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Digital technologies in geometry teaching: a systematic literature review

Tecnologías digitales en la enseñanza de la geometría: una revisión sistemática de la literatura

Les technologies numériques dans l'enseignement de la géométrie : une revue systématique de la littérature

Tecnologias digitais no ensino de geometria: uma revisão sistemática da literatura

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Abstract

Digital technologies (DTs) in geometry teaching are one of the emerging themes in mathematics education. This article aims to present a systematic literature review on the integration of DTs in geometry teaching. Two guiding questions are discussed: What are the main gaps associated with geometry teaching? How can DTs contribute to geometry teaching? To this end, 30 articles selected from national and international journals were analyzed. In the vertical and horizontal analyses, six aspects stood out for understanding each article. Concerning gaps, the results point from teachers' lack of versatility in the face of DTs to the scarcity of subjects that boost geometry teaching during initial education. Among the advantages, it is clear that DTs allow diverse and dynamic approaches, which can contribute to the advancement of students' conceptual understanding of geometric objects.

Keywords: Bibliographic research, Dynamic geometry software, mathematics education.

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Resumen

Las tecnologías digitales (DT) en la enseñanza de la geometría son uno de los temas emergentes en la educación matemática. El objetivo de este artículo es presentar una revisión sistemática de la literatura relacionada con la integración de las DT en la enseñanza de la geometría. Se discuten dos preguntas orientadoras: ¿Cuáles son las principales brechas asociadas a la enseñanza de la geometría? ¿Cómo puede contribuir la DT a la enseñanza de la geometría? Para ello, se analizaron 30 artículos, seleccionados de revistas nacionales e internacionales. En los análisis verticales y horizontales se destacaron seis aspectos para la comprensión de cada artículo. En relación a las fallas, los resultados apuntan desde la falta de polivalencia de los docentes frente a las DT hasta la escasez de asignaturas que potencien la enseñanza de la geometría durante la formación inicial. Entre las ventajas, está claro que las DT permiten enfoques diversos y dinámicos, que pueden contribuir al avance de la comprensión conceptual de los objetos geométricos por parte de los estudiantes.

Palabras clave: Investigación bibliográfica, Software de geometría dinámica, Educación matemática.

Résumé

Les technologies numériques (DT) dans l'enseignement de la géométrie sont l'une des thématiques émergentes dans l'enseignement des mathématiques. L'objectif de cet article est de présenter une revue systématique de la littérature liée à l'intégration du DT dans l'enseignement de la géométrie. Deux questions directrices sont abordées : Quelles sont les principales lacunes associées à l’enseignement de la géométrie ? et Comment DT peut-il contribuer à l'enseignement de la géométrie ?. À cette fin, 30 articles ont été analysés, sélectionnés dans des revues nationales et internationales. Dans les analyses verticales et horizontales, six aspects ressortent pour la compréhension de chaque article. En ce qui concerne les lacunes, les résultats pointent du manque de polyvalence des enseignants face au DT à la rareté des matières qui valorisent l'enseignement de la géométrie lors de la formation initiale. Parmi les avantages, il est clair que le DT permet des approches diverses et dynamiques, qui peuvent contribuer à l'avancement de la compréhension conceptuelle des objets géométriques par les étudiants.

Mots-clés: Recherche bibliographique, Logiciel de géométrie dynamique, Éducation des mathématiques.

Resumo

As tecnologias digitais (TD) no ensino de geometria são um dos temas emergentes em educação matemática. O objetivo deste artigo é *apresentar uma revisão sistemática de literatura relacionada à integração de TD no ensino de geometria*. Discutem-se duas questões norteadoras: *Quais as principais lacunas associadas ao ensino de Geometria? Como as TD podem contribuir com o ensino de Geometria?* Para tanto, analisaram-se 30 artigos, selecionados de periódicos nacionais e internacionais. As análises verticais e horizontais destacaram seis aspectos para compreensão de cada artigo. Em relação às lacunas, os resultados apontam desde a falta de versatilidade dos professores diante das TD até a escassez de disciplinas que potencializam o ensino de geometria durante a formação inicial. Entre as vantagens, evidencia-se que as TD permitem abordagens diversificadas e dinâmicas que podem contribuir para o avanço da compreensão conceitual dos objetos geométricos por parte dos estudantes.

Palavras-chave: Pesquisa bibliográfica, Softwares de geometria dinâmica, Educação matemática.

