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Knowledge and Beliefs of a Teacher Educator: Analysis of her Teaching Practice when Teaching Algebra in Primary Teacher Education

Conhecimentos e Crenças de uma Formadora de Professores: Análise de sua Prática Letiva ao Ensinar Álgebra na Licenciatura em Pedagogia

Saberes y Creencias de una Formadora de Profesores: Análisis de su Práctica Docente al Enseñar Álgebra en la Formación Inicial del Profesorado de Primaria

Connaissances et croyances d'un formateur d'enseignants : Analyse de sa pratique pédagogique lors de l'enseignement de l'algèbre dans le cadre de la formation initiale des enseignants du primaire

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Abstract

This article aims to identify and map beliefs about the teaching and learning of Mathematics and understand their role in the development of a teacher educator's professional knowledge within her teaching practice in primary teacher education. This is a case study carried out from qualitative-interpretative research, in which data of three different natures were collected: documents, interviews and observation. The results point out the incidence of the teacher educator's beliefs about instrumentalist mathematics, its teaching and learning constituted during her academic trajectory and indicate a reorganization of these beliefs from her involvement in a collaborative work aimed at her teaching practice. Finally, results also point out the contributions of the reorganization of these beliefs in the development, mobilization and expansion of their professional knowledge for teaching Algebra in primary teacher education, especially in the domain of pedagogical content knowledge.

Keywords: Teacher Educators, Primary teacher education program, Professional knowledge, Beliefs.

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Resumo

O presente artigo tem como objetivo identificar e mapear as crenças sobre a matemática, seu ensino e aprendizagem, e compreender o papel delas no desenvolvimento de conhecimentos profissionais de uma formadora de professores em sua prática letiva em um curso de Licenciatura em Pedagogia. O estudo é fruto de uma pesquisa qualitativa-interpretativa, do tipo estudo de caso, em que foram recolhidos dados de três naturezas distintas: documentos, entrevista e observação. Os resultados apontam para a incidência das crenças da formadora sobre a matemática instrumentalista, seu ensino e aprendizagem constituídas durante sua trajetória acadêmica e para uma reorganização dessas crenças a partir do envolvimento em um trabalho colaborativo voltado à sua prática letiva. Por fim, aponta-se para as contribuições da reorganização dessas crenças no desenvolvimento, mobilização e ampliação dos seus conhecimentos profissionais para ensinar Álgebra na Licenciatura em Pedagogia, em especial no domínio do conhecimento pedagógico do conteúdo.

Palavras-chave: Formadores de Professores, Licenciatura em Pedagogia, Pensamento Algébrico, Conhecimentos Profissionais, Crenças.

Resumen

Este artículo pretiende identificar y mapear las creencias sobre las matemáticas, su enseñanza y aprendizaje, además comprender su papel en el desarrollo del conocimiento profesional de una formadora de profesores en su práctica docente en la formación inicial de profesores de primaria. El estudio es resultado de una investigación cualitativa-interpretativa, con enfoque de estudio de caso, en la que se recogieron datos de tres naturalezas diferentes: documentos, entrevista y observación. Los resultados señalan la incidencia de las creencias de la formadora de profesores sobre las matemáticas instrumentales, su enseñanza y aprendizaje constituidas durante su trayectoria académica e indican una reorganización de estas creencias a partir de su involucramiento en un trabajo colaborativo dirigido a su práctica docente. Finalmente, los resultados también señalan los aportes de la reorganización de estas creencias en el desarrollo, movilización y ampliación de su conocimiento profesional para la enseñanza del Álgebra en la formación inicial de maestros de la primaria, especialmente en el dominio del conocimiento pedagógico del contenido.

Palabras clave: Formadores de profesores, Programa de formación inicial de profesores de primaria, Conocimientos profesionales, Creencias.

Résumé

Cet article vise à identifier et à cartographier les croyances sur les mathématiques, leur enseignement et leur apprentissage, et à comprendre leur rôle dans le développement des connaissances professionnelles d'une formatrice d'enseignants dans sa pratique d'enseignement dans le cadre de la formation initiale des enseignants du primaire. L'étude résulte d'une recherche qualitative-interprétative, dans une approche d'étude de cas, dans laquelle des données de trois natures différentes ont été collectées : documents, entretiens et observations. Les résultats soulignent l'incidence des croyances de la formatrice d'enseignants sur les mathématiques instrumentalistes, leur enseignement et leur apprentissage constitués au cours de sa trajectoire académique et indiquent une réorganisation de ces croyances à partir de son implication dans un travail collaboratif visant sa pratique d'enseignement. Enfin, les résultats soulignent également les contributions de la réorganisation de ces croyances dans le développement, la mobilisation et l'expansion de leurs connaissances professionnelles pour l'enseignement de l'algèbre dans la formation initiale des enseignants du primaire, en particulier dans le domaine de la connaissance du contenu pédagogique.

Mots-clés: Formateurs d'enseignants, Programme de formation initiale des enseignants du primaire, Connaissances professionnelles, Croyances.

Within the scope of prospective teacher education, based on the approval of the Common National Curriculum Base (BNCC), the National Common Base for the Prospective Teacher Education (BNC) was set up with the aim of reorganizing the pedagogical projects of the country's teaching degrees. This curriculum reorientation of the pedagogy degree (PD) focused, among other aspects, on the reorganization of the area of Mathematics.

In addition to the number of hours allocated to Mathematics teacher education, about ten times lower than in other countries, such as Portugal, as pointed out by the study by Castro and Fiorentini (2021), another aspect that needs to be rethought concerns the distribution of Mathematics content in PD, since, generally, this training is restricted to the contents of numbers and operations, neglecting other aspects of Mathematics, such as algebra (Castro & Fiorentini, 2021).

In this sense, the results of the study by Doná and Ribeiro (2022) suggest that this curriculum reorientation in PD courses may, among other aspects, affect the inclusion of Algebra and Algebraic Thinking as topics for Mathematics subjects. For this, one must also consider the teacher educator's relationship with Mathematics and Algebra as an aspect that can contribute to rethinking how mathematical content is distributed in PD courses (Doná & Ribeiro, 2022).

From our understanding, the points raised above directly affect the teaching of Algebra in basic education. For example, Bortolete, Oliveira and Guaranha (2022) and Jungbluth, Silveira and Grando (2022) highlight the technical view presented by the BNCC itself on Algebra, as well as mentioned the impact of the lack of specific teacher education to work with this content, underscored by teachers who are already working in the early years. To overcome these challenges, even in prospective teache education, Doná and Ribeiro (2022) argue for the possibility of involving teachers who teach Mathematics in PD courses in teacher education experiences that address Algebra as a topic, justifying it by the fact that the quality of education of teacher educators can interfere with the quality of teacher education (Goodwin & Kosnik, 2013).

In this sense, Ping, Schellings and Beijaard (2018) discuss *what, how* and for *what reasons* teacher educators learn. The authors highlight as professional learning contents of the teacher educator, the knowledge base that he must mobilize and develop to teach teachers and, among the means defended by the authors for the teachers' learning, is the collaborative

reflection, a process through which the teacher educator can share personal reflections with pre and in service teachers or interact with peers to reflect together on a critical event.

In addition, we also point to the incipient and need for research on teacher educators who teach mathematics in Brazil (Nacarato et al., 2016; Coura & Passos, 2017; Gatti et al., 2019), the importance of characterizing their knowledge (Coura & Passos, 2021; Almeida & Ribeiro, 2020), the complexity that involves their work through the constitution of their professionality (Gatti et. al., 2019) and for the relationship of their beliefs with their knowledge professional (Carrillo et al., 2019; Ferreti et al., 2021).

Therefore, in this article, we aim to identify and map beliefs about mathematics, its teaching and learning, and understand their role in the development of professional knowledge of a teacher educator in her teaching practice in a pedagogy degree course. To operationalize this objective, we present three research questions in our study: (i) What beliefs about mathematics, its teaching and learning, does a teacher educator express when reflecting on her teaching practice in a pedagogy degree course? (ii) How do beliefs about mathematics, its teaching and learning, interfere in the development of teacher educator's professional knowledge when planning formative tasks to teach Algebra in the pedagogy degree course? (iii) What is the role of reflection, and how does it contribute, for a teacher educator to reorganize her beliefs about mathematics, its teaching and learning, in order to re/construct her knowledge to teach Algebra to future teachers?

Theoretical Framework

Teaching practice of teacher educators

Lesson planning is seen as a development and professional learning opportunity for teacher educators (Superfine & Pitvorec, 2021). This occurs because it is in planning that teachers have the opportunity to confront and anticipate their problems in practice. It is from the planning that a set of contents to be taught and the ways in which to teach them are designed.

