EMP Educação Matemática Pesquisa
ISSN 1983-3156
Qualis A1

http://dx.doi.org/10.23925/1983-3156.2024v26i1p177-207

Articulations between History of Mathematics and Mathematical Modeling: some understandings from a systematic literature review

Articulaciones entre Historia de las Matemáticas y Modelado Matemático: algunas comprensiones a partir de una revisión sistemática de la literatura

Articulations entre histoire des mathématiques et modélisation mathématique : quelques compréhensions à partir d'une revue de littérature systématique

Articulações entre História da Matemática e Modelagem Matemática: algumas compreensões a partir de uma revisão sistemática de literatura

Érica Gambarotto Jardim Bergamim[1]
State University of Maringá (UEM)
Doctor Degree In Education for Science and Mathematics (PCM-UEM)
https://orcid.org/0000-0002-2954-3506

Lilian Akemi Kato[2]
State University of Maringá (UEM)
Doctor in Applied Mathematics (Unicamp)
https://orcid.org/0000-0001-8770-3873

Abstract

In this text we seek understanding, in terms of the teaching and learning process of Mathematics, about the possible articulations between the History of Mathematics (HM) and Mathematical Modeling (MM) revealed in research in the area. To this end, a systematic literature review was carried out, through which we searched for research in databases and national scientific events that addressed topics related to this objective, which led us to the identification of eight works. Through the procedures of Discursive Textual Analysis, initially, we fragmented and unified the data and, later, classified them into categories. From this analytical process, two categories emerged: I – Articulating HM and MM enables teaching and learning processes that generates understanding of the mathematical topics studied and desirable attitudes for the formation of the student and II – Articulating HM and MM makes it possible to highlight the human aspects, social, scientific, and cultural aspects linked to the development of mathematical models and the relationship between mathematics and other areas of knowledge. Based on the emerging categories, potentialities arising from articulations

[1]E-mail: ericagambarotto@hotmail.com.
[2]E-mail: lakato@uem.br.

between HM and MM for the teaching and learning process of Mathematics were evidenced, which justify the reasons why it is relevant to do so.

Keywords: History in mathematics education, Mathematical modeling, Joints. Literature review.

Resumen

En este texto buscamos comprensiones, en términos del proceso de enseñanza y aprendizaje de las Matemáticas, sobre las posibles articulaciones entre la Historia de las Matemáticas (HM) y la Modelación Matemática (MM) reveladas en investigaciones en el área. Para ello, se realizó una revisión sistemática de la literatura, a través de la cual buscamos investigaciones en bases de datos y eventos científicos nacionales que abordaran temas relacionados con este objetivo, lo que nos llevó a la identificación de ocho trabajos. A través de los procedimientos de Análisis Textual Discursivo, inicialmente, fragmentamos y unificamos los datos y, posteriormente, los clasificamos en categorías. De este proceso analítico surgieron dos categorías: I – La articulación de HM y MM posibilita procesos de enseñanza y aprendizaje que genera comprensión de los temas matemáticos estudiados y actitudes deseables para la formación del estudiante y II – La articulación de HM y MM permite resaltar los aspectos humanos, sociales, científicos y culturales, vinculados al desarrollo de modelos matemáticos y la relación entre las matemáticas y otras áreas del conocimiento. A partir de las categorías emergentes, se evidenciaron potencialidades derivadas de las articulaciones entre HM y MM para el proceso de enseñanza y aprendizaje de las Matemáticas, que justifican las razones por las cuales es relevante hacerlo.

Palabras clave: Historia en la educación matemática, Modelo matemático, Articulaciones, Revisión de literatura.

Résumé

Dans ce texte, nous cherchons à comprendre, en termes de processus d’enseignement et d’apprentissage des mathématiques, les articulations possibles entre l’histoire des mathématiques et la modélisation mathématique révélées dans la recherche dans le domaine. À cette fin, une revue systématique de la littérature a été réalisée, à travers laquelle nous avons recherché des recherches dans des bases de données et des événements scientifiques nationaux qui abordaient des sujets liés à cet objectif, ce qui nous a conduit à l’identification de huit travaux. Grâce aux procédures d’analyse textuelle discursive, dans un premier temps, nous avons fragmenté et unifié les données et, plus tard, les avons classées en catégories. De ce

processus d'analyse, deux catégories ont émergé : I – Articuler HM et MM permet processus d'enseignement et d'apprentissage qui génère une compréhension des sujets mathématiques étudiés et des attitudes souhaitables pour la formation de l'élève et II – Articuler HM et MM permet de mettre en évidence les aspects humains, sociaux, scientifiques et culturels liés au développement des modèles mathématiques et la relation entre les mathématiques et les autres domaines du savoir. Sur la base des catégories émergentes, des potentialités découlant des articulations entre HM et MM pour le processus d'enseignement et d'apprentissage des mathématiques ont été mises en évidence, ce qui justifie les raisons pour lesquelles il est pertinent de le faire.

Mots-clés : Histoire de l'enseignement des mathématiques, Modélisation mathématique, Les articulations, Revue de littérature.

Resumo

Neste texto buscamos compreensões, em termos dos processos de ensino e de aprendizagem de Matemática, sobre as possíveis articulações entre a História da Matemática (HM) e a Modelagem Matemática (MM) reveladas nas pesquisas da área. Para tanto, foi realizada uma revisão sistemática de literatura, por meio da qual se buscou por pesquisas em bases de dados e eventos científicos nacionais que abordassem temáticas correlatas a esse objetivo, o que nos levou à identificação de oito trabalhos. Por meio dos procedimentos da Análise Textual Discursiva, inicialmente, fragmentamos e unitarizamos os dados e, posteriormente, os classificamos em categorias. Desse processo analítico, emergiram duas categorias: I – Articular HM e MM possibilita processos de ensino e de aprendizagem que geram compreensão dos tópicos matemáticos estudados e atitudes desejáveis para a formação do aluno; e II – Articular HM e MM possibilita evidenciar os aspectos humanos, sociais, científicos e culturais, atrelados ao desenvolvimento dos modelos matemáticos e as relações da Matemática com outras áreas de conhecimento. Com base nas categorias emergentes, evidenciou-se potencialidades advindas de articulações entre HM e MM para o processo de ensino e de aprendizagem de Matemática, que justificam os motivos pelos quais é relevante fazê-las.

Palavras-chave: História na educação matemática, Modelagem matemática, Articulações, Revisão de literatura.

Articulations between History of Mathematics and Mathematical Modeling: some understanding from a systematic literature review

In this article our focus is tuned to two of the, so called, methodological tendencies of Mathematical Education: The History of Mathematics (HM) and Mathematical Modeling (MM), in what concerns to the possibilities of articulations between them, in view of the teaching and learning processes of Mathematics. One of the reasons for our interest in this subject comes from the articulations between HM and other methodological tendencies, which are objects of study in Brazilian research. As an example, we cite the research by Souza (2020), which aimed to promote alliances between HM, Investigation of Mathematics (IM) and Information and Communication of Digital Technologies (TDIC).

About the existence of connections between HM and MM, and their potential for the teaching and learning processes of Mathematics, we cite the article by Assis (2021), in which the author suggests some articulations with other methodological tendencies in Mathematics Education so the HM can be incorporated as pedagogical resource in the teaching of Mathematics.

Assis (2021, p. 14) points out the need of developing practices of "*epistemological insubordinations*, in which the History of Mathematics make possible cognitive and linguistic experiences in investigations which highlight the intellectual, technical, scientific and cultural characteristics of mathematical traditions" (stress by the author), and for this indicates the MM as one of the "educational trends in which historical evaluations make possible a mathematical experience of teaching" (Assis, 2021, p. 14).

According to this author, the experience of the MM in the education makes it possible to reflect reflections which aid the understanding of the potentialities and limitations of mathematical thinking and discusses "the examples of the known models may not only be adapted for an exposition of historical information, but studied for their theoretical and cognitive contributions in the promotion of a mathematical experience" (Assis, 2021, p. 21). From this reading, we inferred there is a possibility of adapting classic mathematical models, in a manner of valuing the historical aspects involved and, in this sense, Biembengut (2016) also suggests the discussion of known mathematical models. as the logistic model by Pierre Verhulst, in the Mathematics classes. Besides this, by discussing the stages for the development

of Mathematical Modeling, he recommends the discussion of historical aspects in the early stage of the activity.