Digital technologies in geometry teaching: a systematic literature review

Geometry teaching in Brazil went through several phases, including periods of abandonment, particularly among the popular classes. This scenario worsened even more between the 1970s and 1980s due to the influence of the Modern Mathematics Movement. During this period, the approach to mathematical content began to emphasize the language of sets, making it challenging to understand geometric concepts and favoring their disengagement from the classroom. Thus, teachers' little contact with geometry contributed to gaps in the teaching approach at school. This trend prevails to this day, impacting the development of students' geometric thinking (Santos & Nacarato, 2021).

Geometric thinking has been defined as the mental capacity to produce knowledge in geometry. Gravina (2001) states that it develops through the analysis of forms –initially abstracted from our reality– and the deduction of theorems and demonstrations determined based on logical inference. In this sense, Santos and Nacarato (2021) highlight the relevance of teaching resources in developing this way of thinking, among which we situate digital technologies (DTs). These authors believe that the diversity of materials teachers make available to students allows them to reach higher levels of geometric thinking, involving manipulating, drawing, visualizing, and constructing mental images of the objects studied.

In the school context, Krackeker, Burigato, and Santos (2017) see DTs as a resource capable of promoting a more coherent education in light of the needs of today's society, assisting in learning, creating spaces for integration and communication, increasing the possibilities for creative expression, carrying out projects and critical reflections, in addition to being important for problem solving. Thus, linked to these notions, this research brings *a brief overview of the integration of DTs in geometry teaching through a systematic literature review*. The research addresses two guiding questions: *What are the main gaps associated with geometry teaching? How can DTs contribute to geometry teaching?*

A systematic review is a form of review research that uses existing literature on a given topic as a data source. It summarizes the evidence associated with a specific investigation by applying well-defined and systematized search methods, critical assessment, and an overview of selected information (Sampaio & Mancini, 2006). The articles that founded this review were constructed through consultations in national and international journals, using methodological procedures. After this description, we will present vertical analyses of selected articles and horizontal analyses comparing those procedures through six specific aspects as parameters. Finally, we will revisit the research questions to discuss the results, present conclusions, and indicate possibilities for future research.

Methodological procedures

Initially, we decided to conduct the present survey of articles in journals related to mathematics education or those with articles associated with this topic. We should consult national and international journals for a more comprehensive overview of the use and potential associated with digital technologies in mathematics classes, especially when working with plane and spatial geometry.

Domestically, based on the *Qualis* classification[3] from the Coordination of Higher Education Personnel (Capes) in education, we selected 11 journals published in 2013-2016 with strata between A1 and B2 to include at least one representative from each Brazilian region. After that, we searched for two descriptors in the articles, *geometria* [geometry] and *tecnologias digitais* [digital technologies], and researched them separately. Among the available results, we chose those related to geometry that mentioned some digital technology in the title, abstract, and/or keywords. Thus, we obtained 51 articles within the desired time frame, August 2016-August 2021, and excluded those unrelated to continuing teacher education, the final years of elementary or high school. Finally, ten journals and 26 articles remained, distributed in the five Brazilian regions, as shown in Table 1.

Table 1.

Distribution of national articles by region and periodical

Region	Periodical	Number of articles
Midwest	*Perspectivas da Educação Matemática [Perspectives of Mathematics Education]*	4
Northeast Region	*EM TEIA – Revista de Educação Matemática e Tecnológica Iberoamericana [Iberoamerican Journal of Mathematics and Technological Education]*	1
Northeast Region	*Revista Sergipana de Educação Matemática e Tecnológica Iberoamericana [Sergipe Iberoamerican Journal of Mathematics and Technological Education]*	4
North	*Amazônia – Revista de Educação em Ciências e Matemática [Sciences and Mathematics Education Journal]*	1
North	*REMATEC: Revista Matemática, Ensino e Cultura [Mathematics, Teaching, and Culture Journal]*	3

3 The most recent classification available when the search was ongoing and the definition of the *corpus* of articles

Southeast Region	*EMP: Educação Matemática Pesquisa [Mathematics Education Research Journal]*	2
Southeast Region	*Bolema – Boletim de Educação Matemática [Mathematics Education Bulletin]*	2
Southeast Region	*Zetetiké*	1
South	*REVEMAT: Revista Eletrônica de Educação Matemática [Electronic Journal of Mathematics Education]*	4
South	*BOEM: Boletim Online de Educação Matemática [Online Mathematics Education Bulletin]*	4

Internationally, we selected three journals based on the same fundamentals described before. Adopting descriptors *geometry* and *digital technologies* and following the same logic applied to the national articles, we separated 14 papers within the time frame considered. Then, applying the exclusion criteria mentioned before, four articles remained, as shown in Table 2.

Table 2.

