EMP Educação Matemática Pesquisa
ISSN 1983-3156
Qualis A1
http://dx.doi.org/10.23925/1983-3156.2024v26i4p189-210

Continuing teacher education: the impact on students learning mathematics

Formación continua docente: el impacto en el aprendizaje de los estudiantes en matemáticas

Formation continue des enseignants : l'impact sur l'apprentissage des élèves en mathématiques

Formação continuada de professores: o impacto na aprendizagem dos alunos em matemática

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Abstract

This research is the result of the Doctoral thesis entitled “The teaching and learning of Mathematics and the Theory of Conceptual Fields in continuing teacher education.” The topic addresses the use of addition problems from the Theory of Conceptual Fields by Vergnaud and continuing teacher education, based on the RePARe spiral. This quantitative study included 8 continuing education meetings with 20, 3rd-year teachers from the municipality of Canoas/RS. In addition to the meetings with the teachers, we applied a pre-test and post-test with 11 addition problems to 3rd-year students in this municipality. The data gathered demonstrate the importance of continuing education for teacher and student learning, since, upon analyzing the correct and incorrect answers, as well as the strategies used by the students, we perceived that there was a statistically significant impact on student learning in the Experimental Group, which was greater in comparison to the Control Group.

Keywords: Continuing education, Theory of conceptual fields, Teaching and learning mathematics.

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Resumen

Esta investigación es el resultado de la tesis doctoral titulada “La enseñanza y el aprendizaje de las matemáticas y la teoría de campos conceptuales en la formación continua del profesorado”. El tema aborda el uso de las situaciones problema en el Campo Aditivo de la Teoría de los Campos Conceptuales de Vergnaud y la formación permanente de los docentes a partir de la espiral RePARe. La investigación cuanti se basó en la aplicación de 8 encuentros de educación continua con 20 profesores del 3º año del municipio de Canoas/RS. Además de las reuniones con los docentes, se aplicó un pre y post test con 11 situaciones-problema del campo aditivo a los estudiantes de 3° año de este municipio. Los datos encontrados demuestran la importancia de la formación continua para el aprendizaje de docentes y alumnos, ya que, a partir del análisis de aciertos y errores y también de las estrategias utilizadas por los alumnos, se percibió que hubo un impacto estadísticamente significativo en el aprendizaje de los alumnos del Grupo de Clases Experimentales en mayor número que los grupos del Grupo Control.

Palabras clave: Educación continua, Teoría de los campos conceptuales, Enseñanza y aprendizaje de las matemáticas.

Résumé

Cetterechercheest le résultat de la thèse de doctoratintitulée «L'enseignement et l'apprentissage des mathématiques et la théorie des champsconceptuelsdans la formation continue des enseignants». Les sujetssontl'utilisation des situationsproblèmes du domaineadditif de la théorie des champsconceptuels de Vergnaud et la formation continue des enseignantsbasée sur la spiraleRePARe. La recherchequanti-qualitative a étébasée sur l'application: de 8 réunions de formation continue avec 20 enseignants de 3ème année des écoles publiques de la ville de Canoas/RS; et un pré et un post-testsappliquésauxélèves de 3èmeannéeavec 11 situations-problèmes du domaineadditif. Les donnéesdémontrentl'importance de la formation continue pourl'apprentissage des enseignants et des élèves, puisque, à partir de l'analyse des réussites et des erreursdans les pré- et post-tests et ainsi que des stratégiesutilisées par les élèves, il a été constaté qu'il y avait un impactstatistiquementsignificatif sur leurapprentissagesuperieurdans les classes de GroupeExpérimental que danslesclasses de GroupContrôle.

Mots clés : Formation continue, Théorie des champs conceptuels, Enseignement et apprentissage des mathématiques.

Resumo

Esta pesquisa é resultado da tese de Doutorado intitulada “O ensino e a aprendizagem da matemática e a teoria dos campos conceituais na formação continuada de professores”. A temática aborda o uso de situações-problema do Campo Aditivo da Teoria dos Campos Conceituais de Vergnaude a formação continuada de professores com base na espiral RePARe. A pesquisa quantitativa pautou-se na aplicação de 8 encontros de formação continuada com 20 docentes do 3º ano do município de Canoas/RS. Além dos encontros com os professores, aplicou-se um pré e um pós-teste com 11 situações-problema do campo aditivo aos alunos dos 3ºˢ anos desse município. Os dados encontrados demonstram a importância da formação continuada para a aprendizagem docente e discente, pois, a partir da análise dos acertos e erros e também das estratégias utilizadas pelos alunos, percebeu-se que houve impacto estatisticamente significativo na aprendizagem dos estudantes das turmas do Grupo Experimental em maior número do que das turmas do Grupo Controle.

Palavras-chave: Formação continuada, Teoria dos campos conceituais, Ensino e aprendizagem da matemática.

