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The attribution of meaning in mathematical modeling in the early years: a semiotic interpretation of mathematical objects

Atribuição de significado em modelagem matemática nos anos iniciais: uma interpretação semiótica acerca dos objetos matemáticos

Atribución de significado en la modelización matemática en los primeros años: una interpretación semiótica de los objetos matemáticos

Attribution de sens dans la modélisation mathématique des premières années : une interprétation sémiotique des objets mathématiques

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Abstract

The objective of this paper is to look for evidence of the attribution of meaning to mathematical objects through the signs associated with doing mathematical modeling. To do so, we analyzed two mathematical modeling activities developed in the early years of Elementary School, with the theoretical contribution of Mathematical Modeling, with attention to children's cognitive actions and; Peircean semiotics, with regard to signs and the epistemological triangle, which considers them in association with two other elements: the context of reference and its concept. The qualitative methodology is the one that supports this study since we focus on the signs produced by the children, for mathematical objects, when they develop mathematical modeling activities. We interpret them from the epistemological triangles we build. From these triangles, we consider that the attribution of meaning to mathematical objects emerging in mathematical modeling activities developed is imbricated with the situation that originated the activity, signaling that such attribution is a compound that considers the situation and mathematics in

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an articulated way. Furthermore, attributing meaning to mathematical objects gains consistency as the signs change, alter, and complement each other in the face of children's cognitive actions.

Keywords: Cognitive actions in mathematical modeling, Signs, Epistemological triangle.

Resumo

O objetivo deste artigo consiste em buscar indícios de atribuição de significado aos objetos matemáticos por meio dos signos associados ao fazer modelagem matemática. Para tanto, foram analisadas duas atividades de modelagem matemática, desenvolvidas nos anos iniciais do Ensino Fundamental, tendo como aporte teórico a Modelagem Matemática, com atenção às ações cognitivas das crianças, e a semiótica peirceana, no que se refere aos signos e ao triângulo epistemológico, considerados em associação com outros dois elementos: contexto de referência e o conceito. A metodologia qualitativa ampara este estudo, uma vez que foca nos signos produzidos pelas crianças, para os objetos matemáticos, ao desenvolverem atividades de modelagem matemática, e serem interpretados a partir dos triângulos epistemológicos construídos. A atribuição de significado dada aos objetos matemáticos emergentes nas atividades de modelagem matemática desenvolvidas vem imbricada com a situação que originou a atividade, sinalizando que tal atribuição é um composto que leva em conta a situação e a matemática de forma articulada. Ademais, a atribuição de significado aos objetos matemáticos vai ganhando consistência, à medida que os signos vão se modificando, se alterando e se complementando diante das ações cognitivas das crianças.

Palavras-chave: Ações cognitivas em modelagem matemática, Signos, Triângulo epistemológico.

Resumem

Este artículo tiene el objeto de encontrar indicios de atribución de significado a los objetos matemáticos a través de los signos asociados a la modelación matemática. Para ello, se analizaron dos actividades con modelación matemática desarrolladas en los primeros años de la enseñanza primaria. Como soporte teórico, se utilizaron la Modelación Matemática, con atención especial a las acciones cognitivas de los niños, y la semiótica de Peirce, con respecto a los signos y al triángulo epistemológico, en asociación con otros dos elementos: el contexto de referencia y el concepto. Este estudio utiliza la metodología cualitativa, puesto que tiene enfoque en los signos producidos por los niños para los objetos matemáticos durante el desarrollo de las actividades de modelación matemática, y que se interpretaron a partir de los triángulos epistemológicos construidos. La atribución de significado a los objetos matemáticos que surgieron en las actividades de modelación matemática desarrolladas está involucrada en la situación que originó la actividad, lo que indica que tal atribución es un compuesto que tiene en cuenta la situación y las matemáticas de manera articulada. Además, la atribución de significado a los objetos matemáticos adquiere consistencia a medida que los signos se modifican, se alteran y se complementan a consecuencia de las acciones cognitivas de los niños.

Palabras clave: Acciones cognitivas en modelación matemática, Signos, Triángulo epistemológico.

Résumé

Dans cet article, notre objectif est de rechercher des preuves d'attribution de sens aux objets mathématiques à travers les signes associés à la modélisation mathématique. À cette fin, nous avons analysé deux activités de modélisation mathématique développées dans les premières années de l'école élémentaire, ayant la modélisation mathématique comme cadre théorique, en prêtant attention aux actions cognitives des enfants et ; La sémiotique de Peirce, à propos des signes et du triangle épistémologique, qui les considère en association avec deux autres éléments : le contexte de référence et le concept. La méthodologie qualitative est ce qui soutient cette étude puisque nous nous concentrons sur les signes produits par les enfants, pour des objets mathématiques, lors du développement d'activités de modélisation mathématique, et nous les interprétons à partir des triangles épistémologiques que nous construisons. À partir de ces triangles, nous considérons que l'attribution de sens aux objets mathématiques émergeant dans les activités de modélisation mathématique développées est étroitement liée à la situation à l'origine de l'activité, signalant qu'une telle attribution est un composé qui considère la situation et les mathématiques de manière articulée. De plus, l'attribution de sens aux objets mathématiques gagne en cohérence au fur et à mesure que les signes changent, s'altèrent et se complètent en fonction des actions cognitives des enfants.

Mots-clés : Actions cognitives en modélisation mathématique, Signes, Triangle épistémologique.

The attribution of meaning in mathematical modeling in the early years: a semiotic interpretation of mathematical objects

Children, from an early age, learn to communicate mathematically using resources that allow them to express their ideas. In general, the mathematical language they use deposes actions that represent their ability to interpret, question, share and communicate thoughts. We see Mathematical Modeling as a pedagogical alternative possibility to promote such actions since it gives opportunities to those who develop mathematical modeling activities to interpret and question a problem related to reality, share their mathematical and non-mathematical knowledge, and communicate their thoughts on the phenomenon studied.

In this investigation we anchor this communication of thoughts in the semiotic theory and look at it from the signs, which is denoted by Peirce (2005) as a thing that represents something, its object; not in all its dimensions, but it carries characteristics of it. Attention to signs is consolidated in the development of mathematical modeling activities.

In general terms, we understand Mathematical Modeling as a pedagogical alternative that allows exploration/investigation of problem-situations of reality through mathematical tools. In this approach, mathematical modeling activities tend to value different knowledge construction processes, especially mathematical ones, and provide opportunities for children to act as protagonists in these processes. Mathematical Modeling aims to propose solutions to problems arising from real situations based on mathematical models. Thus, the mathematical model is “what 'shapes' the solution to the problem and Mathematical Modeling is the 'activity' of searching for that solution” (Almeida; Silva; Vertuan, 2012, p.15).

Studies that address Mathematical Modeling in the early elementary school years have focused on various approaches. There are studies that discuss the construction of mathematical models (English, 2010; Tortola; Silva, 2021), others discuss aspects of the development of mathematical modeling practices in the classroom (English, 2003; English; Watters, 2004), there are those who look at the speeches produced by children (Luna; Souza, 2009) and others at the training of teachers who work in the early years (Gomes, 2018; Gomes; Silva, 2021; Teodoro, 2018). There are also those who discuss mathematical modeling linked to other theoretical frameworks (Tortola, 2012; 2013; Nunomura, 2021).

Following this line of interlocution between Mathematical Modeling and other theories, the present work aims to seek evidence of the attribution of meaning to mathematical objects through the signs associated with mathematical modeling by children in the early Elementary School years. To do so, we are guided by the assertions of Peirce's semiotics and consider the epistemological triangle proposed by Veronez (2013) in the context of Mathematical Modeling, as a possibility for a semiotic interpretation of the signs associated with mathematical objects raised in mathematical modeling activities developed by children from the fifth year of Elementary School, based on assumptions of qualitative methodology as subsidy for our interpretation.