Distribution of international articles by location and periodical

Local	**Periodical**	**Number of articles**
Europe	*UNIÓN: Revista Iberoamericana de Educación Matemática [Iberoamerican Journal of Mathematics Education]*	1
Europe	*ZDM: Mathematics Education*	2
Europe	*Educational Studies in Mathematics*	1

After the searches, 65 articles remained, downsized to 30 articles that met the criteria for analysis. Based on this selection methodology, we move on to the analysis, which is explained and reported below.

Definition of analysis aspects

This systematic literature review seeks to answer two questions: *What are the main gaps associated with geometry teaching? How can DTs contribute to geometry teaching?* To this end, structurally, two stages will be considered: vertical analysis and horizontal analysis.

In vertical analysis, each article comprising the dataset as a unit of analysis is summarized in a classification system that includes some aspects of interest. In horizontal analysis, the focus is no longer on the article as a whole but on the study of previously defined categories, establishing comparisons between the selected articles (Depaepe, Verschaffel, & Kelchtermans, 2013).

Therefore, this vertical analysis will present the objectives and DTs highlighted throughout the study for the 30 articles of interest. Subsequently, in the horizontal analysis, the corpus will be analyzed based on six categories: (1) The focus of the investigation: the objective of the article or research question, (2) Teaching level, (3) Continuing teacher education, (4) Mathematical content, (5) Gaps associated with geometry teaching and learning, and (6) Results, conclusions, and final considerations (Gumiero & Pazuch, 2020).

Presentation and discussion of materials

This section will bring the results, starting with vertical analysis and moving on to horizontal analysis, seeking to detail the content of the articles selected in this systematic literature review. The corpus of analysis is described in Table 3.

Table 3.

Articles analyzed

Author	Title	Year
Bastos, Galvão, & Souza	O ensino de perspectiva para alunos do ensino médio num ambiente de Geometria Dinâmica	2019
Fassio	Da cartolina ao computador: uma proposta para o estudo de geometria	2020
Rodrigues & Kaiber	Geometria Espacial no Ensino Médio: contribuições da utilização de uma Unidade de Ensino e Aprendizagem (UEA)	2019
Moran & Franco	O uso de tratamentos em registros figurais: representações na forma de material manipulável, *software* e expressão gráfica	2020
Rodrigues & Bellemain	A comparação de áreas de figuras planas em diferentes ambientes: papel e lápis, materiais manipulativos e no *Apprenti Géomètre* 2	2016
Pouzada et al.	Potencialidades, desafios e dificuldades de ensinar Geometria por meio das tecnologias	2020
Cabral & Almeida	Semelhança de triângulos e GeoGebra: uma alternativa de ensino por meio de representações dinâmicas	2020
Jesus & Pereira	Cálculo de áreas de quadriláteros irregulares: uma análise das contribuições de uma sequência didática	2019
Henrique & Bairral	Tecnologias Digitais móveis e metáforas: campos que se encontram em conceitos geométricos	2020
Silva & Bellemain	*Magnitude Studium*: um micromundo para o ensino de área e perímetro	2020
	Reflexão sobre o processo de elaboração de tarefas de	2021

Lecrer & Pazuch	geometria espacial em um movimento formativo de professores	
Sánchez, Castillo, & Luque	Tecnologías Digitales y la Geometria Escolar: El GeoGebra para la enseñanza del teorema de Pitágoras	2021
Batista & Paulo	Ver e visualizar em Geometria: uma experiência com o *software* GeoGebra	2021
Henrique & Bairral	Retas que se cruzam e dedos que se movem com dispositivos de geometria dinâmica	2019
Oliveira & Lima	Estratégias didáticas com tecnologias na formação continuada de professores de Matemática: uma investigação sobre homotetia	2018
Ramirez, Goycochea, & Osorio	Tipificación de argumentos producidos por las prácticas matemáticas de alumnos del nível medio en ambientes de geometria dinâmica	2021
Oliveira & Gonçalves	Construções em Geometria Euclidiana Plana: as perspectivas abertas por estratégias didáticas com tecnologias	2018
Powell & Pazuch	Tarefas e justificativas de professores em ambientes colaborativos de geometria dinâmica	2016
Volpatto, Fortes, & Silveira	Um estudo de caso envolvendo a aplicação do software educacional de geometria espacial	2018
Portella & Leivas	Uso do *software* GeoGebra para desenvolver conhecimentos acerca de algumas propriedades da circunferência	2017
Krakecker, Burigato, & Santos	Uma discussão sobre a definição de trapézio por meio do *software* GeoGebra	2017
Viana & Silva	Raciocínio geométrico e aprendizagem de congruência de triângulos	2020
Gutiérrez-Araujo & Pazuch	Elaboração de objetos de aprendizagem com o *software* GeoGebra para o ensino de geometria	2018
Rovetta & Silva	Potencialidades da rede social *Facebook* como um espaço complementar à sala de aula durante o estudo de sólidos geométricos: discussão de um produto educacional	2018
Ferreira & Scortegagna	Ensinando perímetro e área de figuras geométricas planas usando o software Geogebra	2018
Ballejo & Viali	Aprendizagem de conceitos de área e perímetro com o 6º do ensino fundamental	2018
Guimarães et al.	Uso de teléfonos inteligentes en la investigación sobre las propriedades de los cuadriláteros	2021
Greefrath, Hertleif, & Siller	Mathematical modelling with digital tools - a quantitative study on mathematising with dynamic geometry software	2018
Fujita, Jones, & Miyazaki	Learners' use of domain-specific computer-based feedback to overcome logical circularity in deductive proving in geometry	2018

