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What does art know about (with) mathematics? Visualities that overflow in a group of students in initial training

O que a arte sabe de (com) matemática? Visualidades que transbordam em um grupo de estudantes em formação inicial

¿Qué sabe el arte de (con) las matemáticas? Visualidades que se desbordan en un grupo de estudiantes en formación inicial

Que sait l'art des (avec) mathématiques ? Des visualités qui débordent dans un groupe d'étudiants en formation initiale

Adamo Devi Cuchedza[1]
Federal University of Santa Catarina
Master in Mathematics Education
https://orcid.org/0000-0002-0280-6687

Cláudia Regina Flores [2]
Federal University of Santa Catarina
PhD in Education
https://orcid.org/0000-0003-2351-5712

Débora Regina Wagner[3]
Federal University of Santa Catarina
PhD in Scientific and Technological Education
https://orcid.org/0000-0002-1588-8853

Mônica Maria Kerscher-Franco[4]
Federal University of Santa Catarina
PhD in Scientific and Technological Education
https://orcid.org/0000-0002-1710-1719

Abstract

This article aims to present some analyses about the way of looking at and using art to teach mathematics, implied by the act of visualization and visuality, which put specific ways of learning into operation. To this end, a workshop that used paintings by Mozambican artists is used and was carried out with Mozambican students in a Licentiate course in Mathematics Teaching. The analysis undertaken here is carried out in two movements: in the first one, mathematical thinking is put into practice with Mozambican paintings: symmetry, hyperbole,

[1] acuchedza@gmail.com
[2] claudia.flores@ufsc.br
[3] debora.wagner@ufsc.br
[4] monicakerscher@gmail.com

and parabola. In the second, attention is drawn to truths about the teaching of mathematics with art, for example, the statement that artworks as support contextualizing the learning of mathematics. Finally, we conclude by alluding to a decolonial *ethos*, which highlights a critical and continuous attitude, aligning itself with an exercise of making visualities visible to put art into practice with mathematics for teaching.

Keywords: Art, Mathematics, Visuality, Visualization, Cartography.

Resumo

Este artigo apresenta algumas análises acerca do modo de olhar e usar a arte para ensinar matemática, implicadas pelo ato da visualização e da visualidade, que colocam em operação formas específicas de aprender. Os resultados foram obtidos a partir de uma oficina com estudantes de Moçambique em um curso de Licenciatura em Ensino de Matemática, na qual se fez uso de pinturas de artistas moçambicanos. A análise empreendida foi feita em dois movimentos: no primeiro, colocou-se em exercício o pensar matemática com pinturas moçambicanas: simetria, hipérbole e parábola; no segundo, atentou-se a verdades acerca do ensino da matemática com a arte, como o enunciado de que "a arte funciona como suporte para contextualizar a aprendizagem da matemática". Por fim, a conclusão alude a um *ethos* decolonial, que destaca uma atitude crítica e contínua, alinhando-se a um exercício de fazer ver visualidades para pôr em exercício a arte com a matemática para o ensino.

Palavras-chave: Arte, Matemática, Visualidade, Visualização, Cartografia.

Resumem

Este artículo tiene como objetivo presentar algunos análisis sobre la manera de mirar y utilizar el arte para enseñar matemáticas, implícitas en el acto de visualización y visualidad, que ponen en funcionamiento formas específicas de aprender. Para ello se utiliza un taller que utilizó pinturas de artistas mozambiqueños y se llevó a cabo con estudiantes mozambiqueños en un curso de Licenciatura en Enseñanza de las Matemáticas. El análisis emprendido aquí se lleva a cabo en dos movimientos: en el primero, se pone en práctica el pensamiento matemático con pinturas mozambiqueñas: simetría, hipérbole, parábola. En el segundo, se llama la atención sobre verdades sobre la enseñanza de las matemáticas con el arte, por ejemplo, la afirmación de que el arte funciona como soporte para contextualizar el aprendizaje de las matemáticas. Finalmente, concluimos aludiendo a un *ethos* decolonial, que destaca una actitud crítica y

continua, alineándose con un ejercicio de visibilización de visualidades para poner en práctica el arte con las matemáticas para la enseñanza.

Palabras clave: Arte, Matemáticas, Visualidad, Visualización, Cartografía.

Résumé

Cet article présente quelques analyses sur la manière de regarder et d'utiliser l'art pour enseigner les mathématiques, impliquées par l'acte de visualisation et de visualité, qui fonctionnent dans des manières spécifiques d'opérer l'apprentissage. Les résultats ont été obtenus lors d'un atelier avec des étudiants mozambicains dans un cours de licence d'enseignement des mathématiques, dans lequel des peintures d'artistes mozambicains ont été utilisées. L'analyse effectuée s'est déroulée en deux mouvements : dans le premier, la pensée mathématique a été mise en pratique avec des peintures mozambicaines : symétrie, hyperbole et parabole ; dans le second, une attention a été portée aux vérités sur l'enseignement des mathématiques avec l'art, comme indiqué que “l'art fonctionne comme un support pour contextualiser l'apprentissage des mathématiques”. Enfin, la conclusion fait allusion à un *ethos* décolonial, qui met en exergue une attitude critique et continue, s'alignant sur un exercice de mise en visibilité des visualités pour mettre en pratique l'art avec les mathématiques pour l'enseignement.

Mots-clés : Art, Mathématiques, Visualité, Visualisation, Cartographie.

What Does Art Know of (with) Mathematics? Visualities that Erupt in a Group of Students in Initial Education

Seeing comes before words.
The child looks and recognizes before it can speak.
But there is also another sense in which seeing comes before words.
It is seeing which establishes our place in the surrounding world.
(John Berger)

In Brazil, using art to teach mathematics at school has been a vital and emerging issue. Accordingly, the educational guidelines, the National Curricular Parameters (Parâmetros Curriculares Nacionais - PCNs) and the National Common Core Curriculum (Base Nacional Comum Curricular - BNCC) have driven the issue mainly by promoting interdisciplinarity in teaching. It is noteworthy that since at least 1998, a guideline for the teaching of mathematics has been in force:

> Learning in Mathematics is linked to comprehension, i.e., to the apprehension of meaning; apprehending the meaning of an object or event presupposes seeing it in its relations with other objects and events. Thus, treating contents in tight compartments and a rigid linear succession must give way to an approach in which connections are favored and highlighted. The meaning of Mathematics for students results from the connections they establish between it and the other subjects, between it and their daily lives, and from the connections they establish between the different mathematical themes (Brasil, 1998, p. 15).

