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Elements of the knowledge for teaching arithmetic in the handbook A Aritmética na "Escola Nova", by Everardo Backheuser[1]

Elementos de conocimiento para enseñar aritmética en el manual A Aritmética na "Escola Nova", de Everardo Backheuser

Éléments de connaissance pour enseigner l'arithmétique dans le manuel A Aritmética na "Escola Nova", par Everardo Backheuser

Elementos dos saberes para ensinar aritmética no manual A Aritmética na "Escola Nova", de Everardo Backheuser

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Abstract

In the history of school subjects, pedagogical manuals are considered a relevant source for understanding the knowledge that permeated teacher education in a given historical period. Recognized as a cultural object, disseminators of a theoretical-methodological basis, indispensable for the education of primary school teachers, by providing theoretical foundations and indicating new paths for achieving the educational goals of their time, they became messengers of modern pedagogical ideals. With the aim of understanding the knowledge for teaching arithmetic in primary school, proposed by the Brazilian educator Everardo Adolfo Backheuser, this article analyzes aspects of his handbook, A aritmética na "Escola Nova" [Arithmetic in the "New School"], which, in the 1930s, based on contributions from the education sciences, especially the performance of experimental psychology, disseminated new knowledge for teaching arithmetic in primary school, breaking with the teaching of arithmetic

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that was centered on memorization, excessive repetition of exercises, mechanical practice of arithmetic calculations, and problems unrelated to children's daily lives. The results of this analysis express that the manual analyzed shows how committed the author was to the advances in education sciences and to the updating of professional knowledge provided in normal schools to prospective primary school teachers during the pedagogical wave known as the New School.

Keywords: Teacher education, Primary school, Arithmetic for teaching, Pedagogical handbook, Everardo Backheuser.

Resumen

En la historia de las materias escolares, los manuales pedagógicos son considerados una fuente relevante para comprender los saberes que permearon la formación docente en un determinado período histórico. Reconocidos como un objeto cultural, difusores de una base teórico-metodológica, indispensables para la formación del profesorado de educación primaria, al brindar fundamentos teóricos e indicar nuevos caminos para alcanzar los propósitos educativos de su tiempo, se constituyeron en mensajeros de las ideas pedagógicas modernas. Con el objetivo de comprender los saberes para enseñar aritmética en la escuela primaria, propuestos por el educador brasileño Everardo Adolfo Backheuser, este artículo analiza aspectos del manual, escrito por él, A aritmética na "Nova Escola" [Aritmética en la "Escuela Nueva"] que, en la década de 1930, a partir de los aportes de las ciencias de la educación, especialmente en el desempeño de la psicología experimental, difundieron nuevos conocimientos para la enseñanza de la aritmética en la escuela primaria, rompiendo con la enseñanza de la aritmética, centrada en la decoración, en la repetición excesiva de ejercicios, en la práctica mecánica de los cálculos aritméticos, con problemas no relacionados con la vida diaria de los niños. Los resultados de este análisis expresan que el manual analizado muestra cuánto se comprometió el autor con los avances de las ciencias de la educación y con la actualización de los conocimientos profesionales dispensados en las escuelas normales a los futuros maestros de primaria durante el movimiento pedagógico denominado Escuela Nueva.

Palabras clave: Formación del profesorado, Educación primaria, Aritmética para enseñar, Manual pedagógico, Everardo Backheuser.

Résumé

Dans l'histoire des matières scolaires, les manuels pédagogiques sont considérés comme une source pertinente pour comprendre les connaissances qui ont imprégné la formation des

enseignants à une période historique donnée. Reconnus comme un objet culturel, diffuseurs d'une base théorique et méthodologique, indispensables à la formation des enseignants du primaire, en fournissant des bases théoriques et en indiquant de nouvelles façons d'atteindre les objectifs éducatifs de leur temps, ils se sont constitués en messagers des idées pédagogiques modernes. Dans le but de comprendre les connaissances pour enseigner l'arithmétique à l'école primaire, proposé par l'éducateur brésilien Everardo Adolfo Backheuser, cet article analyse les aspects du manuel, écrit par lui, Arithmétique dans la « Nouvelle École » qui, dans les années 1930, basé sur apports des sciences de l'éducation, notamment dans l'exercice de la Psychologie Expérimentale, ont diffusé de nouveaux savoirs pour enseigner l'arithmétique à l'école primaire, en rupture avec l'enseignement de l'arithmétique, centré sur la décoration, sur la répétition excessive d'exercices, sur la pratique mécanique des calculs arithmétiques, avec problèmes sans rapport avec la vie quotidienne des enfants. Les résultats obtenus à partir de cette analyse expriment que le manuel analysé montre à quel point l'auteur s'est attaché aux avancées des sciences de l'éducation et à l'actualisation des connaissances professionnelles dispensées, dans les écoles normales, aux futurs enseignants du primaire, lors de la vacance pédagogique dite Nouvelle école.

Mots-clés : Formation des enseignants, Enseignement primaire, Arithmétique pour enseigner, Manuel pédagogique, Everardo Backheuser.

Resumo

Na história das disciplinas escolares, os manuais pedagógicos são considerados uma fonte relevante para compreender os saberes que permearam a formação dos professores em determinado período histórico. Reconhecidos como objeto cultural, difusores de uma base teórico-metodológica, indispensáveis à formação do professor do ensino primário, ao disponibilizarem fundamentos teóricos e indicarem novos caminhos para o alcance das finalidades educativas de seu tempo, constituíram-se como mensageiros de modernos ideários pedagógicos. Com o objetivo de compreender os saberes para ensinar aritmética na escola primária, propostos pelo educador brasileiro Everardo Adolfo Backheuser, este artigo analisa aspectos do manual, escrito por ele, A aritmética na “Escola Nova” que, na década de 1930, partindo de contribuições das ciências da educação, especialmente no desempenho da psicologia experimental, divulgou novos saberes para ensinar aritmética na escola primária, rompendo com o ensino da aritmética, centrado na decoração, na excessiva repetição de exercícios, na prática mecânica de cálculos aritméticos, com problemas desvinculados do

cotidiano infantil. Os resultados desta análise expressam que o manual analisado evidencia o quanto o autor estava comprometido com os avanços das ciências da educação e com a atualização dos saberes profissionais dispensados nas escolas normais aos futuros professores do ensino primário durante a vaga pedagógica denominada Escola Nova.