Formação continuada de professores: o impacto na aprendizagem dos alunos em matemática

This article is the result of the doctoral thesis entitled "The Teaching and Learning of Mathematics and the Theory of Conceptual Fields in the Continuing Training of Teachers", whose subject is the Additive Field of Gérard Vergnaud's Conceptual Fields Theory (1996) in problem situations for the teaching of mathematics in the third year of primary school in the municipal network of Canoas, as well as the continuing training of teachers based on the RePARe spiral, by Magina et al. (2018).

This study highlights the problem situations used, based on the conceptual field theory. The Additive Structures Conceptual Field includes problem situations that involve the mathematical operations of addition and subtraction. Learning these concepts is of paramount importance in teacher training, as it expands the learning opportunities for the students of those teachers who understand best these concepts.

Continuing training is understood here as something that teachers seek to continue their studies and become professors during their profession. The RePARe spiral (Magina et al., 2018) supports this, which is a teacher-training model that begins with a diagnostic assessment and moves through theoretical reflection, planning, action, and empirical reflection. This process is cyclical and continues at each training session.

Based on the Conceptual Fields Theory, teacher training, and the RePARE spiral, we present the results of a study based on the training developed in 8 meetings with 20 teachers working in the 3rd year of primary school in the municipality of Canoas. The research included an experimental group and a control group, in which students solved 11 problem situations constructed because of the situations proposed by Vergnaud (1996). This article presents an analysis of the solution to these problem situations.

Mathematics and conceptual fields theory: additive field

To work with the concepts related to the Additive Field, the Continuing Education used the Conceptual Fields Theory, which understands that knowledge is a process of human development that is also social and biological. It is a theory developed in the didactic-pedagogical field and therefore applicable to continuing education.

Vergnaud's (1996) conceptual field theory was developed for the teaching of mathematics, but it can be applied to other fields. In the case of this research, we chose to work with the additive field, focusing on problem situations involving addition and subtraction, knowledge that belongs to the conceptual field of literacy learning.

According to Vergnaud (1996), a conceptual domain makes it possible to analyze the relationship between skills that are developed progressively, involving a set of relationships, situations, thinking operations, and concepts that are articulated. To develop a conceptual field, it is necessary to relate experiences from everyday life and the interaction between the subject and the object of knowledge.

One of the most important aspects of the theory is conceptualization. This is "the identification of objects at different levels, whether directly accessible to perception or not, as well as their properties and relationships" (Vergnaud, 2017, p. 28). According to the author, the construction of a concept requires three elements, namely C = (S, I, R). S is the set of situations that give meaning to the concept; I is the set of operative invariants that organize the schemes to solve the situation; and R is the set of linguistic and symbolic representations that enable the representation of relationships and concepts.

The aforementioned author emphasizes the premise that an individual does not learn by being exposed to a single situation. To learn a particular concept, situations need to be explored in different ways so that new schemas can be created. Therefore, it is important to work with mathematics every day (Boaler, 2018) and under different possibilities in order to consolidate the conceptual field of this specific area, relating new knowledge to that already acquired, since learning is a process that is built over time. Therefore, if concepts are not constantly worked on, this process becomes more difficult and slower.

In this sense, the research focused on logic and numerical structure, giving priority to additive structures. According to Silva and Felicetti (2021, p. 49), "the conceptual field of additive structures includes the additive composition of numbers, basic facts, and operative techniques of addition and subtraction used to solve problem situations". To work with this conceptual field, Vergnaud (2019) presents six basic additive relations:

I. A. The composition of two measures into a third. II. The (quantified) transformation of an initial measure into a final measure. III. The (quantified) relationship of comparison between two measures. IV. The composition of two transformations. V. The transformation of a relationship. VI. The composition of two relations (Vergnaud, 2019, p. 172).

- Magina, Campos, and Gitirana (2001), based on Vergnaud (1996), created an organization that provides didactic and theoretical support to teachers in the use of problem situations, based on the following classification:
- Compositional problem situations: these are made up of parts that make up the whole. The unknown can be in the whole (prototype) or in one of the parts (1st extension).

- Transformation problems: these are characterized by a transformation of the number in the problem situation and by having ternary relations. These situations occur in three different forms: with an unknown initial state (4th extension), an unknown transformation (1st extension), and an unknown final state (prototype).
- Comparison problem situations: they establish comparisons between the referent and the referee through ternary relationships; when the referent is unknown (2nd extension), when the referee (4th extension) is unknown, or when the relationship between referee and referee is unknown (3rd extension).

The problem situations classified as extensions are more complex because they involve changing the unknown and are not used from the beginning of the schooling process. Prototypical situations, on the other hand, are known to be simpler because they are situations that children of 4 or 5 years of age are already exposed to in childhood and are able to find strategies to solve. Borges et al. (2020) confirm: "Thus, a mathematical concept is not isolated from others - since simpler concepts may be necessary for its construction - and also, for its assimilation, it is associated with real or imaginary situations that make sense to the subject" (p.228). By presenting the theoretical support used in the training sessions, the methodology is evident.