On the other hand, Borko et al. (2014) present six practices, divided between planning and orchestrating lessons, which guide the structure of teacher education processes by teacher educators when using class videos as methodological resources. In addition, Ribeiro and Ponte (2020) present three domains (professional learning tasks, role and actions of the teacher educator, and discursive interactions among participants) that, connected and related, can contribute to the elaboration of teacher education programmes that generate opportunities for professional learning to teachers.

Taking the specificities presented by the authors above, we gathered some elements that we consider necessary to compose the teacher educator's actions in the planning of their classes when using formative tasks and foreseeing the exploratory teaching approach in their practice. These actions comprise (i) the definition of the objectives of the class, (ii) the selection of the formative task, (iii) the resolution of the formative task, (iv) the anticipation of possible resolutions of the teachers, (v) the organization of time and of the material (Borko et al., 2014; Ribeiro & Ponte, 2020).

Another component that must be added to the planning and development of a lesson, in order to compose the stages of the teacher's teaching practice, is reflection. In the planning, development and reflection cycle (hereinafter PDR Cycle, Ribeiro, Aguiar & Trevisan, 2020), reflection has the role of reorienting the teacher's actions and contributing to a more refined teaching practice. On the other hand, Schön (2000) presents a model with three types of reflection: reflection in action, reflection on action and reflection on reflection on action. Based on what is presented by Schön (2000), we share the idea that the teacher's reflection can be based on the following questions: What was planned and happened? What was planned and did not happen? What was not planned and happened? What was planned and done differently?

Professional knowledge of teacher educators

Research on teacher knowledge began in the 20th century (Shulman, 1986) and, since then, has gained new and more refined theoretical structures (e.g. Ball, Thames & Phelps, 2008; Carrillo et al., 2018). As a result, studies on teacher educator's knowledge have increased in the early 21st century (Carrillo et al., 2019; Ferreti et al., 2021) and have grown slowly in recent years (Beswick & Goos, 2018). These studies have gained prominence in view of the concern with teacher education and the relationship between these fields - teacher educator's knowledge and teacher knowledge development (Beswick & Goos, 2018). In recent years, research on educator's knowledge has advanced from a perspective of knowledge characterization (Jaworski, 2008; Zopf, 2010; Superfine & Li, 2014; Beswick & Goos, 2018) to the design of more structured theoretical models (Carrillo et al. al., 2019; Ferreti et al., 2021).

Posing that knowledge of pedagogical content should help teachers to make Mathematical Teachers' Specialized Knowledge (MTSK) accessible to teachers, Carrillo et al. (2019) used the MTSK and elevated it to a model that contemplated the knowledge of teacher educators. The authors divided the teachers' knowledge into content knowledge (MK-MTE[3]),

[3] Mathematical Knowledge for Teacher Educators.

pedagogical content knowledge (PCK-MTE) and added Beliefs about the nature of mathematics, its teaching and learning, as an internal process that interferes in the development of the other two domains of the teacher educator's knowledge. Deepening the model of Carrillo et al. (2019), Ferreti et al. (2021) focused their gaze on the PCK-MTE and observed subdomains of this knowledge. To illustrate the subdomains, the authors used the representation of the model referring to the teacher's knowledge (Carrillo et al., 2018), extending it to that of the teacher educator (Figure 1).

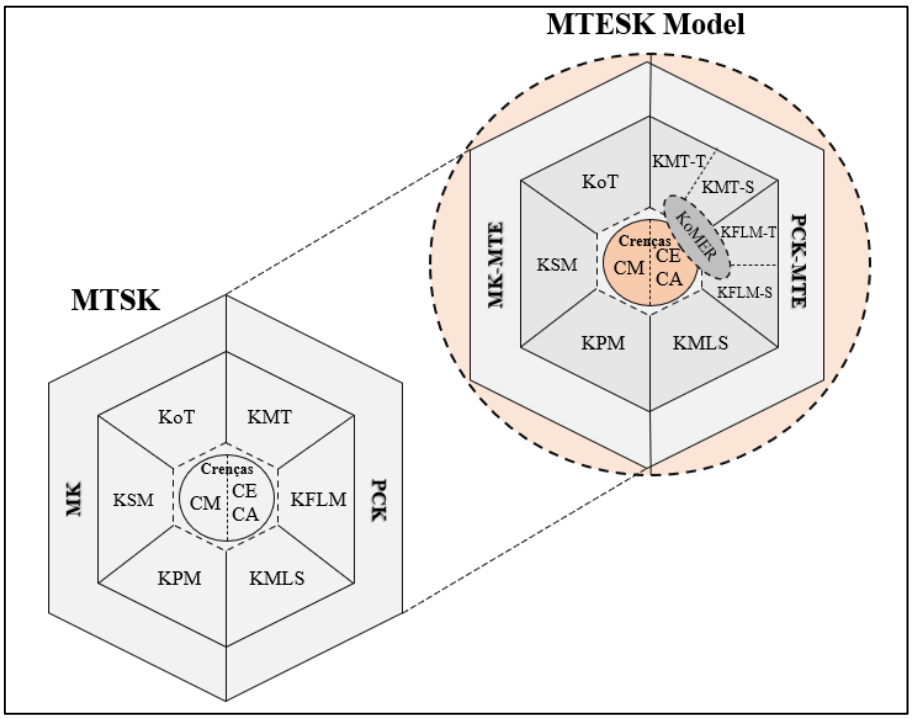

Figure 1.

MTESK Model (Ferreti et al., 2021)

It is noticed that in the PCK-MTE – in comparison to the representation of the teacher's knowledge (MTSK) – the authors included the subdomain of research knowledge in Mathematics Education (KoMER), as well as divided the subdomains of knowledge of teaching mathematics (KMT) and knowledge of the characteristics of learning mathematics (KFLM) in two aspects, one that refers to the context of basic education (KMT-S and KFLM-S[4]) and, the other, to the context of teacher training (KMT-T and KFLM-T[5]). About KoMER, it is worth noting that this knowledge crosses the KMT, KFLM and beliefs subdomains, as KoMER reflects on actions on teaching and learning mathematics and may even contribute to the teacher educator transforming their own beliefs.

[4] We chose to use the "S" after the hyphen representing "students", which refers to basic education students.
[5] We chose to use the "T" after the hyphen representing "teachers", which refers to initial training teachers.

Regarding the division into subdomains of mathematics teaching and learning, Ferreti et al. (2021) report that in MTSK, the KMT-S refers to strategies to teach mathematical skills to children/young people, while in MTESK, the KMT-T encompasses the strategies of teacher educator to develop skills aimed at teaching, skills that allow students to teachers teach math to their students. Knowledge of the characteristics of mathematics learning also differs: in the case of the teacher, the KFLM-S must consider a more contextual and social concern, since children/young people are learning the content for the first time. In the case of the teacher educator, in turn, the KFLM-T considers different learning characteristics, as teachers already have a knowledge of mathematics coming from basic education, which must be deepened and transformed for the teaching function (Ferreti et al., 2021).

Beliefs about mathematics, its teaching and learning

Beliefs are like a kind of subjective knowledge that interferes and that can be measured from action (Beswick, 2012). The author indicates that the view of mathematics as an area of knowledge interferes with the formation of beliefs about school mathematics and its teaching. She uses the existing dichotomy between mathematics "produced" by mathematicians and that "taught" by teachers in basic education schools, to portray that the teacher's views of mathematics may or may not be related to the mathematics he/she teaches, with the way how he'she teaches and how he/she perceives students' learning.

According to Beswick (2012), views of mathematics, as a field of knowledge, can be instrumentalist, platonist or based on problem solving. Relating each of these views to the teaching of mathematics, the teacher with an instrumentalist view sees mathematics as a static body of knowledge, conceives teaching as centered on him/herself and believes that the student will learn by reproducing exercises. Meanwhile, the platonist also sees mathematical knowledge as ready, he/she does not centralize teaching entirely in him/herself, but believes that he learns mathematics when he acquires the conceptual understanding of rules and proofs. Finally, that teacher who has a view of mathematics as a way to solve problems, believes that knowledge can be discovered, its practice is student-centered and mathematics learning is measured procedurally by discovery (Beswick, 2012).

Rodriguez et al. (2018), authors who carried out a comparative study between the beliefs of future teachers and beliefs of their teacher educators about mathematics and its teaching, point out that teacher educators are more likely to believe that mathematics is an investigation process, reflecting the Beswick's vision of resolution of problems (2012). On the other hand, the authors realized that future teachers believe in mathematics as a set of ready-made

knowledge, in line with the instrumentalist and platonist views of Beswick (2012). Regarding mathematics learning, future teachers and teacher educators agreed that this learning should occur through investigation, regardless of who the learner is, whether the student or the teacher (Rodriguez et al., 2018), a view corroborated by Beswick (2012) in his vision of mathematics as problem solving.