Palavras-chave: Formação do Professor, Ensino Primário, Aritmética para ensinar, Manual Pedagógico, Everardo Backheuser.

Elements of the knowledge for teaching arithmetic in the handbook A Aritmética na "Escola Nova", by Everardo Backheuser

Research on teaching knowledge has been a constant debate in the teacher education field in Brazil since the last decades of the 20th century. However, historical studies on *knowledge for teaching arithmetic* in the early school years are recent. This theme has been increasing with the growing interest of mathematics educators in knowing the history of their profession and the subject they teach. This is mainly due to the significant circulation of texts of reference by Chervel (1990), Julia (2001), and Hofstetter and Valente (2017), and the highlighted historical constitution of a school subject within the school and its relationship with the school culture, a space that produces knowledge that marks teaching, in this case arithmetic to teach in primary school.

This type of study implies approaching history with a new look and a new saying that contribute to renewing historiographic practice, emphasizing that the historian's gesture links his/her ideas to the place from which he/she speaks (Certeau, 2007). The constitution of historical aspects starts from a verification of reality and is supported by socioeconomic, political, and cultural production, and the connection between history and place is the premise of a study of society. Certeau (2007) states that "taking your place seriously is not yet explaining history. But it is the condition for something to be said without being either legendary (or 'edifying') or a-topic (without pertinence)" (p. 77).

According to that author, the historian produces their work from the present, from the concerns of their reality, so that they create a specific discourse that has an issuer, the historian, and a recipient, whoever they may be: the academy, society in general, or a specific group. This discussion led to an observation for Certeau (2007): "one cannot speak of a truth, but of truths (in the plural)" (p. 67).

A historiographical research with textbooks should intend to explore them intensively, for example, analyze the comments made by those invited to present the work and check traces of its use. Once the textbook is conceptualized as a pedagogical tool, it is listed within a long history. Thus, structuring this resource is inseparable from the teaching conditions of its time (Bittencourt, 1993). This makes us think that the textbook may present a residue of the pedagogical line prioritized by the author, such as exercises, questionnaires, suggestions for work, in short, activities that students must develop to understand or, most of the time, to retain the contents.

In this article, we do not analyze textbooks but a pedagogical handbook. Knowing that the pedagogical handbooks were intended both for pre-service students (normal courses) and for teachers who, in the demarcated period, already taught mathematics for the first school grades, we posed the following problem to guide our study: Which aspects of arithmetic for teaching are present in the pedagogical handbook *A aritmética na "Escola Nova",* by Everardo Backheuser?

Our research started from the premise that the pedagogical handbooks are so called because they were written to develop the themes foreseen for the teaching of professionalizing subjects of the curricula related to teacher education, in this case, those directly associated with educational issues, namely, didactic pedagogy, methodology and teaching practice.

Knowledge for teaching arithmetic

Concerning teaching knowledge, we rely on the theoretical contributions of Hofstetter and Schneuwly (2017), who conceived two types of knowledge: the *knowledge to teach*, i.e., the knowledge that is the object of teaching work; and the *knowledge for teaching*, i.e., the knowledge that involves the tools used in the mobilization of their work object. The *knowledge to teach* refers to knowledge produced historically by scholars in a particular area of knowledge, such as mathematics, for example, and different scientific fields essential for teacher education. In turn, the *knowledge for teaching* is that of a professional nature, based on educational sciences. In isolation, *the knowledge for teaching* is the work tools, that is, "it is affiliated with pedagogical education subjects that stem from the educational sciences, such as pedagogy and its ramifications" (Pinto & Novaes, 2018, p.140).

When these two types of knowledge are articulated, going through processes of systematization and objectification, as Valente (2019) clarifies, there is new knowledge, recognized and institutionalized knowledge, legitimized as professional knowledge. Hofstetter and Schneuwly (2017) initially used the expression teaching knowledge, considering them differently from those treated in studies that address the point of view of practice, the *action knowledge*, having as a source of research the teachers' experiences.

In the production of professional knowledge (systematized and objectified knowledge), educational sciences offer important contributions, more general pedagogical guidelines that, unfolding in specific didactics, help future teachers to appropriate theoretical-methodological knowledge, knowledge centered on a given subject. Hence, we deal here with the *knowledge for teaching* arithmetic, i.e., professional knowledge that, once objectified, is formalized in

education courses, somehow materialized in normative documents, teaching programs, and pedagogical handbooks.

Based on these contributions, we observed a difference between the terms knowledge *to* teach and knowledge *for* teaching a specific subject, in our case, arithmetic *for* teaching. This is not a wordplay, but an unfolding of fundamental meaning in the historiographical study.

> With that, a new field of investigations refering to the study from a historical perspective of the processes of production of each of these mathematics is installed, as well as the investigation of the dynamics of articulation between mathematics to teach and mathematics for teaching. (Valente, 2019, p. 19)

Thus, we understand the *arithmetic for teaching*, characterized by knowledge arising from disciplines affiliated with the educational sciences which, amalgamated with arithmetic *to* teach, to the knowledge that will support the knowledge to be taught, will allow giving more meaning to teaching objects, linking them to the purposes of education of their time. Finally, the arithmetic *for* teaching is configured as a framework of knowledge, elaborated in the professional scope of teaching, and mobilized based on a theoretical structure, representing, therefore, an *expertise for* teaching arithmetic in a given historical period.