Mithalal & Balacheff	The instrumental deconstruction as a link between drawing and geometrical figure. Educational Studies in Mathematics	2019

Vertical analysis

The table below explains the objective and the DTs highlighted throughout the selected articles.

Table 4.

Objectives and DTs cited in the articles comprising the corpus of analysis

Author	Objective	DTs used
Bastos et al. (2019)	Organize a set of activities that could introduce geometric ideas associated with perspective techniques to high school students, considering their historical path.	GeoGebra
Fassio (2020)	Study students' engagement in the final years of elementary school in a proposal associated with plane geometry that explored different manipulative materials and Geogebra[4].	GeoGebra
Rodrigues & Kaiber (2019)	Analyze the contributions of a teaching and learning unit based on the use of DTs for the development of topics related to spatial geometry.	GeoGebra
Moran & Franco (2020)	Analyze figural treatments throughout the development of a geometry task using three representations: manipulable material, GeoGebra, and graphic expression.	GeoGebra
Rodrigues & Bellemain (2016)	Discuss how elementary school students deal with a task focused on comparing areas of figures in three environments: paper and pencil, manipulative materials, and of dynamic geometry software *Apprenti Géomètre 2*.	*Apprenti Géomètre 2*
Pouzada et al. (2020)	Analyze the speech of basic education mathematics teachers, participants in a specialization class, to understand the potential, challenges, and difficulties associated with the use of DTs in geometry teaching.	-
Cabral & Almeida (2020)	Analyze whether activities focusing on dynamic representations contribute to developing knowledge associated with the theme of triangle similarity.	GeoGebra
Jesus & Pereira (2019)	Analyze the contributions of a didactic sequence based on the area of irregular quadrilaterals to the study of the articulation between the history of mathematics and DTs.	GeoGebra
Henrique & Bairral (2020)	Analyze the use of metaphors in a learning environment that seeks to stimulate conceptual discussions and the use of smartphones in geometry activities.	*Smartphones*

4 From this moment on, we will refer to the GeoGebra software just as GeoGebra.

Silva & Bellemain (2020)	Explain aspects of the conception and development of a microworld called *Magnitude Studio*, created to support teachers in teaching the concepts of area and perimeter.	*Magnitude Studium*
Lecrer & Pazuch (2021)	Analyze the development of investigative tasks related to the teaching of spatial geometry and the use of GeoGebra by mathematics teachers during an extension course.	GeoGebra
Sánchez et al. (2021)	Study the application of two sequences of activities to address the Pythagorean theorem using GeoGebra.	GeoGebra
Batista & Paulo (2021)	Present a report on an experience lived by basic education mathematics teachers, participants in a formative process.	GeoGebra
Henrique & Bairral (2019)	Raise contributions and challenges associated with learning parallel lines cut by a transversal.	GeoGebra for smartphones
Oliveira & Lima (2018)	Create teaching strategies for improving and re-signifying teachers' knowledge about the concept of homothety.	GeoGebra
Ramirez et al. (2021)	Identify and describe the types of arguments used by high school students when carrying out activities using GeoGebra.	GeoGebra
Oliveira & Gonçalves (2018)	Analyze how a teaching approach based on didactic situations linked to the use of the SuperLogo software can contribute to building significant learning associated with geometric constructions.	SuperLogo
Powell & Pazuch (2016)	Reflect on the geometric justifications prepared by mathematics teachers and participants in an extension course and analyze their resolutions of a task related to quadrilaterals and their bisectors in the *Virtual Math Teams with GeoGebra* (VMTwG) environment.	(VMTwG)
Volpatto et al. (2018)	Carry out a case study with high school students, addressing topics pertaining to spatial geometry.	GeoGebra
Portella & Leivas (2017)	Use technology to analyze the appropriation of properties related to the circumference in elementary education.	GeoGebra
Krakecker et al. (2017)	Discuss a study carried out with mathematics teachers doing their master's degrees, which aimed to analyze the learning possibilities provided by a cooperative process based on the use of GeoGebra to solve an activity related to the definition of a trapezoid.	GeoGebra
Viana & Silva (2020)	Identify advances in the geometric reasoning of a group of elementary school students by applying a didactic proposal related to the theme of triangle congruence, based on the manifestation of visual and logical skills.	Slides, animations, and GeoGebra
Gutiérrez-Araujo & Pazuch (2018)	Describe the creation of a learning object using GeoGebra, to facilitate the exploration and validation of geometric properties worked on in the final years of elementary school.	GeoGebra
Rovetta & Silva (2018)	Expose the contributions of an educational product called *Geometry on social media: enjoy this idea!*	*Facebook*