Methodological approach to quantitative research

Based on the results identified in the National Literacy Assessment (ANA) test (Brazil, 2013), this study identified the schools in the municipality of Canoas with their respective results. Based on these, teachers were invited to take part in the research if they met all the assessment results, from the lowest to the highest. These teachers administered pre- and post-tests to their students at the beginning and end of the continuing training. These tests also made up the *corpus of* this research.

This is a quasi-experimental study, i.e. the choice of participants was not entirely random. It has an initial measure and a final measure, expressed in this study through the pre-test and post-test (Laville & Dionne, 2008). The research sought to identify and compare the results of the intervention in two groups, since one group underwent the intervention, i.e., the teachers took part in the continuing training before the post-test, and the other group did not. The intervention was supported by the study and development of activities/materials relating to the teaching and learning processes of mathematics as part of Additive Field problem solving. This study and the development of problem situations related to this mathematical field were provided in eight training meetings created and carried out by the researchers based on the

RePARe model for teachers in the third year of elementary school in the municipality of Canoas. Twenty teachers from 16 schools took part in the continuing training course, totaling 26 classes.

The Municipal Education Department requested that the pre- and post-tests be administered to all 188 third grade classes in the Canoas Municipal Network, or 44 schools. However, only 34 schools administered and delivered the pre-test, for a total of 90 classes. Although 90 classes took the pre-test and post-test, the same number of classes (26) were randomly selected as the experimental group. Thus, the control group consisted of one class from each school that took the pre-test and post-test, i.e., 26 classes constituted the experimental group and 26 made up the control group.

The training sessions were divided into two groups. One attended the training in the morning and the other in the evening. The teachers were divided into two groups: the experimental group and the control group. The training sessions with the Experimental Group were held in 2019, and all the conditions of secrecy and confidentiality were followed, according to the norms and guidelines of the Scientific Committee of the Postgraduate Program and the Institution.

Each 3rd grade class in the Canoas school system has an average of 28 students. A total of 1,252 students participated in the pretest, with 52 classes divided into 26 classes in the experimental group and 26 in the control group. In the post-test, the number of participating students decreased due to transfers and those who did not take any of the assessments. Only the tests of those who took the pre-test and post-test were analyzed, a total of 976, of which 537 were in the experimental group and 439 in the control group.

The pre-test included problem situations from the field of addition and its classifications, namely: composition (prototype, first extension), transformation (prototype, first extension and fourth extension), comparison (second extension, third extension and fourth extension), composition of several transformations, transformation of a relation and composition of static relations, adapted from Silva and Felicetti (2017), and has 11 items covering the following problem situations: Composition with unknown whole, Composition with unknown part, Transformation with unknown initial state, Transformation with unknown final state, Transformation with unknown transformation value, Comparison with unknown referent, Comparison with unknown referent, Comparison with unknown relation, Composition of two transformations, Transformations of a relation and Composition of two relations, as shown in Table 1 below.

Table 1.

Pre-test and post-test problem situations (Silva, 2021)

Type of problem situation		Pre-test and post-test question
Composition	**Prototype**	John has 7 magazines with Monica stories and 9 with Smudge stories. How many magazines with Monica and Smudge stories does John have? (Problem situation D)
	1st extension	John got a bag of 12 cookies from his grandmother. Some were made of cornstarch, and others were sprinkles. Seven cookies were made of cornstarch. How many cookies were starch cookies? (Problem situation H)
Transformation	**Prototype**	Fernanda got three dolls for her birthday. She already had 13 dolls. How many dolls did Fernanda get? (Problem situation A)
	1st extension	Pedro got a box of 14 sweets. He ate some and was left with 8 candies. How chocolates did Pedro eat? (Problem situation K)
	4th extension	At the end of the game of marbles, Pedro had 14 marbles. Pedro lost 6 marbles in the game. How many marbles did Pedro have before he started the game? (Problem situation G)
Comparison	**1st extension**	Amanda is 8 years old, and Carolina is 3 years younger than she is. How old is Carolina? (Problem situation B)
	2nd extension	In the 3rd grade classroom, there are 23 students and 19 chairs. Are there more students or more chairs? How many chairs do you need to find so that all the students can sit down? (Problem situation C)
	4th extension	Camila has a few sweets and Nicolas has 8 more sweets than her. If Nicolas has 15 sweets, how many sweets does Camila have? (Problem situation E)
Composition of two transformations		Maria has colored pens. She got 6 pens from her friend and decided to give 3 to her sister. How many more pens did Maria get? (Problem situation I)
Transforming a relationship		Amanda owed Juliana 9 candies. She paid 2. How many candies does Amanda still owe Juliana? (Problem situation J)
Composition of two relations		Jacqueline owed Rachel 9 candies, and Rachel owes Jacqueline 2. How many bullets does Jacqueline owe Raquel? (Problem situation F)

The evaluation of the students' development in relation to solving problem situations involving the Additive Field was based on the quantitative analysis of the pre-test and post-test. The database was built in Excel[3] using a spreadsheet consisting of a column for each problem situation in the pre- and post-test and a column for each student. The students were identified in the spreadsheet using a code with the initial letter representing the class, the number representing the school and the final letter representing the student.