In face of what was exposed and with the intention of looking at the works which established some articulation between HM and MM in the teaching and learning processes, this research is guided by the following question: **What do the studies which approach possible articulations between HM and MM say in terms of teaching and learning of Mathematics?** We clarified that, according to the Dicionário Michaelis On-Line (2021), **articulating** means: "Making(itself) connected; joining(itself), connecting(itself), relating(itself), unifying(itself)". Thus, in this text, we understand that articulating HM and MM means joining them/relating them in activities turned to the process of teaching and learning of Mathematics, contemplating different manners of interaction and integration between them.

With the objective of answering our guiding question, we conducted a systematic review which searched for theses and dissertations, articles published in journals and in scientific events related to both subjects involved, HM and MM. For organization and data analysis, we considered the methodological procedures of Discursive Textual Analysis (ATD), according to Moraes and Galiazzi (2011).

In the next sections present: the theoretical references which guided the data analysis; the methodological procedures we adopted for the selection, organization and interpretation of the works; the analyzed data; the highlighted results from these processes; some considerations on what was developed and concluded from this process.

Preliminaries on the articulation between HM and MM in the process of teaching and learning

In this topic, we present some aspects related to HM and MM, with sights on possibilities of articulations which may contribute to the processes of teaching and learning of Mathematics. These aspects concern our understanding of HM and MM as areas of knowledge in the scope of Science and in the context of Mathematical Education.

We start, thus, discoursing when we mention History of Science, according to Beltran et al. (2014), the referred science is not citing only Physics, Chemistry, Biology and Mathematics, in an isolated manner, once until the 19th century these were not specialized areas

as we know today. The specialization of areas of scientific knowledge, that is, modern science, occurred between the 16th and 17th centuries.

Having clarified this, we can say "History of Science is the study of the form(s) of elaboration, transformation and transmission of knowledge about nature, techniques and societies in different times and cultures" (Beltran et al., 2014, p. 15). In this assertion we observed the object of study of this area of knowledge is wide and we are according to it due to the fact it contemplates the clarifications on the existence of science since the antiquity and specializations of the science we mentioned. Still, it is important to observe that, according to the authors, "the History of Science acknowledges the existence of Physics, Chemistry, Biology or Mathematics in the past, because the historian of the Science **parts from what are areas of knowledge nowadays**" (Beltran et al., 2014, pp. 80-81, our stress).

Consonant to the exposition in this quote, we understand we deal with HM when the narrated history involves mathematical concepts and procedures, as we know today. In face of this idea, in what refers to HM as an area of knowledge, Miguel and Miorim (2002) indicate it encompasses studies which search to investigate all the dimensions of mathematical activity in history in every social practice which participated in the process of production of mathematical knowledge. Therefore, we acknowledge that everything which involves mathematical knowledge is part of HM.

With the advance of this area, it became possible to evidence the constitution of some autonomous fields of research, which have specificities, but which continue keeping preoccupation with historical nature, reaching relations which may be established between Education, Mathematics and Pedagogy. Between them there is the field of History of Mathematical Education, in which are included those studies which have as object of investigation the problems associated to the effective insertions of history in the initial or continuous training of Mathematics teachers, in the formation of students of any education level, among others (Miguel & Miorim, 2011).

Thus, in the referred field there are the studies which investigate forms of participation of the HM in mathematical education, understood as a pedagogical field of action or as field of investigation (Miguel & Miorim, 2011). In the field of pedagogical action there is the usage of the HM as pedagogical resource, which, according to Mendes (2006, p. 84) "[...] has as its main

finality promoting a teaching-learning of Mathematics which allows a ressignification of mathematical knowledge produced by society along with the time", providing, simultaneously, more motivation and cognitive creativity.

About the use of the HM as a pedagogical resource, Fossa (2011) recommends a thoughtful usage, in which the HM is used to teach the Mathematics concepts themselves. "Thus, the Mathematics content is presented through a historical approach, which generally involves the discussion of interesting themes and not trivial, frequently turning itself to Applied Mathematics or to problems with a strong practical imprint" (Fossa, 2011, p. 64). In relation to the coverage of the historical cut involved in the proposed activities, considering the thoughtful usage, the author indicates that we can settle on the "episodic usage, that is, the usage of History of Mathematics to approach some selected topics inside the discipline" (Fossa, 2011, p. 65). In this bias, for this research, the understanding we take on the usage of HM episodes is in consonance to that is presented by this author, given his conformity with didactic and pedagogical ends for the work in the classroom.

About the aspects related to the MM, its presence in the History of Science, according to Biembengut and Hein (2009), it reassembles to antiquity, characterizing it as an activity as old as Mathematics itself. In this manner, it can be said it is a human activity, because it emerges from the human necessities, as well as Mathematics. This idea is strengthened when Biembengut and Hein (2009, p. 11) affirm that humanity constantly appealed to models, "[…] both to communicate with their similars and to prepare for one action. In this sense, the modeling, [as] the art of modeling, is a procedure which emerges from reason itself and takes part in our lives as a form of constitution and expression of knowledge".

Based on the exposure by Biembengut and Hein (2009), we can observe the origins of the process of modeling are related to the mathematical applications which aided in the problem solving. In this sense, Almeida et al. (2012, p. 12, stress by the authors) stress that "the natural *habitat* of Mathematical Modeling is the area, which was agreed being called Applied Mathematics, and in the interior of which appeared the first concepts and procedures in relation to what characterizes an activity of Mathematical Modeling".

About this characterization of the MM in the field of Applied Mathematics, we stress "the term 'Mathematical Modeling' as process to describe, formulate, model and solve a

situation-problem of some area of knowledge is found in the beginning of the 20th century, in the literatures of Engineering and Economic Sciences" (Biembengut, 2016, p. 161).

In relation to the adaptation of the MM to the context of Applied Mathematics for the scope of Mathematical Education, we have "the first proposals of Modeling in/for Mathematical Education pass to be more present in Congresses of the years of 1970, in many countries, including in Brazil" (Biembengut, 2016, p. 161). This transition, here in Brazil, was demarcated in the decade of 1980, when "the speech of Modeling, by intertwining to the constructivist speech, [...], dislocates in the sense of studying Mathematics from the models, while product for the construction of models while process" (Magnus et al., 2019, p. 1215). It is in this context the MM emerges as an approach to teach Mathematics.

In this manner, from that time the conceptualization and the characterization of the MM in the scope of Mathematical Education "[...] has had different approaches and has been realized according to different assumptions in relation to the pedagogical conceptions which guide the educational practices and the theoretical structuring of scientific researches" (Almeida et al., 2012, p. 12). In face of what was exposed about the MM coming from Applied Mathematics and of the MM in the context of Mathematical Education, we observed the objectives of using modeling in both perspectives are distinct. Therefore, we consider it important to discuss the distinction. For this, we present Table 1 which brings definitions and objectives of Modeling and Modelation[3], according to Biembengut (2016).

[3] Biembengut (2016) uses this term to refer to Modeling in Education, and for this text, we understand it is related to Modeling in Mathematical Education, due to the definition and objectives presented in Table 1.

Table 1.

Characteristics of Modeling and Modelation (Biembengut, 2016)

	Modeling	Modelation
What is it?	"Modeling is a method to solve some situation-problem or to understand a phenomenon by making use of some (mathematical) theory" (p. 98).	"The Modelation is a method of teaching with research in the limitations and school spaces in any discipline and stage of education" (p. 177).
Objective	"Establishing a (mathematical) model of a situation-problem to solve it, understand it, or even modify it if necessary" (p. 175).	"Promoting knowledge to the students in any period of schooling, and teaching them to make their research in this educational structure, that is, in the physical space and in the concerning period to this purpose" (p. 175).

From Table 1, we infer that Modeling, as a method of research, refers to the competence to understand phenomena, while Modelation is more directed to the process of teaching contents of school subjects, in a manner of favoring the students to learn to research. Without belittle other authors who discuss about these distinctions and who presented other conceptions about the MM in the context of Mathematical Education, we clarify the choice for idea presented by Biembengut (2016), in this study, it was given by the fact the author depicts, in her work, activities of Mathematical Modelation with facts of History of Science and Mathematics.

Entering the subject related to the possibility of articulation between HM and MM, we found the Brazilian publication of the theoretical article by Assis (2021), which defends explicitly this articulation, inciting us to the search of other publications which give clues of this possibility. Furthermore, we based on Biembengut (2016), who presents the stages for the development of (Mathematical) Modelation, bringing a discussion on historical aspects involved in the chosen theme for the activity.

For the author, "in the process of Modelation, in particular, different contexts 'rise'. As an example, in relation to the **theme/subject**: It considers that the circumstance, occurrence, and timepassed to be treated by experts and/or researchers (**historical context**)" (Biembengut, 2016, p. 186, our stress). We understood that when it indicates the discussion of these historical aspects in the early stage of a Modelation activity, called **Perception and Apprehension**, the participation of the history in this purpose contributes to awakening the interest of the students for the study theme.