Therefore, we structure the paper into two theoretical sections considering Mathematical Modeling and Semiotics. Next, we bring the methodological aspects. Subsequently, by finalization, we weave some reflections on the investigation.

Mathematical Modeling in the Early Years of Elementary School

Mathematical Modeling from the perspective of Mathematics Education began in the late 1970s and since then many researchers have been dedicated to researching this trend in Mathematics Education at all levels of schooling. However, with regard to the early years of Elementary School, studies date from the year 2000 (English, 2003; 2010; English; Watters, 2004; Luna; Souza; Santiago, 2009; Tortola, 2012; 2016; Teodoro, 2018, Tortola; Silva, 2021; Nunomura, 2021).

For the development of this investigation, we base ourselves on the assertions of Almeida, Silva, and Vertuan (2012) regarding conceiving Mathematical Modeling as a pedagogical alternative, in which a non-essentially mathematical problem is discussed through mathematics, aiming to establish a relation between mathematical knowledge and reality. This characterization is complemented by Tortola (2016) when defending Mathematical Modeling as a pedagogical alternative, it encourages children, under the teacher's guidance, to problematize real situations based on mathematics.

Assuming this configuration of Mathematical Modeling, it is associated with a search for a solution to the problem studied. Thus, the dynamics of classes with mathematical modeling, according to Tortola and Almeida (2013), has the potential to favor learning in multiple aspects: children's autonomy in solving problems; critical appreciation of the use of Mathematics in situations involving reality; reflection while active in society; development of mathematical skills.

Overall, studies in the literature point out that mathematical modeling activities play an important role in mobilizing, building and producing knowledge and “provides children with rich opportunities to experience complex data in challenging and yet meaningful contexts” (ENGLISH, 2010, p. 288, our translation), making it possible to develop critical and reflective thinking.

By developing mathematical modeling activities, children have opportunities to conduct research, debate with colleagues, use their own representations and, in this sense, (re)build diverse knowledge. The opportunity to formulate hypotheses, investigate, develop strategies to solve the problem situation, observe regularities, generalize, synthesize, in order to find the solution in the most appropriate way possible (Tortola, 2016), place children as protagonists of their process of learning and encourage them to learn and pose problems.

The approximation between real situations and mathematics, made possible by Mathematical Modeling from the perspective of Mathematics Education, in addition to considering an initial (problematic) situation and a final situation (which corresponds to a solution for the initial situation), requires a set of actions and necessary procedures to move between these situations. This set of actions and procedures are processed in what the authors Almeida and Silva (2012) denote by students' cognitive actions in mathematical modeling activities. These actions are discussed in the next section.

Cognitive actions in mathematical modeling activities

Discussions about cognitive processes that are associated with the development of mathematical modeling activities have space both in the international scenario, with the work of Ferri (2006) and nationally, with the works developed by Almeida e Silva (2012), Silva (2013), Veronez (2013) and Castro (2017). In general, these works bring aspects related to the students' way of seeing, thinking and acting when they develop mathematical modeling activities.

Ferri (2006) discusses the mathematical modeling cycle from a cognitive perspective. This cycle is based on the students' actions in moving from a real to a mental situation, structuring the problem and simplifying it in order to "filter" information; moving from a real model to a mathematical model, in which mathematical skills are required; identify and interpret the results of this mathematical transition; and, finally, to validate the final results, refer to actions linked to the students' cognitive process. Thus, the author analyzes a mathematical modeling activity from a cycle that she builds, pointing out the procedures carried out by the students.

In Almeida and Silva (2012), the authors investigate relationships between cognitive actions in mathematical modeling activities, denoted by situation comprehension; situation structuring; mathematization; synthesis; interpretation and validation; argumentation and communication, and the modes of inference that relate aspects of semiotics. In their discussions, they bring reflections, in the light of Peirce's semiotics, about the different reasoning associated with the student's cognitive actions.

Silva (2013) and Veronez (2013) consider students' cognitive actions in mathematical modeling in association with Charles Sanders Peirce's Semiotics. Silva (2013) analyzes, semiotically, through the sign triad: sign, object, and interpretant, mathematical modeling activities developed by undergraduate students with the aim of identifying the interpretive signs that emerge in the different phases of a mathematical modeling activity. In particular, the author looks at the students' cognitive actions with the intention of identifying the attribution of meaning to the mathematical object and concludes that the signs associated with the students' cognitive actions give evidence that there was an attribution of meaning to the mathematical object, and also for the problem. Veronez (2013) in turn, discusses the roles of signs, used and/or produced by students in their cognitive actions when developing mathematical modeling activities, and bases his discussions on the semiotic and epistemological functions of signs.

In general, students' cognitive actions in mathematical modeling activities are expressed implicitly, or explicitly, in the transit from the initial situation (problem) to a final situation (response to the initial problem). Understanding the situation, interpreting facts and information, and grouping ideas, are evidences of cognitive action in understanding the situation.

The cognitive action of structuring the situation is linked to the identification of a problem to be solved. However, this identification is related to the definition of goals that are consistent with structuring and/or simplifying the information. The translation of this problem into a mathematical language, in order to recognize a mathematical problem to be solved, is associated with the cognitive mathematization action. In this action, the elaboration of a mathematical model is also processed. The construction and development of a mathematical model serves to enable us to understand, analyze, make predictions, and explain a certain phenomenon from a mathematical point of view. According to Almeida e Silva (2012, p. 7), "the elaboration of a mathematical model is mediated by relations between the characteristics of the situation and the concepts, techniques and mathematical procedures" necessary to solve the problem.

Regarding the early years of elementary school, Tortola e Silva (2021, p. 4); argue that "the mathematical model corresponds to a representation external to the minds of the subjects, whose statements by those involved are in mathematical terms". In this sense, the mathematical model in the early years can be understood as "a natural, numerical, tabular, graphic or figural language, which can serve as a bridge for the use of other languages" (Tortola, 2012, p. 152).

Since the mathematical model is full of the students' intentions and interests, Almeida, Silva and Vertuan (2012, p. 18) argue that its construction “requires mastery of mathematical techniques and procedures and adequate coordination of the different representations associated with mathematical objects”. Thus, in order to find a solution to the problem, it is necessary to use previous knowledge, identify patterns and coordinate mathematical objects in different mathematical representations, based on techniques and concepts. All this, according to Almeida and Silva (2012), is associated with the cognitive synthesis action.

The student's view of the answer obtained, based on the constructed mathematical model, requires that it be judicious and carry out an evaluative and analytical process in the face of such an answer and in relation to the mathematical representation produced. In this process, the student “is faced with the need to compare and distinguish ideas, generalize facts, articulate knowledge from different areas” (Almeida; Silva; Vertuan, 2012, p.18). This process of the student evaluating their answer is recurrent of the cognitive action of interpretation and validation.

Once the answer to the problem has been evaluated, students expose their considerations to others, presenting, justifying, and arguing their choices and ideas based on their knowledge. This communication implies convincing the students, the teacher and themselves that those results are accessible, presentable, and consistent from a mathematical point of view (Almeida; Silva, 2012). This action is understood as a cognitive action of communication and argumentation.

All these cognitive actions are processed during the development of mathematical modeling activities based on reflective processes that lead to ways of seeing, acting and thinking about a given situation. Castro and Veronez (2017) when discussing students' cognitive actions in mathematical modeling activities call attention to the fact that they are interdependent, however, non-linear.

Assuming such characteristics of cognitive actions, we seek to identify the signs that manifest them. It is about the signs that we discuss in the next section.