Julia [4] (2001) indicates that "[...] school culture cannot be studied without analyzing the conflicting or peaceful relationships it maintains, at each period of its history, with the set of cultures that are their contemporary" (p. 10). This means that school practices are innovated according to changes in the target audience and socio-cultural needs that require changes in the content to be taught. Therefore, each new audience, coming from different cultures, influences school contexts and, therefore, is impacted by them.

Choppin (2004) explains that textbooks are the main historical sources constituted by a historian of mathematics education. And he adds:

> The design of a textbook is part of a specific pedagogical environment and a regulatory context that, together with the development of national or regional systems, is, in most cases, characteristic of school productions (state editions, prior approval procedures, freedom of production, etc.). Its production (documentation, writing, pagination, etc.), material realization (typesetting, printing, binding, etc.), commercialization and distribution presuppose substantial forms of public or private financing and the use of techniques and work teams increasingly specialized, therefore, increasingly numerous. (p. 554)

[4]Julia (2001) conceives school culture "as a set of norms that define knowledge to teach and behaviors to inculcate, and a set of practices that allow the transmission of this knowledge and the incorporation of these behaviors; norms and practices coordinated with purposes that may vary from time to time (religious, socio-political, or simply socializing purposes)" (p. 10).

However, being part of the school culture, the textbook is structured, conveyed and used with some intentionality, since it is part of a broader social culture.

This research, which intends to analyze the *arithmetic for teaching*, does not turn to textbooks themselves, but to the pedagogical handbook, written by educator Everardo Backheuser – *A aritmética na "Escola Nova"* – which had a significant circulation in Brazil, mainly in primary teacher education courses

A aritmética na "Escola Nova", by Everardo Backheuser

Everardo Adolfo Backheuser (Figure 1) was born on May 23, 1879, in the city of Niterói, Rio de Janeiro, to João Carlos Backheuser and Joaquina Eugênia de Gouveia Backheuser. He began his primary studies at the age of 7, at the Externato Particular, managed by his sister Evelina. According to Rosa (2017), after three years, he joined Ginásio Nacional, currently Colégio Pedro II. In 1896, he graduated in Letters. Still in 1896, Everardo joined the Niterói literary group called "A Matilha", which became a preparatory course, an opportunity in which he had his first experience in teaching.

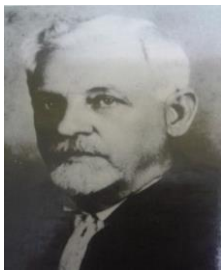


Figure 1.

Everardo Backheuser' photograph (Rosa, 2017, p. 62)

In 1897, he enrolled at the Polytechnic School, obtaining the title of Geographer Engineer, in 1899; Civil Engineer and Bachelor of Physical and Mathematical Sciences, in 1901; and in 1913, the title of PhD in Physical and Natural Sciences. In 1916, he founded the Sociedade Brasileira de Ciências [Brazilian Society of Sciences] along with Henrique Morize, Enes de Sousa, Alberto Betim, José Pantoja Leite, and Roquete Pinto, among others.

Everardo devoted himself to teaching, working at the Normal School of Niterói, at Colégio Pedro II, at the Pedagogy course at the Catholic Faculty of Philosophy, at the Institute of Educational Research of the Federal District and at the Superior Course in Geography at the Geography Society of Rio de Janeiro (Rabelo, 2016).

Beyond his contributions to education, he became involved in political-administrative and journalistic activities. Teacher, geographer and engineer Everardo, according to Rabelo (2016), held some positions by political appointments, such as: Chief Engineer of the Federal District city hall, during the year 1909; director of the Central Pedagogical Museum, from 1929 to 1930; director of the Institute of Educational Research in the years 1936 to 1937; and was also a member of the Legislative Assembly of the state of Rio de Janeiro between 1910 and 1915. Due to his connection with politics and for being publicly opposed to Arthur Bernardes in the search for the presidential succession of Epitácio Pessoa, in 1921/1922 he ended up being imprisoned for four months.

On the international scene, Backheuser also stood out. He was a corresponding member of the Berlin Geographical Society; served on the Paris-based Esperanto Linguistic Committee; was an honorary member of the Society of Geography and Statistics of Frankfurt, and an official representative of the Brazilian Association of Education (ABE[5]) at the Jean-Jacques Rousseau Institute in Geneva.

According to Rosa (2017), Backheuser participated in pedagogical campaigns such as: the "Campaign in favor of education" (1924), "Campaign in favor of the New School" (1926) and the "Campaign in favor of religious teaching" (1928). He showed a lot of commitment, contributing enormously to the reform that Fernando de Azevedo initiated in Rio de Janeiro, in 1927.

Even after his retirement, Everardo took charge of five municipal schools in the Federal District to experiment with some methods and analyze the results obtained, since, in Germany, he intensified studies focused on the principles of the New School. As informed in the Diário de Notícias of Rio de Janeiro, Everardo died on October 10, 1951, in the city of Rio de Janeiro (Rio de Janeiro, 1951).

Everardo Backheuser authored several works within the educational field. Among them, the following stand out: *A aritmética na "Escola Nova"* [Arithmetic in the "New School"], 1933 – chosen for this study – (Figure 2); *Técnica da pedagogia moderna* [Technique of modern pedagogy], 1936; *O trabalho nas escolas experimentais do Distrito Federal* [Work in experimental schools in the Federal District], 1937; *Ensaio de biotipologia educacional* [Educational biotypology essay], 1941; and *Como se ensina a aritmética* [How to teach arithmetic], 1946.

