Ilda: Nine! It's nine...!
Teacher: Are they the boxes or are they balls? **(Informing/Suggesting)**
Ilda: Balls! So the nine balls are in the blue box and zero balls are in the red box.
Teacher: What is that...? [referring to the result] **(Guiding/Supporting)**
Ilda: That's nine!
Teacher: Does everyone agree?
Students: Yes!

This excerpt shows the teacher performing actions from two categories. By Informing/Suggesting, she corrected an inexact answer given by the student, and by Guiding/Supporting, she directed the student's reasoning towards the result of the operation. Luana presented her resolution and an explanation which was accepted by the class.

The teacher went on to call students to the board, in turn, at the end of each presentation, in an Inviting action. Then, she called Maria to show her calculations. On the board, Maria wrote 5+4=9 and explained:

> Maria: I did, 5+4...
> Teacher: Why? **(Challenging)**
> Maria: Because there are five boxes... five *balls* in the blue box and four balls in the red box.
> Teacher: And how many balls are there in total? **(Guiding/Supporting)**
> Maria: Nine!
> Teacher: In the two boxes? **(Guiding/Supporting)**
> Maria: Yes!

Faced with Maria's resolution, the teacher, once again, performed the Challenging action, asking Maria to present reasons for her solution. She also provided a Guiding/Supporting action, guiding the student's reasoning through the questions asked, and calling the attention of the class to important facts.

Another student was invited to present her resolution. This time, Mónica went to the board and wrote 4+5=9. When Mónica wrote her calculation on the board, Maria made the following comment:

> Maria: Mónica is doing the opposite of mine.
> [After Mónica finished writing her calculation on the board, the teacher continued.]
> Teacher: It's the same as Maria's, right?! [referring to the calculation made by Maria] **(Challenging)**
> Mónica: No!
> Teacher: Maria, how is your calculation? **(Guiding/Supporting)**
> Maria: In mine there are five balls in the blue box plus four balls in the red box.
> Teacher: She said there are 5 in the blue and 4 in the red one. And you? How about yours? [asking Mónica]. **(Guiding/Supporting)**
> [As Mónica still took some time to think about the calculations made and written on the board, the teacher once again talked about how Maria's calculation had been done]
> Teacher: 4 in…? [pointing to each element of the calculation]. **(Guiding/Supporting)**
> Mónica: […] blue box and 5 in the red one.
> Teacher: And what's the difference between this one [pointing to Maria's calculation] and this one [pointing to Mónica's]? **(Challenging)**
> Mónica: It's switched! [referring to the amounts contained in the blue and red boxes].

This excerpt shows that the teacher's actions encompassed both Guiding/Supporting and Challenging categories. When questioning the class if the resolution presented by Mónica was the same as Maria's (if 5+4 was the same as 4+5, although nine was the result for both), the teacher encouraged reflection. Then, she performed Guiding/Supporting actions, encouraging explanations and guiding the students' reasoning. After that, the teacher launched the challenge again, to which the students responded promptly, after Guiding/Supporting actions.

It is Paulo’s turn at the board, where he wrote 3+6=9. Resuming the discussion, the teacher reminded the class about Nina's calculation, which was “6+3=9”.

> Teacher: What's the difference between this [pointing to Nina's calculation] and that one [pointing to Paulo's]? **(Challenging)**
> Nina: I put 6 in the blue box and 3 in the red box.
> Teacher: How about yours? [asking Paulo] **(Guiding/Supporting)**
> Paulo: I did... [Paulo did not finish his explanation]
> Teacher: She got 6 in the blue box and 3 in red one, yours is the other way around. [pointing to the calculations, so that Paulo could observe the difference]. **(Informing/Suggesting)**
> [As the teacher also perceived a certain resistance in Paulo while giving an explanation, she continued with the dialogue]
> Teacher: 3 in the blue one... And 6 in...? **(Guiding/Supporting)**
> Paulo: In the red one!
> [Soon after, the teacher asked Paulo to repeat his thoughts in a louder voice so that the classmates could hear].
> Paulo: 3 in the blue box and 6 in the red box.

The same dynamics of the previous section is repeated here. The teacher launched a challenge, encouraging students to reflect on the difference between Paulo’s and Nina’s resolutions; thus a Challenging action. Then, she performed Guiding/Supporting actions, by directing students’ reasoning, and Informing/Suggesting, by providing explanations, in order to encourage them to further their reflection.