Ferreira & Scortegagna (2018)	Present the educational product *Teaching perimeter and area of flat geometric figures using GeoGebra.*	GeoGebra
Ballejo & Viali (2018)	Describe how GeoGebra can contribute to elementary school students' construction of perimeter and area concepts.	GeoGebra
Guimarães et al. (2021)	Investigate whether the use of the GeoGebra mobile app contributes to the process of surveying conjectures and mathematical justifications created by students.	GeoGebra for smartphones
Greefrath et al. (2018)	Answer two questions: (i) "Is there any difference in relation to mathematization competence between students who learn mathematical modeling with or without the use of any dynamic geometry software?"; (ii) "Is there a relationship between self-efficacy related to the program and attitudes towards learning with GeoGebra and the development of mathematization when learning to model using a dynamic geometry software?"	GeoGebra
Fujita et al. (2018)	Study how students can be helped to overcome logical circularity when constructing deductive proofs and investigate how a computer-based feedback supports this process.	Computer-based feedback
Mithalal & Balacheff (2019)	Examine how it is possible to overcome the gap between problems involving drawings and figures in geometry teaching and learning.	*Cabri Géomètre*

Horizontal analysis

In this section, we present the comparisons between the selected articles, sequentially considering the six categories of interest mentioned and explained below.

The focus of the investigation: the objective of the article or research question

Considering the objectives or research question of the articles, we identified five categories: *(A) Students' learning and development*: analysis of the contributions of differentiated didactic interventions in students' process of appropriation of mathematical knowledge; *(B) Teachers' professional learning and development*: exploration of ways to enhance teachers' learning; *(C) Teachers' professional knowledge*[5]*:* approach to knowledge produced by teachers in action, within the scope of their profession *(D) Students' conceptions:* study of students' conceptions of mathematical content; and *(E) Presentation of digital resources*: proposition of materials based on DTs. Table 5 relates the articles to these subcategories.

5 "A knowledge that is in teaching, that is, developed in action (contingent); that is in the profession, that is, defined in a dynamic of sharing and co-construction (collective); a knowledge that is in society, that is, projected outside the professional sphere and asserted in a broader (public) space" (Nóvoa, 2022, p. 8).

Table 5.

Categorization of objectives or research questions of the selected articles

Authors	A	B	C	D	E
Bastos et al. (2019)					
Fassio (2020)					
Rodrigues & Kaiber (2019)					
Moran & Franco (2020)					
Rodrigues & Bellemain (2016)					
Pouzada et al. (2020)					
Cabral & Almeida (2020)					
Jesuz & Pereira (2020)					
Henrique & Bairral (2020)					
Silva & Bellemain (2020)					
Lecrer & Pazuch (2021)					
Sánchez et al. (2021)					
Batista & Paulo (2021)					
Henrique & Bairral (2019)					
Oliveira & Lima (2018)					
Ramírez et al. (2021)					
Oliveira & Gonçalves (2018)					
Powell & Pazuch (2016)					
Volpatto et al. (2018)					
Portella & Leivas (2017)					
Krakecker et al. (2017)					
Viana & Silva (2020)					
Gutiérrez-Araújo & Pazuch (2018)					
Rovetta & Silva (2018)					
Ferreira & Scortegagna (2018)					
Ballejo & Viali (2018)					
Guimarães et al. (2021)					
Greefrath et al. (2018)					

Fujita et al. (2018)					
Mithalal & Balacheff (2018)					

Level of education

Regarding the level of education, 23 of the 30 articles selected presented studies involving students from *(A) elementary education* or *(B) high school* or were intended for them, in the case of educational resources, as can be seen in Table 6.

