ISSN 1983-3156
Qualis A1
http://dx.doi.org/10.23925/1983-3156.2023v25i1p145-166

Didactic-pedagogical guidelines to work with problem solving in mathematics classes

Orientações didático-pedagógicas para o trabalho com resolução de problemas nas aulas de matemática

Lineamientos didáctico-pedagógicos para trabajar la resolución de problemas en las clases de matemáticas

Directices didactiques-pédagogiques pour le travail avec la résolution de problémes dans les cours de mathématiques

Kaique Nascimento Martins[1]
Universidade Estadual de Santa Cruz
Mestrado em Educação em Ciências e Matemática
https://orcid.org/0000-0003-2552-7098

Larissa Pinca Sarro Gomes[2]
Universidade Estadual de Santa Cruz
Doutorado em Educação
https://orcid.org/0000-0001-6839-6927

Marlúbia Corrêa de Paula[3]
Universidade Estadual de Santa Cruz
Doutorado em Educação em Ciências e Matemática
https://orcid.org/0000-0002-3646-8700

Norma Suely Gomes Allevato[4]
Universidade Cruzeiro do Sul
Doutorado em Educação Matemática
https://orcid.org/0000-0001-6892-606x

Abstract

Recent curricular guidelines, such as the National Common Curricular Base and state and local curricula supported by it, emphasize problem solving in their indications, fostering research movements and teaching practices focused on this theme. This article aims to present the understandings that emerged regarding the didactic-pedagogical guidelines to work with problem solving based on the analysis of Brazilian doctoral and master's theses in Mathematics Education produced from 2016 to 2020. A State of Knowledge study was carried out, with a total of 77 productions, 14 doctoral and 63 master's theses located in the Brazilian Digital

[1] knmartins@uesc.br
[2] lpsgomes@uesc.br
[3] mcpaula@uesc.br
[4] normallev@gmail.com

Library of Theses and Dissertations and the doctoral and master’s theses catalog of the Coordination for the Improvement of Higher Education Personnel (Capes). For analysis of the productions, Discursive Textual Analysis and the IRaMuTeQ software were used. The results suggest the predominance of productions that communicate the use of problem solving, as a way to build new mathematical knowledge and/or its re-signification, through investigations that value the process and thinking of students involved in collaborative work. Regarding the teaching stages in which the investigations were inserted or to which they were directed, we established a greater interest in Basic Education, highlighting the stages of High School and Final Years of Elementary School. In teacher education, the largest amount of research is related to initial education, focusing on students of Mathematics Teaching Degree courses.

Keywords: Problem solving, Mathematics education, State of knowledge.

Resumo

Orientações curriculares recentes, como a Base Nacional Comum Curricular e currículos estaduais e locais nela sustentados, enfatizam a resolução de problemas em suas indicações, fomentando movimentos de pesquisa e de práticas de ensino voltados a esta temática. Este artigo tem como objetivo apresentar as compreensões que emergiram a respeito das orientações didático-pedagógicas para o trabalho com resolução de problemas, a partir da análise de teses e dissertações brasileiras do campo da Educação Matemática, produzidas no período de 2016 a 2020. Para isso, foi realizado um estudo do tipo Estado do Conhecimento, com um total de 77 produções, sendo 14 teses e 63 dissertações localizadas na Biblioteca Digital Brasileira de Teses e Dissertações e no catálogo de teses e dissertações da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior. Para análise das produções, foi utilizada a Análise Textual Discursiva e o *software* IRaMuTeQ. Os resultados sugerem a predominância de produções que comunicam o uso da resolução de problemas, como um caminho para a construção de novos conhecimentos matemáticos e/ou a sua ressignificação, por meio de investigações que valorizam o processo e o pensamento dos estudantes, envolvidos em trabalho colaborativo. Com relação às etapas de ensino em que as investigações foram inseridas ou para as quais foram direcionadas, verificamos maior interesse para a Educação Básica, destacando-se as etapas do Ensino Médio e Anos Finais do Ensino Fundamental. Na formação de professores, o maior quantitativo das pesquisas está relacionado à formação inicial, com foco nos estudantes de cursos de Licenciatura em Matemática.

Palavras-chave: Resolução de Problemas, Educação Matemática, Estado do Conhecimento.

Resumen

Directrices curriculares recientes, como la Base Curricular Común Nacional y los currículos estatales y locales sustentados por ella, enfatizan en sus indicaciones la resolución de problemas, fomentando movimientos de investigación y prácticas docentes enfocadas en esa temática. Este artículo tiene como objetivo presentar los entendimientos que surgieron sobre las directrices didáctico-pedagógicas para trabajar con la resolución de problemas a partir del análisis de las tesis de doctorado y maestría brasileñas en Educación Matemática producidas entre 2016 y 2020. Se realizó un estudio del Estado del Conocimiento, con un total de 77 producciones, 14 tesis de doctorado y 63 de maestría ubicadas en la Biblioteca Digital Brasileña de Tesis y Disertaciones y en el catálogo de tesis de doctorado y maestría de la Coordinación de Perfeccionamiento del Personal de Educación Superior (Capes). Para el análisis de las producciones se utilizó el Análisis Textual Discursivo y el software IRaMuTeQ. Los resultados sugieren el predominio de producciones que comunican el uso de la resolución de problemas, como forma de construcción de nuevos conocimientos matemáticos y/o su resignificación, a través de investigaciones que valoran el proceso y el pensamiento de los estudiantes involucrados en el trabajo colaborativo. En cuanto a las etapas de enseñanza en las que se insertaron las investigaciones o a las que fueron dirigidas, establecimos un mayor interés por la Educación Básica, destacándose las etapas de Enseñanza Media y Últimos Años de la Enseñanza Fundamental. En la formación del profesorado, la mayor parte de la investigación está relacionada con la formación inicial, centrándose en los estudiantes de las carreras de Grado en Enseñanza de las Matemáticas.

Palabras clave: Resolución de Problemas, Educación Matemática, Estado del Conocimiento.