Peirce's semiotics and the signs

Charles Sanders Peirce contributed and dedicated a large part of his life to the development of a science that was general and abstract to all methods of investigation used in the most diverse sciences. He called this science Semiotics, the science of signs. A sign, according to Peirce (1995, p. 46) “is something that, under a certain aspect or in some way, represents something to someone; addresses someone, that is, creates in that person's mind an equivalent sign or perhaps a better-developed sign. In other words, a sign is something that stands for something to someone.

For Santaella (2002), one of the followers of Charles Sanders Peirce, the sign can be considered anything, of any kind, such as a word, a book, a museum, a video, etc., that represents a thing (the object of the sign) and produces in an actual or potential mind an interpretive effect (interpretant of the sign).

In Peirce's Semiotics, the sign has a triadic nature (Peirce, 2005). This means that the foundation of the sign (representámen) is associated with an object (what the sign represents) and an interpretant (what the sign represents for someone). This triad of the sign shows that signs play a role in thinking and communication, because, as Peirce (2005) asserts, we are not capable of thinking without signs. Semiotics, therefore, can provide elements to explain and interpret aspects of human cognition.

In the realm of mathematics, aligned with the ideas of Charles Peirce, Steinbring (2006, p. 1, our translation) points out that signs give materiality to thoughts, and that without them "no human thought and no mental generalization would exist". He complements this idea by calling attention to the fact that mathematical knowledge cannot be translated by a mere reading of signs. It needs to be understood from what is implicit in the manifest sign. For this author, the mathematical sign plays a decisive role in the codification, construction, and communication of mathematical knowledge, because mathematical objects need signs (or mathematical symbols) to be represented, coded, and operated.

Hoffmann (2006) also argues that it is impossible to directly understand and operate mathematical objects without signs. For this author, signs are means to think and communicate about mathematical objects, and as signs are interpreted and transformed into more elaborate signs, it can be said that mathematical knowledge is being developed.

Steinbring (2006) argues that for the existence of meaning to mathematical concepts, a connection between signs/symbols and object/context of the reference is necessary. In this sense, Steinbring (2005; 2006) associates two elements to the sign/symbol: object/context of reference and concept.

> All mathematical knowledge needs certain systems of signs or symbols in order to grasp and code the knowledge in question. These signs themselves do not have an isolated meaning; their meaning must be constructed by the learning child. In a general sense, to endow mathematical signs with meaning, one needs an adequate reference context. (Steinbring, 2002, p. 116).

This connection between object/context of reference, sign/symbol, and concept is expressed by Steinbring (2006) through a model that he entitled epistemological triangle (Figure 1). For Steinbring (2006), the epistemological triangle can be used as a theoretical concept to analyze and/or describe some epistemological particularity of mathematical knowledge and also of mathematical communication.

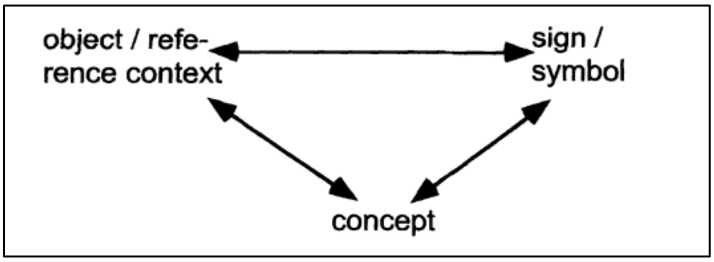

Figura 1.

Epistemological Triangle (Steinbring, 2006, p. 50)

In this epistemological triangle, the sign/symbol is the bearer of mathematical knowledge; it is through it that mathematics is represented. However, this sign/symbol carries characteristics of the reference object/context to which it is associated and also alludes to a concept. Thus, the connections between the three elements of the vertices of this triangle occur in an interdependent manner, which means that the interpretation of signs/symbols in relation to the object/context of reference can be modified, complemented, or generalized. This makes it possible that, with the mediation between these three elements, new mathematical knowledge is constituted (Steinbring, 2002). The concept, in this scenario, is constituted from the balanced relationship between the three elements of the epistemological triangle (context of reference, sign, and concept). Hence Steinbring's statement (2006) that, in terms of the epistemological triangle, it is possible to understand the process of building mathematical knowledge.

Considering that a sequence of epistemological triangles can signal the development of a student's interpretations (Steinbring, 2005) gains space in the works of Farrugia (2007) and Veronez (2013). Farrugia (2007) uses the semiotic model (epistemological triangle) to analyze how children deal with the concepts of multiplication and division. The author points out that the concept of multiplication was being changed by the children and gaining meaning; they were able to identify multiplication as successive sums with equal parts and write, for example, 3 +3 +3 + 3 as being equal to 4 x 3. On the other hand, she draws attention to the fact that children do not proceed analogously for the concept of division. Farrugia (2007), therefore, considers that it is the context of reference that supports the process of construction of meaning to mathematical objects.

Veronez (2013), in turn, analyzes the relationships between the three vertices of the epistemological triangle in the context of mathematical modeling activities and discusses the semiotic and epistemological functions of signs in the process of generating a sequence of epistemological triangles along the development of a mathematical modeling activity that denotes the dynamism embedded in Mathematical Modeling, that is, in the search process for a solution to the problem that originated the mathematical modeling activity.

The study developed also makes up the list of investigations that deal with the epistemological triangle (Steinbring, 2006) to make inferences. However, we use the model proposed by Veronez (2013) to discuss the attribution of meaning to mathematical objects when doing mathematical modeling, since this author explains an epistemological triangle in the context of mathematical modeling. In Veronez (2013), the reference context corresponds to the characteristic elements of a mathematical modeling activity, it is from the students' cognitive actions that the signs emerge, and the concept is associated with the students' knowledge and relates to the reference context. In the next section, we illustrate the model proposed by this author and present our methodological path, as well as the context in which this study was developed.

The research context and its methodological aspects

For this study, which aims to seek evidence of meaning attribution to mathematical objects through the signs associated with doing mathematical modeling, two mathematical modeling activities developed by children from a class of the 5th fifth of Elementary School were considered, from a private school located in the north of Paraná. These activities were developed in line with what is indicated in the third moment of familiarization with mathematical modeling activities suggested by Almeida and Dias (2004). The authors recommend, in this third moment, that the teacher make room for the students to carry out the mathematical modeling activity with more autonomy, that is, they are given the opportunity to choose the topic for study, choose a problem to solve, collect information, simplify the data, define hypotheses and problematize in order to obtain a solution to this problem.

The two mathematical modeling activities addressed, in order to reflect on the objective of this work, were developed over the months of August and September 2021, when the country was still under the consequences of the pandemic caused by the Sars-Cov 2 virus (Covid-19). During this period, at the school where the investigation was carried out, classes had already returned to face-to-face mode, however, parents or guardians could still opt for remote teaching, in which children's access to classes was through digital platforms. Thus, 18 children participated in the present investigation, of which 17 attended classes in person and only 1 child used remote teaching.

The development of this study was designed according to the qualitative approach of research along the lines of Borba and Araújo (2013, p. 25); according to these authors, "research performed according to a qualitative approach provides us with more descriptive information, which excels in the meanings given to actions". This means that this type of research highlights the ways of understanding and interacting with research participants. Goldenberg (2004) adds that there is a dynamic relationship between the investigated context and the researcher in qualitative research. In particular, in this investigation, the researcher had a dual role: he was a teacher and a researcher at the same time.

To ensure that the aspects related to the development of the modeling activities by the children were considered while the children were developing the mathematical modeling activities, the teacher/researcher used the resource of writing down what she considered important and essential. In addition to these field notes, the transcripts of the audio and video recordings of the classes, made possible by the use of the digital platform used due to remote teaching, and the records produced by the children, composed analysis material. In order to preserve the identity of the children, fictitious names will be used to refer to them.