In a similar study, Marshman (2021) investigated how future teachers deal with their teacher educators' beliefs and the impacts these have on the way they plan to teach in the future. The author points out that the teacher educators' beliefs about mathematics and its teaching influence the classroom practice of future teachers (Marshman, 2021). In this sense, Marshman (2021) concludes that the beliefs of future teachers are influenced by several factors, among them, the experiences they had with mathematics as students, the teacher educators who teach them, the curricular assumptions of their courses, and their practical experiences during internships and undergraduate research programs.

Algebra and Algebraic Thinking in the early years

Algebra in the early years is found in Brazilian curriculum documents such as Algebraic Thinking (Doná & Ribeiro, 2022) and this is understood as a means of transforming Algebra into a human activity (Kaput, 2008). Four are the essential ideas linked to Algebra, which must be worked on throughout elementary school: equivalence, variation, interdependence and proportionality. For the early years of elementary school, the focus of this study, the BNCC brings the objects of knowledge "regularity, generalization of standards and properties of equality" (Brasil, 2017, p. 270).

Different authors use a division regarding the aspects of Algebraic Thinking (Blanton & Kaput, 2005; Chimoni, Pitta-Pantazi & Christou, 2021). Blanton and Kaput (2005) divide Algebraic Thinking into two strands, Generalized Arithmetic and Functional Thinking, while Chimoni et al. (2021), in addition to these two strands, add a third, Language Modeling. Generalized Arithmetic is characterized by working with numbers and the properties of operations; Functional Thinking considers the idea of understanding numerical variation; and, Language Modeling, comprises the application of a set of techniques, mathematical and non-mathematical, to model problems aiming at Algebraic Thinking processes (perceive, generalize, represent and justify).

In our study, we delimited the aspect of Generalized Arithmetic, contemplating the meaning of equivalence of the equal sign (Chimoni et al., 2021). In other words, an equivalence relation is one that satisfies three fundamental properties: "reflexive (a = a, for every element

a); symmetric (if a = b then b = a, for any elements a and b); transitive (if a = b and b = c, then a = c for any elements a, b and c)" (Ponte, Branco & Matos, 2009, p. 19).

Understanding the equal sign as an equivalence relation helps to demystify the use of this sign exclusively for its operational meaning, as is common in the early years of elementary school (Trivilin & Ribeiro, 2015). In addition, the meaning of equivalence of the equal sign refers to the multiple ways of representing the same number, using numerical equalities (Barboza, Ribeiro & Pazuch, 2020). The use of the meaning of equivalence of the equal sign allows students to "investigate the different breakdowns of numbers, using numerical expressions to represent them and observing the structure of these expressions" (Ponte et al., 2009, p. 20).

Context of the study

The present study was developed within the scope of a Mathematics Teaching discipline in a PD course at a Brazilian public university, through a partnership between Eduardo (first author of this article) and the teacher educator working in the discipline, who we call Violeta[6]. The design and organization of the class cycles (PDR Cicle, Ribeiro et al, 2020) also involved, albeit indirectly, Alessandro (Eduardo's PhD advisor and second author of this article).

The main objective was to involve the teacher educator in the planning (P), development (D) and reflection (R) of classes that could provide opportunities for professional learning to future teachers for teaching Algebra in the early years. The teacher educator was chosen intentionally, considering her specific profile focused on working with Mathematics and Algebra in PD, and also her availability to carry out the research. Violeta's profile, as well as that of her PD students, will be summarized later.

Methodology

The present study has a qualitative approach, due to the interest in working with the manifestation of data within a certain context, providing the possibility of understanding the research participants within their respective environments (Bogdan & Biklen, 1994); and of an interpretative structure, since it seeks to create ways of understanding a reality as it is, taking into account its subjects, its actors and its events (Moraes, 2018; Ponte, 1994).

We adopted the case study as a research method due to the possibility of studying particular phenomena, such as "a program, an institution, an educational system, a person or a

[6] Codename chosen to keep the teacher educator's identity private.

social unit" (Ponte, 1994). In this sense, a case study from an interpretive perspective seeks to understand how the world is from the participants' point of view (Ponte, 1994). It is worth noting that our study considers the teacher educator and her teaching practice, while she experienced a collaborative work with intentions, in partnership with the authors of this article.

Study design

Before and after the three class cycles, we conducted two semi-structured interviews with Violeta (Boni & Quaresma, 2005) (EI and EF). Throughout the process, there were three plans (P1, P2 and P3), three developments (D1, D2 and D3) and three reflections (R1, R2 and R3) of classes (Table 1), so that, during the meetings for planning and reflection, Eduardo and Violeta worked collaboratively[7]. The development of the classes was carried out by Violeta, in person, in a group of PD students. The classes were recorded on video and made available to Eduardo for the selection of excerpts for reflection. Excerpts for reflection could come both from the classes developed and from the planning meetings, which were also recorded, or even from the resources used in the classroom.

Table 1.

Collaborative work design

EI	P1	D1	R1	P2	D2	R2	P3	D3	R3	EF
	1º *PDR* cicle			2º *PDR* cicle			3º *PDR* cicle			

Among the curriculum materials used by Violeta were the formative tasks, called Professional Learning Tasks (PLT) (Ribeiro & Ponte, 2020). The PLT used by the teacher educator had the following structure: a Mathematical Task (MT) that addressed the contents of equality and equivalence (hereinafter referred to as the meaning of equivalence of the equal sign) and Records of Practice (RP) (Ball, Bem-Peretz & Cohen, 2014) involving resolutions presented by basic education students, accompanied by questions of a didactic-pedagogical nature. The PLT were also used as resources for Eduardo to promote moments of reflection with Violeta. In this case, for the moments of reflection, the RP came from Violeta's own teaching practice in the PD course, accompanied by questions prepared by Eduardo to promote reflection.

It is worth highlighting the intentionality set for the development of the three PDR cycles as a way of progressively giving autonomy and protagonism to the teacher educator for the

[7] The collaborative work between Eduardo and Violeta was made possible by virtual tools (Google Meet and Zoom).

preparation, development and reflection of classes. For example, in the first cycle, Violeta experienced a PLT 1 and organized its development in the PD course, as well as reflected on excerpts selected by Eduardo. In the second cycle, a MT and the previously selected RP were delivered so that Violeta could create the didactic-pedagogical questions that make up PLT 2 and, in addition, think about its subsequent development with the PD students; the reflection was based on excerpts selected by Violeta and Eduardo. Finally, in P3, a MT, a broad set of RP and a file with guidelines were delivered so that Violeta could select which RP she would like to use in PLT 3, and prepare the questions that explored the didactic-pedagogical aspects. The R3 was made from excerpts selected by Violeta. Thus, as Violeta gained prominence in the planning, Eduardo's role was characterized as someone who guided and supported what should be done, however, always asking Violeta to justify all her choices. The same happened in the moments of reflection.

Study participants

The research participants were Violeta and her PD students. Violeta's profile is ideal for working with the subject of Mathematics Teaching in a PD course, as she trained as a teacher educator in the former Magisterium[8], has undergraduate degrees in Mathematics and Pedagogy, a master's in Mathematics Education, and a PhD in Neuroscience and Cognition. Professionally, she has experience in basic education (early years and high school), in school management and in higher education, including working with PD courses. Violeta likes Algebra, as her master's and doctoral research explored this topic.

With regard to PD students, as we did not have direct contact with them, they are presented from Violeta's perspective, who describes them as aged between 20 and 22 years old and reinforces that they have an aversion to mathematics and think of Algebra as "letter manipulation".

Data collection procedures and instruments

The data collected for this research are of three different natures: interviews (initial and final); documents (researcher's notes, formative tasks, curriculum documents, future teachers' practice records, teacher educator's lesson plans, slides used with the teacher); observation (videos of meetings with the teacher and videos of the teacher's PD classes).

[8] This is a course instituted by opinion n. 349/72, approved on April 6, 1972, which had a technical character and was responsible for training teachers to teach basic education in Brazil (Saviani, 2009).

Method of Analysis and construction of categories

Data analysis was carried out in two stages: data exploration (inductive approach) and recurrence to the theoretical framework (deductive approach). We use Content Analysis (Bardin, 2011) through its three processes: (i) pre-analysis, (ii) material exploration and (iii) treatment of results, inference and interpretation.