Résumé

Les directices curriculaires récentes, telles que la Base Nationale Commune des Programmes d’études et les programmes d'Etats et locaux, mettent l'accent sur la résolution de problèmes dans leurs indications, favorisant les mouvements de recherche et les pratiques de l’enseignement axées sur ce thème. Cet article vise à présenter les compréhensions qui ont émergé, concernant les orientations didactiques et pédagogiques pour le travail avec la résolution des problèmes, basées sur l'analyse des thèses et des mémoires brésiliens dans le domaine de l'Education Mathématiques produites dans la période de 2016 à 2020. Pour cela, une étude du type État des Connaissances a été réalisée, avec un total de 77 productions, dont 14 thèses de doctorat et 63 mémoires de masteur, situés dans la Bibliothèque virtuelle brésilienne des thèses et mémoires, aussi bien que dans le catalogue des thèses et mémoires de la Coordination pour l'Amélioration du Personnel de l'Enseignement Supérieur. Pour emmener les analyse des productions ont été utilizes la méthode d'Analyse Textuelle Discursive et le logiciel IRaMuTeQ. Les résultats suggèrent la prédominance des productions qui communiquent l'utilisation de la résolution de problèmes, comme moyen de construire de nouvelles connaissances mathématiques ou sa re-signification, à travers des enquêtes qui valorisent le processus et la pensée des élèves impliqués dans le travail collaboratif. En ce qui concerne les étapes d'enseignement dans lesquelles les enquêtes ont été insérées ou vers lesquelles elles ont été dirigées, nous avons vérifié un intérêt significatif pour l'Éducation de Base, en mettant en évidence les étapes de l'École secondaire et les dernières années de l'École primaire. Dans la formation des enseignants, la plus grande partie de la recherche est liée à la formation initiale, en mettant l'accent sur les étudiants des cours de Licence en Mathématiques.

Mots clés : Résolution de problème, Enseignement des mathématiques, État des connaissances.

Didactic-Pedagogical Guidelines for Working with Problem Solving in Mathematics Classes

Problem solving has been discussed in scientific forums dedicated to studies and research in Mathematics Education. In Brazil, the National Common Core Curriculum (Base Nacional Comum Curricular - BNCC), among other guidelines, emphasizes the importance of problem solving in Mathematics classes, which can be understood and used as a teaching methodology or as content to be taught or even as a combination of these proposals (Brasil, 2018).

In this article, we consider Brazilian master's and doctoral theses in Mathematics Education produced from 2016 to 2020 to show the understandings that emerged from the didactic-pedagogical guidelines to work with problem solving.

This article is part of a broader research (Martins, 2022) which, in one of its phases, sought to analyze how problem solving is used by researchers in the production of *stricto sensu* postgraduate programs in Brazil (2016-2020). Based on this investigation, we could identify three emerging categories: i) didactic-pedagogical guidelines to work with problem solving; ii) trends in Mathematics Education and problem solving; and iii) problem solving in teacher education. Constituted as an excerpt of the research mentioned, this article will focus on the first category.

The categories emerged from the analysis of the corpus of the research, obtained from a mapping of the academic productions available in the Brazilian Digital Library of Theses and Dissertations (Biblioteca Digital Brasileira de Teses e Dissertações - BDTD) and CAPES catalog, in the period of interest of this study. For the analysis, we used the Discursive Textual Analysis (DTA), carried out with the support of the software *Interface de R pour les Analyses Multidimensionnelles de Textes et de Questionnaires* (IRaMuTeQ).

Our interest in the study of problem solving arose from the readings that deal with the historicity of this theme since, throughout the 20th century, research on problem solving and its curricular implications was at the center of discussions, mainly in the United States, influenced by George Polya's studies. His book, entitled *How to solve it*[5], published for the first time in 1944 in English, moved discussions on this topic.

In Brazil, the translation into Portuguese was published in 1975, leading debates about how to guide students in solving mathematical problems. The author considered the need to reframe pedagogical practices to accompany the development of society and reassess proposals

[5] The Portuguese version is titled: *A arte de resolver problemas*.

being discussed in national and international forums since the second half of the 1950s, following the example of the movement that became known as Modern Mathematics (Morais & Onuchic, 2021).

From this perspective, in 1980, the National Council of Teachers of Mathematics[6] (NCTM) published “An Agenda for Action: Recommendations for School Mathematics in the 1980’s”[7]. The main recommendation of this document was to treat Problem Solving as a focus in school mathematics in the 1980s (Onuchic, 1999; Onuchic & Allevato, 2011; Morais & Onuchic, 2021). Morais and Onuchic (2021) point out the search, in the meantime, for making problem solving the focus of school Mathematics, a cause for a series of discussions and disagreements between teachers and researchers in Mathematics Education

Since then, there have been publications on the subject, occupying relevant discussion spaces in national and international scenarios. In Brazil, research groups that are part of several postgraduate programs (PPGs) have been dedicated to this debate. As an example, we mention the Working Group and Studies in Problem Solving (Grupo de Trabalho e Estudos em Resolução de Problemas - GTERP), of the Postgraduate Program in Mathematics Education at the Universidade Estadual Paulista Júlio de Mesquita Filho, campus Rio Claro/SP, coordinated by Professor Dr. Lourdes de la Rosa Onuchic; the Group for Research and Advanced Studies in Mathematics Education (Grupo de Pesquisa e Estudos Avançados em Educação Matemática - GPEAEM), led by Professor Dr. Norma Suely Gomes Allevato; and PPGs spread across the country, such as the Postgraduate Program in Science and Mathematics Education (Programa de Pós-graduação em Educação em Ciências e Matemática - PPGECM), at the Universidade Estadual da Paraíba (UEPB), mainly in research guided by Professor Dr. Silvanio de Andrade (Martins, 2022).

However, although the expression “Problem solving” is frequently used in Mathematics classes, in teacher education and research in Mathematics Education, its use is not always accompanied by reflections on its meaning, causing careless, naive, or mistaken interpretations, which can give rise to practices that do little to explore their potential (Martins, 2022).