In Mozambique[5], the guideline follows a parallel line. In official documents, such as the Secondary Education Curriculum Plan (Plano Curricular do Ensino Secundário - PCESG) and the Secondary Education Mathematics Programs (Programas de Matemática do Ensino Secundário - PMES), guidelines on interdisciplinarity and transversality guarantee the learning of Mathematics content, i.e., Mathematics teaching must "contribute to the development of capacities to use mathematics as an instrument that allows recognizing, interpreting, intervening and solving real problems existing in the various fields of human activity (social, economic and cultural) and the various curricular areas" (Moçambique, 2007, p. 53).

Regarding Mozambican research, several studies developed by Professor Paulus Gerdes from 1970 to 2012 propose a link between mathematics, culture, and history, contributing significantly to the field of mathematics education at national and international levels. In

[5] Mozambique, officially the Republic of Mozambique, is located in the Southeast of Southern Africa, with an area of 801,590 km^2 and is bordered by six countries: to the south and southwest by South Africa and Swaziland, to the west by Zimbabwe, to the northwest by Zambia and Malawi, to the north by Tanzania, and to the east by the Mozambique Channel and the Indian Ocean. It was a Portuguese colony from 1505 to 1975. It became independent as a result of a national liberation war that began in 1964 and culminated in the proclamation of national independence on June 25, 1975.

particular, much of this research takes local art and crafts to explore aspects of mathematics and mathematics education, as is the case with the "sona[6]" tradition." (Gerdes, 2010). Such studies are part of the general field of Ethnomathematics and constitute "a definite way to get Africans to see science as a means of understanding their cultures and as an instrument to serve and make progress in these cultures" (Gerdes, 2012, p. 23).

However, in the teaching field, the eternal search for Western models in curriculum programs, when it comes to Mathematics curriculum, turns towards a continuum in higher education. This continuum sometimes tries to respond to the needs of the job market and Mozambican society, and the globalized world in general because "the mathematics of primary education [functions] mainly as a preparation for secondary education, the mathematics of secondary education mainly as a preparation for higher education" (Gerdes, 2012, p. 32). This kind of education proves that the multiple legacies that cross many Mozambican teaching resources reveal that the decolonization process has not ended (Meneses, 2014).

Although we recognize the role of ethnomathematics research carried out by Gerdes and other researchers, taking into account relationships with the history and culture of African peoples as a driving problem for mathematics teaching in Mozambique, in our study, the way we build the relationships between art and mathematics takes on other contours, moving away from the Ethnomathematics perspective.

In Brazil, the scenario that crosses research that relates art to mathematics teaching has been tracing other paths, pointing to their rowth process (Flores & Wagner, 2014). From this, some trends can be listed. For example, when analyzing masters' and doctoral theses in Capes' national database until 2010, Wagner (2012) highlighted six implicit trends in the relationship between art and mathematics education: valuing the interdisciplinary context between mathematics and art; mathematics as an art form; art as a motivating tool for mathematics learning; the mathematics and art interrelationship; art as an object, an instrument, in the context of mathematics education; and art as a possibility to think about mathematics.

Concerning visualization issues, art has also taken on an interesting role in research, Since most Brazilian research in Mathematics Education has understood it as a skill or a mental scheme, a process of image formation, or as a valuable process for intuition and concept formation that helps people to understand and learn mathematics (Flores, 2012). Hence, art has

[6] Long tradition culture, originally from the Tshokwe people from the northeast of Angola, where people made drawings in the sand. The "sona" is considered a form of writing where a narrator tells a story or a reality of life in which proverbs, tales, fables, games, myths, animals, songs, laws, and enigmas are illustrated, playing an important role in the transmission of the knowing to the new generations.

been used as an object, an instrument that makes it possible to access semiotized or represented knowledge (mathematics). In this case, using this ability and this condition of art means exercising reasoning in the search for mathematical concepts clearly represented in a work of art.

On the other hand, art, and here we are particularly talking about the visual arts, has been considered in its potential to make people think about mathematics (Flores, 2016), in which visualization gives way to the term visuality, starting to consider the perspective of visuality for Mathematics Education (Flores, 2007; 2010; 2012; 2013). By visuality, we understand the discursive ways of looking at the world and everything that surrounds it as modes resulting from games of truth that inform how we see and how everything should be seen, and which are, notably, the result of experiences, of practices of looking historically and culturally constructed (Flores, 2013). Hence, mobilizing this condition of looking means, on the one hand, enabling exercises in looking practices, problematizing mathematics and, on the other hand, showing the discourses that make up our visual training and, in Mathematics Education, showing statements that support types naturalized teaching and learning mathematics.

Amid this problematization, we have been developing a study[7] that raises questions about how or what happens, for example, when pre-service teachers of a Mathematics Teaching Degree in Mozambique exercise the perspectives of visualization and visuality when they are challenged to think about teaching mathematics with art. To this end, we organized and conducted four workshops[8] with a group of pre-service Mathematics teachers attending Universidade Licungo - Extensão da Beira, Mozambique, in 2021. Each workshop considered paintings by Mozambican artists and received a title according to the type of images and the problem that could (or could not) arise from discussions initially carried out by a research group, the GECEM[9]: *Eye, mouth, nose; Forms and abstractions; Body and beauty;* and *What you see and what you think.*

[7] This is a doctoral research in progress, developed by the first author, supervised by the second author and co-supervised by the third author, in the Postgraduate Program in Scientific and Technological Education (Programa de Pós-Graduação em Educação Científica e Tecnológica- PPGECT)-UFSC, Santa Catarina, with financial support from Capes.