The epistemological triangle (Figure 2) proposed by Veronez (2013) is the instrument we use to bring interpretations about the signs associated with students' cognitive actions in mathematical modeling activities developed by the fifth-grade students of Elementary School, with regard to the attribution of meaning to mathematical objects raised in these activities. In this epistemological triangle, the reference context corresponds to the characteristic elements of a mathematical modeling activity; the sign is associated with the manifest signs in the children's cognitive actions, and; the concept is linked to the mathematical objects evoked throughout the development of each of the mathematical modeling activities.

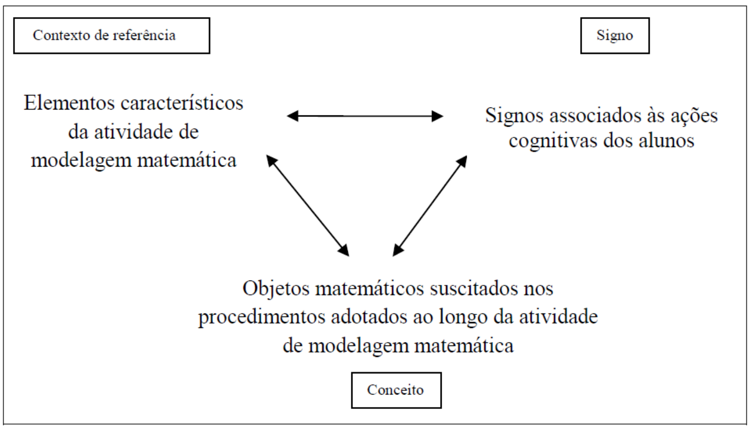

Figure 2.

Epistemological Triangle in the context of mathematical modeling (Veronez, 2013, p. 151)

In this triangle, the signs used and/or produced by students reveal thoughts and communicate knowledge associated with their cognitive actions and, therefore, in addition to representing something, they refer to a context of reference and evoke a concept. Focusing and interpreting these signs in relation to the other two vertices of the triangle is our interest in the study, which we discuss in the next section.

Analysis of mathematical modeling activities from a semiotic perspective

The two mathematical modeling activities that support our analysis have the following themes: *Frames per second* and *Travelling*, both themes chosen by the children. The choice of theme for the first activity was due to another mathematical modeling activity previously developed by the children. It was this activity that triggered their curiosity to investigate about frames per second.

In this first activity, the children had the problem of discovering the number of photos in a video. While watching the constructed videos, they debated about it. In one of these discussions, one of the children, namely Gustavo, made a comment: *Teacher, the difference between a game and a video is due to the amount of FPS that exists there. In the game, there is 120 FPS in each scene that takes place, while in a video it is another FPS*, which triggered the children's interest in investigating aspects of FPS and linking this to the videos they built in the mathematical modeling activity previously developed.

The second topic, called Travelling, resulted from a project that the children were already developing with the class teacher. In this project, the children's objective was to plant and cultivate a vegetable garden at school and then sell the products at a small stand in front of the school. Work on financial education was the purpose of this project. The central idea was to make children realize the real value of money, through effort and reward. The children's reward, in this case, would be to take a trip or tour with the money raised. Hence the reason that led the children to investigate where the group could go with R$300.00 (three hundred reais).

Next, we approach these two mathematical modeling activities by looking at the signs that were produced and mobilized by children in association with their cognitive actions and we make inferences regarding the semiotic interpretation that we perform. That is, we identified the signs produced by the children and analyzed them semiotically, in order to signal their understanding of the phenomenon under study.

How many photos are in a video? - FPS theme analysis

The development of this activity rests on the children's interest in investigating the term FPS enunciated by one of them. The fact that the child enunciates a topic of interest corresponds to what the third moment of familiarization proposed by Almeida and Dias (2004) recommends. Even though there was no stated problem, the children, interested in understanding the theme, produced some signs such as those illustrated in Table 1.

Table 1.

Signs associated with cognitive action understanding of the situation (Authors)

Child	Sign
Gustavo	Frames can be taken as FPS, which is frames per second, or frames per second to keep it simple.
Gina	Guys, from what I understand, the more I draw, the more photos we have.
Samantha	Look, I researched and found that the cameras can shoot 30 or 60 FPS, the difference between one and the other is the video quality.

The children's speeches (Table 1) correspond to signs that act as responsible for understanding the situation (finding out what the FPS is - investigation topic) and that denotes the cognitive action of understanding the situation. From this action, the children recognize that it is "necessary to select elements from the initial situation that are relevant to the intended investigation" (Veronez, 2013) and formulate a problem to investigate: How many photos does the video we created have? This problem is a sign that results from the cognitive action of structuring the situation. The emerging signs of these two cognitive actions have the problem under study as a reference context: the FPS and the concept linked to them refers to what, in fact, means Frames per second.

Once the analyzed situation is understood, the children formulate hypotheses (Table 2), and produce signs in association with the mathematization cognitive action.

Table 2.

Signs associated with mathematization cognitive action (Authors)

Child	Sign

Jeferson	Let's consider that all cell phones have 60 FPS, because everyone'scell phone here in the room is modern. Let's see what math we have here.
Gustavo	We have the time in the video, so I think we are going to use the time, because the FPS is in seconds and most videos are already in seconds and if we don't have it, we can convert it.

The sign "*Let's consider that all cell phones have 60 FPS, because everyone's cell phone here in the room is modern*" is a hypothesis, because children consider it reasonable to accept that their cell phones, when activated to record a video, take 60 photos per second. The hypotheses, in addition to denoting an interpretation of the children, show characteristics that denote certain knowledge about the investigated phenomenon, and are "associated with the ways of seeing the situation" (Almeida; Sousa; Tortola, 2015)

With the hypothesis, Jeferson generates a new sign "*let's see what mathematics we have here*". This sign signals the child's intention to see the problem to be solved through mathematical concepts. This sign, therefore, suggests a language transition; suggests that the problem written in natural language can be taken in mathematical language from the evidence of a mathematical problem to be solved. Such signs is recurrent in the mathematization of cognitive action.

Gustavo's sign: *"we have time in the video, so I think we are going to use time because the FPS is in seconds and most videos are already in seconds and if we don't have it, we can convert"* which is also a hypothesis and is associated with the mathematization cognitive action, it indicates that children believe that using the time in seconds of the videos they created is the process that will lead them to find a solution to the problem under study. The emerging signs of the mathematization cognitive action, although changing the initial reference context, maintain the concept: meaning of the FPS expression. That is, the reference context is now the assumed information that corresponds to the problem under

From these signs, children initiate a process of reflection and produce new signs. Table 3 contains such signs, emerging from the cognitive synthesis action, and which denote the mathematical thinking of children.

Table 3.

Signs associated with the synthesis cognitive action (Authors)

Child	Sign
Jeferson	I think we can use multiplication because if I have 60 photos in 1 second, then I do 60 times 1 which gives us 60 photos per second and then we do with our time in seconds, just replace the 1 in the multiplication.
Samara	Teacher, Jeferson said that we multiply by 1, replace it with 1, but I didn't understand anything, I was talking to Nicoly and we came to a conclusion: if I add 60 when I had 2 seconds, I'll have 120 photos, and if I add another 60 I'll have 180 photos for time 3. Can I think this way?

The sign "*I think we can use multiplication because if I have 60 photos in 1 second, then I do 60 times 1 which gives 60 photos per second and then we do with our time in seconds just replacing the 1 in the multiplication*" evokes a mathematical concept: multiplication. This sign also carries with it the intention to find a solution to the problem and indicates the construction of a mathematical model (even if it is speaking in a mother tongue, or spoken language) associated with the suggestion that it can be multiplied by the time recorded in the video. That is, this sign suggests a possibility of solving the problem based on the multiplicative principle.