Thus, in what follows, we present a discussion about what was shown from the analysis of the productions, aiming to intensify the dialogue about its use in Mathematics classrooms. Therefore, the text that makes up this article will be organized by the following topics: this introduction, the theoretical framework used to present the different ways of approaching

[6] Conselho Nacional de Professores de Matemática.
[7] Uma Agenda para Ação – Recomendações para a Matemática Escolar para a década de 1980.

problem solving, the methodological procedures, the results obtained, and the final considerations.

Some ways of approaching problem solving in school Mathematics

In our opinion, one of the best ways to promote the conscious use of problem solving in Mathematics classes is to reflect on the different possibilities of its use. Like any educational proposal, during its implementation, it is possible to evidence different appropriations and interpretations, some of which are incomplete, superficial, or reductionist. It turns out that the way teachers conduct a problem-solving activity is closely associated with their conceptions about how the mathematical learning process takes place or with the teaching objectives (Martins, 2022).

In this perspective, Schroeder and Lester (1989) present three problem-solving teaching approaches that are widely discussed and problematized in research on the subject (Allevato, 2005; Onuchic & Allevato, 2011; Melo & Justulin, 2019; Morais & Onuchic, 2021), namely, teaching *about* problem solving, teaching Mathematics *for* solving problems and teach Mathematics *via* problem solving. Starting from the *Standards* (NCTM, 2000), this third approach is consolidated as *teaching through problem solving*. In what follows, in Table 1, we present the general lines that configure each of those approaches.

Table 1.

Different ways of approaching problem solving (elaborated by the authors, 2022)

Approaches	**Notes**
Teaching about problem solving	Problem solving is new content and its teaching has as its main focus the identification of patterns to be used in the resolution proposal. It relates to Polya's (2006) model, which is developed from four phases: 1) understanding the problem; 2) designing a plan; 3) executing the plan; 4) hind sighting; or some variation of it.
Teaching Mathematics for problem solving	It differs from the previous one in that the focus is no longer placed on methods and strategies for solving problems and becomes the approach of mathematical content. The student is expected to be able to use/apply mathematical knowledge previously covered in the classroom to solve problems. Problem solving arises to attest to the usefulness of Mathematics.

Approaches	Notes
Teaching Mathematics through problem solving	The problem is considered a starting point for Mathematics teaching and learning. Knowledge production is expected to occur from the student's involvement during the resolution process. The problem is presented to students before they are presented with the appropriate content for its resolution.

Thus, when we opted for the first approach, presented in Table 1, we sought to work, step by step, with rules, phases, and problem solving processes to provide students with general strategies and techniques applicable to any problem, regardless of the mathematical content involved to succeed in resolutions. On the other hand, in the second approach, we assume that Mathematics is utilitarian, so that, although the production of mathematical knowledge is important, the main purpose of learning Mathematics is to be able to use it (Morais & Onuchic, 2021).

The last approach, related to the proposal of Teaching Mathematics via Problem Solving, as discussed by Allevato (2005) and Allevato and Onuchic (2021), currently refers to what is called "Teaching Mathematics through Problem Solving". And yet, Allevato and Onuchic (2021, p. 40) clarify that "the expression through – meaning "along", "during" – emphasizes [izing] that both Mathematics and problem solving are considered simultaneously and are mutually and continuously constructed".

From this perspective, studies such as Van de Walle's (2009) highlight that, when teaching Mathematics through Problem Solving, students must solve problems and, in the process, learn new mathematics. Thus, the author states that "most, if not all, mathematical concepts and procedures can be better taught through problem solving" (Van de Walle, 2009, p. 57).

Thus, based on the approach of teaching Mathematics through problem solving, in Brazil, the GTERP, which develops its activities in the Department of Mathematics of the Universidade Estadual Paulista (UNESP) – Rio Claro, started to do research focused on the methodology of Mathematics Teaching-Learning-Assessment through Problem Solving (Onuchic & Allevato, 2011; Allevato & Onuchic, 2009, 2014, 2021).

The compound word teaching-learning-assessment was created to express a conception in which those actions may/should occur concomitantly. In this proposal, while the teacher teaches, the student, as an active participant, learns, and both carry out the assessment. With this referral, the students analyze their own methods and solutions for the problem aiming to build knowledge. The teacher, in turn, assumes the role of a mediator, evaluating what is happening in the process and reorienting practices, if necessary (Onuchic & Allevato, 2011; Allevato & Onuchic, 2021).

From this perspective, Allevato, Jahn, and Onuchic (2017) clarify that in a Mathematics class held under this conception:

> A problem proposed to the students – the generator problem – will lead to the content the teacher planned to build in that class. It should be reiterated that, in this methodology, problems are proposed to students even before the mathematical content is formally presented to them, which, according to the syllabus of the grade attended, is intended by the teacher, necessary, or more appropriate for solving the problem. In this way, the teaching-learning of a mathematical topic begins with a problem that expresses key aspects of that topic and mathematical techniques must be developed in the search for reasonable answers to the given problem. The assessment of student growth is done continuously during the problem solving process (Allevato, Jahn, & Onuchic, 2017, pp. 255).

Thus, seeking to help teachers to use this methodology in their classes, the GTERP suggests a script, initially presented in Onuchic (1999) and that, over the years, was improved, becoming present in other studies by professors Norma Allevato and Lourdes Onuchic. Next, in Allevato and Onuchic (2021) we present the version used and most discussed. It should be noted that according to the authors, there are no rigid forms to apply the methodology, and the following steps are suggested: 1) Proposition of the generating problem; 2) Individual reading; 3) Reading together; 4) Solving the problem; 5) Observing and encouraging; 6) Recording resolutions on the blackboard; 7) Plenary; 8) Searching for consensus; 9) Formalizing the content; 10) Posing new problems.