[8] Workshops understood as a space that can incite a *experimental attitude of attention* linked to being present in the present, to seeing, hearing, feeling, a space that opens up to experimenting with thinking exercises (Masschelein, 2012).

[9] Group of Contemporary Studies and Mathematics Education (Grupo de Estudos Contemporâneos e Educação Matemática -) GECEM, with Prof. Dr. Cláudia Regina Flores, and vice-leader Prof. Dr. Débora Regina Wagner, at the Federal University of Santa Catarina. Before being developed in Mozambique, the workshops were created and discussed with the group of people that made up the GECEM.

This article, therefore, aims to present some analyses of the discussions undertaken, particularly of the first workshop held with Mozambican students. Here, we do it in two moves. The first puts mathematical thinking into practice with Mozambican paintings: symmetry, hyperbola, and parabola. The second draws attention to truths about teaching mathematics with art, for example, the statement that art works as a support to contextualize mathematics learning.

The text is organized in three stages: initially, we discuss ways of looking implied by visualization and visuality, which put into operation specific ways of learning. Afterwards, we present the workshop organization, bringing the images, the artists that composed it, and the dynamics on which it was developed. Then, highlighting visualities that sprout from and with the pre-service teachers involved in the research, we do an analysis to define visual practices that exhibit knowledge that we summon and by which we are summoned when we look at the image of a work of art. Finally, we conclude by alluding to a decolonial ethos, which highlights a critical and continuous attitude, aligning itself with an exercise of making visualities visible to put art into practice with mathematics for teaching.

On the ways of seeing and learning mathematics

Let us think about the following situation: we are all students and we are in a classroom looking at some images that are arranged on a table, supported by a blue cardboard (Fig. 1). The images are reproductions of works of art by different artists and nationalities. We are summoned to look at them.

Figure 1.

Artwork images on a table (GECEM Image File)

This situation was proposed to a group of people and discussed by Débora Wagner, Cássia Schuck, and Cláudia Flores (2016). The group was asked: Do you see mathematics in the images? When looking at the images, do you immediately see mathematics? Now, it seems to us that these questions, in short, summon us in the following way: visualize the image, find, and identify mathematical concepts. What concepts can we see in them? Perhaps in some, i.e., in the most obvious ones, we will soon be able to find what is being asked; in others not so much. However, we see a geometry that insists on shining in the eyes. We see a circle, a rectangle, triangles, lines, rounded shapes, parallel lines, perspectives, etc. What else do we see? Volume, depth, asymmetries... We seek to see from an art that elaborates an idea of knowing and knowledge, which shapes it and informs the eyes of what can be seen, but also from an eye that knows how to see by recognizing forms and mathematical and mainly geometric concepts. Therefore, from this incitement, we highlight a practice on the use of art to teach mathematics that excels in the exercise of a visual and mental skill, i.e., visualization. In an appropriate abridgment, visualization is:

> a way to stimulate thought, imagination, intuition, and sensitivity. It is the expression mechanism of a visual language and visual reasoning. It can be considered the main mechanism for making one 'see' a mathematical result without resorting to proof in its strict sense of logical deduction. Visualizing is singularizing, exemplifying, and maintaining universality. It is being able to formulate mental images and is at the beginning of every process of abstraction (Cifuentes, 2005, p. 46, emphasis added).

In this way, looking at the art image means performing a mental, cognitive operation, while the eye, as an organ, performs its function that was once shaped. A way of looking from a specific point, under conditions and appropriations already naturalized, rooted, fixed as rules. We can say that it is a way of looking exercised by the eye of the mind, the eye of what is visible, an eye of presence. Hence, in this way of visualization, "it is the Cartesian, perspective, geometric eye" [that is put into action]. And the better the mind's eye sees it that way, the better it will learn or know mathematics" (Flores, Machado & Wagner, 2018, p. 131), for example.

A way of seeing, then, that seeks in the image, in the case of the art work, a represented knowledge, a knowledge that is recognized and displayed, and that, in the end, seems to favor the communication of a truth, of collectively constructed knowledge. Art is taken as an object, a place of registration or applicability of knowledge and geometric shapes that, in turn, appear to us as 'inherent' to art itself. While mathematics appears to us represented and, by this bias, "learning has usually been a repetition of experiences already carried out, a contact with the discoveries of science, a study of the rules and postulates" (Flores, 2016, p. 505).

However, it seems that in this movement of visualizing images, "we never, or almost never, ask ourselves where our truths come from" (Flores, 2007, p. 30). On the contrary, what is shown is the need to train, educate the look to know how to see things, the world, and things in the world in a specific way, promoting and practicing discourses that, as true and hegemonic, were forged in history and by culture. In this interstice, it is up to us to question whether visualizing would be the only way of looking at art images to learn mathematics.

So, let us look again at the set of images represented in Figure 1. And if we ask ourselves, what can we think about mathematics with them, instead of identifying or recognizing mathematics in them? Or, who knows, as Foucault (2008, p. 217) suggests, question whether "in at least one of its dimensions, it [painting] is a discursive practice that takes shape in techniques and effects" in which the knowledge of mathematics manifests itself? Hence, the term visuality has functioned as a strategy to show a set of discourses, of visual practices that have shaped how we see, can, and know how to see things in the world (Flores, 2013).

The term visuality has been exercised in Mathematics Education (Flores, 2010; 2013) and, mainly, with the work with art in the classroom, in which art no longer appears as a simple object that holds knowings but as a thought, instigating us to exercise thinking about mathematics and its teaching (Flores, 2016). That said, with visuality, the eye that is put into operation is the eye of thought, the eye that affects something and is affected by something, the sensitive eye, the eye of experience and experimentation. In this case, learning mathematics with art "is not limited to just recognitive acts, as one understands that they are only one of the occupations of thought" (Flores & Kerscher, 2021, p. 27), but "to experience ways of seeing with art, in which mathematics operates as an organizing element in the process of connecting and disconnecting, producing new possibilities" (Flores & Kerscher, 2021, p. 29).