This sign produced by Jeferson generated estrangement in other children and led to the enunciation of another sign, portraying another possibility of resolution. The sign "*teacher, Jeferson said that we multiply by 1, replace it with 1, but I didn't understand anything, I was talking to Nicoly and we came to a conclusion: if I add 60 in the moment I had 2 seconds I I'm going to have 120 photos, and if I add another 60 I'm going to have 180 photos for time 3. Can I think of it that way*?" manifested by Samara, although different from Jeferson's sign, raises the same mathematical concept: multiplication. However, the multiplication process indicated in this sign is linked to the additive principle. Other signs associated with the cognitive synthesis action were also produced by the children, as illustrated in Figure 3.

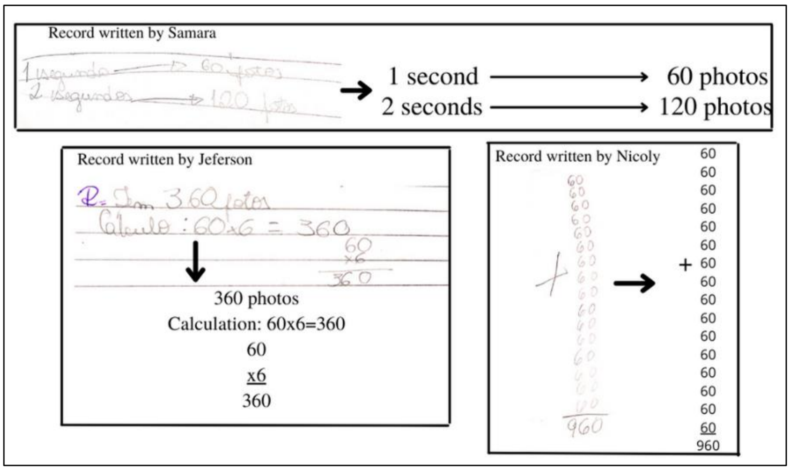

Figure 3.

Signs, of cognitive action synthesis, produced by Samara, Jeferson e Nicoly (Autores)

The signs produced by Samara and Nicoly indicate the use of the additive principle, albeit in different ways, in solving a problem involving the concept of multiplication. Samara's sign refers to an interpretation that lays out, in lines, how the problem is being solved. That is, she understands that 1 second corresponds to 60 photos and 2 seconds corresponds to 120 photos (twice as much as 1 second), and implies in her way of representing that 3 seconds would be 180 photos, that is, 60+60+60, and so on. Nicoly, on the other hand, explains its resolution using the sum of equal parts. This way of writing multiplication is also found in Farrugia (2007) when children use the repetition of grouping 3 units as 3+3+3+... to indicate the multiplication process used by them.

In Jeferson's sign there is an indication of the multiplication algorithm, suggesting that he understands that the problem in question can be solved by performing a multiplication that considers the number of seconds times 60, where 60 represents the number of photos per second.

These three constructions, signs by Samara, Nicoly, and Jeferson, are mathematical models, resulting from the cognitive synthesis action, which indicate a solution to the problem in focus. However, Gina realizes that performing sums of repeated terms, 60+60+60..., or multiplying the time by 60 did not solve her problem. The sign "*Look Samara, Jeferson made his video in 6 seconds so he multiplied it, but mine took 1 minute and 58 seconds, I don't know what to do here*" indicates that Gina managed to realize that she needs to do something more to find out the number of photos from the video.

Such a sign, therefore, leads Gina to carry out a reflection process before the mathematical model constructed by Samara, Nicoly, and Jeferson (Figure 3), and leads to the production of a new sign: "*If they used the seconds, and I already have 58 seconds, what do I do with that 1-minute, teacher? Can I put 60 because 60 is 1 minute?*". This sign, even if it directs a question to the teacher, indicates that Gina recognizes that there is a need to transform the time of her video into seconds (Figure 4).

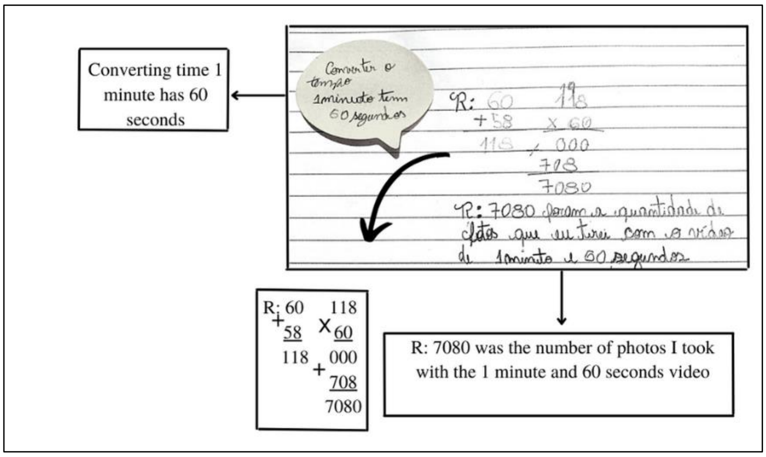

Figure 4.

Signs produced by Gina in the cognitive synthesis action (Authors)

The mathematical notes referring to the calculations performed by Gina assume the function of mathematical models and denote the use of different languages (mathematical language and spoken language) in an attempt to indicate a solution to the problem. The sign: "*Converting time 1 minute has 60 seconds*" indicates that she knows that to convert 1 minute and 58 seconds to seconds she needs to add 60 seconds to 58 seconds. Thus, the result of adding 60+58 = 118 is assumed to be the time spent on your video and, accordingly, it multiplies this result by 60 (number of photos per second). Nevertheless, Gina still feels the need to record the result in natural language, producing the sign: "*7080 was the number of photos I took with the 1 minute and 60 seconds video*". Although there is a mistake in this sign, as her video time is 1 minute and 58 seconds, we understand that she just got confused in the writing, as the mathematical calculations are correct, 1 minute and 58 seconds were considered in the multiplication.

All the signs associated with the cognitive synthesis action (Figure 3 and Figure 4) have as a reference context the investigative process linked to the search for a solution to the problem and the concept is associated with the unit of time measurement, multiplication, and addition.

The acceptance of these solutions by all children is the result of the cognitive action of interpretation and validation (Table 4) and the arguments and discussions rehearsed by them when justifying the solutions obtained result from the cognitive action of communication and argumentation.

Table 4

Signs associated with cognitive action, interpretation and validation (Authors)

Child	Sign
Gina	I added 1 minute to 58 seconds and then I multiplied by 60.
Samara	I understood. It was similar to what we had done before, but without adding the time, just multiplying.
Jeferson	So this means that when there is more than a minute, we need to convert the minute into a second and then multiply by 60.

These signs in Table 4 show the analysis of the answers obtained as a reference context and the concept corresponds to the mathematical objects emerging in the process of solving the problem.

The signs resulting from the students' cognitive actions caused an alternation of both the reference context and the concept. In this sense, the three elements of the epistemological triangle were changing and expanding. However, it was the signs of each of the children's cognitive actions that brought to light new interpretations and, therefore, the production of new signs, causing the context of reference to be modified and evoking concepts that were linked to them. The epistemological triangle of this activity, which we built and which denotes the alternation of its three vertices, is illustrated in Figure 5.

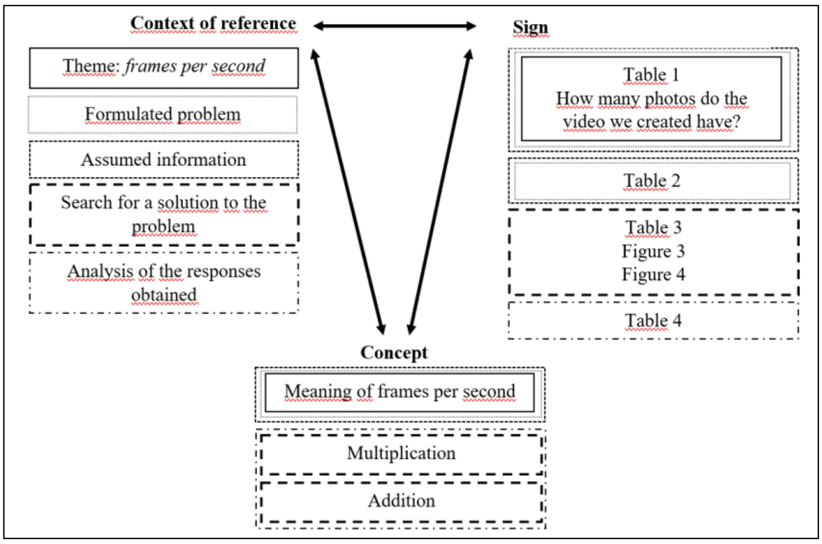

Figure 5.