We also highlight the Exploration, Resolution, and Posing of Problems, or just Problem Exploration, as an approach discussed in research carried out within the scope of the Postgraduate Program in Teaching Science and Mathematics Education (Programa de Pós-Graduação em Ensino de Ciências e Educação Matemática - PPGECM) at the State University of Paraíba (UEPB), under the guidance of Prof. Dr. Silvanio de Andrade. This practice and the methodology of Mathematics Teaching-Learning-Assessment through Problem Solving begin with the idea that the problem situation is the starting point. However, in Problem Exploration, there is no established set of steps. Andrade (2017) points out that in this conception:

> [...] Initially, the teacher or the students can propose a problem or problem situation. The students work on it and, together, the teacher and students discuss the work developed in a process of reflections and condensations. Thus, they possibly reach the resolution, new content, new problems, and the accomplishment of new works, new reflections, and new condensations. In this process, the work of exploring problems is unfinished, and may go beyond the search for a solution to the problem and refers to everything that is done in it from the P-W-RC movement (Problem – Work – Reflections and Condensations) (Andrade, 2017, pp. 365-366).

We emphasize that when using the term "exploration", this practice covers posing and solving problems. About the problem posing, we set out from the idea that the student can/should elaborate and/or (re)formulate their own problems. This practice can also be used in problem solving, i.e., during the process, students can be invited to pose problems about the mathematical concept/content addressed (Martins, 2022).

Regarding official guidelines, the BNCC (Brasil, 2018) presents notes for work with problem solving and students' formulation/elaboration of problems. We justify the indication of the use of both terms, as

> [...] they should develop the ability to abstract the context, apprehending relationships and meanings to apply them in other contexts. To favor that abstraction, it is important that students re-elaborate on the proposed problems after having solved them. For this reason, in the various skills related to problem solving, there is also the elaboration of problems (Brasil, 2018, pp. 299).

In addition, the document points to the mathematical process of problem solving as a privileged form of mathematical activity, thus it is "at the same time an object and strategy for learning throughout Elementary School (Brasil, 2018, p. 266)". However, based on studies that have already been developed and are known to us, we understand that when properly used, problem solving can enhance the teaching and learning process not only in Elementary School but in all stages of teaching that involve the teaching and learning of Mathematics. It can also be important in teacher education to enhance their practices. Thus, having presented the theoretical contribution, in the next section, we will explain the methodological procedures that guided the research.

Methodological procedures

As explained in the introduction, this article aims to present understandings that emerged regarding the didactic-pedagogical guidelines to work with problem solving from the analysis of 77 Brazilian productions, 14 of which doctoral and 63 master's degree theses in Mathematics Education, developed in *stricto sensu* postgraduate programs from 2016 to 2020. For this, we conducted a study based on the State of Knowledge (Romanowski & Ens, 2006).

The searches were done exclusively in digital media, in the Brazilian Digital Library of Theses and Dissertations (Biblioteca Digital Brasileira de Teses e Dissertações - BDTD) and in the Catalog of Theses and Dissertations of the Coordination for the Improvement of Higher Education Personnel (Catálogo de Teses e Dissertações da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES), based on the terms "Problem Solving" and "Mathematics Education" [resolução de problemas; educação matemática]. The operator was the Boolean[8] operator AND between the terms. Regarding the period, considering that the present investigation was mostly carried out in the year 2021, we chose to consider the productions defended from 2016 to 2020, covering the last five full years.

From the results, we located 236 productions, 111 from the CAPES catalog and 125 from the BDTD. Initially, we sought to carry out an analysis by crossing existing information in titles, abstracts, keywords, and the postgraduate programs in which the productions were developed. This analytical phase resulted in the definition of 77 productions that, in the eyes of the researchers, were in accordance with the research objectives.

Having defined the corpus, we start the analytical process itself. For this, we resort to the assumptions of the Discursive Textual Analysis (DTA) with the help of the IRaMuTeQ software (Martins, 2022; Martins, Gomes & Paula, 2022; Martins et al., 2022). DTA is a qualitative data analysis methodology structured in three recursive procedures: i) Unitization; ii) Categorization; and iii) Metatext (Moraes & Galiazzi, 2016).

In unitization, the process of fragmentation of the texts of the corpus into units of meaning occurs according to the researcher's interpretations. It is also up to the researcher to define the size of the units and how to organize them. For this, we suggest using colors and codes to recognize their origins (Paula, 2018). Furthermore, we understand that, in the first unitizations, we must preserve the original writing of the authors, and, as the level of impregnation with the data increases, such units can be rewritten (Moraes & Galiazzi, 2016).

With regard to categorization, Moraes and Galiazzi (2016) highlight that it can occur

[8] The Boolean operators can be added between the search terms to define for the system how the combination between the search terms should be done. The Boolean operator AND, for example, aims to restrict results to searches that contain one term *and* the other.

under at least three methods, depending on the researcher's choice: i) inductive method: categories can emerge from contact with the corpus; ii) deductive method: categories are defined a priori; and iii) mixed method: combines the two previously mentioned methods. From this perspective, we should note that we used the inductive method in this study, and categories emerged from using the IRaMuTeQ software. When one adopts this method, the process of emergence of categories follows the perspective of gradual –initial, intermediate, and final– construction.

Finally, the movement carried out in the unitization and categorization subsidized the researcher in the construction of the metatext, in which:

> [...] there is no intention to return to the original texts, but to the construction of a new text, a metatext that has its origin in the original texts, expressing a researcher's look at the meanings and senses perceived in them. This metatext constitutes a set of descriptive-interpretative arguments capable of expressing the understanding reached by the researcher in relation to the researched phenomenon, always based on the corpus of analysis (Moraes, 2003, pp. 201-202).

Thus, as previously explained, associated with DTA procedures, we use IRaMuTeQ, a free software created by the Frenchman Pierry Ratinaud, anchored by R statistical software and the programming language Python. That software has a Portuguese version and enables different types of analysis. In this article, we resort to the Descending Hierarchical Classification (DHC) and Factorial Correspondence Analysis (FCA) (Martins, 2022; Martins, Gomes & Paula, 2022; Martins et al., 2022).