[5] According to Rabelo (2016), "Backheuser was a co-founder of ABE, along with Heitor Lyra, in 1924. He was active in the creation of Associations of Catholic Teachers (APCs) nationwide. He was founder and president of the APC of the Federal District (1928) and of the Brazilian Catholic Confederation of Education (1933)" (p. 201).

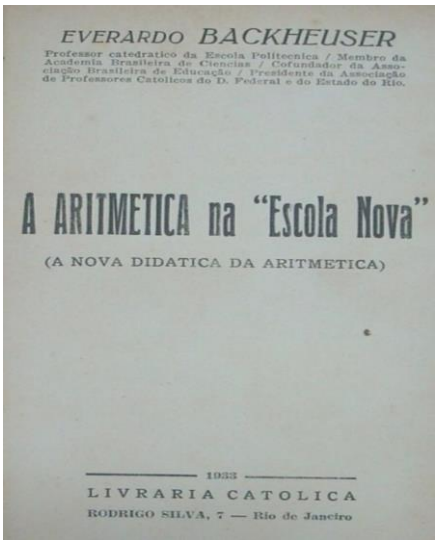


Figure 2.

Handbook: A aritmética na "Escola Nova", by Everardo Backheuser (1933)

A brief reading of the title page immediately reveals Backheuser's notoriety. It contains the following information about the author: full professor at the Polytechnic School; member of the Brazilian Academy of Sciences; co-founder of the Brazilian Association of Education; president of the Association of Catholic Teachers of the Federal District and the state of Rio. (Figure 2).

This pedagogical handbook was implicitly divided into two parts. The first, divided into six chapters, presents a correlation of psychological, philosophical, and historical concepts of teaching and learning arithmetic in Brazil. The second, divided into two chapters, indicates more specifically the guidelines to be followed by would-be primary teachers to teach arithmetic.

Table 1.

Structuring of the book A aritmética na “Escola Nova”

Chapter	Title	Pages
First Part		
-	Preamble	9-12
I	The didactics of arithmetic in the light of psychology	13-18
II	Psychological types	19-44
III	Variation of child psychology with age	45-65
IV	End of primary school term	66-70
V	The teaching of arithmetic in Brazil	71-80
VI	The main factors of teaching arithmetic	81-106
-	Conclusions	107-108
Second Part		
I	Some suggestions for practice	109-138
II	Mental calculation	139-152

The first chapter begins with an introduction to the importance of some psychology, sociology, and philosophy concepts in the teaching and learning of arithmetic in the early years of elementary school. According to Backheuser (1933), these sciences can be considered as girders for the process of understanding the concepts of arithmetic.

With regard to teaching trends in the New School, Backheuser (1933) states that:

> Particularly in the 'new school', as observed by Lorenço Filho (25, p. 14), 'two trends emerged in the education renovation movement: that of social and philosophical criticism, and that of pure psychological criticism; that is, that of criticism of the purposes of *old* education, and the criticism of *means* which, in school technique, can be employed, for readjustment to those purposes'. Thus, it seems impossible to address, nowadays, pedagogical problems without placing them on the powerful girder of psychology. More than any others, those of didactics. Therefore, we will have to rely on it in this essay on didatics of arithmetic [emphasis in the original]. (pp. 13-14)

From a sociological point of view, according to Backheuser (1933, p. 14), “the need for knowledge of arithmetic is obvious”, since everyday life requires mastery of concepts for commercial processes, such as the purchase and sale of products. Psychology does not act only in educational methods, “but together with physiology, the way of knowing the reality of the student” (p. 13). The philosophy, on the other hand, comes from the teacher's need to make the teaching process individual or different,i.e., to characterize, differentiate, and particularize teaching to all students in the classroom.

Thus, these sciences Backheuser considered fundamental to the process of teaching and learning arithmetic, must remain harmonized in the education and performance of teachers who would teach in primary schools.

In the second chapter, the author presents some psychological concepts or types that he deems relevant to the didactics of arithmetic. Backheuser (1933) highlights the types: mnemonic, intelligence, and mathematics, understood as a set of techniques to be used to help the memorization process, associated with the production of supports such as schemes, graphics, images, or phrases related to the content one intends to teach. In the three types, there are subgroups relevant to learning, which are the elements or psychological factors, said to be visual, auditory, and motor.

> The *visual* types apprehend the facts, retaining them with the aid of the sight; the *auditory*, with ear, and the *motor*, with the movements perceived. For example: the visual type remembers the *number* by the digit that they *saw in writing*; the auditory by the name that they *heard*, and the motor, by the *hand movement* used in writing the digit or the *mouth movement* used to pronounce the word indicating that number [emphasis in the original]. (Backheuser, 1933, p. 20)

According to the author, in learning the concepts of arithmetic, the student needs to see some physical elements, such as fruits, toys, and school materials to associate the quantity with the written numbers. But for memorization, there is a need to hear what is being seen, when then a mathematical language will be introduced. And finally, the doing, which can be carried out by writing and developing calculations and/or playing educational games that may or may not have manipulative materials.

To specifically characterize the psychological type, called mathematicians, the author subdivides it into groups, namely: theoretical, composed of those students who like to delve into handbook calculations; mechanized, by those who normally prefer to develop mental calculations, often without worrying about their conceptualization; and the active, those who understand the arithmetic concepts that are associated or contextualized with objects or situations of their daily life, for example in operations carried out in local businesses (Backheuser, 1933).

In the third chapter, the author discusses the process of children's arithmetic learning, supported by some educators, among them the German psychologist Ernst Friedrich Wilhelm Meumann (1862-1915). Backheuser (1933) states that this process should be divided into two phases. For the first, aimed at children up to 9 years old, he indicates four guidelines for teaching arithmetic in primary school, as follows:

> 1) – TEACHING MUST BE DONE TOGETHER, without specialization of resources, that is, it must be done *globally*, in the expression of Ferrière and Decroly, who thus clearly translated into Latin languages the *Gesamtuntericht* of the Germans, so advocated since Berthold Otto.
> 2) - TEACHING MUST BE DONE THROUGH GAMES, especially in kindergarten (and in the 1st primary grade for children who have not attended kindergarten).