Hence, as Jorge Coli (2006) shows, a painting, a sculpture, an architecture, or a theater play triggers thoughts about the world, things, women and men, and societies. With that, we wonder whether a work of art also triggers thoughts about knowledge, in this case, about mathematics and its teaching. Thus, our question could be redirected to: What enhances the encounter with mathematics and art? What is summoned, and what summons us to think when we look at a painting, for example? And what do we make of this experience?

Thus, on the one hand, we seek a critical exercise about visual practices that place art under the gaze of mathematics and, on the other, the experimentation of thought, abstraction, spatial distribution, and calculations, among other possibilities, by which we summon and are summoned when we look at a work of art.

Organizing the workshops

We are guided by cartography as a research and data production strategy. "The cartographic guideline is based on clues that guide the research path, always considering the effects of the researcher's process on the research object, the researcher, and their results" (Passos, Kastrup, & Escóssia, 2012, p. 17). From this, the production and cultivation of data that make up the research followed a rhizomatic path, characterized by several routes of comings and goings, always open, making first a curatorship[10] or selection of works of art by Mozambican artists.

[10] "The curator is the one who selects and chooses their images among their 'store drawers' as a 'bricoleur' who works with the available means and as a proposer who invents and reinvents, enhancing aesthetic experiences" (Martins, 2006, p. 9, emphasis added). Thus, a curatorship occurs in the curator's encounter with the images, with what they provoke and trigger in their thinking, with the possibility that something will also happen to those who will encounter their proposition.

We chose Mozambican art, on the one hand, to show and value the country's culture and, on the other hand, because it constituted an existential territory (of thought) of Mozambique - it is there, and from there, subjectivities are placed or can be created and reverberated at school as well. Moreover, by art, we aim only paintings, even though we have a vast artistic field in Mozambique.

Thus, first, to find the artists, one path followed was to search the web through Google, using the following keywords: "Mozambican plastic arts", "Mozambican plastic artists", and "names of several Mozambican plastic artists". Then, among the research found on this topic on Mozambican art, we highlight Costa (2018), entitled Artistas de Moçambique olhando para si próprios e para o mundo" [Artists from Mozambique looking at themselves and the world], and Meigos (2018), entitled "Dinâmicas das Artes Plásticas em Moçambique" [Dynamics of Plastic Arts in Mozambique], to understand some of the characteristics of Mozambican art. While the first describes modern and contemporary art in Mozambique and the different generations of artists who embodied it, their techniques used, styles, contents, materials, trends, and the new forms of art practiced by them, the second, inserted in the field of sociology, aims to account for the structuring, dissemination, and legitimation of the plastic arts in Mozambique, capturing its nuances and structuring marks in two different periods, 1975-1986 and 1987-2016, the post-colonial period. This chronological division of the post-colonial period chosen by the author has to do with the political, economic, and cultural history of Mozambique after independence.

To Costa (2018) and Meigos (2018), what characterizes Mozambican art, especially plastic art, is the portrait *of everyday life*, the *corporeality*, *the zoomorphization*[11] and *the story-telling*[12], narrating suffering, slavery, colonization, hunger, floods, debauchery, desire for freedom, nudity, voluptuousness, beauty, sexuality, eroticism, anxiety, and even the complex relationship between traditional and modernity revealed in beliefs, practices, and everyday social assumptions. That art gives greater emphasis on the body, particularly the female body, meaning procreation, domesticity, sexual nature, sensuality, voluptuousness, marriage, and family, among others. The male body represents virility, the phallic, domination and, therefore, the supremacy of the heterosexual, that is, "the normative", as opposed to homosexual. Human corporeity is a social and cultural phenomenon par excellence.

[11] Attribution of human qualities, actions, or feelings to inanimate or irrational beings, animism, personification which, incidentally, is common in local beliefs (Meigos, 2018, p. 125).
[12] Transmigration of the current orality tradition, the stories, the ghosts, the spirits (djines of Makonde sculpture) and a whole surrealist imagery. (Ibid.)

After that, we highlighted a universe of several works of art which, after a previous exercise, together with GECEM, of looking at them and realizing one's thoughts on mathematics, we finally selected twelve images by six Mozambican artists: Naguib Elias Abdala, João, ivane, Victor Souza, Silva Dunduro, Pedro Jeremias Tembe (Dito) and Ernesto Shikhani. We selected works that could somehow create possibilities for thinking and exercising the eye both through visualization and visuality, considering the power to meet our objective amid so many equally significant images. The chosen images are representations of self-portraits, fabulation, zoomorphization, human faces, geometrization, and everyday scenarios represented by bodies, especially the female body, and were distributed in four workshops called: *Eye, mouth, nose; Forms and abstractions; Body and beauty;* and *What you see and what you think*. Eight prospective teachers from the Mathematics Teaching Degree course at the Universidade Licungo - Extensão da Beira, Mozambique, participated in the four workshops. All were attending the last phase of the course.

The workshops were developed between March and May 2021, on Fridays afternoon, obeying the time interval from 1 pm to 5 pm, in a room identified as the Mathematics Laboratory. At the first moment, to safeguard the participants' identities, each chose a fictitious name that would serve to be presented in the research, as an example: Triângulo, Segredo, Fermat, Matato. Thus, it is with these fictitious names that we will present the speeches and analyses.

Each workshop was divided into two moments: the first consisted of exploring the images and the second, an open discussion or the performance of some activity involving a production, a workshop. The images were made available in two ways, projected and in replicas in small sizes and colored, that were given to the participants. With them, some questions served to incite the activity, for example: What, when looking at the image, do you identify from mathematics, mathematical concept, mathematical symbol? What maths does this picture make you think about? Such questions were not posed directly to the participants but guided the research in the context of the development of the workshops and, which, after all, had the intention of provoking aspects undertaken by visualization and visuality.

As we need to limit this article, we will focus on the first workshop: Eye, mouth, nose. It was created from four images of works of art (Fig. 2): 1 and 4 are by the artist João Tivane, untitled, (undated); 2 is by artist Shikhani, untitled, 1979; 3 is by the artist Victor Souza, untitled, (undated).