At the DHC, the submitted texts are grouped based on their terminologies, and classes represented by dendrograms are formed[9], in which it is possible to verify some similarities and differences between them, sizes, and themes that compose the classification. On the other hand, the FCA is a kind of internal function of the DHC that allows a different possibility of visualization of the corpus, through information made available in a Cartesian plan (Martins, 2022; Martins et al., 2022a).

The classes represented by the IRaMuTeQ are equivalent to the DTA categories. However, it is the researcher's responsibility, based on their interpretations, to define the level of the classes (initial, intermediate, final), assign titles, and identify whether the way the classes are distributed makes sense in light of the objectives or they need to be adjusted and/or

[9]Tree diagram.

subdivided. In addition, in this process, the unitizations occur automatically when one runs the DHC. However, the text segments highlighted in each class can be retrieved and interpreted (Martins, 2022; Martins et al., 2022a).

The organization and codification of the corpus subjected to the IRaMuTeQ software is the researcher's responsibility. They must define the texts that will be submitted and organize them according to the norms stipulated in the software manual. This movement is verified in detail by Martins, Gomes, and Paula (2022), who explain the codification processes.

In this article, the abstracts that presented the essence of the productions were submitted to the software. Thus, according to Pedruzzi et al. (2015), we defined some mandatory elements: objectives and/or research questions, theoretical-methodological aspects, main results, and conclusions. In cases where the abstracts did not present one of those elements, we carried out a directed reading, aiming to identify such aspects and complete the texts, always conserving the author's original writing.

After submitting the corpus to the IRaMuTeQ and analyzing the information that emerged, we reached three final categories, already mentioned in the introduction. In what follows, we chose to present the understandings that emerged in one of them, entitled Orientações didático-pedagógicas para o trabalho com resolução de problemas [Didactic-pedagogical guidelines to work with problem solving]. This final category resulted from five intermediate categories: i) the teaching-learning-assessment methodology; ii) problem solving as a methodology; iii) George Polya's steps; iv) exploration, resolution, and posing of problems; and v) proposition, formulation, and/or elaboration of problems (Martins, 2022; Martins et al., 2022a).

Through the expressive quantity of productions that make up the corpus, in this article we do not bring all the productions, nor made explicit the condensations with the descriptions of the process of constitution of the final category. To the interested reader, we suggest reading Martins' master's thesis (2022) to observe this movement. In what follows, we present the Metatext, which communicates the elements that emerged for the constitution of the State of Knowledge regarding the didactic-pedagogical guidelines to work with problem solving.

Didactic-pedagogical guidelines to work with problem solving: what is evidenced in research?

The expression "problem solving" is constantly used in classrooms and constitutes a pillar of all school Mathematics (NCTM, 2007)[10]. However, its conception and use in these

[10]It is a Portuguese version of the *Standards* 2000 (NCTM, 2000).

contexts may vary according to teachers' choices. Such choices are directly linked to the objectives of its use, which guide the selection of problems, the moment in which they will be used, and teachers', and consequently, students' posture.

In this context, a characteristic evidenced in the corpus analyzed was the concern with the construction and/or re-signification of mathematical knowledge from the work with problem solving. This may be demonstrated by the number of productions that use/approach the concept of teaching Mathematics through Problem Solving (Schroeder & Lester, 1989; Allevato, 2005). To confirm this fact, it is possible to state that there is an occurrence of at least 60% of the productions in which problem solving was used for this purpose. Thus, we can infer that, in these contexts, the vision of work with problem solving goes beyond practices that encourage the reproduction of techniques previously presented by the teacher. Thus, when approached from this perspective, problem solving assumes different nomenclatures: Methodology for Mathematics Teaching-Learning-Assessment through Problem Solving; Exploration, Resolution, and Posing of Problems; Teaching Strategy; and Problem Solving Methodology. Using different terminologies is justified, mainly, by the theoretical support utilized by the authors of the productions considered here because, over the years, research works that start from teaching through problem solving but with some specificities have been developed.

Justulin and Noguti (2017), for example, claim that problem solving came to be seen as a teaching methodology from the late 1980s onwards. Onuchic (1999) incorporates this nomenclature and presents a set of seven stages to guide the work in the classroom with the topic. However, later, the studies conducted by that author began to use the term Methodology for Mathematics Teaching-Learning-Assessment through Problem Solving. The stages that lead to problem solving were expanded to nine, ten, and even eleven, presented, for example, in Onuchic and Allevato (2011), Allevato and Onuchic (2014, 2021), and Andrade and Onuchic (2017), as shown in Table 2:

Table 2.

Script of stages of the Teaching-Learning-Assessment methodology (prepared by the authors)

Stage script		
Onuchic (1999)	Onuchic and Allevato (2011)	Allevato and Onuchic (2014, 2021)
1. Forming groups –	1. Preparing the problem;	1. Posing the problem;

delivering an activity; 2. The teacher's role; 3. Results on the board; 4. Plenary; 5. Analyzing the results; 6. Consensus; 7. Formalizing.	2. Individual reading; 3. Reading together; 4. Solving the problem; 5. Observing and encouraging; 6. Recording resolutions on the blackboard; 7. Plenary; 8. Searching for consensus; 9. Formalizing the content.	2. Individual reading; 3. Reading together; 4. Solving the problem; 5. Observing and encouraging; 6. Recording resolutions on the blackboard; 7. Plenary; 8. Searching for consensus; 9. Formalizing the content; 10. Posing and solving new problems.

The difference identified in the proposed scripts, especially when compared with the one presented by Onuchic (1999), for the script explained by Onuchic and Allevato (2011) occurs, according to the authors, due to the need to better explore the steps and make them clearer to teachers. According to Onuchic and Allevato (2011, p. 83), "trying to meet the demand of providing students with the prior knowledge necessary for the more productive development of the methodology, we slightly changed the first script, including new elements and creating the second". Furthermore, we also verified that the main change that occurred from the second script, explained in Onuchic and Allevato (2011), to the most recent ones, is the addition of the posing of problems.