> 3) - TEACHING MUST BE GIVEN IN AN INTUITIVE CHARACTER, because in this infant period the child is essentially *objective*.
> 4) EDUCATION MUST BE GIVEN ENTIRELY ON AUTHORITY, that is, the teacher must never enunciate his thought but proclaim it with perfect security, thus avoiding saying anything about which the child may not find confirmation inside or outside of school [emphasis in the original]. (p. 48)

In the first guideline, he indicates that teaching should be done together. The teachers must plan a teaching in which all disciplines are coordinated, which he calls global teaching. In the second guideline, he points out the intermediation of teaching through games and manipulative materials, as these can represent a fantasized synthesis of reality for the child, which is an achievement of the educational sciences. Besides arithmetic games, some objects can be adapted for teaching, such as clocks, dice, dominoes, and playing cards, as they can create "dramatizations of scenes from domestic life, buying and selling objects, and especially the street-fair game, this one of movement and an unusual interest, known to all the teachers who tried it" (p. 57). We can see from these indications that he defended not only the use of games, but also the contextualization of teaching arithmetic, with daily situations experienced by students, exploring materials they knew.

The third guideline points to the use of intuitive teaching[6], advocated by Pestallozzi and Comenius in the 19th century, but not as the only or main way of teaching; rather, as another methodological tool that primary school teachers could use in classes where they were teaching arithmetic concepts. And the last instruction concerns the authority that the teacher should have, not only regarding the students' discipline in the classroom, but also regarding what he is going to teach. That is, the teacher must master what he is going to teach, must obtain full knowledge of the subject taught and contextualized with the students, so as not to run the risk of discovering untruths in the contents worked in the classes. Backheuser (1933, p. 66) called this competence "authority teaching", which would not be in disagreement with the principles of the new school, as it was already advocated by it.

In the next chapters (fourth and fifth), Backheuser briefly discusses how arithmetic was taught in primary schools in Brazil and the trends in arithmetic teaching, which were conceived through reasoning, memorization, and mental calculation, considered a utilitarian aspect of arithmetic.

[6] "The emphasis given by the intuitive teaching method to the empirical one, to direct observation, to seeing, feeling, and touching is, therefore, based on the assumption that knowledge begins with the operation of the senses on the outside world, from which sensations and perceptions and facts and objects would be produced, transformed into raw material for ideas, which, added from imagination and reasoning, would enable the development of the capacity for judgment and discernment" (Carneiro, 2014, p. 31).

After the conclusions, Backheuser (1933) begins the second part of the book, in which he presents some practical suggestions for teaching arithmetic, which were intended for primary teachers in training. These suggestions were divided into topics, as follows: The notion of number; The digits; Connection of arithmetic to other basic subjects; Games; Addition and subtraction; Multiplication; Division and fraction.

Considering that not all children learn the notions of numbers equally, the author advises that diversified methodological tools should be used, so all in the same classroom achieve learning. Among them, intuitive learning is highlighted, through observation and comparison of images and/or objects, as can be seen in the following practical example of the notion of numbers:

> - One of the ways of presenting the notion of number is to consider it as a *collection* of a specific number of objects to which one more is added. *4 oranges* (already known collection) with more *1 orange* form *5 oranges*, quantity corresponding to the number acquired 5. It will always be easy to repeat essays of this type in class, as there is no shortage of *object collections* [emphasis in the original]. (pp. 111-112)

Thus, in addition to the notion of number, the student would be led to understand counting, through the addition of new objects presented to the group. This practice could privilege children who are more visual, those who are able to learn easily by seeing the process; those who are more auditory, who find it easy to learn by associating it with what is being explained by the teacher during classes; and those who are more tactile, who learn better by handling the material.

Backheuser (1933) brings to the fore the need to make clear the difference between numbers and digits. This could be done with examples during teaching practice. Let us see it in the differentiation between numbers and digits:

> - As, for various reasons of practical utility, the notion of 'number' is accompanied by knowledge of the respective 'digit', it happens that many children end up confusing the two notions, and instead of saying 'digit', they say 'number'. It is convenient – as soon as possible – to solidify the 'notion of number' through suitable exercises, showing, for example, how the same number is generated in different ways. 6, for example, equals 5 plus 1, but also 7 minus 1; a 4 plus 2; 8 minus 2; a 2 times 3; etc. [emphasis in the original] (p. 113)

Among other examples presented, the author also emphasizes the importance of writing on the "blackboard", associated with the correct pronunciation of mathematical language, because "in addition to the need, as we have just shown, to distinguish them from *numbers*, the

teacher needs to see how much easier it will be for the student to connect to the *symbol* what the number is, the notion it represents [emphasis in the original]" (p. 113).

The author emphasizes the importance of relating the teaching of arithmetic to other subjects in the curriculum. He stresses the relevance of an interdisciplinary work with other subjects, exemplifying, for example, drawing, music, and physical education, and even with those that do not have explicit common points with arithmetic, but could favor the development of some concepts of arithmetic with the analysis of dates, as is the case with history.

The author suggests the use of manipulatives and games that can be used in teaching arithmetic, from those that were already designed to be applied in the classroom, to those that may be suitable for teaching, as is the case of dominoes.

> - *Domino* provides more relevant services than the dice. It facilitates the learning of the most diverse sum counts. *Domino* it is, therefore, one of the most advisable games. We can modify it seeking an educational advantage, making one of the 'dominoes' pictures have digits instead of dots. In this way, the child, in the course of the game, when putting the stones together, will try to combine 'numbers' with 'dots' and vice versa [emphasis in the original]. (p. 117)

According to the author, games should be used in the development of exercises, rarely to introduce content, as advocated nowadays. Thus, Backheuser (1933) states that the games would aim to train students during the process of fixing the concepts worked on by primary teachers. In some passages, the author emphasizes the importance of adapting the game to the student's level: "the game must be at the student's mental level, maybe very slightly above, but normally a little below" *sic* (p. 119).