Table 6.

Level of education explored in research

Authors	A	B
Bastos et al. (2019)		
Fassio (2020)		
Rodrigues & Kaiber (2019)		
Rodrigues & Bellemain (2016)		
Cabral & Almeida (2020)		
Jesuz & Pereira (2020)		
Henrique & Bairral (2020)		
Silva & Bellemain (2020)		
Sánchez et al. (2021)		
Henrique & Bairral (2019)		
Ramírez et al. (2021)		
Oliveira & Gonçalves (2018)		
Volpatto et al. (2018)		
Portella & Leivas (2017)		
Viana & Silva (2020)		
Gutiérrez-Araújo & Pazuch (2018)		
Rovetta & Silva (2018)		
Ferreira & Scortegagna (2018)		
Ballejo & Viali (2018)		
Guimarães et al. (2021)		

Greefrath et al. (2018)		
Fujita et al. (2018)		
Mithalal & Balacheff (2018)		

Continuing teacher education

Six of the articles presented research developed with teachers who were participating in continuing education courses, which we grouped into three subcategories: *(A) Specialization, (B) Qualification course, and (C) Master's degree.* Only two studies (Moran & Franco, 2020; Fujita et al., 2018) had as participants teachers working in basic education or supervised practicum students who were not part of the continuing education process.

Table 7.

Continuing education stages explored in research

Authors	A	B	C
Pouzada et al. (2020)			
Lecrer & Pazuch (2021)			
Batista & Paulo (2021)			
Oliveira & Lima (2018)			
Powell & Pazuch (2016)			
Krakecker et al. (2017)			

Mathematical content

The articles were separated into two subcategories regarding the mathematical content covered: *(A) Plane geometry* and *(B) Spatial geometry*. Two papers (Greefrath et al., 2018; Pouzada et al., 2020) did not specifically refer to the geometric content at issue.

Table 8.

Mathematical content explored in research

Authors	A	B
Bastos et al. (2019)		
Fassio (2020)		
Rodrigues & Kaiber (2019)		

Moran & Franco (2020)		
Rodrigues & Bellemain (2016)		
Cabral & Almeida (2020)		
Jesuz & Pereira (2020)		
Henrique & Bairral (2020)		
Silva & Bellemain (2020)		
Lecrer & Pazuch (2021)		
Sánchez et al. (2021)		
Batista & Paulo (2021)		
Henrique & Bairral (2019)		
Oliveira & Lima (2018)		
Ramírez et al. (2021)		
Oliveira & Gonçalves (2018)		
Powell & Pazuch (2016)		
Volpatto et al. (2018)		
Portella & Leivas (2017)		
Krakecker et al. (2017)		
Viana & Silva (2020)		
Gutiérrez-Araújo & Pazuch (2018)		
Rovetta & Silva (2018)		
Ferreira & Scortegagna (2018)		
Ballejo & Viali (2018)		
Guimarães et al. (2021)		
Fujita et al. (2018)		
Mithalal & Balacheff (2018)		

Gaps associated with geometry teaching and learning

Firstly, not all selected articles explicitly presented geometry teaching and learning gaps. However, we identified six subcategories based on the analysis of the eight works that addressed this information.

Table 9.

Gaps associated with geometry teaching and learning identified in research

Categories	Authors
Mismatch in integration between DTs and the classroom.	Guimarães et al. (2021)
Teachers' low versatility in the face of DTs.	Fassio (2021). Pouzada et al. (2020) Guimarães et al. (2021)
Absence of DTs in school environments.	Pouzada et al. (2020) Guimarães et al. (2021)
Prevalence of monotonous and decontextualized teaching approaches.	Rodrigues & Kaiber (2019) Cabral & Almeida (2020) Jesuz & Pereira (2020) Henrique & Bairral (2019) Ballejo & Viali (2018)
Lack of improvement in spatial thinking and exploratory reasoning.	Cabral & Almeida (2020)
Shortage of subjects in the teaching degree that enhance geometry teaching.	Pouzada et al. (2020) Rodrigues & Kaiber (2019)

Results, conclusions, and final considerations

Analyzing the results, conclusions, and final considerations of the articles in question, we found that only Greefrath et al.'s (2018) work states that dynamic geometry software did not significantly change the students' learning in their intervention. Among the other works, we highlight ten categories resulting from the researchers' considerations about using technological resources and geometry teaching and learning processes.

Table 10.