In addition to the variables represented by the questions, the Experimental Group and Control Group variables were created. The information about the variable is in a column in the Excel spreadsheet. Based on this categorization, it was possible to analyze the strategies used by the students in the pre-test and post-test. These were organized in separate tables by class so that each teacher participating in the Experimental Group could analyze their students' performance during the continuing training. The post-diagnostic evaluation was based on the

[3] Excel is a spreadsheet editor and was chosen because it is easy to access and free, belonging to the Office package.

post-test, whose problem situations had the same structure as the pre-test. The post-test was corrected in the same way as the pre-test and formed the database for the quantitative analysis.

A comparative analysis was then carried out between the pre-test and the post-test, as well as comparisons between the classes that took part in the study and the classes in the Control Group, which provided an intra-group and inter-group comparison: inter-group because the pre-test and post-test of the students in the same class were compared, and intra-group because of the comparison between the Experimental Group and the Control Group.

To analyze the pretest and post-test, descriptive statistics were used to indicate the types of strategies used by each group in the tests applied. Furthermore, for each question and for the total of the questions, the mean difference between the weighted means in the pretest and the posttest was calculated; the Student's (T) test for paired samples was used to calculate the level of significance between the means of the groups. In this sense, the following criteria for statistical significance were adopted: values for (p) less than 0.05 were considered statistically significant (Bos, 2004), according to the analysis presented below.

Categorization of students' problem-solving strategies

According to Vergnaud (2014), situations need to be worked with in terms of the referent, the signified, and the signifier. The referent is the student's context and how the student represents it; the signifier recognizes operative invariants; and the signified is related to the symbolic system. According to Santana (2010), all types of symbolic representation are important and useful because in every situation there is a choice of representation for the schema.

Fingers are therefore easily accessible to children, and from an early age, they are taught by their families to use them to make numerical representations. Although, according to Muniz et al. (2014b), the use of fingers has long been forbidden in mathematics education, the authors show that their use can be one of the first tools used by the child in the construction of numbers, dispelling the idea of prohibition:

> It is important that the school emphasize the use of the fingers in counting and calculating small amounts in the literacy cycle. Counting with the fingers can involve the child discovering the five fingers on each hand, as well as the two sets of five that make up ten. More than that, it means discovering the quantities larger and smaller than five, how much is missing from five, and how much is missing from ten (p. 10).

For the aforementioned authors, the use of the body in learning mathematics is to be valued because, in addition to beginning the construction of the symbolic base, it allows for the "development of the first counting strategy and mathematical operationalization" (p. 10). It also allows for bidirectional correspondence and the discovery of practical addition procedures. For this reason, the use of fingers in solving the problems was given a score of 1, as it is an initial strategy used by children both for number construction and for the operations that make up the additive field.

Regarding the use of pictograms, Muniz et al. (2014a) emphasize the need to encourage the use of this strategy, as it helps to record the concrete situation and facilitates the solution of problem situations. In addition, according to the authors, pictorial processes can be worked on in accordance with the first and second years of primary school. In this sense, the rating for the use of pictograms was given a value of 2.

The use of pictograms and algorithms is rated 3, as it combines two strategies that support each other. It is not possible to determine which of the strategies was used first in the pre- and post-test recordings, but the use of two strategies may indicate that the child needs confirmation of his solution or that he thinks that he cannot just record the answer pictorially to solve correctly.

The problems solved using mental arithmetic were classified as 4 and are related to rounding and estimation procedures:

> The set of procedures in which, after analyzing the data to be processed, it is articulated without resorting to a predetermined algorithm to obtain exact or approximate results. Mental arithmetic procedures are based on the properties of the decimal numbering system and the properties of operations [...] (Parra, 2009, p. 195).

The process of mental calculation, according to Parra (2009), also needs to be taught, but it is still a particular path. Using it requires knowledge of operations and the decimal numbering system itself.

An algorithm was considered, according to Vergnaud (2014):

> Algorithm: Arrange the two numbers one under the other, the units' digit of the second number under the units' digit of the first, the tens digit under the tens' digit, and so on until the two numbers are completely written out. Calculate the sum of the two digits in the units' column [...]. If the sum is less than ten, write the number with the unit digit of the number to be obtained. If the sum exceeds ten, move the remainder of ten to the tens column and write the remainder (less than ten) as the unit digit of the number to be obtained. Do the same for the tens' column [...] (p.310).

Vergnaud (2014) presents the above algorithm for solving classes of problems involving any two integers, where the goal is summation. The same algorithm can be used for subtraction with appropriate modifications, such as the use of transport instead of reservation. The author points out that the sequence of actions to be performed in the algorithm is long and that the rules, although simple, may not be fully understood. To be able to use the algorithm competently, students need to develop a clear understanding of the addition operation, and for this to happen, they need exercises designed appropriately by teachers. In this sense, the algorithm was classified in this research with the maximum value for correct strategies, 5, because, according to Vergnaud (2014), it is a procedure that consists of several steps and requires a consistent understanding of the concepts of the addition field.