However, we consider the participation of history in Modelation activities also may assume a relevant role about understanding of concepts and mathematical procedures involved in the referred activities. This is because the author recommends the use of classic models (as an example, those based on the ideas of Isaac Newton and Pierre Verhulst) for the development

of Modelation activities. This recommendation was also done in Biembengut and Hein (2009, p. 30), when the authors affirmed that "to those who want to make a work using the Modelation, but do not feel properly safe, we orient: – knowing some classic models by mean of literature about the History of Science of contemporary Science, adapting them to the classroom [...]".

Analyzing the exposure by Biembengut and Hein (2009), Biembengut (2016) and Assis (2021), we noted they give us clues about the articulations between History of Science (in particular, HM), which contribute to the teaching and learning processes of Mathematics. From this, we decided to search for national and international research which directly or indirectly approaches this theme. The procedure to do this research and analysis of the results found are presented in the next sections.

Methodological aspects

By searching to answer the question **"What do the studies which approach possible articulations between HM and MM say in terms of teaching and learning of Mathematics?"**, we started through research in many databases, which led us to the development of a systematic literature review.

According to Mendes and Pereira (2020), the interest for the systematic review comes from the clarity of the methodological procedures adopted and of the criteria well defined besides the systematized stages. To do it, these authors propose stages for the realization of systematic reviews for the area of Teaching and Mathematical Education, from a study in many researches.

For the development of a systematic review, the authors identified "five stages which are constituted as common to this kind of research, to be known: objective and question; search of works; study selection; analysis of the productions; presentation of the systematic review" (Mendes & Pereira, 2020, p. 196) and describe how to realize each stage. With the ends of clarifying how these stages were contemplated in our systematic review, we present the Table 2, in which we expose them and present how they were developed, considering the specificities of our investigation.

Table 2.

Description of the procedures adopted for the realization of the systematic review (Adapted from Mendes & Pereira, 2020).

Stages for the elaboration of the systematic review, according to Mendes and Pereira (2020)	**Description of the development of the systematic review, considering the particularities of our investigation**

Objective and Question Stage in which the purposes to realize the systematic review must be clear. Thus, it must be defined as a question of research.	**Question**: What do the studies which approach possible articulations between HM and MM say in terms of teaching and learning of Mathematics?
Search of works[4] Stage in which are defined more adequate keywords, the databases and the specificities of search used, kind of materials and time period.	**Keywords**: "History of História da Matemática" AND "Modelagem Matemática"; "História da Matemática" AND "Modelo Matemático"; "History of Mathematics" AND "Mathematical Modeling"; "History of Mathematics" AND "Mathematical Model"; (História da Matemática) (Modelagem Matemática); (História da Matemática) (Modelo Matemático); **Kinds of work**: articles from journals and of scientific communications of events; dissertations. **Sources of search**: Catalog of Theses and Dissertations from CAPES (Catálogo de Teses e Dissertações da CAPES); Biblioteca Digital Brasileira de Teses e Dissertações; Periódicos da CAPES; Google Academics; *Scopus*; *Web of Science* and Annals of ENEM editions, from SNHM and of CNMEM. **Time period**: there was no time limit.
Selection of studies Stage in which are defined the criteria for the selection of works. Usually it is done in two moments: in the first it is aimed to read titles, abstracts, keywords; in the second it is aimed to read the complete work to verify if the work will be useful to compose the *corpus* of the research	**Criteria for inclusion and exclusion**: <u>Exclusion:</u> – Works which did not mention to the MM or to the HM when listing the trends in Mathematical Education. – Works which only bring in some sections the historical aspects on the developed subject, but which did not articulate these information with the MM. – Works which only bring in some sections related to the historical development of a mathematical model without establishing the connection with the processes of teaching and learning of Mathematics. <u>Inclusion:</u> – Works which discuss some kind of articulation between HM and MM and which present discussions which could contribute to the formation of students and teachers. – Works which bring proposals already implemented or not. – Works which did not bring proposals for the classroom, but which brought reflections about the processes of teaching and learning.
Analysis of productions Stage in which is searched to extract the pertinent data to the purposes of the research.	We analyzed in qualitative manner the eight works, selected according to the procedures of the ATD (Moraes & Galiazzi, 2011).
Presentation of the systematic review Every previous stage must be presented, searching to answer the proposed question of the research.	It was presented in all sections of this work. In particular, in the section presenting the discussion of results, we searched to make explicit how our guiding question was answered, as well as we bring reflections about possible articulations between HM and MM and their potentialities for the processes of teaching and learning.

As it can be observed in Table 2, in our systematic review, having as the basis our guiding question, we established keywords in Portuguese and English languages. The combinations of keywords in Portuguese language between double quotation marks and with the boolean operator AND were used in the search field of the CAPES Catalog, of the BDTD,

[4]The search in the databases was carried out in the second half of February 2022.

the CAPES periodicals and Google Academics. The combinations of keywords in the English language were used in the search field of the *Scopus* website and the *Web of Science*. This action made it possible for us to widen the data *corpus,* counting with international articles. After the realization of the search in national and international websites, we found five works which met our selection criteria (presented in Table 2).

Facing this scenario, we opted to realize searches in national events annals, which also could indicate us writings which would be in consonance with our question. In this perspective, the chosen events were: National Encounter of Mathematics Education (Encontro Nacional de Educação Matemática - ENEM), National Conference on Modeling in Mathematical Education (Conferência Nacional sobre Modelagem na Educação Matemática - CNMEM) and National Seminar of History of Mathematics (Seminário Nacional de História da Matemática - SNHM). For the selection of the works found in the annals of each ENEM, SNHM and CNMEM edition, due to not having the option of searching for keywords in specific search fields of websites in each edition, we drew up specific strategies to identify the works.

In relation to the ENEM, in possession of the annals of the 13 editions of this event, initially, searched to look the texts corresponding to the experience reports, to the scientific communications and posters, which in brought in the title the words related to the History of Mathematics and Mathematical Modeling or Mathematical Model. We read the abstract of those which had these words and, when in doubt, we did a floating reading of the complete work. The same procedure was adopted for the works published in the annals of 10 editions of the CNMEM[5] and the annals of the 14 editions of SNHM.

With the searches in the annals of the events mentioned we found three more jobs, in a manner that our *corpus* passed to contain eight works. The stages and procedures for identification and selection of these works can be observed in Figure 1, which was elaborated based on PRISMA (Principais Itens para Relatar Revisões Sistemáticas e Meta-análises) recommendations, and which has as objective "[...] helping the authors to improve the reports of systematic reviews and meta-analysis" (Moher et al., 2015, p. 336). To do so, it presents a checklist and a flow Table with four stages, which aid in the process of reporting the stages roamed in a systematic review.

[5] This event had 11 editions, however for this research we had access to the annals of 10 editions, missing the one corresponding to the second edition.

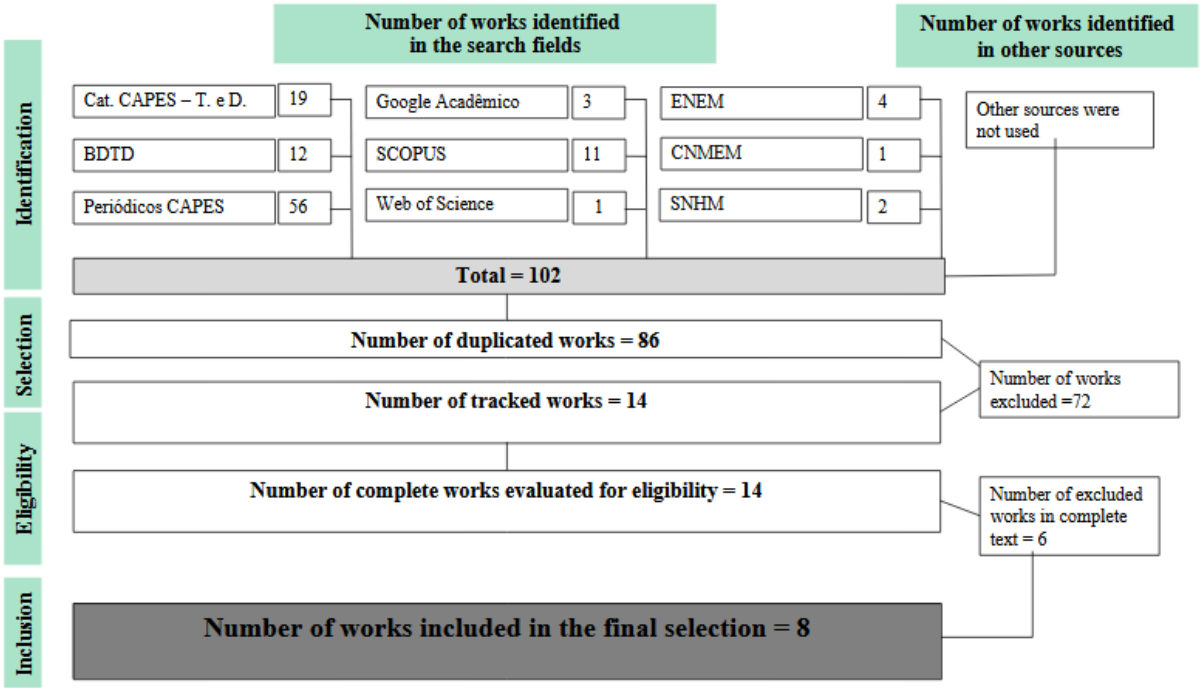

Figure 1.