Distribution of research results, conclusions, and final considerations

Subcategories	Authors
Students' interest.	Bastos et al. (2019) Sánchez et al. (2021) Henrique & Bairral (2019) Volpatto et al. (2018) Ballejo & Viali (2018)
Improved learning of geometric concepts.	Bastos et al. (2019) Fassio (2020) Rodrigues & Kaiber (2019) Moran & Franco (2020) Rodrigues & Bellemain (2016) Pouzada et al. (2020) Cabral & Almeida (2020) Jesuz & Pereira (2020) Henrique & Bairral (2020) Silva & Bellemain (2020) Lecrer & Pazuch (2021) Sánchez et al. (2021) Batista & Paulo (2021)

	Henrique & Bairral (2019) Oliveira & Lima (2018) Oliveira & Gonçalves (2018) Powell & Pazuch (2016) Volpatto et al. (2018) Portella & Leivas (2017) Krakecker et al. (2017) Gutiérrez-Araujo & Pazuch (2018) Rovetta & Silva (2018) Ferreira & Scortegagna (2018) Ballejo & Viali (2018) Guimarães et al. (2021) Fujita et al. (2018) Mithalal & Balacheff (2018)
Increasing dynamism in mathematics classes	Henrique & Bairral (2020) Batista & Paulo (2021) Henrique & Bairral (2019) Volpatto et al. (2018)
Preparation of varied learning situations.	Bastos et al. (2019) Silva & Bellemain (2020)
Possibility of construction and advancement in the visualization of geometric solids.	Rodrigues & Kaiber (2019) Moran & Franco (2020) Rodrigues & Bellemain (2016) Pouzada et al. (2020) Silva & Bellemain (2020) Lecrer & Pazuch (2021) Sánchez et al. (2021) Batista & Paulo (2021) Ramirez et al. (2021) Oliveira & Gonçalves (2018) Powell & Pazuch (2016) Volpatto et al. (2018) Portella & Leivas (2017) Krakecker et al. (2017) Viana & Silva (2020) Gutiérrez-Araujo & Pazuch (2018) Rovetta & Silva (2018) Ferreira & Scortegagna (2018) Guimarães et al. (2021)
Enhancement of exploration and validation of geometric properties.	Lecrer & Pazuch (2021) Batista & Paulo (2021) Henrique & Bairral (2019) Oliveira & Lima (2018) Oliveira & Gonçalves (2018) Powell & Pazuch (2016) Volpatto et al. (2018) Portella & Leivas (2017) Krakecker et al. (2017) Gutiérrez-Araujo & Pazuch (2018) Rovetta & Silva (2018) Ferreira & Scortegagna (2018) Guimarães et al. (2021) Mithalal & Balacheff (2018)
	Rodrigues & Kaiber (2019) Cabral & Almeida (2020)

Assistance in argumentation and development of conjectures.	Jesuz & Pereira (2020) Henrique & Bairral (2020) Lecrer & Pazuch (2021) Sánchez et al. (2021) Powell & Pazuch (2016) Krakecker et al. (2017) Gutiérrez-Araujo & Pazuch (2018) Rovetta & Silva (2018) Ferreira & Scortegagna (2018) Guimarães et al. (2021) Fujita et al. (2018)
Assistance in the transition from concrete to abstract mental schemes.	Bastos et al. (2019) Volpatto et al. (2018) Portella & Leivas (2017) Mithalal & Balacheff (2018)
Exemption from drawing skills.	Bastos et al. (2019) Fassio (2020)
Approach to reality problems.	Pouzada et al. (2020) Ferreira & Scortegagna (2018)

Analysis of results: resumption of research questions

What are the main gaps associated with geometry teaching?

Research indicates that the main gaps associated with geometry teaching fall into six factors: mismatch in integration between DTs and the classroom; low versatility of teachers when using DTs; the prevalence of monotonous and decontextualized teaching approaches; the lack of improvement in spatial thinking and exploratory reasoning; scarcity of subjects that enhance the teaching of geometry during initial education and the absence of DTs in school environments.

Regarding the *mismatch in integration between DTs and the classroom*, Guimarães et al. (2021) point out that although discussions about DTs in education are increasingly present in the academic environment, the resources are still not actively inserted in basic education mathematics classes. This lack stops approaches from boosting learning, particularly geometry content learning, in which conventional resources hamper visualization and experimentation. The authors believe the precariousness observed in school computer laboratories and the lack of teacher training to work with DTs are some of the possible explanations for this reality.

Pouzada et al. (2020) state that teachers' lack of preparation for using DTs arises from the fact that many do not use them in their daily lives, added to the fact that such technologies are absent from the school environment. Fassio (2021) highlights that educational innovations take the teacher to a risk zone, causing the need to rethink the content to be taught and the ways to do it. As this situation generates discomfort, teachers tend to crystallize practices, especially in geometry teaching, a field still little explored throughout initial education.