The next students at the board were Nádia and João, who wrote 7+2=9. With the following explanation:

> Nadia: seven in the blue box and two in the red box, which equals nine.
> Teacher: Does everyone agree?
> Students: Yes!
> [Another group, formed by Marta and Aline, went to the board, and wrote 2+7=9 and immediately began explaining]
> Marta: 2 in the red box and 7 in the blue box.
> [The teacher followed an order of the boxes in which the first element of the sum is for the blue box and the second element is for the red box (as shown in the illustration of the proposed task)]
> Teacher: Two in the red box? But the one on top [pointing to the previous calculation] said there were seven in the blue box and two in the red box… So, two in the blue box and seven in the ….. box? **(Informing/Suggesting)**
> Marta: Red one!
> Teacher: Do you have any more? **(Challenging)**
> Luis: It seems so! Looks like there’s still one left… More than one, two!
> Students: Two, Two!

In view of the resolutions presented by the peers, the teacher observed that there is confusion between 7 + 2 = 9 (seven balls in the blue box plus two balls in the red box) and 2 + 7 = 9 (two balls in the blue box plus seven balls in the red box). Then, she intervened in order to correct the student's answer, an action in the Informing/Suggesting category. Note that, in addition to the commutative property of addition, the teacher wanted them to realize that in the context of the task, 7 + 2 is different from 2 + 7, even though both add up to 9. Faced with the several possibilities already discussed by the class, she encouraged students to reflect by asking if there were still other possibilities, an action from the Challenging category.

The students started saying that there were two of them and that they would be the other way around, so the teacher asked Luís:

> Teacher: And how might this be done? [pointing to the expression 8+1=9] **(Guiding/Supporting)**.
> Luís: Eight in the red box and one in the blue box.
> Teacher: Eight in the red one and one in the blue! **(Informing/Suggesting)**.
> [A new pair goes to the board and shows 1+8=9]
> Teacher: So you did it the other way around? [referring to the previous and current expression 8+1=9 and 1+8=9] **(Guiding/Supporting)**.
> Luana: One in the red box and eight in the blue box.
> Teacher: Does everyone agree with Luana?
> Students: Yes
> Teacher: Look at Marta's group here.
> [A new pair, Marta and Alexandre, went to the board and wrote 0+9=9]
> Alexandre: Zero in one box and nine in the other.
> Teacher: And how is it up here? [pointing to the expression 9+0=9] **(Guiding/Supporting)**.
> Alexandre: Nine in one and zero in the other.
> Teacher: You put zero in the blue one and where are these nine? [pointing to expressions] **(Guiding/Supporting)**.
> [Alexander did not answer.]
> Teacher: If zero is in the blue box, does it have to be nine?
> Student: In the blue one!
> Teacher: Blue, if zero is in the red one, the opposite has to be nine in the red one. **(Guiding/Supporting)**.

By presenting more possibilities for resolutions (8 + 1 and 1 + 8, 9 +0 and 0 + 9), the teacher wanted the students to perceive the difference between them. In order to do that, she performed Guiding/Supporting actions, focusing students' attention on important facts. In Figure 2, it is possible to observe on the board the resolutions presented by students.

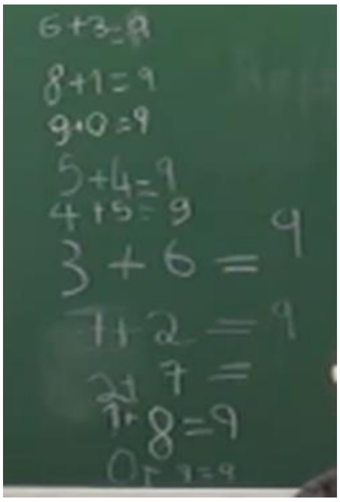

Figure 2.

Resolutions presented by students (Research data)

The teacher continued the discussion:

Teacher: How many hypotheses do we have here? **(Guiding/Supporting)**.
Students: Ten!
Teacher: Does anyone have any more hypotheses to present? (Guiding/Supporting).
[No response from students, so she continued.]
Teacher: Which groups did not have ten hypotheses?
[Some students raised their hands]
Teacher: Gil's group has more, this group has more [points to a new group], Alexandre and João's sheet has more, right? How many balls are there? **(Guiding/Supporting)**.
Students: Nine!
Teacher: How many hypotheses were most indicated? **(Guiding/Supporting)**.
Students: Ten!
Teacher: What is the number of hypotheses in relation to the number of balls? **(Guiding/Supporting)**.
Student: It's one more.
Teacher: It's one more! **(Informing/Suggesting)**.