Having data of different natures, we started the pre-analysis with the "floating" reading (Bardin, 2011). It was through this reading that we highlighted the existence of two important epistemological aspects: the teacher's knowledge and her beliefs. After noticing the incidence of the teacher's beliefs and their importance in the development/mobilization of her knowledge, we decided to carry out a search for theoretical references on the theme "beliefs". The studies by Carrillo et al. (2019) and Ferreti et al. (2021) present beliefs as a central element in the teacher educator's knowledge structure. To establish possible relationships between beliefs and knowledge (Figure 2), we look at beliefs separately and resort to the divisions presented by Beswick (2012):

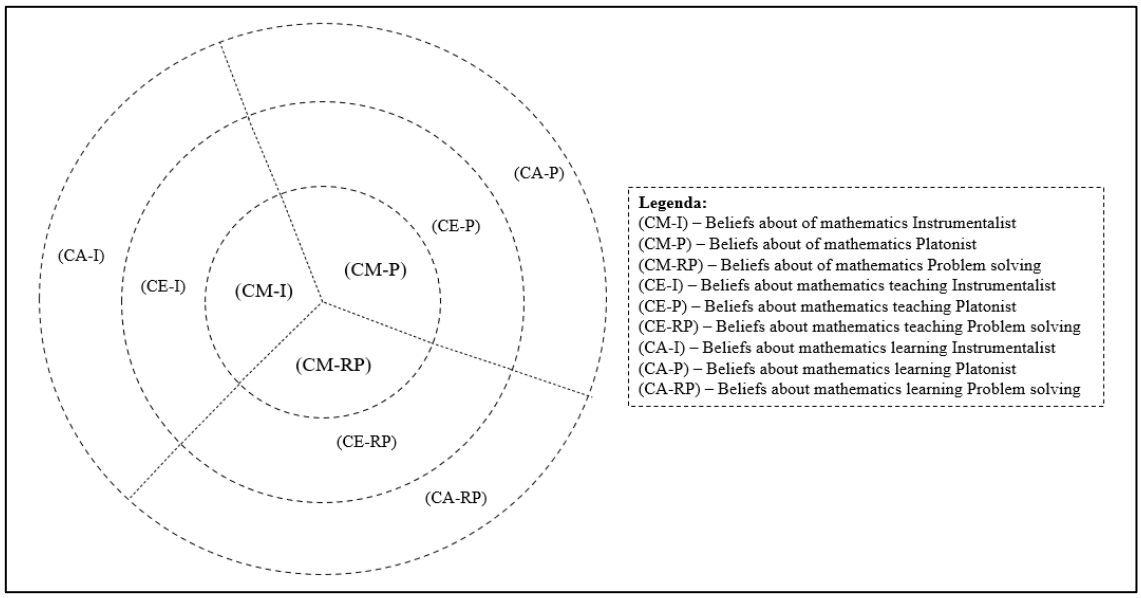

Figure 2.

Beliefs about Mathematics, Teaching and Learning (CMEA), inspired by Beswick (2012)

The elaborate structure presents three circles that border with sectioned lines, the first representing beliefs about the nature of mathematics, the second of beliefs about teaching mathematics and, finally, the third representing beliefs about learning mathematics. The broken lines that divide the circles indicate the relationship between the beliefs, and the position of the circles represents the influence that certain beliefs have on each other.

We analytically considered the PCK-MTE domain (Ferreti et al., 2021), since we understand that this domain is more related to the stages of the teacher educator's teaching practice, and we based ourselves on its six subdomains to build the analytical model presented in the Figure 3:

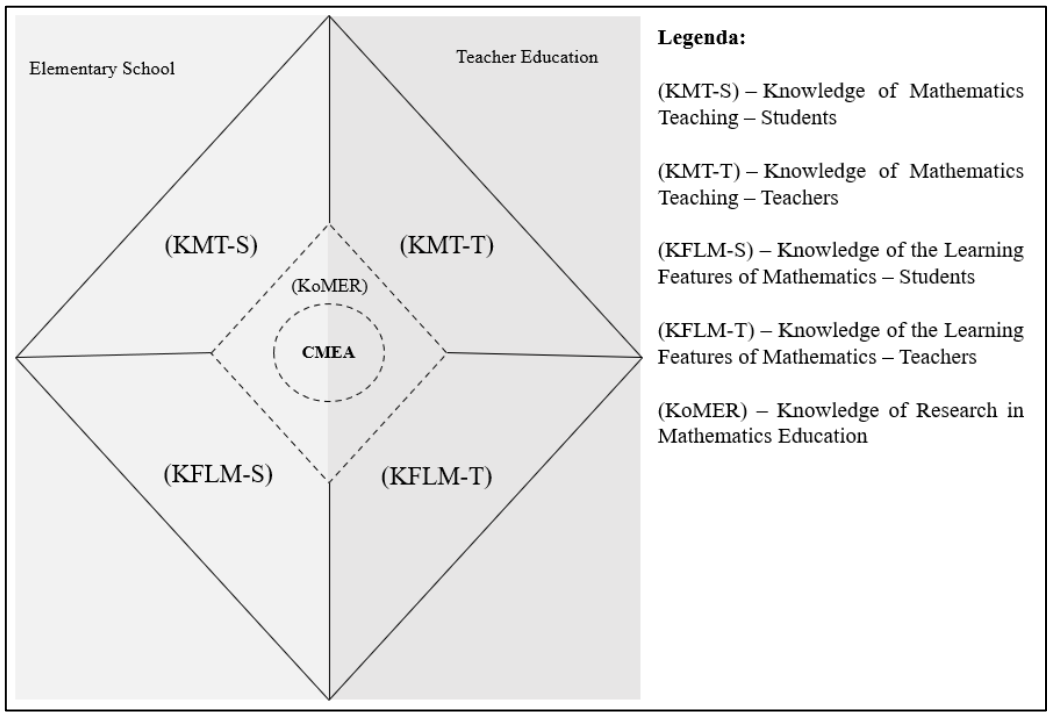

Figure 3.

PCK-MTE subdomains, inspired by Ferreti et al. (2021)

The model was divided into two spaces (basic education and initial teacher education) to accommodate the KMT and KFLM subdomains. The inner diamond represents the KoMER, which is sectioned because we share the idea that it is not a specific subdomain, but an additional knowledge of the teacher educator, which can influence the other subdomains, including their beliefs (Ferreti et al. al., 2021). The beliefs, represented in Figure 2, occupy the central circle in the model in Figure 3, since they influence and are influenced by the other subdomains of the PCK-MTE and by KoMER.

Another important point to be highlighted for the construction of analysis categories is the characterization of mathematical content. The Algebraic Thinking, delimited in this research through Generalized Arithmetic – is what characterizes the mathematical knowledge explored with and by the teacher educator Violeta. Among the contents that belong to the branch of Generalized Arithmetic, the property of operations, the property of numbers and the meaning of equivalence of the equal sign stand out (Chimoni et al., 2021).

With that, using Figures 2 and 3, and the contents of Generalized Arithmetic (GA), we present a detailed description of the categories and subcategories that are used in the data analysis of our study (Figure 4).

Category	Subcategory	Description
CMEA*	CM-I	Mathematics is ready-made knowledge, based on rules and procedures.
	CM-P	Mathematics is a static set of knowledge, ready to be taught/learned.
	CM-RP	Mathematics is creation, reflection and can be transformed.
	CE-I	The teaching of mathematics is centered on who teaches and is based on the application of rules and procedures.
	CE-P	Mathematics teaching can have the learner as protagonist and is based on the understanding of rules and procedures.
	CE-RP	The teaching of mathematics has the learner as its protagonist and is based on the possibility of mathematical discoveries.
	CA-I	Mathematics learning is measured by the subjects' skill level.
	CA-P	Learning is measured by the level of knowledge and understanding of mathematical ideas.
	CA-RP	Learning is measured in the discovery process provided through the autonomy given to subjects to build their mathematical knowledge.
PCK-MTE	KAGT-S	In this domain, the teacher educator knowledge is delimited by the strategies used in teacher training and by their ability to provide future teachers with tools to deal with the teaching of Generalized Arithmetic in Elementary School.
	KAGT-T	In this domain, the teacher educator knowledge is delimited by the strategies, tools, approaches and methodologies used by him to teach Generalized Arithmetic and to teach about Teaching Generalized Arithmetic to future teachers.
	KFLAG-S	In this domain, the teacher educator knowledge refers to predictions about the possible difficulties and challenges that elementary school students will face in learning Generalized Arithmetic or solving tasks related to GA contents.
	KFLAG-T	In this domain, the teacher educator knowledge refers to the anticipations of the challenges that future teachers will face to learn Generalized Arithmetic, teach Generalized Arithmetic or solve formative tasks related to GA contents.
	KoMER	In this domain, the teacher educator knowledge is delimited by his involvement with research activities (groups, publication of articles, participation and organization of events) and by the use of literature to guide the planning, development and reflection of his classes in teacher training.