For the correction, the use of fingers was considered when the children wrote or drew their fingers in the resolution, as well as mental calculation when they wrote, "I did it with my head", "mental", "head", or drew. Pictograms and algorithms were obvious in the correction and did not need to be explicitly identified by the children.

Calculation errors were considered when children added or subtracted incorrectly, when they forgot a unit or a ten, or when the positional value of the number was not used correctly. Operations were swapped, i.e., when subtraction should have been used, addition was used, and when addition should have been used, subtraction was used. According to Huete and Bravo (2006), this occurs when the child doesn't make the connection between the data and the question and ends up swapping operations.

Misinterpretation occurred when children wrote, "I don't know," "I don't understand," or used random numbers that were not in the problem situation. According to Huete and Bravo (2006), failure to understand the problem occurs when the problem is unfamiliar to the student or when the specific vocabulary is unfamiliar to the student, which leads to misunderstanding.

Therefore, for the correct strategies, according to Silva and Felicetti (2017), the following were analyzed: fingers (1), pictograms (2), use of pictograms and numbers (3), mental calculation (4) and use of numbers (5). For the incorrect strategies: error in setting up the calculation (-1), use of an interchanged operation (-2), error in interpretation (-3), and blank response (-4). The analysis based on this categorization is presented below.

Analysis of the performance of the experimental and control groups in additive field problem situations

When analyzing the results of the 11 problem situations included in the pre- and post-test, we highlight the difficulties presented by the students in both groups in problem situations C, E, F, G, and H, whose incorrect strategies accounted for about 50% of the answers. As mentioned in the analysis of each table, four of the five problem situations are more complex (comparison - 2nd extension, comparison - 4th extension, composition of two relations, and transformation - 4th extension) and may have been less worked on by the teachers in both groups (Silva, 2021). Since the teachers in their classes do not normally use these situations, it may be difficult for them to include them in the context of a plan. If the theory of conceptual fields (Vergnaud, 1996) states that a given conceptual field needs time to be understood by the student, then the teachers, as students who need to articulate different skills and abilities, also need time to do so. Therefore, the time of professional development was considered.

Problem situation F (composition - 1st extension) was worked on with the teachers at the second meeting in April, problem situation G (transformation - 4th extension) at the third meeting in May, problem situations C and E (comparison - 2nd extension, comparison - 4th extension) at the fourth meeting in June, and problem situation F in August at the fifth meeting. Although the problem situations were worked on gradually and returned to at each meeting, the teachers expressed that they had difficulty remembering each one. In this case, the teachers' own learning and mastery of the conceptual field might be unfinished and might interfere with their students' learning.

Another hypothesis relates to the complexity of the problem situations themselves. As seen above, the situations in which the students showed an error rate of around 50% do not only involve the operation itself (addition or subtraction). To solve them, it is necessary to read, understand, interpret, activate a schema, adapt it to the situation, or create a new one. To build schemes, they need operative invariants that organize and structure them, and representation is also part of the solution, as are situations that need to be linked to the concepts being worked on. In this hypothesis, the students may be in the process of constructing these schemata, which would be more efficient if they were stimulated by different situations from kindergarten, which may not have occurred with the students in the classes in question. Furthermore, as Vergnaud (1986, p. 79) states about the didactics of problem situations, "we must always keep this idea in mind and be able to provide students with situations that aim to broaden the meaning of a concept and to test their skills and conceptions".

Regarding the percentage of correct answers in each problem situation, some of these situations are highlighted in Table 2 below.

Table 2.

Results of the increase in the percentage of correct answers in each problem situation in the pre-test and post-test in the Experimental Group and the Control Group (Silva, 2021).

Problem situation	Control Group	Experimental Group
A	8.8	6
B	15.1	17.3
C	22.3	11.8
D	14%	15.4%
E	11.2%	10.3%
F	8.1%	15.3%
G	10.4%	15.5%
H	14.1%	11.6%
I	4.3%	10.3%
J	6.1%	17.2%
K	6.1%	19.1%

The Experimental Group showed higher results than the Control Group in 7 of the 11 problem situations when comparing the pre- and post-test (problem situations B, D, F, G, I, J and K), and the Control Group in only four (problem situations A, C, E, and H). This data may show that those teachers who seek continuing training develop more efficient plans and act in a way that promotes teaching, learning, and problem situations more effectively.

Regarding the percentage of correct answers in the post-test, the Experimental Group had a higher percentage of correct answers in nine situations (problem situations A, B, C, D, F, G, I, J and K). Only in problem situations E and H did the Control Group have a higher percentage than the Experimental Group, which may show that the training meetings had a positive impact on the teachers' practice and, consequently, on the learning of the students in their classes.