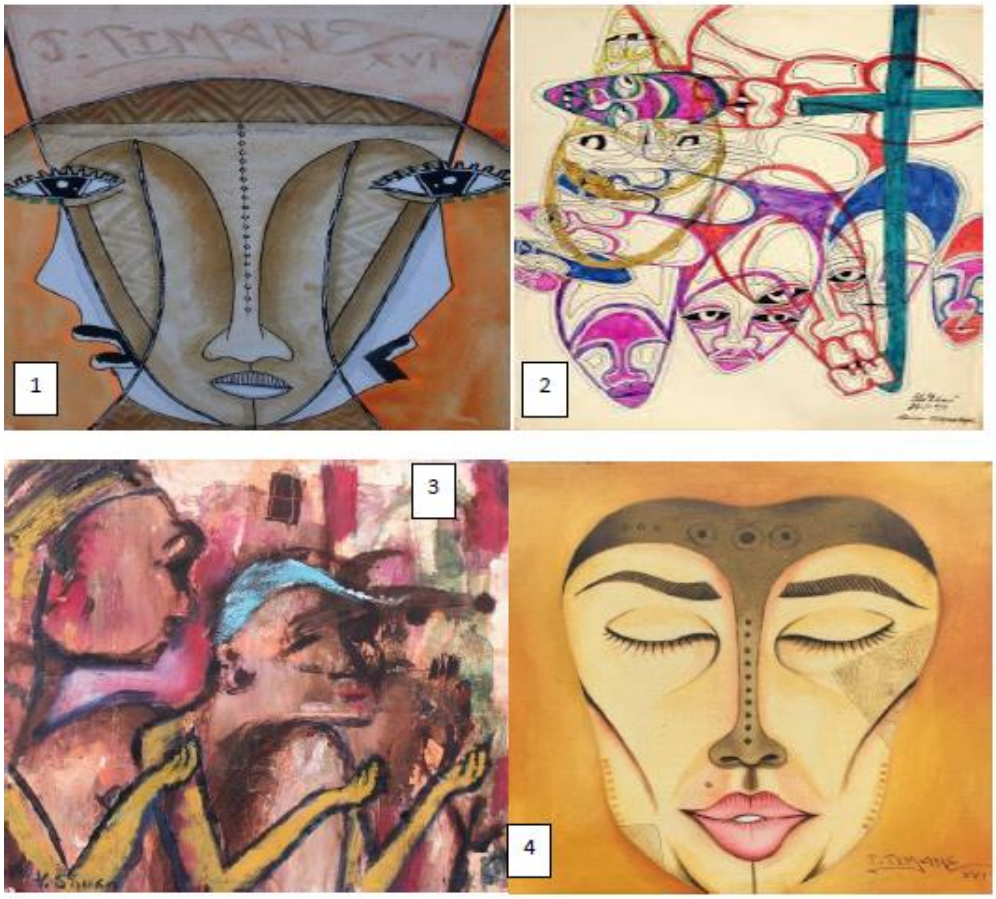

Figure 2.

Images presented at the workshop Eye, mouth, nose *(Research file)*

With them, as said, a first exercise was carried out with the students. We started by asking the participants to answer in writing the triggering questions, and then we opened a space for dialogue where each one could expose what they felt about the exercise and what they could add or think together with the group. Next, the researcher[13] wrote on a logbook[14] his impressions, meanings, and experience in the group and notes on the participants' speeches. The speeches were recorded and later transcribed, seeking to outline a[15] shared experience plan.

From one exercise, many events

Before the images (Fig. 2), at first, what emerged in common in the participants' view was the attempt to search for relationships between paintings and mathematics, more evidently,

[13] It refers to the first author of this article.
[14] By logbook we mean the notes made each day by the researcher in a notebook as raw material, as suggested by Barros & Passos (2012).
[15] At the *ethos* of the cartography, the experience plan refers to collective production coming from a decentralization in research practices in favor of a collective and shared process of knowledge production (Sade, Ferraz, & Rocha, 2016).

making the image the place to identify mathematical concepts, and then, what of them could be suggestive for them to think about mathematics. When asked whether these images provoke, awaken, or suggest something, they answered:

> *Triângulo*: Symmetries, functions, areas [...]. With image 1, first, we have graphs of functions, parabola, hyperbola, and more. Many symmetries are still represented, for example: in the same image 1, in relation to the eyes, there is an axis that separates them at an equal distance. The same can be seen in Figure 4.
>
> *Segredo*: Going with the idea of a triangle, in these two images (1 and 4), if we pass a vertical line through the nose, we conclude that the face will be divided into two equal parts. Hence, on the one hand, we have the idea of equality and, on the other, of symmetry.
>
> *Fermat:* The images also make me think of symmetry, hyperbola, despite being figures that describe a given critical situation, I understand what the author intends to convey. Figures 2 and 3 make me think of plane and projective geometry. For example, figure 2 illustrates figures that are being projected by the lines and figure 3 makes me think of parallel lines, squares, perpendicular lines, subjects related to plane geometry.

Now, the research participants and the researchers, in front of these paintings, see symmetries, for example. But, after all, to which symmetries are they referring? Are there symmetries? They claim to see plots of graphs of functions, parallel lines, and perpendicular lines. Is mathematics actually present, or does it emerge in the thought urged by the artist's practice, leading the painting to exhibit specific knowledge? Well, let us see what this could be, after all, with the image.

Let us take image 1 from Figure 2. As stated by the participants, the eye is led to see symmetries in the image. Although the specificity of the type of symmetry does not appear in the speeches, it is clear that it is a reflection or specular symmetry[16], which we sketched on the image taking as an axis the vertical line shown in green (Fig. 3).

[16] Reflection symmetry is that which makes an object resemble its image in relation to a given axis, while maintaining its shape, angle, and size.