Regarding the *prevalence of monotonous and decontextualized teaching approaches*, Rodrigues and Kaiber (2019) discuss the need to rethink geometry teaching. The authors warn that traditional methods have been producing unsatisfactory results, mainly because they are based on approaches that do not adapt to students' realities. Agreeing with these ideas, Cabral and Almeida (2020) emphasize that nowadays, geometry topics are worked on with an emphasis on formulas and procedures, negatively impacting the development of students' geometric thinking.

As for the *lack of improvement in spatial thinking and exploratory reasoning,* Cabral and Almeida (2020) say that developing geometry teaching that values those skills is essential so that it is not limited to automation, memorization, and operational techniques, which contribute little to students' development.

Regarding the *scarcity of subjects that enhance the teaching of geometry during initial education*, Rodrigues and Kaiber (2019) affirm that, in many cases, teachers do not address geometry topics in class because they find the content difficult, partly because they did not adequately appropriate it during their teaching degrees. Complementing this perception, Pouzada et al. (2020) indicate that initial education courses have ignored the educational addition technologies can offer for mathematics teaching.

Finally, regarding the *absence of DTs in school environments*, Pouzada et al. (2020) and Guimarães et al. (2021) highlight the precariousness of computer labs in Brazilian schools, which makes it hard to develop pedagogical practices to enhance geometry understanding.

How can DTs contribute to geometry teaching?

From Pouzada et al.'s (2020) perspective, DTs can enhance geometry teaching by allowing students to carry out and manipulate geometric constructions, expanding the visualization and experimentation of the concepts addressed. They complement the most common technologies, such as pencils, rulers, and compasses, as their ability to provide greater manipulative interactivity helps to improve students' understanding, overcoming the limitations associated with concrete materials, such as, for example, drawing skills.

Geometric investigations facilitated by DTs, particularly by dynamic geometry software, highlight important aspects of mathematical activity, such as the formulation and testing of conjectures and the search for generalizations (Fassio, 2020). The software arouses students' interest by making their active interaction with geometric objects possible, enabling their movement and the construction of variations that can corroborate ideas in favor of establishing properties, contributing to the development of geometric thinking.

In convergence with these ideas, Portella and Leivas (2017, p.136) emphasize that the clear visualization of geometric objects provided by dynamic geometry software facilitates the transition from concrete to abstract mental schemes. In their words, "Visualization, a characteristic of human thinking, contributes to the development of intuitive reasoning, triggering the ability to represent objects mentally."

Furthermore, DTs allow the diversification of pedagogical practices, favor situations based on experimentation, emphasize students' protagonism, and make geometry studies more effective and meaningful.

Conclusions and contributions to mathematics education

This article presented a *systematic literature review on integrating DTs in geometry teaching.* The research addresses two guiding questions: *What are the main gaps associated with geometry teaching? How can DTs contribute to geometry teaching?* In this quest, we selected 30 articles published between 2016 and 2021 in national and international journals per the criteria presented in the sections above. The corpus was scrutinized through vertical and horizontal analyses.

The study highlighted six subcategories regarding the gaps associated with geometry teaching: mismatch in the integration between DTs and the classroom; teachers' low versatility in using DTs; the prevalence of monotonous and decontextualized teaching approaches; the lack of improvement in spatial thinking and exploratory reasoning; scarcity of subjects that enhance the teaching of geometry during initial education; and the absence of DTs in school environments.

We observed that DTs in geometry teaching enhance the teaching of content belonging to this mathematics field, diversify educational practices, allow students to carry out and manipulate geometric constructions, and expand visualization and experimentation opportunities with the concepts addressed. Furthermore, it is noteworthy that geometric investigations facilitated by DTs, especially by dynamic geometry software, privilege crucial aspects of mathematical activity, such as the formulation and testing of conjectures and the search for generalizations, promoting the transition from concrete to abstract mental schemes.

Based on these results, we recommend that future studies develop initiatives in which teachers' continuing education changes the focus of formative courses offered by the university on DTs in geometry for school teachers to continuing education actions with university professors and in-service school teachers. In this scenario, discussing the specificities of the school context is crucial for developing joint intervention strategies in teachers' pedagogical practices, aiming to integrate DTs to approach geometry concepts.

Finally, we must highlight that we understand that a systematic literature review presents a section of the analyzed topic, among other possible ones, considering the choices made while preparing the corpus of analysis, from determining the journals consulted and descriptors to the languages considered.

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