From this perspective, we highlight that the approach entitled Methodology for Mathematics Teaching-Learning-Assessment through Problem Solving (Onuchic & Allevato, 2011; Allevato & Onuchic, 2014) was predominant in the category discussed here, considering that it was identified in at least 23 productions[11]. Such productions use/address aspects related to the methodology above and describe investigations carried out at different levels of education, with emphasis on Basic Education, with 14 productions, two of which are directed to the Early Years of Elementary School, five to the Final Years and seven to High School.

Therefore, what enhances this approach and can facilitate its use in classrooms is the definition of a set of steps communicated in the scripts previously presented. Therefore, we understand, based on our perception of the corpus analyzed, that the teacher can have guidance,

[11] Pagani (2016), Domingos (2016), Rocha (2016), Lago (2016), Brasil (2017), Ferreira (2017), Pereira (2018), Assis (2018), Silva (2018), Vallilo (2018), Lima (2018), Binotto (2019), Martins (2019), Sá (2019), Pironel (2019), Martins (2019b), Silva (2019), Vargas (2019), Araújo (2020), Carvalho (2020), Fernandes (2020), Andreatta (2020), and Duarte (2020). Such references can be consulted in Martins (2022).

organized in stages, to put the methodology into practice. In the most recent approaches, in which the expression teaching-learning-assessment is used, it is vehemently indicated that Assessment be integrated with teaching and learning, carried out continuously during problem solving. In this sense:

> The aim is that, while the teacher teaches, the student, as an active participant, learns, and that the assessment is carried out by both. Students analyze their own methods and solutions for the problems, always aiming at constructing knowledge. [...] The teacher reviews what is happening and the results of the process, to reorient classroom practices when necessary (Onuchic & Allevato, 2011, p. 81).

On the other hand, the Exploration, Resolution, and Proposition of Problems approach (Andrade, 1998, 2017) is also an alternative to the teacher's pedagogical practice, which starts from teaching through Problem Solving, explained in 11 productions[12] that were developed exclusively in the UEPB.

Regarding this approach, we highlight, among other things, the concern with the posing of problems. As Andrade (2017, p. 357) points out, "in recent years, in this proposal, we have also given strong attention to work with Problem Posing, in which, in a practical way, we have used the expression: Exploration, Resolution, and Problem Solving".

From this perspective, we realize that both the insertion of the problem-posing stage in the Mathematics Teaching-Learning-Assessment through Problem Solving methodology, and the indications for its use, present in the perspective of Exploration, Resolution, and Problem Posing, indicate a growing interest in the subject in research that deals with problem solving, in particular, as a teaching methodology. However, we emphasize that there is still no consensus regarding the term used, considering that we perceive the use of the terms formulation and/or elaboration of problems, or even the term design (Figueiredo, 2017), the latter being used to refer to the movement of creating problems with the help of Digital Technologies.

Therefore, we understand that those terms are interconnected and relate to providing students with the opportunity to elaborate and/or (re)formulate problems. We believe that such practice can be seen as a way to teach and give meaning to Mathematics; or even as a way for the teacher to assess whether the student has understood certain content, given that it is unlikely that this student can elaborate and/or (re)formulate a problem about content that he or she knows

[12] Santos (2016), Araújo (2016), Silveira (2016), Silva (2016a), Bezerra (2017), Costa (2019), Silva (2020), Martins (2019a), Santos (2019a), Santos (2019), and Lins (2019). Such references can be consulted in Martins (2022).

little or nothing about.

We also believe it is relevant to work with this practice in teacher education because, when working with Problem Solving in the classroom using it as a starting point, it is important that the problem posed starts from where the students are. For this, the teacher can create or (re)formulate the problems to bring them closer to the class context.

On the other hand, we ratify that George Polya's studies, with an emphasis on the book *How to Solve It*, which boosted the development of the theme, still guide most of the productions investigated here. From this perspective, when dealing with Polya's work, the main point highlighted in the productions are the phases created by the author to help solve problems: understanding the problem; establishing a plan; executing the plan; doing a retrospective of the resolution (Polya, 2006).

However, dealing with the author's ideas does not mean we are just theorizing about problem solving and/or using the four steps in isolation. When work with those steps is accompanied by moments of discussion and reflection among peers, the practice of problem solving can provide students with opportunities to construct and/or reconstruct mathematical knowledge. Such discussions can be constituted by the teacher's feedback to the students, or even through the dialogue between them after the resolutions, explaining the paths to reach the final answer.

From this, we understand that, when we use such steps with the aim of not only theorizing about problem solving but also developing an understanding of mathematical content, such practice can also be seen as a teaching methodology. When this occurs, we can even see that the authors may resort to studies that deal with Problem Solving to build knowledge (Valério, 2016). To develop her investigation, the author relied on Polya (2006), Dante (1991), and Onuchic and Allevato (2004), among other authors. In line with this statement, we found that the term *teaching methodology* was used to address Polya's steps (1995), or Mason, Burton and Stacey's (1982) stages, created from Polya's steps:

> Mason, Burton, and Stacey (1982) state that, to think mathematically in an effective way while solving a problem, it is necessary to test ideas and discuss them. Therefore, the resolution of mathematical problems is a dynamic process that allows increasing the complexity of ideas and expanding the ability to understand. Based on George Polya's ideas and four steps for problem solving, these authors established three phases for a problem solver to succeed in their task: Entry, Attack, and Review (Pita, 2016, pp. 53).

Therefore, we noticed that the discussions revolved around Problem Solving to build mathematical knowledge. Our understanding led us to consider that this practice, in Brazil, is strongly influenced by the studies by Onuchic (1999), Onuchic and Allevato (2011), Allevato and Onuchic (2014), and Andrade (1998, 2017).

On the other hand, we also located studies that aim to problematize issues related to students' strategies using Polya (1995), which can be evidenced in Muniz (2017), for example. The author justifies her theoretical choice by stating that, about problem solving, the research is closer to Polya's (1995) than to post-Polya's ideas. From this perspective:

> Scholars who follow Polya's (1995) line were divided between "teaching about Problem Solving" and "teaching to solve problems". "Teaching through Problem Solving" is seen as a post-Polya work of facing Problem Solving (Muniz, 2017, pp. 33).