Epistemological triangle of the mathematical modeling activity Frames per second (Authors)

To illustrate the association between the three elements of the epistemological triangle, we chose to present text boxes of different formats. Thus, the sign, the context of reference, and the concept that has the same text box format are in association, that is, they make up an epistemological triangle. For example, the sign “How many photos do the video we created have?” is associated with the reference context theme: *frames per second,* and the concept: *meaning of frames per second at one moment in the mathematical modeling activity* and, at another moment, only one of the vertices of the triangle is changed, namely, the reference context: *formulated problem*.

Where to travel? – Analysis of mathematical modeling activity which theme is Travelling

The “Vegetable Garden at school” project was the impulse that made this mathematical modeling activity happen since, from this project, children choose as an interest to investigate places where they can travel with the money collected from the sale of products from the garden. The understanding of the initial situation, the interpretation of the facts, and the understanding of the dimension of the project are related to the cognitive action of understanding the situation, and, from this, the children signal a problem to be solved: Where to travel with the money collected? This question is a sign that was indicated by Jeferson, and this sign is the result of the cognitive action structuring the situation.

To solve the problem under study, the children delimit strategies for verifying what would be necessary to investigate in order to execute a trip. In Table 5 we present signs that are associated with the children's ways of understanding and thinking about the phenomenon under study (the trip). These signs, corresponding to the cognitive action of understanding the situation, communicate children's interpretations of it.

Table 5.

Signs associated with cognitive action understanding the situation (Authors)

Child	Sign
Jeferson	Where are we staying?
Gina	We need to check about the gas. Transportation.
Gustavo	Gas and toll. No, the first thing is to see how much money we are going to raise and the teacher had already mentioned it at another time because I think there is little money for all that they are talking about.

Nicoly	There is the food.
Samara	Hygiene stuff.

These signs "*Where are we going to stay*?", "*Transportation*", "*We need to check about gas*", "*Gas and tolls*", "*There is the food*", and "*Hygiene stuff*" indicate a direction for the children about what should be considered and what paths children can take, with that the reference context linked to these signs is the theme, and the concept corresponds to the value to be raised in the garden.

However, when they realized the number of things needed to carry out a trip, a new sign emerged: "*No, people, the first thing is to see how much money we are going to raise and the teacher had already mentioned it at another time because I think it is little money for all that they are talking about*" and indicates that the child understands that in order to make a trip it is necessary to know how much money is available and then analyze other elements. Such a sign signals that the amount of money is a hypothesis and is the result of the mathematization cognitive action. In Figure 6 we present information that was shared with the children regarding the school vegetable garden project.

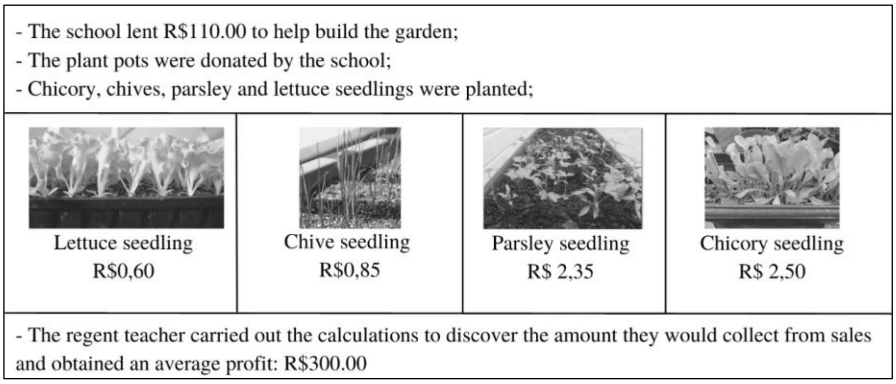

Figure 6.
Information related to the Vegetable Garden project (Authors)

The information about the amount they could possibly raise, which was provided by the teacher at another time, served as a hypothesis for the children that developed the mathematical modeling activity and provoked Gustavo to signal another problem, now showing the average amount of money that they would have raised from the garden: "*Teacher, so if the amount we are going to collect is R$300.00, then our problem will be: Where will we be able to travel with 300 reais*?". This problem corresponds to a sign that indicates a possibility of investigation on the subject and results from the cognitive action structuring the situation. Such a sign has as a context of reference the problem formulated in other ways, that is, considering the hypothesis, and the concept refers to the average value they are assuming.

Considering this new problem, the children devise strategies to identify the amount that each child will receive from the sale of products from the garden. Such strategies (Table 6) correspond to signs associated sometimes with the mathematization cognitive action, sometimes with the synthesis cognitive action and direct the children towards the construction of a mathematical model.

Table 6.

Signs associated with the cognitive action mathematization and synthesis (Authors)

Child	Sign
Samara	If we have $300, how much will each of us have?
Jeferson	We must divide 300 by 18. We shall about the entrance fee for our tour, because we don't have much money and then comes what we were talking about before, our parents pay and we pay the entrance fee.
Gina	It's not working, I don't know how to continue. It resulted 16 under the key and remained 12. But there's no way! 12 doesn't divide 18, and 18 is greater than 12 Jeferson, I'm irritaded. I don't want to do this anymore.

It was the sign "*If we have R$300.00, how much will each of us have*?" manifested by Samara, who referred to the idea of the need to determine the amount that each child will receive with the sale of seedlings, which indicated the recognition of a language transition between the formulated problem and the translation of this problem into a mathematical problem, characteristic of the action cognitive mathematization. This sign represents that the child thought mathematically about the formulated problem and established relationships with the number of children in the class since the amount collected from the garden would be used by all.

In addition to this sign, the sign "*We will have to divide 300 by 18*", manifested by Jeferson, indicates a mathematical concept: division. This sign carries the child's intention to identify how much each one will receive from the sale of products from the garden and signals the search for an answer to the problem, this sign suggests a way to reach the solution to the problem and signals the recognition of that in order to choose the place to travel, it is first necessary to know how much money each person will have at the end.

From this sign manifested by Jeferson, another sign was generated: “*It's not working, I don't know how to continue*. It resulted 16 under the key and 12 was left "and" *But there's no way! 12 didn't divide 18, 18 is bigger than 12 Jeferson, I'm irritated, I don't want to do the math anymore*” manifested by Gina, who reveals that she felt uncomfortable (irritated) because of not being able to divide one number by the other. We consider that this discomfort refers to the division having a decimal number as a quotient. On the other hand, this discomfort favored the researcher teacher to guide the children on how to proceed in relation to mathematical operations that have a decimal quotient. Such orientation is associated with a characteristic of mathematical modeling, which is the opportunity to study mathematical concepts that have not yet been seen or revisit concepts already studied (Veronez, 2013, p. 87). The signs emerging from these cognitive actions, mathematization, and synthesis, have as a reference context the search for a solution to the problem, and the concept is linked to division.