Flow Table of the information with different stages of the developed systematic review (Adapted from: Moher (et al., 2015, p. 338) and Mendes and Pereira (2020, p. 207)).

Of the eight works found, five are national, being three published in events (an article of scientific communication, a poster and an experience report) and two Professional Master's Degree dissertations. The other three works are international and were published in English language and are from journal publications. With the characteristics "kind of work" and "year of publication" we constituted representative codes for each work. In Tablet 3 are presented the titles of the works, information on the authors, year, kind of work and each corresponding code.

Table 3.

List of selected works(The authors)

Title	Authors – Year of publication	Kind of work	Work Code
Employing genetic 'moments' in the history of mathematics in classroom activities [6]	Farmaki and Paschos (2007)	Article	AR.2007
Functions in Highschool: History and Modeling	Souza (2011)	Dissertation	D.2011
Developing Students' Reflections on the Function and Status of Mathematical Modeling in Different Scientific Practices: History as a Provider of Cases [7]	Kjeldsen and Blomhøj (2012)	Article	AR.2012

[6] Translation: Tradução: Empregando 'momentos' genéticos na história da matemática em atividades de sala de aula.

[7] Translation: Desenvolvendo as Reflexões dos Alunos sobre a Função e o Status da Modelagem Matemática em Diferentes Práticas Científicas: A História como Provedora de Casos.

The Mathematical Modeling during the history and appearance of Modelation in Brazil	Ferreira et al., (2013)	Article	CC.2013
The Screw of Archimedes: an innovation in Mathematics teaching under the perspective of Mathematical Modeling at IFPA	Matos et al. (2013)	Experience report	RE.2013
Complex Numbers: a didactic proposal based on Mathematical Modeling in historical contexts	Paes (2013)	Dissertation	D.2013
A historical contextualization for the classic model by Malthus	Silva et al., (2017)	Poster	PO.2017
How history of mathematics can help to face a crisis situation: the case of the polemic between Bernoulli and d'Alembert about the smallpox epidemic [8]	Gosztonyi (2021)	Article	AR.2021

From the identification of these works, to analyze the data we used the procedures from ATD. For Moraes and Galiazzi (2011, p. 7), "the Discursive Textual Analysis corresponds to a data analysis methodology and information of qualitative nature with the finality of producing new understandings about phenomena and speeches". For the ATD usage it was necessary to guide the organization of the analysis, in conformity with the four focuses described by the authors, to be known: **disassembly of texts** or process of **disassembly** and **unitarization**; **establishment of relations** or process of **categorization**; **capturing the new emerging** and **self-organized process** (Moraes & Galiazzi, 2011).

The **textual disassembly** "implies examining the texts in their details, fragmenting them in the sense of reaching constituent units, enunciations referring to the studied phenomena" (Moraes & Galiazzi, 2011, p. 11). In this article, to identify the units of meaning, first we selected in each work fragments of text -paragraphs- in which the authors did some affirmation about the possible relation between HM and MM, connecting to the process of teaching and learning[9]. With this action were found 34 fragments and from them we realized a new reading, searching to identify units of meaning. For such identification, we highlighted parts of the fragments which made us constitute the titles of the units of meaning, with typographic resources, considering parts highlighted with distinct typographic resources generated distinct units of meaning.

Since our selected fragments were composed of paragraphs, in some cases a fragment generated more than a unit of meaning. On the other hand, we stress that in this process,

[8] Translation: Como a história da matemática pode ajudar a enfrentar uma situação de crise: o caso da polêmica entre Bernoulli e d'Alembert sobre a epidemia de varíola.

[9] We searched for parts of texts which approached History of Mathematics (or related words: historical context, historical facts, historical episode, among others) and mathematical modeling or mathematical model and which made some kind of affirmation, relating to the processes of teaching and learning of Mathematics. These affirmations, generally, were found when the authors tried to justify the research (presented in the section of introduction) or when arguing about the results of the research (in the section of discussion of the results or final considerations).

sometimes, in different fragments of the same work we identify equal units of meaning; when this occurred, we chose the fragment which best represented the unit of meaning in question. By concluding the process of unitarization we identified 48 distinct units of meaning. Table 4 presents the number of fragments and units of meaning which were extracted from each work.

Table 4.

Quantity of fragments and units of meaning extracted from each work (The authors)

Code of the work	**Number of fragments**	**Number of units of meaning**
AR.2007	3	3
D.2011	8	10
AR.2012	7	11
CC.2013	2	2
RE.2013	2	2
D.2013	3	5
PO.2017	4	5
AR.2021	5	10
Total	**34**	**48**

The **establishment of relations** "involves building relations between the units of basis, combining them and classifying them, gathering these unitary elements in the formation of groups which congregate proximal elements, resulting from this category systems". This process was done by mean of the inductive method "[...] which implies producing categories from the analysis units constructed from the *corpus*" (Moraes & Galiazzi, 2011, p. 23), that is, our categories emerged from the process of analysis and were not defined *a priori*. In a first movement of classification of the 48 units of meaning 13 categories emerged.

We clarified the part formatted with the bold typographic resource in the units of meaning, which led us to establish meaning similarities to classify in an initial category. This choice guided us to look at the potentialities coming from the possibilities of articulation between HM and MM and understood that this is in agreement with the inherent ATD processes, having in sight our intention of leaving the phenomenon show itself. Table 5 shows how we gathered some units of meaning in an initial category.

Table 5.

Example of classification of units of meaning in an initial category (The authors)

Title of the units of meaning	Unity codes[10]	Initial Category
The mathematical models and the historical context **contribute to significant learning**.	D.2011.F3.U1	Articulating HM and MM **contributes to the learning of mathematical contents**.
The junction of historical study and the method of Mathematical Modeling constitutes a **tool to learn some concepts**.	D.2011.F6.U1	
A sequence of activities lined on Mathematical Modeling and History of Mathematics might **contribute for a quality learning**.	D.2013.F2.U2	
Analyzing historical episodes which contain Mathematical Modeling **aids in the learning of specific topics**.	AR.2021.F5.U1	

In this example, the parts formed with the bold typographic resources of the units of meaning from Table 5 are directed to the subject learning of mathematical contents. Having explained this, we clarified the titles of the initial categories (and also, intermediate and final) were constructed in a manner to initiate by "*Articulating HM and MM...*", because we understand that, first this articulation was the object of our investigation and, second, in our interpretation of this verb it contemplates every possibility of interaction between HM and MM, with view to the teaching and learning of Mathematics which were approached in the selected works. We highlight that, in a similar manner to the exemplified, the initial categories were grouped by similarity of meaning and generated five intermediate categories, which were grouped again, considering the proximity of ideas, which resulted in two final categories.

In the focus **capturing the new emergent** it is considered that "the intense impregnation in the materials of analysis unchained in both previous focuses makes it possible the emergency of a new renewed understanding from the whole" (Moraes & Galiazzi, 2011, p. 12). We intend to evidence from this renewed understanding the construction of metatexts of each of the final categories. We emphasize the **self-organized process** perpassed all the mentioned focuses, once it was necessary a process of impregnation with the data, constant review of categories and metatext reconstruction, which will be presented in the next section.