In this excerpt, the teacher, knowing that some students solved the task in other ways, but incorrectly, wanted to take advantage of these other resolutions for a class discussion. Then, through Guiding/Supporting actions, she led the students' reasoning in order to steer the discussion towards other possibilities that were not valid for the task, but of which the students were not aware, yet. In the end, she performed an action in the Informing/Suggesting category, while validating a correct answer provided by a student.

A new pair of students was invited to come to the board.

Teacher: Present the ones you got, Alexandre. **(Inviting)**
Alexandre: I made three! One times nine.
Teacher: Do you have any more? [she said moving to Alexandre's table]. Let's see your sheet! [holding the sheet in his hands, she told Alexandre while he finished writing 9x1=9 on the board] **(Inviting)**

Teacher: Tell me, Alexandre. Why did you do these? Explain, please, Alexande why did you add these? Explain Alexandre, why did you write 1x9=9, 9x1=9 and 1+1+1... how many times one? **(Guiding/Supporting)**
Alexander: Nine.

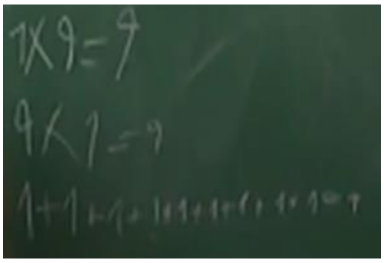

Figure 3.

Resolutions presented by Alexandre and his pair (Research data)

Alexandre and his classmate presented three expressions on the board, as seen in Figure 3. The teacher, faced with the perception that those solutions were not adequate and could trigger a good discussion, encouraged students to present their resolutions and explain what they had done, actions included in the Inviting and Guiding/Supporting category. She continued questioning.

Teacher: Let's see how many colleagues agree with this hypothesis. **(Guiding/Supporting)**
Luana: Yes.
Teacher: Why? Luana? **(Challenging)**
Teacher: Look, we have to discuss whether this makes sense or not!
Teacher: Luís, tell me why you said it makes sense. [while all the students were speaking at the same time, Luís had spoken]. You said it makes sense. So, Luís, why did you say it makes sense? **(Challenging)**
Teacher: Look, what's below, where are the boxes? No boxes here? How many boxes do you have to have in this part?
Students: Nine!
Teacher: Says Luana, how many boxes appear here? **(Guiding/Supporting)**
Students: Nine!
Teacher: And here? Nine times one. [pointing to the 9x1=9 expression] **(Guiding/Supporting)**
Students: Two!
Teacher: which representation is that? [still pointing to 9x1=9] **(Guiding/Supporting)**
Luan: 1+1+1+1+1+1+1+1+1=9.
Teacher: So how many boxes are there? [pointing to 9x1=9] **(Guiding/Supporting)**
Teacher: So; Nina, do nine boxes make sense? **(Challenging)**
Students: No!
Teacher: And this? [points to 1x9=9] **(Guiding/Supporting)**
Nina: This one does.
Teacher: Why? **(Challenging)**
Nina: Because it's nine only once.

Teacher: Once the nine **(Informing/Suggesting)** and where is “nine”? **(Guiding/Supporting)**
Nina: What?
Teacher: Where are the boxes here? Where are the boxes? **(Guiding/Supporting)**
Guilherme: At the nine.
Teacher: One times nine? That is how it is? [points to 1+1+...+1=9] **(Guiding/Supporting)**
Guilherme: No, it's the other way around.
Teacher: So, how many boxes are there [points to 1x9=9]. Guilherme?! Guilherme, how many boxes do we have here? Tell us Guilherme! **(Guiding/Supporting)**
Guilherme: One.
Teacher: One box **(Informing/Suggesting)**, and how many balls are in that box? [pointing to the nine in the 1x9=9 expression] **(Guiding/Supporting)**
Guilherme : Nine.
Teacher: Nine! **(Informing/Suggesting)** [asking all students] Do we have only one box? **(Guiding/Supporting)**
Students: No.
Teacher: But all this makes nine, why doesn't this make sense in this challenge? **(Challenging)**
Maria: We tried to do like the number of the day... we did the math the way we could. [student was referring to another exploratory task called “*number of the day*”]
Teacher: To get to nine? Why? What we have here is important. [shows activity sheet] **(Challenging)**
Maria: We have to do it with the two boxes and with the balls.
Teacher: With the nine balls? **(Guiding/Supporting)**
Mary agreed.
Teacher: Can we do it any way we want, to get to nine? Does that make sense for this challenge? **(Challenging)**