As for the increase in the percentage of correct answers between the pre- and post-tests, the Experimental Group also outperformed the Control Group, as it showed a higher percentage of correct answers in seven problem situations (B, D, F, G, I, J and K), while the Control Group showed a higher percentage in four problem situations (A, C, E and H). Table 3 and the results of the increase in the percentage of algorithm use are shown below.

Table 3.

Results of the increase in the percentage of algorithm use in each problem situation in the pre-test and post-test in the Experimental Group and Control Group (Silva, 2021).

Problem situation	Control Group	Experimental Group
A	14.1%	35%
B	13.6%	36.7%
C	9.6%	23%
D	14.3%	36.9%
E	6.1%	16.4%
F	9.5%	24.8%
G	7.6%	21.6%
H	8.7%	23.5%
I	7.6%	31.8%
J	10%	34.7%
K	9.1%	26.8%

The increase in the rate of algorithm use in the experimental group in all questions is also worth noting. The use of the algorithm as a strategy for solving problem situations occurred in both groups, but the experimental group outperformed the control group in the 11 problem situations proposed.

This is not to imply that the use of algorithms is a strategy that should be taught at the expense of others. On the contrary, all strategies were encouraged during the training sessions. As Kamii (1991) points out, algorithms cannot be taught as the only way to solve a calculation, nor can they be deprived of any sense of their steps or the positional value of numbers (Humphreys & Parker, 2019).

Vergnaud (2014) states that the algorithm is a complex strategy because it has many steps and can lead to errors. In addition, the algorithm requires abstraction. To use it, one must understand that it is a remarkable and efficient tool. The positional value of the number must be understood and consciously used in the digits through the relationship between quantities (Humphreys & Parker, 2019).

In this sense, the use of algorithms stands out as a complex and abstract solution strategy. The results of the experimental group show that the training sessions had an impact on the practice of the teachers, who began to promote different types of solution strategies, re-signifying the algorithm and its real functioning and purpose (Silva, 2021), which is consistent with the Conceptual Fields Theory, which states that problem situations must evolve in terms of complexity and ways of solving (Vergnaud, 1986). Teachers stated that students were unaware of the relationship between the algorithm and the positional value of the number and had difficulty using this strategy. The same result is not observed in the control group; therefore, it can be concluded that the results obtained by the students in the pretest are the result of the intervention carried out in the training sessions. It is therefore a question of the change shown by the experimental group from concrete strategies (use of fingers, pictograms, and algorithms) to strategies based on abstraction (use of algorithms). It can be seen that in both groups there was a percentage increase in the post-test compared to the pre-test, which means that the algorithm was worked on in both groups but more satisfactorily in the experimental group.

It is important to note the low percentage of mental calculation (a maximum of 4.7%) in all problem situations and in both groups. It should be noted that mental calculation was considered when the students wrote, "I did it in my head", "I calculated it in my head", or drew their heads, indicating that they had been instructed to do so. Although the strategy was worked on during the training, these results may show that the teachers in both groups do not work with mental computation in the classroom and do not encourage this strategy when solving problem situations and those they have not developed some theorems in action, according to Vergnaud (1996). This may be because the teachers themselves did not learn mental arithmetic in school and may reproduce the way they learned it, which, according to Nóvoa (2004), is repeated in their teaching practice.

Table 4 below shows the results of the statistical analysis of the successes and errors in each problem situation in the pretest and posttest in the experimental group, where successes are scored from 1 to 5 and errors from -1 to -4). The difference between successes and failures thus shows how much the child has or has not progressed in his or her learning between the pre-test and the post-test.

Table 4.

Results of hits and errors in each problem situation in the pre-test and post-test in the Experimental Group (Silva, 2021).

	Pre-test				**Post-test**					
Problem situation	**Average**	**Standard Deviation**	**Median**	**Fashion**	**Average**	**Standard Deviation**	**Median**	**Fashion**	**Difference**	**P***
A	2.6	2.5	3.0	5.0	3.8	2.2	5.0	5.0	1.2	< 0.0001
B	1.3	3.1	2.0	5.0	3.0	3.0	5.0	5.0	1.7	< 0.0001
C	0.0	2.8	-1.0	-2.0	1.3	3.3	1.0	5.0	1.3	< 0.0001
D	1.7	2.8	2.0	5.0	3.4	2.7	5.0	5.0	1.7	< 0.0001
E	-0.7	2.6	-2.0	-2.0	0.2	3.2	-2.0	-2.0	1.0	< 0.0001
F	-0.2	3.0	-2.0	-2.0	1.1	3.5	-1.0	5.0	1.4	< 0.0001
G	-0.9	2.6	-2.0	-2.0	0.5	3.4	-2.0	-2.0	1.4	< 0.0001
H	-0.2	2.9	-1.0	-2.0	1.0	3.4	-1.0	5.0	1.2	< 0.0001
I	1.5	3.1	3.0	5.0	2.8	3.2	5.0	5.0	1.3	< 0.0001
J	1.1	3.2	2.0	5.0	2.8	3.2	5.0	5.0	1.7	< 0.0001
K	0.2	2.9	-1.0	-2.0	1.8	3.3	3.0	5.0	1.6	< 0.0001

Total	0.6	1.8	0.5	2.5	2.0	2.0	2.0	4.4	1.4	< 0.0001

* Student's paired t-test in each problem situation.