Presentation and result analysis

We initiate this section highlighting it was not established the time limit for the works, but by ending the search and selection of those we noted they were published between 2007

[10] The codification followed the pattern ahead: kind of work, year of publication, order of the fragment and order of the unit of meaning. Thus, the example D.2011.F3.U1 refers to the first unit of meaning of the third fragment of text extracted from the dissertation published in 2011.

and 2011, with an interval of 15 years, being this a considerable gap. Another result observed concerns the fact that from the eight selected works, five report the development of activities involving HM and MM, being three international and two national works. These activities were developed with students from High School (D.2011; RE.2013; AR.2007), students from Graduation (AR.2012), and teachers in continued education (AR.2021).

About the HM and MM aspects involved in the works found in our systematic review, we identified the approach of diverse themes of Science being used in different contexts. About these themes, as an example, in two works – AR.2007; D.2011 – were presented the ideas of the geometric model of movement by Nicole Oresme, developed during the 14th century; in AR.2012 it was discussed the cellular division model by Nicolas Rashevsky, from the decade of 1930, with a discussion of disagreement between him and some biologists; in D.2013 it was presented the method of cubic equation solving proposed by Cardano in the 16th century; in CC.2013 it was discussed the Principle of Archimedes[11] present in the Tractate *The floating bodies*, about the planetary model by Ptolemy in the 2nd century BCE. and, also, about the Problem of the Bridges by Königsberg, in which Leonard Euler solves the problem by mean of a mathematical model; in RE.2013 it was presented the mechanism by Archimedes (287 BCE.-212 BCE), one of the oldest hydraulic machines, used for water pumping; in PO.2017 it was discussed the Classic Model by Malthus, which treated about the relation between food production and population growth in the 18th century, about statistical models developed for the prevention of smallpox pandemics.

Succinctly, we present that in relation to the articulation modes, that is, how Hm and MM appear integrated and/or related in the proposals presented by the authors, we found distinctions, in the sense that they occurred in didactic sequences in separated manner – being used in different moments, starting with HM and ending with MM, as an example – (D.2011); they occurred in a more integrated way by mean of analysis of historic episodes which treat about situations which involve MM (AR.2012; AR.2021; PO.2017); they might occur by mean of the usage of the idea of a model depicted in the HM, but without mentioning about historical aspects with the students (AR.2007). This was visible when we analyzed the works D.2011 and AR.2007, which treat about the same historical subject related to the geometric model by Oresme, but the approach which they propose are different. These presented general results point to some indicative which allow us to answer our question.

[11] In this tractate, he affirms that every body dipped in a fluid receives buoyancy, being this buoyancy is the strength, with upwards direction, which the liquid exercises on the body dipped in a liquid.

Next we present the categories which emerged from our analytical process, initiating by the title, then the Table which illustrates the process by means of which such category emerged and, lastly, the corresponding metatex, which exposes the understandings the authors's speeches made possible to develop.

First Final Category – Articulating HM and MM makes possible processes of teaching and learning which generate understanding of studied mathematical subjects and desirable attitudes for the student's formation.

Table 6.

Units of meaning, initial and intermediate categories which generated the First Final Category (The authors)

Unity of meaning codes	Initial Cat.	Intermediate Cat.	Final Cat.
D.2013.F1.U1; D.2013.F3.U1 AR.2007.F3.U1; AR.2021.F3.U1 (4)	Articulating HM and MM **contributes to the teaching of mathematical contents**.	Articulating HM and MM **makes it possible to establish new strategies for teaching**.	**Articulating HM and MM makes possible the processes of teaching and learning which generate understanding of the studied mathematical topics and desirable attitudes for the formation of the student.**
D.2011.F5.U1; AR.2012.F2.U1 (2)	Articulating HM and MM **makes it possible to identify mathematical models developed in the past**.		
D.2011.F1.U1; PO.2017.F3.U1 (2)	Articulating HM and MM **makes it possible to develop strategies for teaching**.		
D.2011.F3.U1; D.2011.F6.U1; D.2013.F2.U2; AR.2021.F5.U1 (4)	Articulating HM and MM **contributes to the learning of mathematical contents**.	Articulating HM and MM **makes it possible to learn attributing meanings and understanding mathematical contents**.	
D.2011.F7.U1; AR.2007.F2.U1 AR.2021.F3.U2; AR.2021.F5.U2 AR.2012.F4.U2; AR.2012.F7.U1 RE.2013.F2.U1 (7)	Articulating HM and MM **aids in the understanding of mathematical procedures and contents**.		
D.2011.F2.U2; AR.2012.F1.U; AR.2012.F3.U1; AR.2021.F2.U2 (4)	Articulating HM and MM **makes it possible for the development of reflexive and critical thought**.	Articulating HM and MM **makes it possible for the development of desirable attitudes from the students**.	
D.2013.F2.U1; D.2013.F3.U2 RE.2013.F1.U1 (3)	Articulating HM and MM **makes it possible for the students to develop autonomy**.		

The process of modeling, utilizing mathematical tools to solve real problems identified by the men, has occurred since antiquity. This idea is corroborated by Biembengut and Hein (2009, p. 7), when they affirm "the mathematical modeling[12] [...] has been present since the

[12]We clarify the notion of mathematical modeling credit here is related to the "art of expressing by means of mathematical language situation-problem of our midst" (Biembengut, 2016. p. 7), what is more turned to the modeling as a competence which the human being acquires to interpret phenomena, and not to our process of modeling as it is understood in the field of Mathematical MOdeling in Mathematical Education with ends of approaching teaching.

most primitive times. That is, the Modeling is as old as Mathematics itself, appearing from applications in daily routines of ancient peoplest". Concerning this, the MM, understood as an alternative to teach Mathematics, originated from contexts from Applied Mathematics. Therefore, we consider, in some way, knowing and utilizing processes of mathematical modeling used in the past might contribute to the processes of teaching and learning of Mathematics, according to the current purposes of Mathematical Education.

Based on it, we defend that the articulation between HM and MM makes possible processes of teaching and learning which generate understanding of concepts and studied mathematical procedures, as well as the students manifest attitudes which are essential during this formation process. To support this argument we discussed three aspects manifested in the speech of the selected works and which emerged in the intermediary categories; they are: establishing of new strategies to teach Mathematics; learning with understanding and attribution of meaning to the mathematical contents; manifestation of desirable attitudes by the students.

In what refers to the first aspect, related to the idea that articulating HM and MM might contribute with new strategies to teach Mathematics, we express that HM makes it possible to identify problems which were resolved by means of models and which contribute to teaching, in the sense of explaining the development of a concept. This is supported, as an example, in the excerpt: "historical facts make possible finding modeled problems, from the use of the relation and interdependence between variables, ideas which led to the development of function" (D.2011.F5.U1, our underlining). In this excerpt, we observed the idea that the modeled problems are related to the contents of functions, however we understand it can be extended to other contents, in a manner which the HM might be a source of resources to find mathematical models.

We are able to verify that relative to teaching was cited an idea which is in consonance to what Biembengut and Hein (2009, p. 30) already had presented, when they affirmed "those who want to do a job using the modeling, but do not feel properly safe, we orient: – know some classic models by mean of literature about History of Science or Contemporary Science, adapting for the classroom [...]". This search for classic models and their usage may help in the process of teaching in another bias, which is related to preparing the teacher to work with MM, suggesting in an early work with modeling it is aimed to first look for known mathematical models and, consequently, known processes of modeling and which may be discussed with the students to teach them the content.

Also in agreement with this idea suggested by Biembengut and Hein (2009), author of other work discuss that

> Returning to our considerations for aspects related to teaching and learning, making usage of classic mathematical models, we can suggest that to employ this situation in the classroom, it could be made a proposal to analyze the hypotheses considered by Malthus, who motivated by a problem for which he did not find an answer which satisfied him, elaborated a model for it (PO.2017.F3.U1, our underlining).

We stress the suggestion of teaching by mean of the Classic Model by Malthus, analyzing the hypothesis considered for the construction of this model, contributes for the students to relate the role of each mathematical component present in the model in correspondence with the hypotheses took into consideration, what is also defended by Biembengut (2016). Still, we consider this kind of approach of classic mathematical models is related to the Model Analysis, according to what is proposed by Sousa (2021, p. 319), according to whom, "even making usage of ready models, it can transcend their simple illustrative role, and gains more reflexive weight, what makes it closer to the MM".

Still in relation to teaching, it appeared in some works the idea of which analyzing historical episodes which narrate the construction or usage of mathematical models is relevant to teach specific mathematical topics, in the sense of being a teaching support, as it can be noticed in the following excerpt:

> In the first place, the debate might be relevant as support for the teaching of specific mathematical domains [...] The critic by D'Alembert offers an initial example of a subjective approach of the probability (Colombo & Diamanti, 2015; Rohrbasser, 2011), and the reading of his arguments might encourage a discussion on different interpretations of the probability (AR.2021.F3.U1, our underlining).