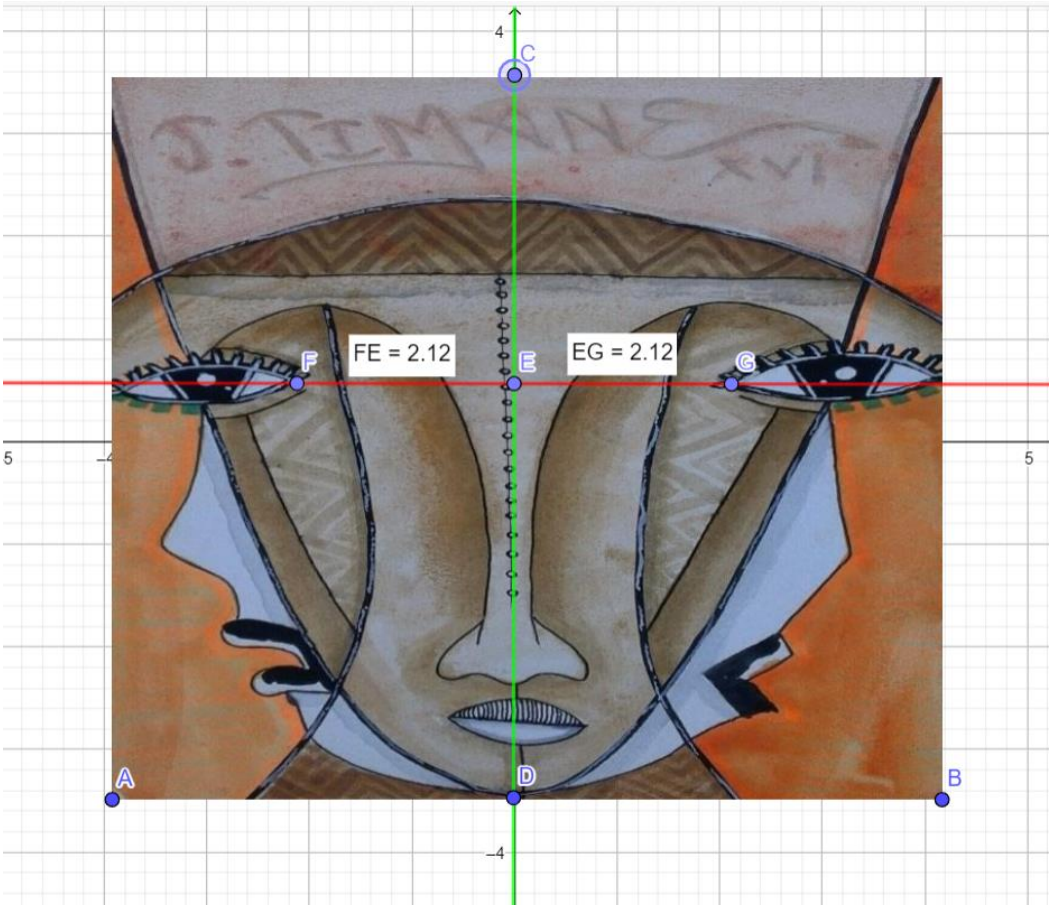

Figure 3.

Symmetries exercised with image 1 (Authors' production)

When looking at the image, one of the symmetries pointed out by the participants was about the distance of the vertical axis in relation to the eyes. Let us see: with the help of GeoGebra[17], we draw a line (red) perpendicular to the vertical axis line (green) and inserted the points F and G corresponding to the inner extremity of the eyes. In doing so, we checked the measurements between the outlined points, which correspond to a symmetry that separates the eyes at an equal distance in relation to the vertical line; in this case, $d_{FE} = d_{EG} = 2{,}12$ (Fig. 3). The arrangement of some elements in the painting takes us to an organization and harmony in the look, and, from that, a notion of equality, as suggested by the workshop participants.

It is said that the idea of symmetry dates back to ancient times, a concept in which man appropriated nature to apply in the arts in general (Contador, 2011). The fact is that this idea has been with us for a long time, and it has certainly formed our view, our practice of drawing and representing. This aesthetic practice that appears to our eyes, would it also be the practice

[17] GeoGebra is a dynamic mathematics computer program that combines geometry, algebra, statistics, and calculus concepts.

exercised by the artist who, in turn, provides the conditions for the image to display a particular idea of knowing and knowledge? But, after all, is there symmetry, from the point of view of mathematics, in the image? In addition to the symmetries presented, are there others to see? Is the image, from the vertical axis traced, symmetrical in all its elements? Why, as soon as we look at the image, do we think of symmetry? All this summons us to an exercise: that of looking at the image and thinking about what is summoned and whether it is summoned by it.

We proceed to make another composition of the look with image 1 of Figure 2. Participants say they see the graphic representation of functions in the image, including the graph of a parabola (Fig. 4).