* The acronyms used in the Figures have already been presented in the models of Figures 2 and 3, with the addition of "AG" to represent "Generalized Arithmetic".

Figure 4.

Teacher's knowledge and beliefs

Considering that the overall study relied on a variety of data collection instruments/procedures, the "data choice" stage that is the focus of analysis in this article culminated in the "constitution of the corpus" (Bardin, 2011) and implied a sequence of rules that were followed. The use of the "rule of completeness", "rule of representativeness", "rule of homogeneity" and "rule of pertinence" (Bardin, 2011) stands out. The initial interview, parts of the PLT, parts of the Slides and the Videos of the planning and reflection meetings with the teacher educator are sources of data for analysis in this article[9], thus safeguarding data of the three natures already indicated above.

With the "corpus" selected, in the "formulation of hypotheses and objectives" stage (Bardin, 2011) we reformulated the objective and questions of the study, in order to include the beliefs that emerged from the data. In the "preparation of the material" we highlighted "evident

[9] To identify the source of this data during the analysis, we use: (InI) for Initial Interview; (PLT) for Professional Learning Tasks; (VP1, VP2... VR1) for the planning and reflection videos with the teacher; (SP1, SP2... SP1) for Slides.

excerpts[10]" and, based on some rules already used, we selected what we call episodes. In total, three episodes were selected, one related to each study question.

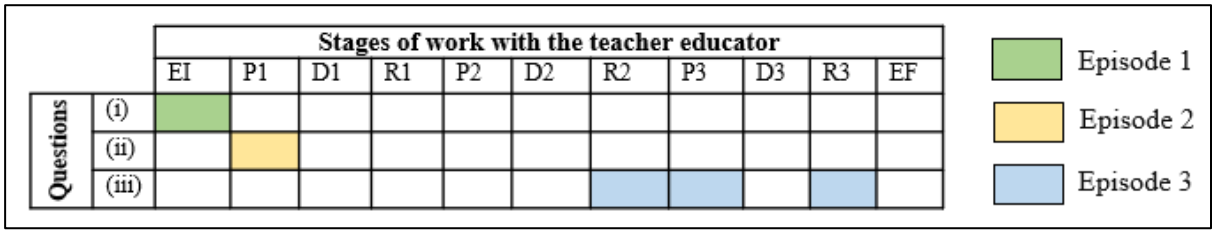

Figure 5.

Composition of the episodes and their relationship with the stages of work with the teacher and with the study questions

In the "exploration of the material" we relate what we call "evident excerpts" with the categories of analysis, similarly to what Bardin (2011) calls "coding". Finally, in the "treatment of the obtained results and interpretation" (Bardin, 2011), we gathered the coded data and selected some evident excerpts to narrate the context of the episodes. A summary of the analysis design is presented in Figure 6.

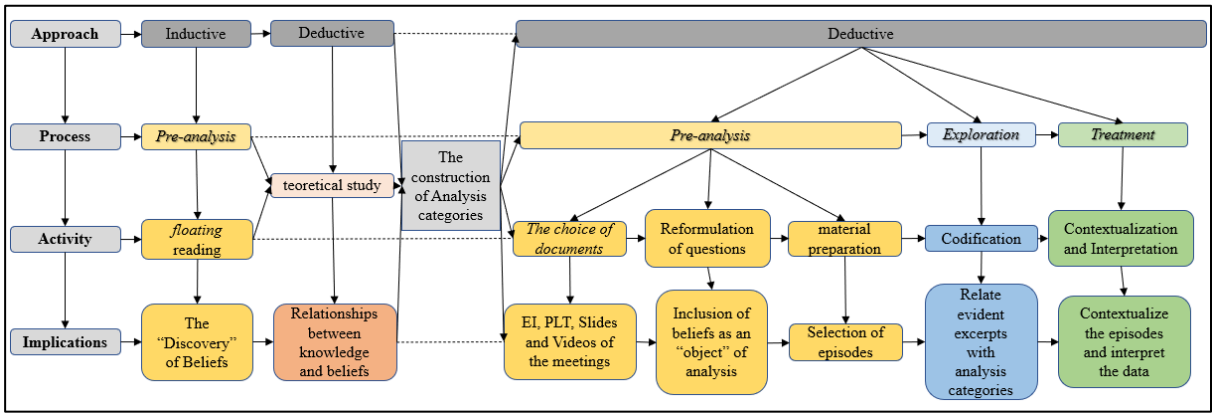

Figure 6.

Construction scheme of categories and data analysis

Results

Episode 1: "life was like that [with] numbers I count, [with] letters I write"

With the aim of getting to know Violeta's academic/professional trajectory, during the initial interview, after 47 minutes of conversation, what was most striking was the way in which she rescued her memories as a basic education student to tell her trajectory. The teacher

[10] Evident excerpts are clippings of transcribed speeches, documents, parts of the PLT that highlight or connect with the research questions. When we refer to "evident excerpts" we are not concerned with the context, we only highlight statements that show knowledge and beliefs.

educator's relationship with teaching, mathematics and Algebra led us to dedicate an episode to these memories.

Violeta narrated that she was never good at math, even during the early years she had difficulty solving tasks that involved problem situations. During her schooling, already in her final years, she highlighted that her relationship with mathematics went from a mere difficulty to hatred: "my hatred for mathematics arises from the moment letters begin to be part of this scenario, because until then, I remembered that life was like this [with] numbers I do math, [with] letters I write" (Violeta, InI).

The way in which Violeta related to Mathematics in elementary school reflects a mixture of instrumentalist mathematics (CM-I) and platonist mathematics (CM-P), since she seems to recognize that mathematical knowledge was static and that's it, just waiting to be learned through rules and procedures. When the teacher says that "[with] numbers I do math, [with] letters I write", she reinforces a view that mathematics necessarily involves numbers, and that it is a consequence of formal rules and procedures, unaware of the use of other languages and symbols for the development of mathematical knowledge.

During the initial interview, Violeta narrated several challenges she faced with Mathematics and Algebra while studying Teaching, among them, she highlighted an event that occurred during the resolution of a test:

> (...) he [his professor] gave a test [and asked] to sketch the graph of the function (...) five functions, I spent the fifty minutes of the class trying to find the value of x (...) mine conception was that in mathematics you could not put a random number, you had to have a certain number (...) (Violeta, InI).

The situation narrated by Violeta reinforces her belief in instrumentalist mathematics (CM-I) and instrumentalist mathematical learning (CA-I), which seems to us to be a consequence of the type of teaching carried out by her teacher. Tasks like "sketch the graph of the function" represent the reproduction of a resolution procedure based on assigning values to "x", finding Cartesian coordinates (x,y) and sketching the graph (reproduction).

After completing the Magisterium, accompanied by the demerit in the medical entrance exams, which were Violeta's first option, the teacher educator decides to take a Licentiate Degree in Mathematics to study its contents and, thus, prepare for new entrance exams. In our interpretation, taking a degree in Mathematics was essential for a change in Violeta's relationship with Mathematics and Algebra, because, according to her, "only when I went to college did things start to make sense" (Violeta, InI). Violeta pointed out that all of her (academic) education was based on "Pure and Applied Mathematics" (Violeta, InI), and her

relationship with Mathematics Education began when she decided to pursue postgraduate studies:

> (...) my proposal in 2010 [for the master's] was how to teach Algebra, because I overcame this hatred, this dislike for mathematics I had as a student (...) But Algebra ceased [had ceased] to be a frustration as I learned to be how I teach Algebra (...) (Violeta, InI).

At that moment, we noticed a change in Violeta's speech about her relationship with Mathematics and Algebra. This change seems to have occurred due to the teacher starting to feel more secure with the content, after having taken a course "based on pure and applied Mathematics", as she herself highlighted. However, we are not saying that the change in Violeta's relationship with Mathematics and Algebra implies a change in the way she sees Mathematics, she conceives its teaching and learning (CMEA). In other words, we assume that Violeta has, in fact, learned Mathematics and Algebra and that this has positively interfered in her relationship with them, but we still cannot say that this has transformed her beliefs about Mathematics, its teaching and learning (CMEA).

When the teacher educator says that her frustration became "how am I going to teach Algebra", we can see that she still had strong tendencies to carry with her the beliefs about mathematics, its teaching and learning that she expressed having experienced during Basic Education (CM- I/CM-P/CA-I). To this end, Violeta sought support in Mathematics Education for her anxieties in relation to the teaching of Algebra, therefore, we assume that these beliefs become objects of reflection by the teacher.