This excerpt illustrates the support for the teaching is revealed in the possibility that an episode of the HM, which narrates the construction and the usage of mathematical models, supports discussions related to the different interpretations associated to the application of a mathematical content, which is an important factor when considering the teaching of determined mathematical contents, because it makes possible the attribution of meaning to what is thought.

In encounter to this last idea, but turning, now, to the second aspect contemplated in this category, it is observed the articulation between HM and MM, in the speech of the authors, has potential to contribute with the learning of mathematical concepts and procedures and, intertwined to this, providing their comprehension. This appears, for example, in the excerpt "Therefore, this work will have as finality aiding the students in the significant learning of the

polynomial functions by mean of the historical context and of mathematical models" (D.2011.F3.U1, our underlining), and also in this other

> We perceived projecting the activities and the objectives of our educational approach around the [model] ideas by Oresme and of the structure by Duval can be considered excessively ambitious. However, our experiments with a small group of students relatively young/of initial grades demonstrated this activities might lead directly and simply to the students and understanding of concepts and representations which are at the core of the advanced mathematical thought (AR.2007.F2.U1, our underlining).

We highlight that the two excerpts are coming from different works which use the geometrical model of variation of movement by Oresme (developed during the 14th century), in order to contribute with the learning and understanding of mathematical ideas related to the content polynomial function. However, the modes to articulate HM and MM are different. As an example, in D.2011 there is a separation between the two, once it starts the sequence of activities recurring to the HM, by mean of the presentation of ideas related to the model cited above, in a manner to awaken the interest of the students by the subject, and finishes with the activity of MM involving the subject of functions, but which do not resume the model discussed in the beginning of the sequence. Thus, the historical informations discussed in the beginning were not incorporated to the MM activity proposed later, but, in the author's view, once historical aspects related to the subject of functions were approached and then it was developed a modeling activity involving the subject functions, both, in an equal sequence of activities, contributed to provide the students learned the content.

On the other hand, in AR.2007, historical aspects related to the geometric model by Oresme were not presented to the students, that is, the HM is not directly explained in the activities. However, this model, understood as a genetic moment (genisys) of the concept of function, was used to show a new manner to calculate distance, speed and time, based on the graphic representation speed-time proposed by the referred mathematician. In order to do it, were proposed problems related to the uniform movement, which may be solved by mean of the geometric interpretation by Oresme (for him, in an axis it was represented the speed, in the other axis the time and the distance were given by the area of the rectangle limited by these axes), and are explored with the students the many algebraic modes of representing the problem, including the algebraic by mean of an equation.

In this manner, we understand the emphasis was, starting from the knowledge of this geometric model from the pas, exploring the mathematical ideas related to it in view to favor the learning, once it explores calculation of distances of a different manner and this might favor

the understanding and establishment of connections with the many modes of representing the same problem. Thus, the model which guided the development of the activities assumed a role of tool which aids in the learning, in conformity to what Jankvist (2009) affirms about arguments referring to the history as tool, being that, in this case, a mathematical model from the past is constituted from the clipping of the history used.

Still in this aspect related to the learning and understanding, turning to the mathematical procedures, in particular procedures associated to the process of modeling mathematically, we observe the appearance of the idea that when the episodes from HM are discussed/analyzed, which report modeling processes, this might subside the understanding of the process of modeling. This idea is corroborated by the excerpts: "I analyzed the potential interests of this polemic for the mathematical education which go beyond the current crisis: I insisted the subject might support [...], the understanding of the processes of mathematical modeling (AR.2021.F5.U2, our underlining) and "[...] supported the competence of modeling of the students in relation to the interpretation and validity of results from the model (AR.2012.F7.U1, our underlining).

These excerpts corroborate the affirmation by Biembengut (2016), that to learn to model the students need to know examples of models. But yet, it is needed to "understand the contexts and the respective languages of each model; knowing what mean the variables and constants; identifying concepts, theories and techniques involved; electing which languages using and which contexts to support on" (Biembengut, 2016, p. 183).

In this bias, we consider the mathematical models the scientists/mathematicians developed during the history have potential to aid the students to learn to model and develop competence in modeling, which according to Blomhøj and Jensem (2003, p. 127, our translation), regards to "the insightful readiness of a person to realize in autonomous manner every aspect of a mathematical modeling process in a determined context, and reflecting on the modeling process and the usage of the model". We understood this competence might be favored in measure that mathematical models are analyzed/discussed during history and which might be worked in the perspective of Model Analysis, according to the proposition by Sousa (2021), before proposing to the students creating their own models.

Related to the two aspects present, we discuss, now, about the third aspect which emerged from the authors' speech, about the manifestation of desirable attitudes by the students, which refer to the idea that the articulation HM and MM may provide the students to develop a reflexive and critical thought, as well as autonomy. We stress that these attitudes are present in the *Base Nacional Comum Curricular* – BNCC (2018), by indicating that in the final grades of

Elementary School it is important to provide the students to learn and evaluate mathematical argumentation, which involves reading of mathematical texts and development of critical sense in what refers to the argumentation they present. By their turn, in High School, it is defended the formation of young people as critical, creative, autonomous and responsible subjects, fitting to the schools of such levels providing experiences which guarantee to them the necessary learning for the reading of reality, facing current challenges and ethical and fundamental decision making (Base Nacional Comum Curricular, 2018).

We consider this can be contemplated in the curricular component of Mathematics, when the processes of modeling mathematically problems coming from the society from the past are discussed with the students, looking both for aspects of argumentation related to mathematical concepts and procedures related to these processes. From these considerations, we explain that, in relation to critical and reflexive thought, the excerpt ahead illustrates how this idea appears in the speech of the authors.

> The students started realizing the importance of a mathematical model in the problem solving coming from reality and applied to the classroom. [...], the students started asking how the scholars in the medieval period solved problems which appeared at that time with so little mathematical resources. What procedures were found to solve matters of practical order and obtain satisfactory results (D.2011.F2.U2, our underlining).

We observed that it is exemplified how the students themselves thought in a reflexive and critical manner by asking questions about the mathematical methods/models used in the Middle Ages. Such fragment comes from the early moment of the sequence of activities in which were discussed historical aspects related to the geometric model proposed by Oresme, but he highlights that in the manner to introduce the study of functions awoke interest of the students, instigating them to ask questions which reflect a critical sight, factors which might help in the learning.

In relation to autonomy, in activities which articulate HM and MM, we noticed it appears in the speech of authors connected to the idea of the students being more responsible for the construction of their knowledge and having initiative to look for answers for questions they use to do, as exemplified in the excerpt:

> In this way, we elaborated our didactic sequence guided by Mathematical MOdeling and recurring to the History of Complex Numbers so the process of teaching/learning happen in a satisfactory manner, with meanings and applications,, leading the students to answer questions raised by themselves (D.2013.F3.U2, our underlining).

We stress this excerpt was taken from a work in which the sequence of activities starts, in the view of the author, with an MM activity in measure the sequence is developed it recurs to historical knowledge about the appearing of complex numbers, in the attempt of attributing meaning to this concept, and answering questions which the students frequently raise related to the genisys and applicability of this mathematical content.

In this sense, according to Miguel and Miorim (2011), implicit in every teaching-learning process which has as objective the understanding and signification, it is the gathering and discussion of the questions for the acceptance of certain facts, reasons and procedures which parts from the student. These questions, according to the authors, can be answered by mean of the discussion of aspects of the HM related to the subject in study. In encounter to this, we stress that D.2013 gives us an example of articulation in which it starts with an MM activity MM, for which is convenient/necessary recurring to the historical knowledge related to the mathematical content in study, in order to solve the problematization proposed and/or attributing meaning to the referred content.

From this discussion which we realized around the three aspects intertwined to this category, we consider that **articulating HM and MM makes possible processes of teaching and learning which generate understanding of studied Mathematics topics and desirable attitudes for the student's formation**, corroborating what Assis (2021) had proposed, mainly, when the author defends the examples of known models might be studied for being theoretical and cognitive contributions in the promotion of a mathematical experience.

Second Final Category – Articulating HM and MM makes possible evidence of the human, social, scientific and cultural aspects, intertwined to the development of mathematical models and the relations of Mathematics with other areas of knowledge.

Table 7.