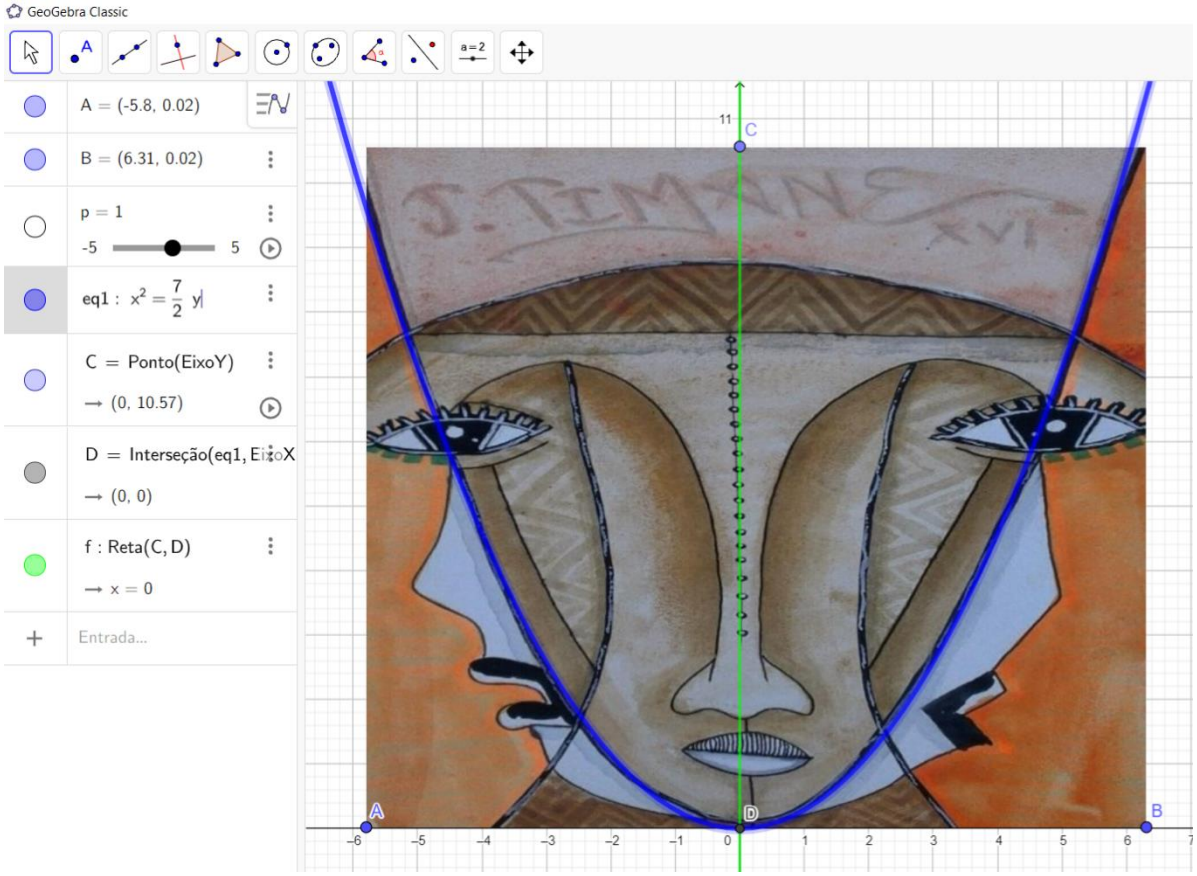

Figure 4.

Provocation in the eye: a parabola with the image (Authors' production)

That said, let us do the following exercise: we position the image x on the axis in the Cartesian plane and the vertical line of symmetry on y, the axis in GeoGebra. To identify the graph of a parabola with the concavity facing upwards on the lines that form the central face in the painting, we found a variation of the quadratic function on the image, given by $f(x) = \frac{2}{7}x^2$ (blue contour) (Fig. 4), in which we observe an approximation with the contours of the represented face.

The law of formation of the quadratic function is given by the expression: $f(x) = ax^2 + bx + c$. Thus, to reach the similarity with the image, a variation of the reduced form of the 2nd-degree equation was used: $x^2 = \frac{7}{2}y$, which can be written in terms of x, as outlined earlier.

With this, the elements of the image that are closest to a symmetry between its components become evident to our eyes. 'Within' the concavity of the parabola, we notice the regularity and similarity of the lines that make up the face, for example.

Let us move on to another exercise with this same image. From the idea of symmetry, another relation that emerged from the look and that was posed by the participants, refers to the graph of a hyperbola in the painting. To draw on image 1 from figure 2, the graph of the hyperbolic function, also with the aid of the GeoGebra, we find equation $\frac{3x^2}{1^2} - \frac{y^2}{1^2} = 1$, which can be written in terms of x: $f(x) = \sqrt{3x^2 - 1}$Thus, the red lines that refer to the graph of this hyperbola are sketched, overlapping in an approximation of the contours presented by the figure (Fig. 5).

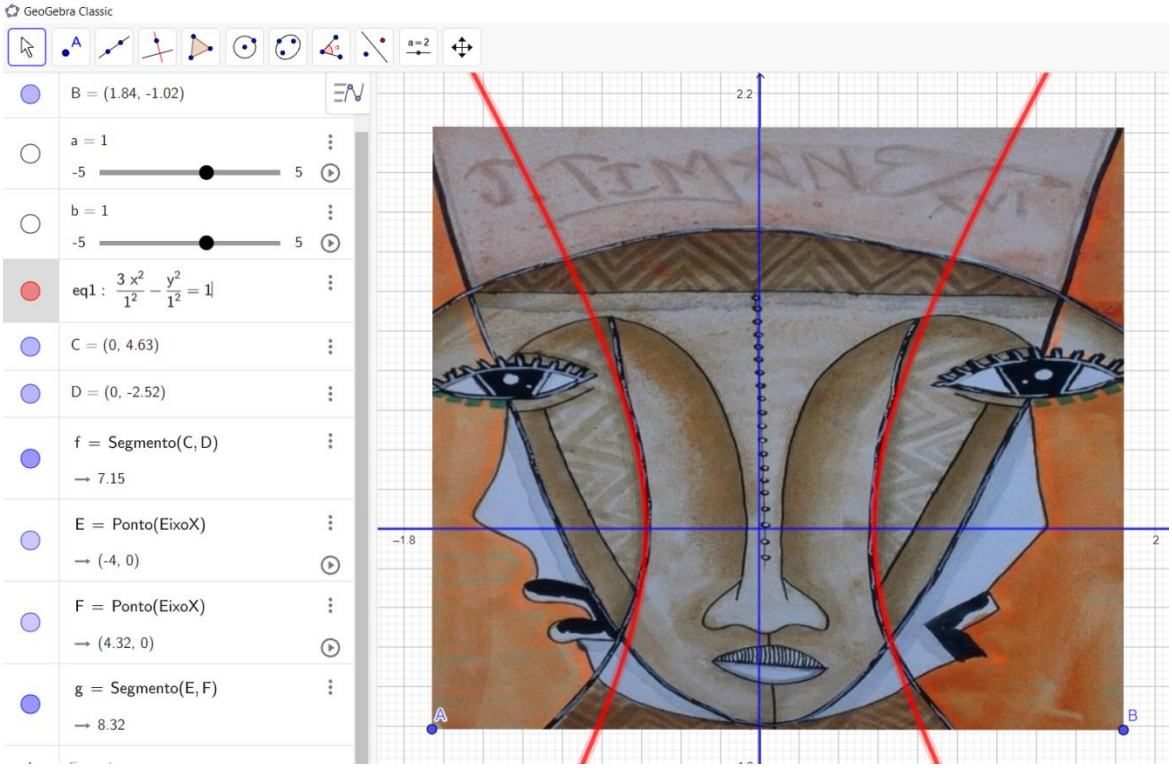

Figure 5.