Currently, she works in initial teacher education and is responsible for offering support to future teachers so that they can teach Algebra in the early years of elementary school. In this sense, we take a small leap forward in the analyses of the initial interview and direct the conversation to Violeta's practice as a teacher. She seemed very confident when narrating what she does in her Mathematics Teaching PD courses.

The teacher educator used research data to guide her narratives, as can be seen in this excerpt: "if you take any macro evaluation that talks about teaching mathematics, you will see that teaching mathematics in Brazil is inefficient, right?" (Violeta, InI). She went on to say that Brazil produces a lot of research in the area of Mathematics Education, but that this research does not reach Basic Education. Her use of research data to support her claims reflects her knowledge of research in Mathematics Education (KoMER).

Using this information, Violeta said that her current PD students were trained in Basic Education based on the guidelines contained in the National Curriculum Parameters (PCN).

However, even so, they reproduce an idea of "Algebra based on the manipulation of letters" (Violeta, InI). In this sense, she concludes:

> Why do today's Pedagogy students still hold this conception of Algebra? So, this forced me to look at their beliefs, what do they believe mathematics is, what do they believe learning mathematics is, what does mathematics mean in teaching? (Violeta, InI).

Based on this statement, the teacher educator seems to understand that future teachers have a view on instrumentalist mathematics (CM-I), which can be reflected in the obvious passage "Algebra based on the manipulation of letters". In addition, at the same time, she demonstrates concern about the beliefs that future teachers manifest about mathematics and algebra, which seems to indicate that she considers in her teaching practice the role of these beliefs in and for her students' learning. However, Violeta does not seem to believe in the transformation of these beliefs even during initial teacher education:

> (...) You can say in the Degree that he [the student] is impermeable, he will reproduce what he believes. So, you can come here, you can research, you can do whatever you want, but when the teacher goes to the classroom, he will reproduce what he believes in (Violeta, InI).

We assume that the importance that the teacher educator attributes to the beliefs of future teachers remains in the field of ideas, since she herself demonstrates that she does not believe in the transformation of these beliefs when she says that the future teacher "is impermeable" and that he will "reproduce what he believes". The use of the term "reproduce", used by Violeta to refer to the way future teachers teach mathematics, reflects a platonist learning belief (CA-P), in which knowledge is ready and static, waiting only to be taught and/or learned.

In this episode, we seek to highlight Violeta's trajectory and the transformation of her relationship with Mathematics and Algebra. In Basic Education, the teacher educator's relationship with Mathematics and Algebra was conflicting, but, during undergraduate and graduate courses, she apparently strengthened her relationship with Mathematics/Algebra, which may have contributed to her reorganizing their beliefs about mathematics, its teaching and learning.

As a way to complement and further the interpretations put forward here, the next episode shows how Violeta's beliefs are manifested in her teaching practice. Episode 2 was extracted from P1 (first planning moment) and presents the teacher educator experiencing a PLT and thinking about issues related to the development of the PLT, which would take place during her PD teaching.

Episode 2: "They will hardly notice this issue of equivalence."

In episode 2, Violeta was invited to solve a PLT that aimed to connect mathematical and didactical knowledge related to Generalized Arithmetic. The PLT that Violeta solved contained the mathematical task (MT) entitled "Carolina and João's allowance" (Figure 7).

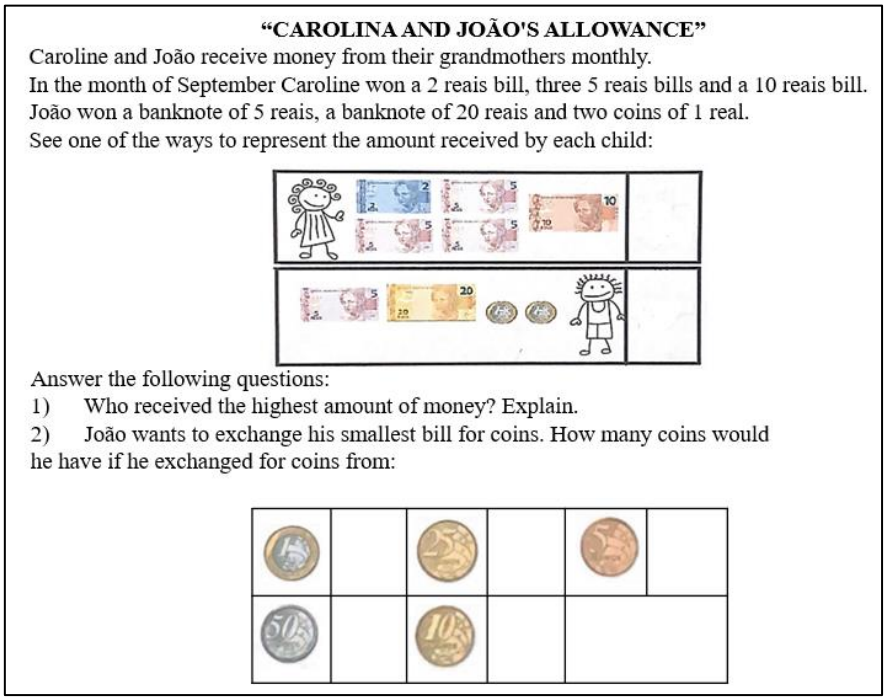

Figure 7.

MT - "Carolina and João's allowance", inspired by Barboza (2019)

By proposing that the teacher educator solve the MT, our intention was to mobilize mathematical knowledge (MK-MTE) in order to lead her to reflect on issues related to the teaching of mathematics in basic education and teacher education (PCK-MTE). For this, after Violeta had solved the MT, Eduardo asked her some questions to direct her gaze to the challenges of developing this MT in a PD class:

> Eduardo: So, I'm going to pose you an "invitation". Let's think of you as a teacher. If you use this task, what difficulty do you think the future teacher might have?
> Violeta: Look, I believe that there are future teachers who will make the same mistakes as students [in basic education] (...) I think that future educators, from what I'm working with them, have this difficulty [referring to the concept of proportion], even more so in making this relationship: fifty cents is half a Brazilian real. So, if I have five one-real coins, I will have ten fifty-cent coins. Reason and proportion. (...) No one will notice that I'm touching on a matter of ratio and proportion (VP1).

The teacher educator seems to believe that, even after completing basic education, future teachers may face the same difficulties that basic education students would. In this sense, Violeta believes that future teachers can perceive the existing relationship between the

currencies, but they would not realize that this relationship is a proportionality. The teacher educator attributes the perception of the relationship with money to something they deal with on a daily basis. Relating future teachers' learning to the perception of proportionality seems to us to refer to an instrumentalist learning belief (CA-I), based on a passive posture of application of rules without reflection on processes. That is, valuing knowledge of the context for solving the task was apparently not considered by the teacher educator when raising the challenges of future teachers, because she was restricted to her belief in instrumentalist mathematics learning (CA-I).

Then, records of practice (RP) of basic education students who solved the same MT were presented, accompanied by questions directed to mathematical and didactical knowledge regarding the MT and the RP, so that Violeta could think about it. The RP with the questions made up the second part of the PLT, which contained two types of RP, a written one, solved by the students (Figure 8), and a video episode (from a class in which this MT was developed).

a) Based on the solution presented below, describe how you interpret the reasoning used by the students to discover the total number of 10 and 5 cent coins that were needed to compose 5 reais.

2) John wants to exchange his smallest bill for coins. How many coins would he have if he exchanged for coins from:

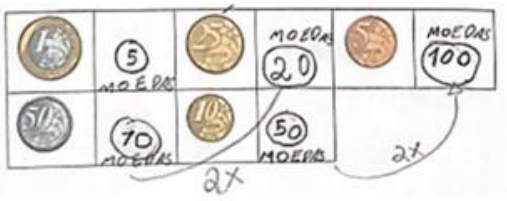

Figure 8.

Excerpt from PLT that includes the written PR accompanied by a question, inspired by Barboza (2019)

The purpose of question (a) constant in the PLT was to mobilize the teacher educator's pedagogical content knowledge (PCK-MTE), through the interpretation of the reasoning used by the basic education student to solve the MT (KFLAG-S). In addition, after solving question (a), Eduardo presented questions to the teacher educator with the intention that she: (i) anticipate possible challenges future teachers would face when analyzing the RP (KFLAG-T), (ii) think about strategies she would use to develop this PLT in teacher education (KAGT-T), (iii) analyze how it would address strategies on teaching mathematics in basic education, using this PLT in teacher education (KAGT-S).