The author begins the addition and subtraction section by stating that, "these two operations must be taught so that the child learns, at the *same time*, to 'join units' and in 'decrease units' [emphasis in the original]" (p. 120). For this, he is categorical in saying that the necessary time should be spent teaching this content, as it is fundamental for the learning of others, both in primary and secondary education. The author highlights that, in the teaching of addition,

> the only difficulty in teaching this operation lies in the sum of the digits. When the ten appears, the new case is assimilated to the well-known old case – Adding 27 to 5 is reduced to remembering the sum of 7 to 5 [emphasis in the original]. (p. 121)

He suggests adding numbers in order, units with units, tens with tens. Even with this difficulty in making students see the sums, there is no indication for the development through an "assembled" algorithm, as currently carried out, in which the numbers are written in such a way that the order is superimposed, which may facilitate the students view.

The author points out that contextualized problems should be used, as already defended in previous chapters of his pedagogical handbook. For subtraction, Backheuser (1933) presents two ways of teaching, "[...] removing units from the largest number until obtaining the smallest or, on the contrary, adding units to it until reaching the largest. 8 minus 5 equals 3, or 5 to 8 is 3 short" (p. 121).

However, he states that teachers and would-be teachers should adopt the second method, as it would represent the very definition of "difference", which was usual in commercial practice, in the purchase and sale of goods, in which the seller would need to pass change to the buyer. Besides these arguments, the author points out that it would be better for students if there were a standardization in the way of teaching subtraction because, if they changed schools, it would be possible to continue what they were already learning. To justify his position, he says that in other countries, such as Germany and Austria, it was done that way.

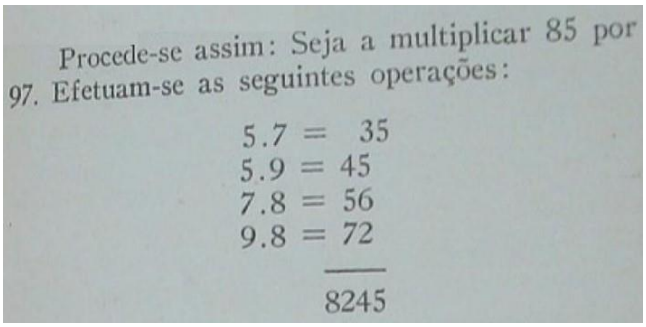
The author begins by stating that the teaching of multiplication should not follow the natural order of numbers, that is, by 2, by 3, by 4 and so on, but by an organization that would give meaning to students, starting with 2, because the students would already have the notion of a pair, which would be easy to contextualize with everyday objects, such as a pair of shoes, two pairs of shoes. Afterwards, the teacher would introduce the multiplication by 4, which would already use the base learned in the multiplication by two and could be associated with parts, or characteristics, of some objects, such as the number of legs of chairs, tables, or the corners of a door. Next, the 10 and 5 would be the focus, as Backheuser (1933) judges them to be the easiest.

Only then would the teacher return to multiplication by 3, which would not be so simple to contextualize, but could be associated with the triangle of pennants, with the persons of the Holy Trinity. It should be noted that, at various times, the author uses examples and biblical contexts, as is the case of the animals in Noah's Ark, which would be in pairs.

Then, multiplication by 6 should be taught, which could be associated with half a dozen, later with 12, because it has already been correlated with a dozen, a language widely used in the marketing of various products at the time.

And finally, multiplication by 7 should be taught, which could be related to the days of the week, followed by 8 and 9, which would be the most difficult to contextualize with everyday practice.

In addition to an indication of order, the author also suggests a method: cross multiplication.



Procede-se assim: Seja a multiplicar 85 por 97. Efetuam-se as seguintes operações:

5.7 = 35
5.9 = 45
7.8 = 56
9.8 = 72

8245

Figure 3

Cross multiplication. (Backheuser, 1933, p. 125)

The cross multiplication method, presented in Figure 4, according to the author, could be advantageous for primary school students who were learning multiplication, since its tuition was simple and visual. In this case, it is recommended to multiply all the digits of one factor with those of the other. In the example illustrated in Figure 4, we have: the unit of the first would be multiplied by the unit and ten of the second, just as the ten of the first would be multiplied by the unit and ten of the second, the result of the product would be the sum of the results obtained in the partial multiplications.

He also points out that, although this method was very effective, it would be very laborious, if used, when the numbers, the factors, had a large number of digits.

As for division and fraction, the author, in just three paragraphs, explains briefly "how" the primary teacher should teach the content. He only indicates the sequence to be adopted, which is similar to what had already been presented in the multiplication, but with much less, or almost no detail.

Backheuser (1933) still presents other topics that would be correlated to the teaching of arithmetic, such as what he called dressing problems.

> For example: (9 + 6 – 3) : 2 would be *dressed* by the students with the following statement: 'Alvaro had 9 pencils; after winning half a dozen, he gave 3 to Alberto; of those remaining to him, he decided to divide equally between himself and his brother. How many pencils did each one have? [emphasis in the original]. (p. 134)

The author reinforces the importance of contextualized problems. Among the justifications presented by him is that of awakening the students' non-mechanical reasoning, as this would make them think about which of the arithmetic operations should be used, how to develop them, and in what order the resolution would take place.

According to the author, these are problems that, apparently, seem to be difficult, but, when well analyzed and the data collected, students would realize that they are easy to solve.

In addition to improving students' reasoning, these problems would pique their interest, making them more motivated.