Provocation in the look: a hyperbola with the image (Authors' production)

The graph generated by the function circumscribes other planes in the image, such as the side views of the faces, which provoke us to see other details in the painting. As a result,

despite the highlighted symmetry in relation to the distance from the vertical axis, the faces in the profile gain other lines, for example, different shapes in the mouth and nose in relation to the right and left side views in the image.

So, in this movement of looking at the image and doing those exercises with mathematics, we want to raise, on the one hand, the idea of considering that the image is, in fact, representing some mathematical knowledge, which arises through the cognitive process of adjusting the view to knowledge, i.e., through the exercise of visualization. However, it also seems to us that when we look at it, what we do is not just understand or identify mathematical knowledge in it but feel that something is being formulated by it and with it, and that, therefore, we ourselves are participating in this formulation. Let us remember that "We never look at one thing only, we are always looking at the relationship between things and ourselves" (Berger, 1999, p.11). In short, looking at the image also implies realizing the visual discourses that form and guide our view, our gaze.

In another way, the participants saw in the images a potentiality for teaching and learning. They said, for example:

> *Matato:* The images make me think about how good it would be to work with them for the study of functions, the transformations that a function can undergo, and also its superimposition applied in the arts. [...] I would do [it] with them as an example of *applied mathematics*.

In this passage, we underline the expression applied mathematics in the participants' speech. We did so because when it comes to transcribing the audio from the recordings and, assuming a cartographic ethos with inspirations in the cartographic method clues (Passos, Kastrup, & Escóssia, 2012; Passos, Kastrup, & Tedesco, 2016), we observed repeated statements in favor of mathematics teaching and learning, i.e., that *art works as a support to contextualize mathematics learning*[18]. This means that from working with art images, besides identification, conceptualization, and the exercise of thinking about mathematics, there is the possibility of making habits and naturalities emerge about teaching and learning with art.

Of all this, therefore, not only the artistic activity is event, in the sense that art brings out as much knowings as possible –the previously thought, but also those of the order of the unthought, of the unspecific– as it is for the possibility of exercising thinking about mathematics. Now, this last case leads us to reiterate, once again, that learning It is an *event,*

[18] Under the title "*Contextualizar a matemática pela arte? Problemáticas junto a futuros professores da Universidade Licungo, Moçambique"* [Contextualize mathematics through art? Problems along with pre-service professors at Licungo University, Mozambique], part of this analysis was presented at the IX CIBEM, at PUC-SP, in December 2022.

since it is engendered by creation, by invention with art (Flores & Kerscher, 2021), in which "learning is of the order of the sensitive (becoming incarnated) more than the intelligible, simply" (Gallo, 2017, p. 111).

For a decolonial ethos: as conclusions

The threads that weave and move the writing of this article make connections with what erupts from the thought of a group of pre-service teachers attending a Mathematics Teaching Degree course in Mozambique, in a research work that considers the perspectives of visualization and visuality to mathematics teaching with art through workshops. In this article, therefore, we outline an exercise of the look: based on our ways of seeing and of and with the research participants, we problematize the naturalness of thought in work with art and mathematics that, in turn, almost always maintains a single referential as valid, i.e., the visualization. Hence, we resume the question: What knowings do we summon and which we are summoned by when we look at a work of art?

Well, with this, we do not want to give answers but rather exercise an attitude, an ethos, which puts us in a position of continuous criticism of the relations of knowledge and power that make up an epistemic colonial matrix. Having covered the research, although still in progress, we verified that despite looking at an image of art to teach mathematics, images of Mozambican artists, from different perspectives from other places, from other cultures, it always implies the duty to populate ourselves with knowledge so, from them, we can deduce everything that is already there, defined, represented, by recognition: a symmetry, an equality, a parabola, a hyperbola, etc.

From the workshop in focus here, we challenge ourselves, in a way, to face traps in which thoughts have already been imprisoned, captured, to put ourselves in motion towards problems, concerns, tremors, assuming, on the one hand, that ways of relating mathematics to art show teaching practices that silently produce ways of existing, believing, and building worlds (Wagner & Flores, 2020). On the other hand, we open ourselves to possibilities to create conditions for thinking differently, sheltering other existences in terms of the forms and ways in which we place ourselves as subjects before other subjects, before teaching, mathematics, art, and the world.

Finally, we invite you to ask yourself: How to promote encounters between art and mathematics, between learning and teaching, things that are already there, familiar, naturalized, colonized, but that with art and mathematics and learning can also be interrogated and placed in a movement of becoming? In this case, no longer the so invested experience in locating in

art the mathematics that the person knows, but the experience of thinking with art the mathematics that is displayed and exercised in practices, the experience that we have, that we know, that forms and informs us about the world, about us, but also about teaching processes and learning. More than that, it would be to operate:

> a shift in the look, a different way of thinking, allowing oneself to be seen in other ways and letting oneself be affected by other ways of seeing the world [...] inverting the look from where we are used to thinking, impregnated with rationality forged in the relationships between modernity and coloniality (Tamayo & Mendes, 2021, p. 5).

Authorship Contribution Statement

ADC delivered the workshops, collected, and registered the data, and made the present data available for this article. ADC also treated figures 3, 4, and 5 mathematically. CRF conceived the idea for the article, analyzed the data, and wrote the text. DRW discussed the data analysis and conclusions of the article. For the final version of the text, the content was revised as requested by the reviewers. MMKF treated and produced the mathematics in Geogebra, creating figures 3, 4, and 5. All authors contributed to the writing and final version of the article and proofread the text after the reviewer's feedback.

Data Availability Statement

Os dados que suportam os resultados deste estudo serão disponibilizados pelo autor correspondente, Adamo Devi Cuchedza, mediante solicitação razoável.

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Tradutor do português para o inglês e revisor do inglês:

Maria Isabel de Castro Lima

E-mail: baulima@gmail.com