When reflecting on the result obtained from dividing 300 by 18 (300÷18=16.75), which is the value assumed as the amount allocated to each child, Jeferson argues: "*Let's think about free tickets for our trip, because we don't have much money and then comes what we were talking about before our parents pay and we pay the entrance*", this speech is a sign, because it indicates that he knows that the amount of money they have is little and reveals that the child recognizes that money that they will have would only pay the entrance to the place chosen. This sign, associated with the cognitive action of interpretation and validation, signals the child's understanding of the phenomenon under study and its interpretation in the face of the mathematical representation (the value that each child will have) and of their own situation.

As a result of this conclusion, the children consider it necessary to investigate the ticket price (admission) of some places that are close to the city where they live, namely: water parks and cinemas. This investigation provokes the production of signs (Figure 7) that are associated with the cognitive action of understanding the situation since the children seek to know the cost of each of these places.

Ody Park water park Maringá (Paraná)	Solar das Águas Quentes water park Pitangueira (Paraná)	Pôr do Sol water park Pitangueira (Paraná)	Cinemas
Values without lunch:	Values:	Values:	Values:
Wednesday to Friday: Adult: R$ 45.00 Child: R$ 40.00	Adult: R$ 50.00 Child: R$ 30.00	Adult: R$ 50.00 Child: R$ 30.00	3D - Full: R$ 26.00 3D - Half: R$ 13.00
Saturday, Sunday and public holidays: Adult: R$ 60.00 Child: R$ 50.00	Discount for 22 people Adult: R$ 40.00	Discount over 20 people is 10%	3D - Full: R$ 24.00 3D - Half: R$ 12.00

Figure 7.

Signs associated with cognitive action understanding the situation (Authors)

From the analysis of these signs, the children choose to investigate the discounts granted by Pôr do Sol water park, since they realize that there is a possibility of discounting the entrance fees, considering it is a group of people. Table 7 illustrates the signs produced by children resulting from the cognitive synthesis action.

Table 7.

Signs associated with synthesis cognitive action (Authors)

Child	Sign
Jeferson	If it was 50% would it be 25, teacher? Because half of 50 is 25 and half of 100 is 50. 10% for R$100 reais, I don't know. The seller gives you a 10% discount, you take R$10 off it, but R$50.00 is R$5.00 because it is half of R$100.00. Guys, think, R$5.00 is 10% of R$50.00 because 5 times 1 is equal to 5, and 20% is 5 times 2, how much is that? 10 right? 30% is 5 times 3 equals 15, that is, 30% of 50 is 15, 40% is R$20.00, 50% is R$25.00 which is half of R$50.00, 60% is R$30.00, 70% is R$35.00, 80% is R$40.00, 90% is R$45.00, 100% is R$50.00. For every R$3.00 we will earn R$3.00. It's the same reasoning. We have R$30.00 for the passport and we will also have a 10% discount, 10% discount is R$3.00. Because we take the 0 out of 30, we have 3 so let's look at the 3 times table, 3 times 1 equals 3, so we'll have a R$3.00 discount. How much is 30 minus 3, Samara? R$27.00, right
Gina	So we have R$100.00 for the ticket and we are going to do it for a 10% discount, so we take the first two numbers and do 10 times 10 which is equal to 100, so 10% of R$100.00 would be 10 times 1 which equals 10, i.e. 10% of $100.00 is $10.00. I understand, so on the 5 times table, every 10% is R$5.00.
Samara	Okay, but people, what about the ticket for children? Because that was an adult, how will we think, how much will 10% of 30.00 be?

The signs "*If it were 50% would it be 25 a teacher? Because half of 50 is 25 and half of 100 is 50*" and "*10% for R$100 reais, I don't know. The seller gives you a 10% discount, you take R$10 out of it, but R$50.00 is R$5.00 because it's half of the R$100.00*" that was manifested by Jeferson evokes the mathematical concept: percentage. This sign indicates that he is strategically thinking about the concept of percentages and relating it to the division of fractions, as he tries to relate half of 100% (which is the whole). This sign, in addition to carrying the child's interpretation, denotes that he seeks to identify how much discount would be granted considering the minimum group of people. This interpretation corresponds to the cognitive synthesis action, an action in which “the use of concepts, techniques, methods, and representations is necessary” (Almeida; Silva; Vertuan, 2012, p. 18).

As this sign caused strangeness in the children, Jeferson manifested a new sign: “*Ok, think like this, R$5.00 is 10% of R$50.00 because 5 times 1 equals 5, 20% is 5 times 2, how much is it? 10 right? 30% is 5 times 3 equals 15, that is, 30% of 50 is 15, 40% is R$20.00, 50% is R$25.00 which is half of R$50.00, 60% is R$30.00, 70% is R$35.00, 80% is R$40.00, 90% is R$45.00, 100% is R$50.00*”. Such a sign indicates that it is relating the 5 times table with the calculation of the percentage. This strategy of the child was due to him realizing the relation between the percentage and 100%, inferring that it would be enough to remove the zeros and associate the number with the multiplication (table).

This new concept mobilized by the children when developing this mathematical modeling activity favored the production of new signs. This generation of new signs deposes students’ (lack of) knowledge about mathematical objects (Veronez; Chulek, 2020), which means that the composition of new interpretations takes place through the actions that children have when they are trying to solve the problem. The sign “*So we have R$100.00 for the ticket and we are going to do it for a 10% discount, so we take the first two numbers and do 10 times 10 which equals 100, so 10% of R$100, 00 would be 10 times 1 which is equal to 10, that is, 10% of R$100.00 is R$10.00*” indicated by Gina, communicates her understanding from the sign manifested by Jeferson (discussed in the previous paragraphs).

The possibility of the researcher teacher intervening in the discussions and explaining to the children what the percentage is and when they are and can be used is due to the fact that in mathematical modeling the teacher can act “as a guide, directing, explaining, suggest With the solution on the adult entrance fee, Jeferson indicates how much children would pay with the discount: “Every R$3.00 we will earn R$3.00. It's the same reasoning. We have R$30.00 for the passport and we will also have a 10% discount, 10% discount is R$3.00. Because we take the 0 out of 30, then we have 3 so we must look at the 3 times table, 3 times 1 equals 3, so we'll have a R$3.00 discount. How much is 30 minus 3 Samara? R$27.00 right”. This sign indicates the concept: percentage and multiplication and signals the park entrance fees with discount. This way of writing the percentage by the child is configured as an answer to know the value of the percentage, however, it does not correspond to the correct representation of the percentage, since the association made by Jeferson enables only tens that have zero as a unit.ing, exposing and helping students during the activities" (Lima, 2020, p. 32).

Thus, all the signs associated with the cognitive synthesis action (Table 7) have as a reference context the investigative process linked to the search for a solution to the problem and the concept is associated with multiplication (table), division, and percentage.

Understanding the amount paid for the ticket to the water park, the children consider analyzing the response obtained. This process recurs in the cognitive action of interpretation and validation (Table 8).

Table 8.

Signs associated with cognitive action, interpretation, and validation (Authors)

Child	Sign
Jeferson	It won't be possible to go to Pôr do Sol, R$10.40 to R$27.00 is missing.
Gustavo	Let's think, the teacher said that we will be able to raise 300 reais with our garden until the month of November, we are in September. So September, October, November, 2 to 3 months for the vegetable garden to grow and we sell the stuff, and if we do a little more in the teacher garden? We could make it until December. Let's think of a plan B, our plan A is to travel to Pôr do Sol, so let's see plan B. Teacher, we have an answer, with R$300.00 we can go to the cinema and if we work a little more to raise it, let me see here, it is R$187.20 more for we can go to Pôr do Sol.

After identifying the value of the water park ticket, the sign "*It won't be possible to go to Pôr do Sol, R$10.40 to R$27.00 is missing*" manifested by Jeferson signals that the child reflected that the amount of money to be collected from the garden is not enough to pay it. In addition, this sign brings up this child's use of the concept of subtraction.