Furthermore, we finalized this metatext evidencing the interest of the authors of the analyzed theses for research aimed at Basic Education, emphasizing the stages of High School and Final Years of Elementary School. In a teacher education context, the main target audience is Mathematics teaching degree students.

Final considerations

The investigation portrayed in this article aimed to present the understandings that emerged from the didactic-pedagogical guidelines for working with problem solving, based on the analysis of Brazilian doctoral and master's theses in Mathematics Education, produced from 2016 to 2020 in a search in BDTD and the CAPES catalog of doctoral and master's theses.

Based on the results obtained, we can identify that in the analyzed period, the didactic-pedagogical guidelines for working with problem solving strongly influenced research carried out in Brazil, among which we highlight Onuchic (1999), Onuchic and Allevato (2011), Allevato and Onuchic (2014), and Andrade (2017). We also evidenced the prevalence of productions that use/approach Problem Solving to build mathematical knowledge and that there are different possibilities for work with problem solving in Mathematics classes from this perspective, which can be ratified, for example, by the different nomenclatures assumed. In this context, the Methodology Mathematics Teaching-Learning-Assessment through Problem Solving and Exploration, Resolution, and Problem Posing stand out.

On the other hand, we understand that in the context of Mathematics classrooms, there can be discussions about the strategies students use to solve problems, in the light of Polya's steps (2006), for example. We believe that when developed from dialog and information sharing between peers, such a practice can provide opportunities for reflection, construction of resolution strategies, and production of mathematical knowledge.

In this way, this article ratifies that problem solving has been establishing itself as a trend in Mathematics Education, being discussed and problematized in postgraduate programs, and that the others have been following the transformations of society that are reflected directly and indirectly in the school organization and, particularly, in the curriculum. In this regard, it is enough to observe the remarkable movement of recent research that deals with problem posing (Cai & Hwang, 2020; Allevato & Possamai, 2022; Possamai & Allevato, 2022), following the guidelines provided for in the BNCC.

We also identified the need for more studies inserted and/or directed to pedagogy courses and the Early Years of Elementary Education. In addition, about continuing education, we point out the need for investigations that include in-service Higher Education teachers.

Finally, we hope that this study can contribute to the development of future research that deals with problem solving in Mathematics Education and to subsidize pedagogical practices in classrooms, accompanied by reflections and exploration of the possibilities of its use, so that the teaching, learning, and assessment processes can be leveraged, improving our students' Mathematics learning.

References

Allevato, N. S. G. Resolução de problemas. In: **Associando o Computador à Resolução de Problemas Fechados:** Análise de uma Experiência. 2005. 370 f. Tese (Doutorado em Educação Matemática) Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista Júlio de Mesquita Filho, Rio Claro, 2005.

Allevato, N. S. G.; Onuchic, L.R. Ensinando matemática na sala de aula através da resolução de problemas. **Boletim GEPEM**, n.55, n. 55, jul./dez. 2009.

Allevato, N. S. G.; Onuchic, L. R. Ensino-Aprendizagem-Avaliação de Matemática: Por que através da Resolução de Problemas? In: ONUCHIC, L. R.; ALLEVATO, N. S. G.; NOGUTI, F. C. H.; JUSTULIN, A. M. (Orgs). **Resolução de Problemas:** Teoria e Pratica. Jundiai: Paco Editorial, 2014, p. 35-52.

Allevato, N. S. G.; Onuchic, L.R. Ensino-Aprendizagem-Avaliação de Matemática: Por que através da Resolução de Problemas? In: ONUCHIC, L. R.; ALLEVATO, N. S. G.; NOGUTI, F. C. H.; JUSTULIN, A. M. (Orgs). **Resolução de Problemas:** Teoria e Pratica. 2. ed. Jundiai: Paco Editorial, 2021, p. 37-58.

Allevato, N. S. G.; Jahn, A. P.; Onuchic, L. R. O computador no Ensino e Aprendizagem de Matemática: reflexões sob a perspectiva da Resolução de Problemas. In: ONUCHIC, L. R.; LEAL JR, L. C.; PIRONEL, M. (orgs). **Perspectivas para a Resolução de Problemas.** São Paulo: Editora Livraria da Física, 2017. p. 247-278.

Allevato, N. S. G.; PossamaI, J. P. Proposição de Problemas: possibilidades e relações com o trabalho através da Resolução de Problemas. **Com a Palavra o Professor**. UESB: Vitória da Conquista. v.7, n. 18, 2022. p. 153 – 172.

Andrade, S. **Ensino-aprendizagem de Matemática via resolução, exploração, codificação e decodificação de problemas**. Dissertação (Mestrado em Educação Matemática) – Universidade Estadual Paulista, Rio Claro, 1998.

Andrade, S. Um caminhar crítico reflexivo sobre Resolução, Exploração e Proposição de Problemas Matemáticos no Cotidiano da Sala de Aula. In: ONUCHIC, L. R; LEAL JR, L. C; PIRONEL, M (orgs). **Perspectivas para a Resolução de Problemas.** São Paulo: Editora Livraria da Física, 2017. p. 355-396.

Andrade, C. P.; OnuchiC, L. R. Perspectivas para a Resolução de Problemas no GTERP. In: Onuchic, L. R; Leal, L. C; Pironel, M (orgs). **Perspectivas para a Resolução de Problemas.** São Paulo: Editora Livraria da Física, 2017. p. 433-466.

Brasil. Ministério da Educação. Base Nacional Comum Curricular. Educação Infantil, Ensino Fundamental e Ensino Médio. (versão final). Brasília: MEC, 2018.

Cai, J.; Hwang, S. Learning to teach through mathematical problem posing: theoretical considerations, methodology, and directions for future research. **International Journal of Educational Research**, v. 102, p. 1-8, 2020.