In this part of the discussion, the teacher resorted to several actions in order to help students realize, through a process of validating the conjectures presented, that, although those operations result in 9, they do not satisfy the task conditions. At times, she summarized the correct answer provided by the student (Informing/Suggesting category), and in several other instances she took Guiding/Supporting actions, asking a series of questions that steered the student's thinking, and focus onto the aspects she intended emphasize, such as the number of boxes. In addition, at other times she challenged students by questioning whether, in a particular case, the resolutions presented by a pair made sense and why (actions in the Challenging category).

In response to the teacher, some students said that it did not make sense. The teacher invited João to the board to continue the presentations and asked him to present a different expression from those already presented.

João and Armando went to the board and wrote “3+4+2=9 and 2+4+3=9”. Then, the teacher asked everyone if that expression made sense. Figure 4 illustrates what they wrote:

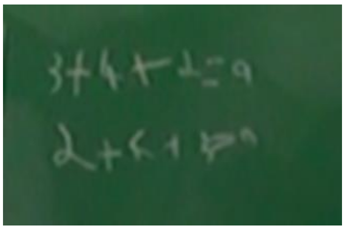

Figure 4.

Resolutions presented by João and Armando (Research data)

Teacher: Look, how many boxes appear here? [indicating the expression 3+4+2=9]. **(Guiding/Supporting)**
Marta: Three!
Teacher: Tell me, Marta. Why? **(Challenging)**
[Marta did not answer]
Teacher: These three...? These three balls will go...? To one of the boxes! Could it be the...? [pointing to the number three in the expression]. **(Guiding/Supporting)**
Marta: Blue. [referring to one of the boxes].
Teacher: And these four balls? [pointing to the number four in the expression]. **(Guiding/Supporting)**
Marta: Into the red one!
Teacher: And these two? To the yellow one? Is there a yellow box? [indicating the number two in the expression]. **(Guiding/Supporting)**
Students: No!
Teacher: But that makes nine! **(Informing/Suggesting)**
Guilherme: Also nine, but another way!
Teacher: Guilherme, what do you mean by “but it's another way”? **(Guiding/Supporting)**
Guilherme: You can get to nine any way!
Teacher: But is that what the problem called for? Armando and João, was it? Do we have three boxes? **(Challenging)** [Students shake their heads in response].

At this stage of the discussion, the teacher calls the students' attention to a solution presented by João and Armando, where they present two expressions with a sum of three numbers: 3 + 4 + 2 = 9 and 2 + 4 + 3 =9. In order to lead the students' thoughts towards the perception that those resolutions are not valid (validation process), the teacher performs actions in the categories of Guiding/Supporting, Informing/Suggesting and Challenging. The excerpt exemplifies the amalgamation afforded by the teacher's actions that, however analyzed under separate categories, will contribute in concert to the students' mathematical reasoning. First, the teacher questions Marta (Why?). Given her difficulty justifying her answer, the teacher resets the discussion through Guiding/Supporting and Informing/Suggesting actions. Together such actions help students realize that what is at stake is not just getting to the result; nine.

Then, the teacher invited the pair Gil and Mónica to present their resolution. They, in turn, wrote the following on the board: 10 – 1 = 9, 11 – 2 = 9, 12 – 3 = 9 and 13 – 4 = 9, as shown in Figure 5, below:

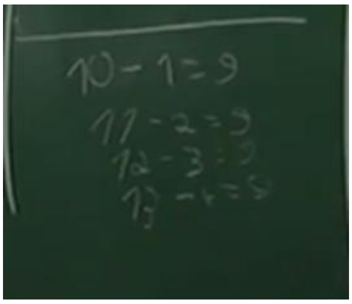

Figure 5.