When reflecting on these issues, Violeta reinforces again that she believes that future teachers would not notice the idea of equivalence that would be present in the reasoning contained in the PLT's RP: "(...) with the mathematical knowledge of the pedagogy students, I have evidence of gaps there, they will hardly notice this issue of equivalence, or even talk about proportionality, I don't see that they will notice it" (Violeta, VP1). At that moment, it is noted that Violeta and Eduardo also focus their attention on the equivalence relation of equality. This is because there was a negotiation of the content explored by the PLT and both came to the conclusion that there is exploitation of equivalence in equality (or the meaning of equivalence of the equal sign), when money is exchanged (a five reais bill is equal to five coins of one real, or symbolically, $5 = 1 + 1 + 1 + 1 + 1$), and there is the exploration of proportionality, when relating the coins (the number of coins of twenty-five cents corresponds to twice the number of fifty cent coins) (Figure 8).

Violeta's statement "it is difficult for them to perceive this issue of equivalence" reinforces her belief, already mentioned above, about the learning of future teachers (CA-I), since the perception of equivalence is already conditioned to the use of money in everyday life (e.g. when we exchange a five-real note for five one-real coins). What should be explored is that it was an equivalence relation based on the concept of equality. In view of this, Eduardo takes the opportunity to question her about her role as a teacher educator, and the need to articulate her class with overcoming what she believes to be a challenge for future teachers:

> Eduardo: Let's think as a teacher educator, taking into account these difficulties that you raised, do you think that, even working in a group, [the future teachers] would not even be able to describe the reasoning used [Figure 8] with a mother tongue, less mathematics?
> Violeta: If I took this issue of describing thought with a simpler and less mathematical question, they might even (...) he [future teacher] will have to have an interpretation (...) (VP1)

Eduardo sought to draw Violeta's attention to group work as a teaching strategy to address Generalized Arithmetic in PL (KAGT-T). In addition, he tried to promote a break with Violeta's belief about future teachers' mathematical learning (CA-I). However, Violeta's statements that the future teacher "will have to have an interpretation" and will describe thinking in a "simpler and less mathematical" way, seems to continue to reflect a view that mathematics should be rigorous, based on rules and procedures (CM-I). If we go back to the excerpt from the initial interview, in which the teacher educator indicates "[with] numbers I do math, [with] letters I write", this only reinforces the role of mathematical language and a low appreciation of mother tongue that are still present in the speech from the teacher educator.

Therefore, if we relate what was discussed in episode 2 to what was presented in the previous episode, it seems to us that the teacher educator still carries with her some beliefs resulting from her memories about mathematics experienced in Basic Education (CM-I). It is assumed that, even after going through graduation and post-graduation, in which other views of mathematics were apparently presented to her (CM-P; CM-RP), some of her memories, as a basic education student, were as marks that remain reverberating in her teaching practice.

Until now, we have seen remnants of Violeta's beliefs that are based on an instrumentalist view of mathematics (CM-I), which leads to a similar learning belief (CA-I), as shown in the model shown in Figure 3. In fact, these beliefs were present during the planning of the first PDR cycle (especially in P1, as we saw in this episode) and, directly, we conjecture that such beliefs interfered in the mobilization of the teacher educator's pedagogical content knowledge (PCK-MTE). We assume that such interference is due to her instrumentalist view of mathematics and her consequent view of how future teachers' learning takes place (CA-I).

Thus, the teacher educator stopped reflecting on other strategies she could use with future teachers to teach Generalized Arithmetic in basic education (KAGT-S). In addition, it missed the opportunity to anticipate strategies and difficulties of basic education students (KFLAG-S), which could be discussed later with future teachers, and contributed to the articulation of strategies to teach in teacher education (KAGT-T) and in overcoming the challenges that future teachers will face in teaching Generalized Arithmetic (KFLAG-T). We identify here the relationships between knowledge and beliefs that are pointed out in the model shown in Figure 2.

Episode 3: "Finally, equivalence!"

Episode 3 begins with an excerpt from the reflection meeting of class 2 (R2), in which Violeta was invited to bring two excerpts from the class (D2) to add to the excerpts that Eduardo had selected, in order to compose the reflection about the second class. To subsidize the reflection process, we elaborated some questions that guided the work of/with the teacher educator. The first questions were aimed at the excerpts brought by her, and asked her to justify her choices, and the others focused on the excerpts that Eduardo had selected.

The clipping chosen to compose this episode portrays an excerpt selected by Eduardo and emphasized how the mathematical content (and the referred class) was planned, and how such content was explored by the teacher educator during the class. To that end, we showed Violeta a clipping of the planning video (P2), in which we discussed the proposal to explore the concept of equivalence. In addition, Violeta also watched a video clip of the class moment (D2)

in which she, theoretically, explored the mathematical contents contained in the PLT. The following dialogue occurred after Violeta watched the videos:

> Eduardo: Do you consider that you explored all the planned mathematical contents in order to reach the objective [of the class]?
> Violeta: (...) We [she and the her future teachers] even extrapolated a little.
> Eduardo: I was selecting the contents that you approached, the issue of units and tens, the issue of associativity, the issue of equations and functions (...) and the icing on the cake (...) our objective [of the class] which was the equivalence? (...) (VR2).

The highlighted passage shows Eduardo's concern for the fact that the equality equivalence (the focus of the class) seemed not to have been addressed with the emphasis that was planned. However, according to the teacher educator's statement, "we even extrapolated a little." She portrays her understanding that perhaps there was a lack of focus on the planned content. Then, the discussion was directed towards trying to understand why equality equivalence was not addressed in the PD class as had been planned:

> Eduardo: Why are they "stuck" still discussing addition, units and tens, issues that would be simpler and can't get to Algebra?
> Violeta: (...) They realize that 5=5 and that 2+3 equals 5. This they can already perceive (...) they have this [algebraic] thought, it's in the unconscious, and I need them to become aware that what they do is Algebra.
> Eduardo: And what can we do to make them see Algebra differently or at least see Algebra? (VR2)

At that moment, Eduardo puts under "assessment" the fact that she was not able to take the discussion on equality equivalence to the future teachers. When answering "they have this thought, it's in the unconscious", Violeta states that they recognize equivalence in equality, unlike what was expressed by the teacher educator in the previous episode. Let's remember: there, at that moment, Violeta believed that future teachers would not see equivalence and proportionality in the proposed MT: "they will hardly notice this question of equivalence, or even talk about proportionality, I don't see that they will notice that" (Violeta, VP1). By stating now, in episode 3, that future teachers perceive equality equivalence, albeit in an "unconventional" way, and that they just need to become aware that what they do is Algebra, Violeta seems to be transforming her belief on mathematical learning from instrumentalist (CA-I) to platonist (CA-P), in which he believes in the ability of future teachers to become aware of an "already existing" mathematical knowledge that they "already do", Algebra.

To make sure if this reorganization of Violeta's beliefs is really taking place, as well as to try to better understand how this interferes with her teaching practice and the development of her own professional knowledge, we bring a clipping of the planning of the third class (P3).

It is worth mentioning that at that moment, Violeta already had the autonomy to plan the entire PLT and, even during the discussions with Eduardo throughout the planning, she retrieved excerpts from the past reflection (R2) to support the justifications for her choices.

The planning of class 3 (P3) started with the teacher educator presenting her choices regarding the thematic unit, the object of knowledge and the skills she selected to be addressed in the PLT. Then, Eduardo takes the moment to ask her about how she intended to approach what was being planned during the class:

> Eduardo: How to support the discussion with future teachers for the thematic unit, object of knowledge and selected skills?
> Violeta: I have to lead questions, I have to let them freely, but I have to ask good questions (...) think of a strategy in which students [future teachers] are led to look and not give the answer (...) (VP3)

The excerpt shows Violeta thinking about her practice of teaching equality equivalence, a topic selected by her. By exposing how she would conduct her class, emphasizing "leave it freely", "ask good questions" and "they are led to look and not give the answer", Violeta reorganizes her speech if we compare it to those presented in the previous meeting (R2), in which she seemed to conceive mathematical knowledge as something ready, finished. Thus, it seems to us that Violeta starts to manifest a teaching belief based on problem solving (CE-RP), which leads to the construction of knowledge through "discovery".

It is possible to see that, based on this teaching belief (CE-RP), Violeta made anticipations about challenges that future teachers could face when solving the MT in two different ways, as well as anticipated strategies and resources that could help them to overcome these challenges. Violeta suggests that future teachers use cardboard and brushes to expose the different resolutions for the MT on the blackboard, in order to make the different strategies more visible to the whole class.