Units of meaning, initial and intermediate categories which generated the Second Final Category (The authors)

Unit of meaning codes	Initial Cat.	Intermediate Cat.	Final Cat.
AR.2021.F5.U3; PO.2017.F1.U1 PO.2017.F3.U2 (3)	Articulating HM and MM **makes it possible perceiving the human character related to the development of Mathematics**.	Articulating HM and MM **makes it possible to know the influences of human, social, cultural and scientific aspects for the obtention of mathematical models and development of Mathematics**.	**Articulating HM and MM makes it possible to evidence human, social, scientific and cultural aspects, intertwined to the development of mathematical**
AR.2012.F4.U1; AR.2012.F6.U1 AR.2012.F6.U2; AR.2021.F4.U2 PO.2017.F4.U1 (5)	Articulating HM and MM **makes it possible to perceive the influence of social, cultural and scientific aspects for the obtention of mathematical models**.		

<table>
<tr><td>D.2011.F8.U2; AR.2012.F5.U1
AR.2012.F6.U3;
AR.2021.F4.U1
(4)</td><td colspan="2">Articulating HM and MM makes it possible discussing interdisciplinary aspects inherent to the development of Mathematics</td><td rowspan="4">models in relation to Mathematics with other areas of knowledge.</td></tr>
<tr><td>D.2011.F4.U1; R.2012.F6.U4
CC.2013.F2.U1
(3)</td><td>Articulating HM and MM makes it possible to understand the influence of modeling in society.</td><td rowspan="3">Articulating HM and MM makes it possible to know the role of the mathematical models in society.</td></tr>
<tr><td>D.2011.F2.U1; AR.2007.F1.U1
CC.2013.F1.U1
(3)</td><td>Articulating HM and MM makes it possible to understand the role of the models in the problem solving.</td></tr>
<tr><td>D.2011.F8.U1; AR.2021.F2.U1
PO.2017.F2.U1;
AR.2021.F1.U1
(4)</td><td>Articulating HM and MM makes it possible knowing the Mathematics applications.</td></tr>
</table>

The form as the mathematical content is normally presented to the students does not illustrate the manner how this knowledge was historically produced (Miguel & Miorim, 2011). This is because “instead of departing from the manner in which a mathematical concept was developed, showing the questions which it answers, we take this concept as something ready” (Roque, 2012, p. 21). Furthermore, this form of presenting mathematical content “[...] first the definitions, then the theorems and the demonstrations which use these definitions and, finally, the application of the theorems to some particular situation considered a problem” (Roque, 2012, p. 21), does not encompass the doubts, hesitations, inherent flaws to the production of mathematical knowledges. We understand this causes a distorted view of the development of Mathematics in the students, which may make it difficult to understand mathematical ideas.

In this sense, we consider the mathematical models coming from processes of modeling mathematically, present in the HM, may inspire didactic situations in which it is plausible discussing/questioning with the students about this order of exposition, when adapted in consonance to the didactic ends. It is in this perspective that we defend articulating HM and MM makes possible evidence of human, social, scientific and cultural aspects connected to the development of mathematical models and the relations of Mathematics with other areas of knowledge, evidencing how it develops.

In order to corroborate this argument we discuss three aspects manifested in the speeches of the authors of the selected works and which emerged in the intermediate categories, to be known: the influences of human aspect (among others) for obtaining models and development of Mathematics; the interdisciplinary aspects, coming from relations of Mathematics with other areas of knowledge; and the role of mathematical models in society.

About the first aspect - the influences of human, social, scientific and cultural aspects for the obtention of mathematical models and development of Mathematics – it brings ideas

related to how society in its more diverse branches influences in the process of obtaining and using a mathematical model, as illustrated in the excerpt ahead:

> The students acquired many important insights studying the past episode in the failed attempt by Rashevsky in 1934, of interesting the biologists in the mathematical modeling as a method to obtain knowledge on biological phenomena. They perceived that different scientific cultures have different opinions about the value of a model as an instrument to obtain scientific knowledge; and that the explaining power of a model is connected not only to the context of its usage, but also to the philosophical and theoretical positions of the scientists which discuss the model and its usage (AR.2012.F6.U2, our underlining).

It is explained in it the students might perceive that different scientific areas understand the mathematical models under different scopes, and these different views also influence the development of such models, illustrating the production of mathematical knowledge many times needs the viewpoint of other sciences. Therefore, with these discussions the students are able to demystify some ideas related to how the Mathematics develops itself, as an example: the neutral and selfish characters which are attributed to it, what was already pointed as a potentiality of HM as pedagogical resource (Miguel & Miorim, 2011). In this way, we consider the study of mathematical models produced in the past makes it possible for this potentiality to occur in classroom activities.

In this first aspect, the idea of the humanist character connected to the development of Mathematics also appeared, both in relation to the knowledge of human flaws for the obtention and usage of a mathematical model and to what refers to how mathematical models are part of human culture and how they are constructed and perfected in a manner to contribute to humanity, according to the following excerpt:

> I analyzed the potential interests of this polemic for the mathematical education which go beyond the current crisis: I insisted the subject might support the learning of specific mathematical topics, the understanding of mathematical learning processes and the view of Mathematics as part of human culture and as a tool to answer the social crises, including pandemics (AR.2021.F5.U3, our underlining).

This fragment brings an example that by means of the proposed approach the students might notice the social crises influence in the development of mathematical models, which, by their turn, are elaborated with the intention of minimi\ing them, evidencing the human influence related to the development of Mathematics and that, many times, propels the development of this area. In this manner, we consider this first aspect points out a possibility to discuss the order of exposition of the mathematical content and also goes in encounter to one of the specific competences of the Mathematics subject for Elementary School, listed in the BNCC (2018),

because this document indicates the students must acknowledge Mathematics as a human science, which is developed according to the needs and preoccupations of different cultures and historical moments, besides being a living science which collaborates for the solution of scientific and technological problems which are basis for discoveries and constructions.

In what refers to the second aspect, this exposes the articulation between HM and MM proportionate evidencing the relations and of Mathematics with other sciences, both in what refers to observe and analyze critically the function of modeling in other areas, and in what is related to noticing what kinds of problem emerge in interdisciplinary fields which Mathematics is constituted as a solving tool, providing the understanding the relation between Mathematics and them and this relationship with other areas of knowledge is part of the nature of the development of some mathematical objects.

As an example, in the excerpt: "In this article, we focused on the history as a provider of cases through which the students are able to try the meaning of different practices and scientific cultures for the function and the status of mathematical modeling in other sciences.*"* (AR.2012.F5.U1, our underlining), the relation with other sciences appears by affirming that by means of the history, more specifically of determined episodes of HM which approach modeling, it is possible proposing that the students analyze its function in scientific practices of other sciences.

It is worthy to stress the two aspects cited above go in encounter to other specific competence of the subject of Mathematics for Elementary School, listed in the BNCC, to be known: "Understanding the relations between these concepts and procedures of the different fields of Mathematics [...] and of other areas of knowledge, feeling safe about its own capacity of building and applying mathematical knowledge, developing self esteem and perseverance in the search for solutions" (Brasil, 2018, p. 267).

About the third aspect, the role of mathematical models in society, we highlight it emerged in the speeches of authors in a manner to encompass the referred role, demonstrating its influence in society in the sense of aiding in the problem solving which they solve and their applications. In CC.2013, as an example, are cited three problems which were solved by mean of the MM, according to this excerpt:

> To attempt to demonstrate the truth of the proposition made by Maria Salet Biembengut and Nelson Hein, we are going to present three situations from the History of Mathematics (and why not saying: of humanity) in which we can observe clearly the usage of mathematical modeling to solve a situation-problem. The first happens during Antiquity, with the participation of Archimedes of Syracuse; the second, in the early Christian Era, but still in Antiquity (2nd Century), involves Klaudius Ptolemaios; and

> the third, during the Modern Age, has, in its cast, the unique figure in the History of Mathematics of Leonhard Euler (CC.2013.F1.U1, our underlining).

Such situations illustrate the role of modeling by solving problems which are of daily nature and are related to the description of natural phenomena (Roque, 2012), what also contributes for the discussion on order of exposition of mathematical contents, because it evidences that in the development of mathematical ideas, the starting point comes from a problem.

Still in relation to the role of the models in society appeared the idea about how modeling can lead to changes in the scientific practices and its efficiency. One example of this is in the debate between Bernoulli and D'Alembert on the statistical model proposed by Bernoulli, who helped demonstrate the efficiency of the method of inoculation for the prevention of a smallpox pandemic.

> Through an analysis of the polemic and of the resources related, I demonstrate this historical debate has many interests for mathematical education; and, more specifically, it might help the teachers to face the dilemmas which appear with the COVID-19 pandemic (as an example, if a teacher must talk about it in the classroom or not, how to help the students to interpret the statistical data and the mathematical models linked to the pandemic and in a more general way, how to handle the confusion and preoccupations which appear in connection to the pandemic) (AR.2021.F1.U1, our underlining).