Another highlight given by Backheuser (1933) in his pedagogical handbook refers to mental calculation. The author says that, until then, it was considered recommendable for primary education, but, from the experience of the New School, it became mandatory practice in arithmetic classes. “Right, as an exercise of intellectual functions, all arithmetic calculations are mental. Commonly, however (and in this sense we take it), only numerical exercises done orally are considered as mental calculations, carried out, as they say, 'off the top of one's head' [emphasis in the original]. (p. 141)

The author's eloquent defense of mental calculations in teaching arithmetic permeates almost all chapters of the analyzed work. Much of the justification for such inclusion is based on its practical use in the daily lives of students and people, not only during the period in which they would be attending primary school, but in all stages of their student, professional, and personal lives.

Another factor for the use of mental calculations in arithmetic classes is the need to use different methodological tools, a fact pointed out by some of the psychological foundations that the author describes in the first chapters of the pedagogical handbook. That is, there must be a balance in didactics for arithmetic teaching to contemplate the different types of learning neglected by students: the visual ones, which would need to see the written counting; the auditory ones, who would need to listen and mentally construct the utterance; and the motor ones, who would usually need to write the problem statements themselves and their resolution and who would do better with handling manipulative materials and/or games aimed at arithmetic.

Backheuser (1933) offers a brief outline of the “how” and “when” primary school teachers should include mental calculations in their arithmetic classes, what he called “precepts in the march of drilling concrete mental exercises”[7]. Let us see the guidelines for mental calculations with concrete exercises:

> 1) Only start the concrete exercises after the class is well exercised in the corresponding abstracts;
> 2) The concrete exercises must be easier than the class's level of knowledge in arithmetic, so that the 'operation itself' does not pose any difficulty. The difficulty will reside solely in the reasoning to be put into play;

[7] According to Backheuser (1933), concrete mental exercises are small arithmetic problems, contextualized by students' everyday situations, which require simple calculations to obtain their solutions.

> 3) They must be formulated on subjects that really interest the class, the practical character that is asked of them must not be understood as useful in the practical life of the adult, but, on the contrary, in the students' lives. They will deal with the games, the purchase of objects used by them, domestic matters, etc.;
> 4) Not presenting the problem and demanding an immediate response; allowing time for the child to understand what is being asked of him/her;
> 5) Encouraging the class to formulate their own problems, in the same paradigm of others already solved, not, however, with the mere substitution of numerical data. (pp. 144-145)

The practice of mental calculations in primary school arithmetic classes should aim at safety and speed in solving activities and problems. The problems could be abstract, which would serve to transpose the concepts studied for the development of mental calculations. They would be more direct exercises or activities without many variables; and the concrete ones would be the more elaborate problems, whose resolutions would require understanding them to carry out the data collection. In short, abstracts would represent a preparatory course for concrete problems.

A systematization of arithmetic for teaching in Everardo Backheuser's pedagogical manual

The analysis of *A aritmética na "Escola Nova"* made us realize that the arithmetic to teach –the contents to be worked on– articulates with the arithmetic for teaching –the tools to carry out teaching. According to Hofstetter and Valente (2017), the knowledge intended that can be taught in the first years of school is revealed as new knowledge.

The analysis of the handbook, based on references to the constitution of professional teaching knowledge, allows us to compare elements resulting from the articulation between arithmetic to teach and elements of arithmetic for teaching. Preliminarily, we prepared Table 2 to highlight the contents to be taught and the tools used to teach arithmetic in the first years of school.

Table 2.

Teaching objects and teaching working tools highlighted in teaching manuals (Prepared by the authors)

Teaching objects	Working tools
Notions of number Numbers Even and odd numbers Addition Subtraction Multiplication Division Fractions Rule of three Percentages Notions of financial mathematics	Co-teaching Teaching through games and manipulative materials Contextualization of teaching with everyday situations Mental calculation Teaching by observation (intuitive) Problem solving

By listing teaching objects and teacher working tools, highlighting arithmetic to teach and arithmetic for teaching present in each of the manuals analyzed, Table 2 allows us to advance in the understanding of how the two arithmetics (to teach and for teaching) placed two disciplinary fields in relation, mathematics and education sciences, aligning them with the field of teaching for the production of professional knowledge, the evidence of which is present in the pedagogical handbooks of the New School period.

According to Bertini, Morais, and Valente (2017), connecting knowledge to teach with knowledge for teaching aims to produce professional knowledge, which is fundamental for teachers to teach arithmetic in the early years of school. This leads us to understand that the manual analyzed here provides evidence that its author defends the importance of training primary school teachers, indicating arithmetic for teaching, since they highlighted the need for skills beyond mastery of the subject's content.

When characterizing teaching objects and teaching tools, indicating content and teaching strategies mobilized by the authors in manuals intended for the training of primary school teachers, Table 2 signals the next step in the process of producing professional knowledge. A movement toward a systematization of arithmetic for teaching, resulting from the fusion of teaching objects with didactic strategies, giving a broader meaning to knowledge that, beyond the didactic content, assumes a professional status, being indispensable knowledge for the teacher who teaches arithmetic in the first years of school. Beyond solving questions, these guidelines are committed to contextualized teaching aligned with students' everyday situations. They are intuitive and reflective teaching that recognizes the student as the subject of their

learning. A teaching that uses not only the observation of objects and images but also a variety of games and manipulable materials, systematically presented in the pedagogical manuals analyzed.

Table 3 shows that knowledge for teaching, here called arithmetic for teaching, when observed comparatively, presents some similar and some different elements. In a more advanced comparison, these contrasts allowed us to consider the systematizations present in each pedagogical manual.

Table 3.