Table 4, which shows the quantitative analysis carried out using Student's paired t-test, shows the means, standard deviations, median, and mode of the pretest and posttest for the experimental group, as well as the difference between the means of the two tests and the significance level (p) between them. It can also be seen that the question with the most correct answers was the one with an average of 2.6 points among all the students; the median of this question was 3 and the mode was 5. This confirms the fact that problem situation A was not so complex, remembering that it is important to work with different levels of problem situations (Vergnaud, 2017) so that the students can progress in their learning.

The overall average of the pre-test problem situations was 0.6. Five of these problems had an average above the total (problem situations A, B, D, I, and J), and six had an average below the total (problem situations C, E, F, G, H, and K). Six of the problem situations had a median below 0.5 points (total median) (problem situations C, E, F, G, H, and K), and six questions had a mode below the total, which was 2.5 (problem situations C, E, F, G, H, and K). Thus, problem situations C, E, F, G, H, and K were below the mean, median, and mode in all three dimensions. These situations are complex and unknown to the teachers who attended the training sessions (Comparison - 2nd extension, Comparison - 4th extension, Composition of two relations, Transformation - 4th extension, and Composition - 1st extension) and may not have been worked on with the students in previous years, which helps to explain the data below the average in relation to the other problem situations. The results of problem situation H may show that, although it is of low complexity, the students have not had to deal with different types of problem situations and the displacement of the unknown because they are used to the unknown being in the whole, and it may have presented itself as something complex for them, i.e.,

> We can indeed find gaps of several years between children for the same skill, and the behaviors observed for a problem only acquire their meaning if we can relate them to behaviors observed in other problems (of the same or a different category) (Vergnaud, 1986, p. 80-81).

In the post-test, the average of the mean and median was 2.0, and problem situations C, E, F, G, H and K, although showing an increase remained below the average. As for the mode, whose average was 4.4, only problem situations E and G were below this average, and the other situations reached 5.0. This indicates that the teachers' work with the different types of problem situations produced results with their students. The questions with the greatest increase in scores were B, D, and J, with an increase of 1.7.

All questions improved significantly in the post-test (p<0.0001). The overall mean of all questions was 2, the same as the median, and the overall mode was 4.4. Question A remained the best-performing question, with a higher mean score than in the pre-test, and this difference was significant (p<0.001).

Table 5.

Results of hits and errors in each problem situation in the pre-test and post-test in the Control Group, considering hits valued from 1 to 5 and errors from -1 to -4 (Silva, 2021).

	Pre-test				**Post-test**					
Problem situation	**Average**	**Standard Deviation**	**Median**	**Fashion**	**Average**	**Standard Deviation**	**Median**	**Fashion**	**Difference**	**P***
A	1.9	2.5	2.0	3.0	2.6	2.4	3.0	5.0	0.8	< 0.0001
B	0.5	2.9	1.0	-2.0	1.7	3.0	3.0	5.0	1.2	< 0.0001
C	-0.3	2.5	-1.0	-2.0	0.5	2.9	-1.0	-2.0	0.8	< 0.0001
D	1.1	2.7	1.0	-1.0	2.1	2.7	3.0	5.0	1.0	< 0.0001
E	-0.9	2.3	-2.0	-2.0	-0.1	2.7	-1.0	-2.0	0.7	< 0.0001
F	-0.5	2.7	-2.0	-2.0	0.4	3.1	-2.0	-2.0	0.8	< 0.0001
G	-0.8	2.4	-2.0	-2.0	-0.1	2.9	-2.0	-2.0	0.7	< 0.0001
H	-0.5	2.6	-2.0	-2.0	0.4	3.0	-1.0	-2.0	0.9	< 0.0001
I	0.9	2.8	1.0	3.0	1.4	3.1	3.0	3.0	0.4	=0.0068
J	0.9	2.9	1.0	3.0	1.5	3.0	2.0	5.0	0.6	=0.0003
K	-0.1	2.6	-1.0	-2.0	0.8	2.8	1.0	-2.0	0.9	< 0.0001
Total	0.2	1.7	0.2	0.5	1.0	1.8	1.0	2.5	0.8	< 0.0001

* Student's paired t-test in each problem situation.

Table 5, which shows the quantitative analysis carried out using Student's t-test, shows the means, standard deviations[4], and maximum and minimum values of the pre-test and post-test for the control group, as well as the difference between the means of the two tests and the significance level (p) between them. The overall mean for all questions in the pretest was 0.2. Half of the students had an overall average score above or below 0.2 (median). The most common average among the students was 0.5 (mode). These data differ from those of the experimental group, which had an overall average of 0.6, a median of 0.5, and a mode of 0.5,

[4] The standard deviation is used to indicate how uniform a set of data is. It is a measure of dispersion and checks the distribution of data around an arithmetic mean.

indicating that the students in the experimental group outperformed those in the control group on the pretest.