In our understanding, the anticipation of challenges and possible difficulties related to the learning of Generalized Arithmetic (KFLAG-T), as well as the planning of strategies on the subject (KAGT-T), are knowledge mobilized by Violeta and that seem to have occurred in as a result of possible reorganizations of their beliefs about teaching mathematics (from CE-I to CE-RP), as well as beliefs about learning (CA-I to CA-RP).

During the reflection of class 3 (R3), Violeta was invited to present, now autonomously, four excerpts from class 3 (D3) that she would like to reflect on, based on a set of questions that we prepared to guide this reflection (Figure 9).

"THE TEACHER EDUCATOR REFLECTION"

Answer the following questions by thinking about the episodes separately:

1. Choose a name for the chosen episode.

2. Identify and narrate the episode from your class that you would like to reflect on.

3. Why did you choose this episode for your reflection?

4. What questions would you ask yourself to carry out a self-reflection about this episode?

5. What would you do differently in your practice in the classroom, in order to overcome/solve the situation narrated above?

Figure 9.

Questions for reflection by the teacher (SP3)

Among the excerpts selected by Violeta, one of them caught our attention. The teacher herself titled this excerpt as: "Finally, equivalence!". When justifying the given name, Violeta says: "after two PLTs explaining the fundamental ideas of mathematics, when I asked what mathematical idea was present there, the whole room, like a choir, told me: equivalence. Hallelujah, equivalence showed up!" (Violeta, VR3).

At that moment, the teacher educator emphasizes the work developed during the three PLTs, praising the fact that future teachers have perceived the equivalence relationship. However, as the three PLTs addressed the equality equivalence, we believed that the future teachers pointed out the content automatically and we had doubts if they really understood and advanced in relation to what Violeta had pointed out in the reflection meeting of class 2 (R2): "they have this thought, it's in the unconscious" (Violeta, VR2).

To this end, Violeta highlights the understanding of the content saying that, in the first class (D1), the future teachers involved several fundamental ideas in the discussion, while in the development of the third class (D3), they showed the equivalence, allowing her to even do other relations, in addition to equality, such as, for example, the equivalence of fractions. According to the teacher educator, she was surprised by the way equivalence was mentioned by future teachers, who justified and managed to show that they understood the existence, in that PLT, of equivalence in equality.

The acknowledgment by Violeta herself seems to signal another important step towards the reorganization of her beliefs about the learning among future teachers, because, if during episode 2 she believed that future teachers would not be able to perceive equality equivalence, in episode 3, we have as evidence the teacher verbalizing that future teachers, through discovery, perceived equivalence (CA-RP).

It should be noted that the manifestation of the teaching belief based on problem solving (CE-RP) during lesson planning, precedes Violeta's recognition of future teachers' learning about equality equivalence (CA-RP). The planning meeting (P3) was held on the same day as the reflection meeting (R2), in which we discussed why we were unable to teach the meaning of equivalence of the equal sign in the PL. In this case, it seems important to us to reinforce the role that reflection played in the teacher educator's teaching practice, as it was from that moment of reflection that Violeta started to look retrospectively at her classes and, therefore, plan the next one.

During the reflection meetings, we could see that Violeta's beliefs limited the stages of her teaching practice, as well as, it seemed to us, that they also affected the mobilization (or development) of her professional knowledge. As evidenced in episode 2, by manifesting beliefs of instrumentalist learning on the part of future teachers (CA-I), Violeta failed to mobilize/develop important knowledge for her class, becoming hostage to the events of the moment, as when she says that the future teachers would face the same difficulties as basic education students. In this sense, showing her excerpts about how she planned and how the class actually took place, seems to have contributed not only to her thinking about her next class, but also to her reorganizing her beliefs about the learning of future teachers, and direct special attention to anticipating the challenges of teaching Generalized Arithmetic in teacher education (KFLAG-T) and planning strategies to teach Generalized Arithmetic in this environment (KAGT-T).

Discussion of results

It is possible to perceive that Violeta's beliefs were constituted from her experience with mathematics in basic education, permeated all the teacher educator's schooling and, finally, interfered in her choice for the Teaching education course. Beswick (2012) states that beliefs are built over time and some of them remain so rooted that they are manifested in teaching action. In this sense, we could see that Violeta brought with her beliefs established in her academic/professional trajectory.

But, after all, how did these beliefs interfere in the teacher educator's teaching practice and in the development/mobilization/expansion of her professional knowledge? Taking what Carrillo et al. (2019) and Ferreti et al. (2021) point out, our results strengthen the relationship between beliefs and knowledge, these two matters of an epistemological nature, as it was noticed that, by manifesting beliefs on mathematics and instrumentalist learning, Violeta failed to mobilize important domains of her professional knowledge, such as the PCK-MTE (Carrillo

et al., 2019; Ferreti et al., 2021). For example, when anticipating that future teachers would not be able to identify the meaning of equivalence in equality when working with PLT, Violeta stopped thinking about strategies, resources, tools and approaches to teach future teachers the meaning of equivalence of the equal sign, a topic within Generalized Arithmetic (Chimoni, Pitta-Pantazi, Christou, 2021), as well as not discussing the teaching of this content in basic education and not anticipating the challenges of learning this content in a PD course.

In addition, as Marshman (2021) points out, we identified in our study how the teacher educators' beliefs, in this case Violeta's, directly impact the way future teachers see and conceive mathematics for teaching and, sometimes, teacher educators are responsible for providing positive experiences that contribute to the reorganization of future teachers' beliefs about mathematics, its teaching and learning, as we saw happening with Violeta.

Finally, the reorganization of Violeta's beliefs became more evident during the third PDR cycle (Ribeiro, Aguiar, Trevisan, 2020) and seems to have been possible due to reflections on the classes. As pointed out by Schön (2000), reflection in action (during the meetings with Eduardo) played a realigning role for a new planning based on issues that did not work out and could be noticed and rethought by the teacher (Superfine & Pitvorec, 2021). Reflection on the class (on action) also enabled the teacher educator to reorganize her beliefs about mathematics and its learning, contributing to the direction of teaching based on the view of mathematics as problem solving (Beswick, 2012).

Still as a result of what we noticed in episode 3, the teacher educator starts to recognize the possibility of working with equivalence in teacher education and, from there, considers the strategies, resources and approaches to teach Generalized Arithmetic to teachers (Barboza, Ribeiro & Pazuch, 2020), so that it anticipates the challenges for learning this content and establishes relationships with basic education. This, in our understanding, leads Violeta to mobilize/expand/develop her PCK-MTE domains (Carrillo et al., 2019; Ferreti et al., 2021).

Conclusions

In terms of conclusions, we resumed the questions proposed in our study to indicate that, before the beginning of the collaborative work, the teacher educator manifested instrumentalist beliefs of Mathematics and, from the involvement with the collaborative work in her own teaching practice, the teacher educator reorganized her beliefs about Mathematics, its teaching and learning for problem solving. This reorganization of beliefs allowed the teacher educator to mobilize/expand/develop aspects of her PCK-MTE aimed at planning her classes to teach Algebra in a PD course.

In this sense, future teachers were given opportunities, through the teacher educator's practice, to engage with Algebra, which is pointed out as a gap, for example, by Castro and Fiorentini (2021) and by Jungbluth et al. (2022). In addition, PD students were given opportunities to break with a technicist view of Mathematics/Algebra indicated in the BNCC (Bortolete et al., 2022). Finally, the results highlight the possibilities of involving teacher educators in collaborative work (Doná & Ribeiro, 2022), enabling the improvement of their education and, consequently, the education of future teachers (Goodwin & Kosnik, 2013).

In addition to all this, we understand that the study points to results that contribute to minimize the lack of research on teacher educators who teach Mathematics in Brazil (Nacarato et al., 2016; Coura & Passos, 2017; Gatti et al., 2019) and, furthermore, we highlight the important role that beliefs play in expanding/mobilizing/developing the professional knowledge of teacher educators, contributing to the characterization of this knowledge (Coura & Passos, 2021; Almeida & Ribeiro, 2020) and to the deepening the studies by Carrillo et al. (2019) and Ferreti et al. (2021).

As limitations of our study, we indicate the fact that it was carried out with only one teacher educator and in only one branch of Algebraic Thinking (Generalized Arithmetic). However, we believe that formative processes with the structure that we developed with Violeta can be reproduced with a larger number of teacher educator, even enabling processes of reflection among peers in a more in-depth way and based on different experiences. In this way, we invite other researchers to develop new studies that explore each of the stages of the teacher educator's teaching practice in a collaborative way, involving other Mathematics contents and other aspects of Algebraic Thinking, as well as studies that explore the relationships between the beliefs and mathematical knowledge of the teacher educators (MK-MTE) and/or the beliefs and their relationship with each of the subdomains of the PCK-MTE.

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