Systematizations of arithmetic for teaching (Prepared by the authors)

Arithmetic for teaching	THORNDIKE (1936)
Co-teaching	Coordinated teaching of all subjects
Teaching through games and manipulative materials	Games understood as an achievement of pedagogy, a pedagogical translation of the child's fantasy phase
Contextualization of teaching with everyday situations	Problems should be linked to life, leading students to the necessary solutions based on their interest in various daily situations
Mental calculation	To be carried out in groups, with individual questions, aiming at safety and speed in operations
Problem solving	Teaching through writings that stimulate students' mnemonic capacity to perform operations
Teaching by observation (intuitive)	Teaching through observation allows visual learners to learn better.

Seen with a "generalization character" (Lima & Valente, 2019, p. 941), arithmetic for teaching is professional knowledge from the New School. With different systematizations, numerous similarities can be observed in Tables 2 and 3 in relation to the teaching objects and the tools to be mobilized by teachers. However, in the hanbooks analyzed, there is a notable conformity with the New School ideology, whether about teaching methods or the close relationship between psychology and pedagogy, which conditions activities to the development of students' potential, with the use of different material resources in the classroom to assist in learning arithmetic.

Final considerations

Considering the importance of pedagogical handbooks for the teachers' professionalization, this study sought to show how the didactic work of Everardo Backheuser,

A aritmética na "Escola Nova", contributed with new knowledge for the teacher to teach in primary school.

Thus, we guided our research with the following question: What aspects of arithmetic for teaching are present in the pedagogical handbook *A aritmética na "Escola Nova",* by Everardo Backheuser? This analysis allowed us to see the author's guidelines tfor the teaching of school arithmetic regarding methods, processes, forms, and ways of planning, conducting, and evaluating teaching, knowledge of a general didactics. The author indicated more specialized knowledge, resulting from the dialogue between the reference science (mathematics) and the educational sciences, especially regarding the contributions arising from the psychology of education.

In this way, we understand how much the guidelines from the analyzed handbook were committed to the advances in educational sciences, especially in the performance of the experimental psychology in the constitution of professional knowledge of the future teachers of primary education in normal schools during the pedagogical wave called New School.

The work analyzed revealed constant elements for the training and guidance of teachers who would teach arithmetic in primary schools, such as co-teaching, teaching through games and manipulative materials, contextualization of teaching with everyday situations, mental calculation, teaching through observation (intuitive), and problem solving, These elements characterize professional knowledge, that is, arithmetic for teaching in the first years of school.

So the arithmetic *for* teaching constituted in the pedagogical handbook *A aritmética na "Escola Nova*", by Everardo Backheuser, intended for the education of primary school teachers and used in normal schools during the period of the New School pedagogical wave, shows, in essence, professional knowledge of teaching, systematized and objectified for teaching arithmetic in primary school.

References

Backheuser, E. (1933). *A aritmética na "Escola Nova"*: *A nova didática da aritmética.* Livraria Católica. https://repositorio.ufsc.br/xmlui/handle/123456789/116434.

Bittencourt, C. M. F. (1993). *Livro didático e conhecimento histórico*: *uma história do saber escolar.* 1993. [Tese de Doutorado em História Social]. Faculdade de Filosofia, Letras e Ciências Humanas da Universidade de São Paulo, São Paulo.

Carneiro, R. S. (2014). *O método intuitivo na aritmética de Calkins e Trajano*. [Dissertação de Mestrado em Educação Matemática]. Universidade Severino Sombra, Vassouras-RJ.

Certeau, M. (2007). *A escrita da história* (Trad. Maria de Lourdes Menezes, 2. ed.). Forense Universitária.

Chervel, A. (1990). História das disciplinas escolares: reflexões sobre um campo de pesquisa. *Teoria & Educação*, 2, 177-229.

Choppin, A. (2004, set./dez.). História dos livros e das edições didáticas: sobre o estado da arte. *Educação e Pesquisa*, *30*(3), 549-566.

Hofstetter, R., & Schneuwly, B. (2017). Saberes: um tema central para as profissões do ensino e da formação. In R. Hofstetter, & W.R. Valente (Org.), *Saberes em (trans) formação: tema central a formação de professores* (pp. 113-172, 1. ed.). Editora da Física.

Hofstetter, R., & Valente, W. R. (2017). *Saberes em (trans) formação: tema central a formação de professores* (1 ed.). Editora da Física.

Julia, D. (2001, jan./jun.). A cultura escolar como objeto histórico (Trad. Gizele de Souza). *Revista Brasileira de História da Educação*, 1, 9-44.

Pinto, N. B., & Novaes, B. W. D. (2018). Caracterização de saberes profissionais da matemática para ensinar nos primeiros anos escolares: anotações metodológicas. *HISTEMAT*, ano 4, 1, 139-153.

Rabelo, R. S. (2016). *Destinos e trajetos*: *Edward Lee Thorndike e John Dewey na formação matemática do professor primário no Brasil (1920-1960).* [Tese de Doutorado em Educação]. Faculdade de Educação da Universidade de São Paulo, São Paulo.

Rio de Janeiro (Estado). (1951). *Diário de notícias*. Edição 08884, segunda seção, quarta página, 11 de outubro de 1951. http://memoria.bn.br/docreader/DocReader.aspx?bib=093718_03&Pesq=Everardo%20Adolfo%20Backheuser&pagfis=12452.

Rosa, M. (2017). *Escolanovismo católico Backheusiano*: *apropriações e representações da Escola Nova tecidas em manuais pedagógicos (1930-1940).* [Tese de Doutorado em Educação, Centro de Ciências Humanas e da Educação, Programa de Pós-Graduação em Educação]. Universidade do Estado de Santa Catarina, Florianópolis.

Valente, W. R. (2019). Saber objetivado e formação de professores: reflexões pedagógico-epistemológicas. *Revista História da Educação (online)*, 23, 1-12. e77747. UFRGS. https://seer.ufrgs.br/asphe/article/view/77747/pdf.

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