This last excerpt brings us the idea that we can study mathematical models developed throughout history, in a manner to look at the past to understand aspects of the present, what was defended by Fauvel (1991), when the author presented reasons to use HM, saying that "it contributes for the students to search in the past mathematical solutions for the present and project their results in the future" (Fauvel, 1991 apud Mendes, 2006, p. 86). We understand that this excerpt indicates to us the historical cut, which might encourage this referred reason, comes from an episode from history which approaches the process of obtention and usage of mathematical models.

The three aspects which constituted this category **"articulating HM and MM makes it possible to evidence the human, social, scientific and cultural aspects, intertwined to the development of mathematical models and the relations of Mathematics with other areas of knowledge"**, showed themselves in consonance with what Barbosa (2009, p. 82) affirmed about the role of the mathematical models in scientific education, "mathematical models cannot be presented as entities themselves, [...]. It is necessary that the students have the opportunity

to discuss (and experiment, when it is the case) the circumstances which led human thought to such mathematical representation".

By doing it, we will contribute to demystify "certain conceptions [which] seemed to firmly established, which are admitted as true in the light of good sense, without a clearer critical analysis analysis", according to what was denounced by Machado (1998), when discussing slogans related to Mathematics, which might influence its teaching (and, consequently, its learning).

Some considerations

In this text we searched for comprehension, in terms of teaching and learning processes of Mathematics, about the possible articulations between HM and MM revealed in the research from the area. In order to do it, following procedures of systematic literature review, we identified and selected eight works which made it possible to reach this goal. After this, by analyzing them in the light of our guiding question and based on ATD procedures, two categories were built, which reveal our understanding on the articulations between HM and MM approached in the referred works.

We resume that in this study we assume articulating HM and MM means joining/redirecting them in activities turned to the teaching and learning processes of Mathematics, that is, it contemplates different forms of interaction and integration of both trends. Based on this understanding, the built categories reveal potentialities coming from possibility of articulations between HM and MM, once they demonstrate "**articulating HM and MM makes possible teaching and learning processes which generate understanding of the mathematical topics studied and desirable attitudes for the formation of the students**" and "**articulating HM and MM makes it possible highlighting the human, social, scientific and cultural aspects, intertwined to the development of mathematical models and the relations of Mathematics with other areas of knowledge**".

It is worth stressing that both categories found do not preclude or deplete understandings on the articulations between both trends. However, based on the adopted methodological course of research, we understood these categories depict interactions and integrations which were possible to be built based on works which approach diverse possibilities of articulating HM and MM in the process of teaching and learning of Mathematics, then bringing representative results on the subject.

Based on these results, we considered the presented categories encourage studies and practices which make it viable the realization of activities which articulate HM and MM in the

process of teaching and learning of Mathematics, with sight on the development of a formation which prioritizes knowledge as instrument of action and decision. Furthermore, this study points out other research possibilities, one of them refers to how to make articulations between HM and MM, in a manner in which the potentialities pointed in these categories are contemplated, among others.

Acknowledgements

The first author thanks, for the doctorate scholarship, the *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior* – CAPES, financing code 001.

References

Almeida, L. W., Silva, K. P., & Vertuan, R. E. (2012). *Modelagem Matemática na Educação Básica*. Editora Contexto.

Assis, J. F. (2021). História, Matemática e Ensino: tradições, harmonizações e insubordinações necessárias**.** *Cadernos do IME* **–** Série Matemática, 17.

Barbosa, J. C. (2009). Modelagem e Modelos Matemáticos na Educação Científica. *Alexandria, 2*(2), 69-85.

Base Nacional Comum Curricular (BNCC). (2018). Ministério da Educação.

Beltran, M. H. R., Saito, F., & Trindade, L. S. P. (2014). *História da Ciência para formação de professores.* Editora Livraria da Física.

Biembengut, M. S. (2016). *Modelagem na Educação Matemática e na Ciência*. Livraria da Física.

Biembengut, M. S., & Hein, N. (2009). *Modelagem matemática no ensino*. Editora Contexto.

Blomhøj, M., & Jensen, T. H. (2003). Desenvolvendo a competência de modelagem matemática: Clarificação conceitual e planejamento educacional. *Ensino de matemática e suas aplicações, 22*(3), 123-139.

Dicionário Michaelis On-Line (2021). *Articular*. Editora Melhoramentos. https://michaelis.uol.com.br/.

Farmaki, V., & Paschos, T. (2007). Employing genetic 'moments' in the history of mathematics in classroom activites. *Educational Studies in Mathematics*, 66, 83--106.

Ferreira, G. P., Silveira, A., & Silva, L. A. (2013). A Modelagem Matemática ao longo da história e o surgimento da Modelação Matemática no Brasil. *Anais do 11º Encontro Nacional de Educação Matemática*.

Fossa, J. A. (2011). *Ensaios sobre a Educação Matemática*. Editora Livraria da Física.

Gosztonyi, K. (2021). How history of mathematics can help to face a crisis situation: The case of the polemic between Bernoulli and d'Alembert about the smallpox epidemic. *Educational Studies in Mathematics*, 108, 105-122.

Jankvist, U. T. (2009). A categorization of the “whys” and “hows” of using history in mathematics education. *Educational Studies in Mathematics, 71* (3), 235-261.

Kjeldsen, T. H., & Blomhøj, M. (2012). Developing Students’ Reflections about the Function and Status of Mathematical Modeling in Different Scientific Practices: History as a Provider of Cases. *Science & Education, 22* (9), 2157-2171.

Machado, N. J. (1998). *Matemática e língua materna: análise de uma impregnação mútua*. Editora Cortez.

Magnus, M. C. M., Caldeira, A. D., & Duarte, C. G. (2019). Do Modelo Matemático à Modelagem Matemática: descontinuidades históricas. *Bolema, 33* (65), 1215-1232.

Matos, F. C., Ribeiro, J. L., Machado, Y. A. P., Ferreira, R. S., Matos, S. P. M., & Silva, C. I. S. (2013). O parafuso de Arquimedes: uma inovação no ensino de matemática sob a perspectiva da modelagem matemática no IFPA. *Anais do 11º Encontro Nacional de Educação Matemática.*

Mendes, I. A. (2006). A investigação histórica como agente de cognição matemática na sala de aula. In Mendes, I. A., Fossa, J. A., & Valdés, J. E. N. *A História como um agente de cognição na Educação Matemática*. Editora Sulina. p. 79-136.

Mendes, L. O. R., & Pereira, A. L. (2020). Revisão sistemática na área de Ensino e Educação Matemática: análise do processo e proposição de etapas. *Educação Matemática Pesquisa*, *22*(3), 196-228.

Miguel, A., & Miorim, A. (2002). História da Matemática: uma prática social de investigação em construção. *Educação em Revista*, 36, 177-203.

Miguel, A., & Miorim, A. (2011). *História na Educação Matemática: propostas e desafios*. Editora Autêntica.

Moher, D., Liberati, A., Tetzlaff, J., & Altman, D. G. (2015). Principais itens para relatar revisões sistemáticas e meta-análises: a recomendação PRISMA. Tradução: Galvão, T. F., & Pansani, T. S. A., *Epidemiol. e Serviços de Saúde*, *v. 24*(2), 335-342.

Moraes, R., & Galiazzi, M. C. (2011). *Análise Textual Discursiva*. Editora Unijuí.

Paes, L. A. A. (2013). *Números complexos: uma proposta didática baseada na modelagem matemática e em contextos históricos.* [Dissertação de Mestrado Profissional em Matemática em Rede Nacional, Universidade Estadual de Londrina].

Roque, T. (2012). *História da matemática: uma visão crítica, desfazendo mitos e lendas*. Editora Zahar.

Silva, B. G., Biffi, L. C. R., & Trivizoli, L. M. (2017). Uma contextualização histórica para o modelo clássico de Malthus. *Anais da 10ª Conferência Nacional sobre Modelagem na Educação Matemática*.

Sousa, E. S. (2021). Análise de modelos como um método de ensino de matemática na educação básica. *Revista Práxis Educacional*, *17*(45), 316-337.

Souza, G. C. (2020). Aliança entre HM, TDIC e IM: fundamentos e aplicações. *REMATEC, 15*, 117-136.

Souza, V. R. (2011). *Funções no Ensino Médio: história e modelagem*. [Dissertação de Mestrado Profissional em Ensino de Matemática, Pontifícia Universidade Católica de São Paulo].