Resolutions presented by Gil and Mónica (Research data)

The teacher asked the pair:

Teacher: Ok, Gil, please, explain! It was your idea, and Monica says she didn't agree. **(inviting)**
Mónica: We have nine balls on the sheet, but where are we going to get another ball? [question asked by the student to classmates, regarding the number of balls numerically described by Gil, the colleague].
Teacher: Where is this “extra ball” here? [referring to the expression “10 – 1=9”]. **(Guiding/Supporting)**
Mónica: In 10 – 1...! Now, how can we get one out of the “thing”?
Teacher: You're saying we don't have 10 balls! Here the balls were bouncing, they bounced from side to side. [indicating the expression 10 – 1 = 9]. **(Informing/Suggesting)**
Mónica: And also 11 – 2, how come we have 11...? And 12...? And 13...?
Teacher: What did Gil try to do? Maria! **(Guiding/Supporting)**
Maria: Gil tried to make us try to have more balls, but that doesn't make sense, we don't even have 13 balls, how are we going to do these calculations?!!
Teacher: But what did he do? [asking the class] **(Guiding/Supporting)**
Students: He used “less” instead of “more” [referring to the signs used in the expressions].
Teacher: He used a...? **(Guiding/Supporting)**
Students: A difference! [referring to the minus sign]
Teacher: A subtraction! **(Informing/Suggesting)**

At this point in the discussion, the teacher chose the resolution made by Gil and Mónica to be discussed. Gil presented possibilities that, despite not being adequate in relation to what the task required, had a potential for discussion by the students. To maintain the dynamics of this discussion, the teacher conducted the actions of Inviting, Guiding/Supporting, and Informing/Suggesting. In this excerpt, Gil did not justify the way he did the task. Still regarding the dynamics of the discussion, Mónica mobilized the validation process, since she did not agree with the resolution presented by her peer and presented a justification in the form of “we don’t even have 13 balls, how are we going to do these calculations?”. The students realized that Gil's conjecture is not valid as subtraction does not satisfy the task.

Finally, for the last presentation, the teacher called on Dario and Lúcia to show how they solved the task. The students immediately went to the board and wrote the possibilities organized as shown in Figure 6, below:

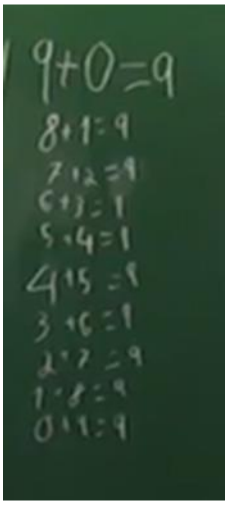

Figure 6.

Resolutions presented by Dario and Lúcia (Research data)

Then, the teacher started asking questions:

Teacher: Lucia, please, explain why you erased everything, and why you decided to organize your work again? **(Inviting)** [They had already done the work, but had erased everything...]
Dario: We had already done some, but we erased everything because I organized this way! First we did... We were doing another organization, first we did one calculation [referring to just one of the calculations made] and then we did the opposite.
Teacher: But what kind of organization is this? I had not seen it yet...! **(Guiding/Supporting)**
Dario: It is... a Sequence!
Teacher: Sequence? How? Please, explain... Explain what happens in the sequence. **(Guiding/Supporting)**
Dario: Here it will increase [referring to the elements on the right side of the calculation in relation to the plus sign and from top to bottom].
Teacher: Increase by how much? **(Guiding/Supporting)**
Dario: By one! And here it's going to decrease by one [referring to the elements on the left side of the calculation in relation to the plus sign, and from bottom to top].
Teacher: Look, and what happens in addition? **(Guiding/Supporting)**
Students: When we decrease 1, we must add 1...
Teacher: When we take one... If we take a ball out of a box...? What would we have to do? **(Guiding/Supporting)**
Students: Put another...!
Teacher: We would have to put it in the other box! **(Informing/Suggesting)** To keep the nine...? **(Guiding/Supporting)**
Students: Balls!

Teacher: What is happening here? [indicating the calculation 9 + 0 = 9]. Nine in one box and zero in the other, if I take one of the nine balls how many will be left? **(Guiding/Supporting)**
Students: eight!
Teacher: And what do I have to do with this ball that I took out of that box? **(Guiding/Supporting)**
Students: Put it in the other box!
Teacher: Do you think this organization makes sense?
Students: Yes!

At this stage of the discussion, Guiding/Supporting actions predominated. In view of the resolution presented by these students, the teacher asked a series of questions so that the other students would discern an existing pattern in the task, in a process of generalization: if I take a ball from a box I have to put it in the other one to keep the same amount. Although this relationship has not been generalized to other quantities, such perception can be used by the teacher in future tasks, contributing to the understanding of the meaning of addition.