As in the experimental group, the question with the most correct answers was A, with an average of 1.9 points across all students. The median for this question was 2 and the mode was 3, which differs from the experimental group, which had 2.5, 2, and 5, respectively. Questions C, E, F, G, H, and K had negative averages in the pretest, with negative medians and modes. The experimental group had no negative averages in problem situations C and K. Problem situation C is a comparison - 2nd extension, and problem situation K is a transformation of the 1st extension, being of high and low complexity, respectively. The results for problem situations C, E, F, G and H are related to the high complexity of the types of problem situations. Problem situation H and the one with low complexity, like the experimental group, may show a lack of exposure to different types of problem situations, which, according to conceptual field theory, can make it difficult to learn mathematical skills and knowledge (Vergnaud, 1986).

[1] The standard deviation is used to indicate how uniform a set of data is. It is a measure of dispersion and examines the distribution of data around an arithmetic mean.

All questions in the post-test improved significantly ($p<0.0001$) in both groups. In the post-test, the overall mean of all questions was 1, with a median of 1 and a mode of 2.5. These data show that the control group also performed less well on the post-test than the experimental group, which had a total mean of 2, a median of 2, and a mode of 5.4.

As in the Experimental Group's post-test, Question A continued to be the best performing question, with a mean score higher than the pre-test, and this difference was significant ($p<0.001$). Questions C, F, H, and K, which had negative averages in the pretest, now had positive averages. Questions E and G, however, continued to have negative scores, which differed from the experimental group, which achieved positive averages in all problem situations that were negative in the pretest.

Table 6.

Chi-squared test of the Control and Experimental Groups (Silva, 2021)

Improvement	**Control Group F. (%)**	**Experimental Group F. (%)**	**Total**
No	146(33.3%)	103(19.2%)	249(25.5%)
Yes	293(66.74%)	434(80.82%)	727(74.49%)
TOTAL	439(44.98%)	537(55.02%)	976(100%)
Chi-square – uncorrected 0.0000005206			

Table 5 shows the percentage of improvement or no improvement in the answers given in the post-test in both groups. In the control group, 33.3% of the students showed no improvement in their answers to the problem situations, instead of 19.2% in the experimental group. The indices show that 66.74% of the answers to the problem situations in the control group showed improvement in the post-test and 80.82% in the experimental group, i.e., the experimental group showed a difference in improvement of 14.08% compared to the control group. The χ^2 test proved to be statistically significant, with $p=0.0000005206$. This means that the experimental group was more likely to improve their learning, which shows that the training given to the teachers contributed to this. It can be seen that both groups had an increase in the number of correct answers. This is because there is a natural progression in the teaching and learning process, and it is normal for students to develop over the course of a school year (Silva & Felicetti, 2021). However, it is clear that the results presented in this analysis show the differentiated learning achieved by the students in the experimental group, highlighting the importance of continuous training that goes beyond lectures and conferences, training in which teachers can discuss and learn about specific content.

Final considerations

The research demonstrated the importance of professional development in teaching practice and, consequently, its impact on student learning. The eight training sessions covered Conceptual Fields Theory and an adaptation of the RePARe methodological model, and these methodological choices facilitated the emergence of new problem-solving strategies for the students.

The results of the quantitative analysis show the importance of exploring different types of problem situations to develop strategies, skills, and competencies in the additive domain. Through the application of the pre- and post-test in the experimental group and the control group, a total of 52 3rd grade classes, it was noticed that through the evaluation listed for each strategy, the students in the experimental group showed better results.

Within each group, the quantitative analysis indicates that there was an increase in the number of correct answers, as the experimental group had more correct answers than the control group. Similarly, the experimental group made changes that are more significant in the strategies used to solve the problems, moving from more concrete strategies, such as using fingers and pictograms, to more abstract strategies, such as using algorithms. These results prove that there was growth in learning in both groups, although it is clear that the growth was greater in the experimental group; in other words, when teachers learn the content to teach better, they teach it better.

In the results shown by the Chi-square test, the experimental group showed improvement in 434 (80.82%) students, while the control group showed improvement in 293 (66.74%), which means that the experimental group had an impact related to the training sessions. The control group also improved, as it was expected that students would improve over the course of a school year. What is different is that 14.08% of the students in the experimental group improved more, which shows the impact of the professional development on the teachers' practice and consequently on the students' learning.

This shows that professional development based on reflection and developed within the context of classroom practice not only changes the way teachers work in the classroom, but also has a positive impact on solving problem situations in terms of the strategies used by students. Understanding concepts, analyzing, discussing, sharing proposals, and reviewing one's own practice are premises that must accompany teachers throughout their professional careers, which is why analyzing how their students learn and solve situations can be used to promote both teacher and student learning.

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