Figueiredo, F. F. **Design de problemas com a utilização das tecnologias digitais na formação inicial de professores de Matemática.** 2017, 276f. Tese (Doutorado em Ensino de Ciências e Matemática) Universidade Luterana do Brasil, Canoas, 2017.

Justulin, A. M.; Noguti, F. C. H. Formação de professores e Resolução de Problemas: um estudo a partir de teses e dissertações brasileiras. In: ONUCHIC, L. R.; LEAL, C. P.; PIRONEL, M (orgs). **Perspectivas para a Resolução de Problemas.** São Paulo: Editora Livraria da Física, 2017. p. 21-54.

Martins, K. N. **A pesquisa brasileira em resolução de problemas na educação matemática: um estudo a partir de teses e dissertações (2016-2020)**. 2022, 161f. Dissertação (mestrado em educação em ciências e matemática) Universidade Estadual de Santa Cruz, Ilhéus, 2022.

Martins, K. N.; Gomes, L. P. S.; Paula, M. C. de. *Software* IRaMuTeQ: uma ferramenta auxiliar na Análise Textual Discursiva. **Revista Paradigma**, Vol. XLIII, Edición Temática: Pesquisa Qualitativa em Educação Matemática, pp 205-227, mayo, 2022.

Martins, K. N.; Paula, M. C.; Gomes, L. P. S.; Santos, J. E. O software IRaMuTeQ como um recurso para a análise textual discursiva. **Revista Pesquisa Qualitativa**. v.10, n. 24, p. 213-232, abr./ago. 2022.

Melo, M. C. P.; Justulin, A. M. Resolução de problemas: um caminho para o ensino de matemática. **Ensino e Tecnologia em Revista**. v.3, n.1, p. 112-128, jan/jun. 2019.

Moraes, R.; Galiazzi, M. C. **Análise Textual Discursiva**. 3. ed. Ijuí: Unijuí, 2016.

Moraes, R. Uma tempestade de luz: a compreensão possibilitada pela Análise Textual Discursiva. **Ciência & Educação**. v.9, n.2, p. 191-211, 2003.

Morais, R. S.; Onuchic, L. R. Uma abordagem histórica da Resolução de Problemas. In: Onuchic, R. L.; Allevato, N. S. G.; Noguti, F. C. H.; Justulin, A. M. (orgs), **Resolução de problemas, teoria e prática.** Jundiaí, Paco Editora: 2014. p.17-34.

Morais, R. S.; Onuchic, L. R. Uma abordagem histórica da resolução de problemas. In: Onuchic, L. R.; Allevato, N. S. G.; Noguti, F. C. H.; Justulin, A. M. (Orgs). **Resolução de Problemas:** Teoria e Prática. 2. ed. Jundiai: Paco Editorial, 2021, p. 19-36.

Muniz, A. M. C. **Procedimentos utilizados por estudantes do nível médio técnico em problemas de semelhanças de triângulos contextualizados e não contextualizados.** 2017, 114f. Dissertação (Mestrado profissional em Matemática em Rede Nacional) Universidade Federal Rural de Pernambuco, Recife, 2017.

NCTM. **Princípios e Normas para a Matemática Escolar**. 1. ed. Lisboa: APM, 2007.

Onuchic, L. R. Ensino-aprendizagem de Matemática através da Resolução de Problemas. In: Bicudo, M. A. V. (Org) **Pesquisa em Educação Matemática:** Concepções e perspectivas. São Paulo, Editora Unesp, 1999. P.129-218.

Onuchic, L. R.; Allevato, N. S. G. Pesquisa em Resolução de Problemas: caminhos, avanços e novas perspectivas. **Bolema** - Boletim de Educação Matemática, Rio Claro, SP, v. 25, n. 41, p. 73-98, dez. 2011.

Paula, M.C. de. **A prática pedagógica na formação de professores com o uso de TDIC sob o foco das objetivações de Agnes Heller**: Brasil e Portugal num estudo de caso múltiplo integrado. 2018. 317f. Tese (Doutorado em Educação em Ciências e Matemática) – Pontifícia Universidade Católica do Rio Grande do Sul, Porto Alegre, 2018.

Pedruzzi, A. S.; Schimidt, E. B.; Galiazzi, M. C.; Podewils, T. L. Análise Textual Discursiva: Os movimentos da metodologia de pesquisa. **Atos da Pesquisa em Educação**, Blumenau, v. 10, n. 2, p. 584-604, 2015.

Pita, A. P. G. **A ideia de função por meio da resolução de problemas: narrativas da educação de jovens e adultos**. 2016, 165f. Dissertação (Mestrado em Educação Matemática) Universidade Anhanguera de São Paulo, São Paulo, 2016.

Polya, G. **A arte de resolver problemas.** Trad. Heitor Lisboa de Araújo. Rio de Janeiro: Interciência, 2006.

Possamai, J. P.; Allevato, N. S. G. Elaboração/Formulação/Proposição de Problemas em Matemática: percepções a partir de pesquisas envolvendo práticas de ensino. **Educação Matemática Debate.** Montes Claros (MG), Brasil, v. 6, n. 12, p. 1-28, 2022.

Romanowski, J. P.; ENS, R. T. As pesquisas denominadas do tipo “Estado da arte” em Educação. **Diálogo e Educação,** Curitiba, v. 6, n. 19, p. 37-50, 2006.

Schroeder, T. L., Lester Jr., F. K. Developing Understanding in Mathematics via Problem Solving. In: TRAFTON, P. R.; SHULTE, A. P. (Ed.). **New Directions for Elementary School Mathematics**. Reston: NCTM, 1989 (Year Book).

Valério, W. **Resolução de problemas, uma abordagem com questões da OBMEP em sala**

de aula . 2016, 89f. Dissertação (Programa de mestrado profissional em Matemática) Universidade de São Paulo, São Carlos, 2016.

Van De Walle, J. A. **Matemática no Ensino Fundamental**: Formação de Professores e aplicação em sala de aula. 6. ed. Porto Alegre: Penso Editora LTDA, 2009.

Revisado e traduzido por: Maria Isabel de Castro Lima