Discussion and final considerations

At the beginning of the discussion, being invited by the teacher, Nina presented her resolution for the task, (6 + 3 = 9), which evidenced the elaboration of a conjecture on her part. We agree with Araman and Serrazina (2020a), when they regard each resolution strategy presented by students as a conjecture elaborated by them, as, when students elaborate a strategy, define a procedure to be used, albeit unconsciously, they presume that this path leads them to a probable or possible result.

After that, in a justification process, the teacher asked Nina to give reasons in support of her conjecture. When asking for reasons for the solution presented by the student, the teacher’s actions fall into the category of Challenging. Actions in this category have a high potential for fostering the development of mathematical reasoning (Ponte et al., 2013).

However, that student was unable to justify her decision, which led the teacher to carry out a series Guiding/Supporting, and Informing/Suggesting actions, aimed towards that student as well as other students in the class, with the intention that they would be able to formulate a justification for their colleague's resolution. According to Araman et al. (2019), the actions provided for in these categories are integral for the development of mathematical reasoning, as they support the discussion initiated by the teacher, and provide elements for students to further their understandings, thus functioning as a framework for perceptions which lead to new understandings. All such actions converged, and a student called Renato was able to formulate a possible justification (If the 6 goes into the blue box, the 3 goes into the red one).

Then, the discussion continued; the teacher called on other students to present their resolutions, through actions of Inviting. In face of the resolution 3 + 6 = 9, the teacher again challenged the students to justify whether it was the same as the resolution previously presented by Nina and why. A new round of actions in the Guiding/Supporting, and Informing/Suggesting categories then took place, with the objective of leading students to realize and justify that, even though the two resolutions yield the same result, they express different situations, expanding their understandings regarding the meaning of addition, and the commutative property. The same occurred regarding resolutions 7 + 2 and 2 + 7; 8+1 and 1+8; etc.

Once again we observed that the actions of the teacher comprise a cycle which begins with Inviting, followed by Challenging actions. At some point, she took several actions regarding the Guiding/Supporting, and Informing/Suggesting categories, which, as well as playing a role in sustaining the discussion and fostering understandings, provoked students to justify their answers. Araman et al. (2020) state that “such actions permeate the entire discussion, as they start from specific issues and evolve throughout the discussion, leading students' thinking where the teacher wants” (p. 457).

Realizing that some students presented resolutions that were not valid for the task, the teacher invited them to share their resolutions with colleagues. Resolutions such as 1 X 9 = 9; 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9; 3 + 4 + 2 = 9; 10 – 1 = 9, among others, were presented. In order to guide the students in a process of validating such resolutions, once again the teacher performed several actions from the four categories, at all times, attempting to help students reach the realization that such resolutions were not valid, and were able to give reasons.

During the discussion, mediated by the teacher's actions, through a validation process, and by elaborating justifications, the students were able to understand that, for the task at hand, obtaining the result “nine” alone did not yield a correct solution. According to Ellis et al. (2018), these actions have a high potential for promoting mathematical reasoning, as they help students to justify their thinking, offering underlying reasons.

Finally, the teacher invites to the board a pair of students (Dario and Lúcia), who used a representation that helped other students understand. And, once again, through different actions, the teacher elicits a justification for the resolution presented, expanding their understanding of the meaning of addition, which can be seen when the students conclude “when we take 1 (from a box), we have to put 1 (in the other box)”, in a process of generalization.

It should be noted that the teacher's actions were fundamental to the discussion of the task. Actions in the Inviting category led students to explain how they solved the task. Such actions do not lead directly to the development of mathematical reasoning; however, they foster the students' involvement in the discussion. It is worth noting that, in this case, by inviting students to report inadequate resolutions for the context, the teacher initiated relevant discussions that contributed to the validation process. Actions in the Guiding/Supporting and Informing/Suggesting categories led the students to formulate resolution strategies, identify patterns, and elaborate conjectures. These actions combined supported more elaborate reasoning processes that occurred as a result of Challenging actions, such as the validation of conjectures, through justification and generalization.

The results obtained in this study show the importance of developing mathematical reasoning, based on its processes, with students in the early years of schooling. They also show the relevance of teachers' actions and "the way through which such actions open possibilities for mathematical reasoning processes in the early years of education" (Araman et al. 2020, p. 459).

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