From the evaluation of the obtained solutions and tested conclusions, a new sign was generated: "*Let's think, the teacher said that we will be able to raise R$300 reais with our garden until the month of November, we are in September. So September, October, November, give 2 to 3 months for the vegetable garden to grow and sell, and if we do a little more in the teacher garden*?". This sign, manifested by Gustavo, in addition to indicating that he reflected on the obtained solutions, suggests a possibility to raise the money that is missing for them to go to the water park. Gustavo also realizes that postponing the trip could be a possibility, as they could also consider another location. The signs: "*Let's think of a plan B, our plan A is to go traveling at sunset so, let's see the plan B.*" and "*Teacher, we have an answer, with R$300.00 we can go to the cinema and if we work a little more to raise money, let me see here, R$187.20 more we can go to Pôr do Sol.*" indicate these new possibilities: going to the movies or extending the trip and raising more money.

Therefore, the children conclude that to pay for the ticket to the water park it is necessary to raise more money, and to go to the cinema they will have enough money. These signs associated with the cognitive action, interpretation, and validation (Table 8) have as a reference context the analysis of the responses obtained and the concept corresponds to the mathematical objects used to solve and the possibilities indicated as planes A and B.

The signs produced in this mathematical modeling activity caused an alternation of reference contexts and also of concepts. The association between these three elements is illustrated in the epistemological triangle of Figure 8.

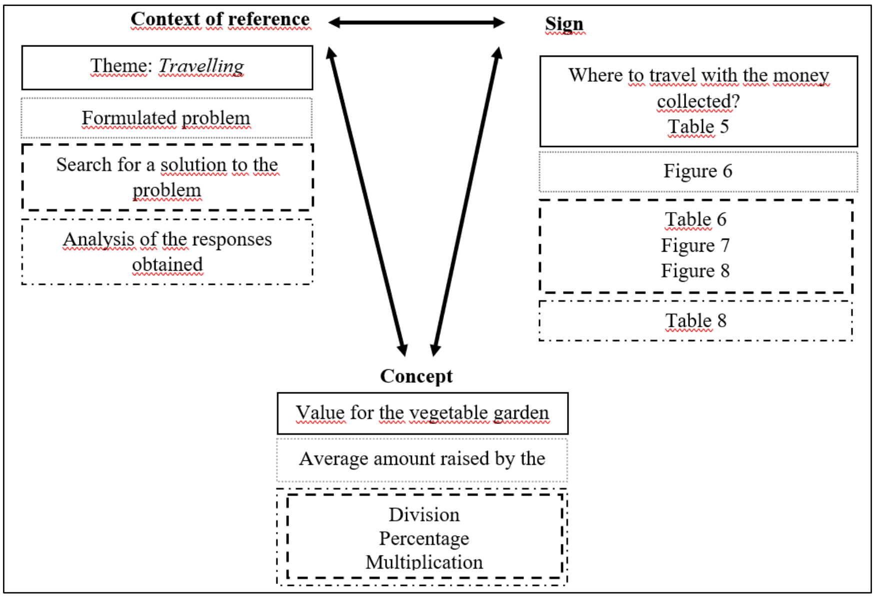

Figure 8.

Epistemological triangle of mathematical modeling activity Travelling (Authors)

In this epistemological triangle, the text boxes also have different shapes, that together illustrate the alternation between the three elements of the epistemological triangle. Thus, the signs: Where to travel with the money raised? and Table 5 that are inside a text box with a continuous outline, as well as the reference context (Theme: travelling) and the concept (value for the garden) that are with the same outline and are associated with each other. Thus, these three elements constitute an epistemological triangle of this mathematical modeling activity, among many others that appear constructed in Figure 8.

In the next section we discuss about the objective of this study based on the epistemological triangles we constructed, which correspond to our semiotic interpretation of the two mathematical modeling activities described and analyzed here.

Results and discussion

The search for evidence of meaning attribution to mathematical objects through the signs associated with mathematical modeling is based on the signs produced by children throughout the development of the two mathematical modeling activities addressed in the previous sections. Our semiotic interpretation is therefore illustrated in the epistemological triangles we construct for each of these activities.

In our interpretation, the signs, associated with children's cognitive actions, carry, in general, characteristics of the situation under study and, more specifically, knowledge about mathematical concepts. With each sign produced by the children, we identified relations with the other two vertices of the triangle: the context of reference and the concept. Thus, such signs changed both the context of reference and the concept. The reference context was altered within the context of the characteristic elements of Mathematical Modeling and the concept alternated according to the concepts, both of the situation and of Mathematics, that were raised.

The fact that the signs alter, modify and, in a way, complement each other denotes that children carry out different interpretations, which are sometimes related to the situation, sometimes with Mathematics and sometimes considering articulations between situation and Mathematics. It also signals that the signs correspond to what Peirce (2005) points out when he emphasizes that the signs represent something that one wants to communicate.

The dynamism of the signs produced by the children indicates that their interpretations are being altered, rethought and modified as they find out new interpretations, become familiar with the subject under study and consider its characteristics and the mathematical concepts that are used in the search for a solution. Hence the fact that this dynamism of signs favors the alternation of the other two elements of the epistemological triangle: context of reference and concept.

In particular, the signs produced by the children suggest clues of attributing of meaning to mathematical objects when, in the FPS-themed activity, the chain of signs produced by them alters the context of reference and the concept linked to them. The fact that children produce signs in a dynamic process, which portrays their interpretations and familiarity with the subject under study, and in relation to what they are analyzing, denotes that they attribute meaning both to what they do and to the mathematical objects they bring through their interpretations.

Regardless the nature of the signs produced by the children (statements, questions, conjectures, considerations), in association with their cognitive actions, we evidenced that the attribution of meaning to mathematical objects does not occur in a tight way. On the contrary, it happens in articulating these signs with the other two elements of the epistemological triangle: context of reference and concept, and cannot be recognized separately from them. Thus, we consider that the attribution of meaning to mathematical objects happens in the association between the signs produced by children, the reference context that elucidates aspects of mathematical modeling and the concepts raised in the process of understanding the situation and in the process of solving the study problem.

In the case of the activity whose theme is Travelling, the attribution of meaning to the mathematical objects is intertwined with all the signs that the children produced and that, in a certain way, refer to the situation that originated the activity, that is, the interest of the children for investigating where they could travel to with the money raised through the school vegetable garden project. After all, in fact the attribution of meaning is a compound that considers the situation and mathematics in articulate ways, manifested by signs.

As a whole, we can infer that the signs produced changed as they gained consistency and complemented each other in view of the children's cognitive actions in relation to specific aspects of the situation under study. This alternation of signs brings out the dynamic character of Mathematical Modeling and expresses, in the constructed epistemological triangles, that such signs cannot be analyzed in a dissociated way, since they portray intentions, familiarities and interpretations of children: with the topic, with the problem, with the mathematical object and with the answer to the problem under study.

Other considerations

Considering the interest in seeking evidence of meaning attribution to mathematical objects through the signs associated with mathematical modeling, we turn our attention to two mathematical modeling activities, in which, when looking at each activity in particular, we seek from the collected data and from the analysis conducted, to discuss, semiotically (by means of epistemological triangles), about the signs produced by the children.

In such triangles, the vertices changed, alternated or evolved according to the children's interpretation throughout the activities. The signs, associated with cognitive actions, had different natures and were wether more associated with aspects of the situation or with mathematical characteristics. However, it was the children's cognitive actions that favored the production of signs that, consequently, altered the context of reference and the concept. The reference context, most of the time, was linked to specific aspects of mathematical modeling, however, the concept sometimes referred to Mathematics, sometimes to aspects of the situation.

This alternation of the vertices of the constructed epistemological triangles, in addition to highlighting a characteristic proposed by Steinbring (2005), that the relation between the three vertices is flexible and that they can never be analyzed in isolation, enhances the dynamicity of mathematical modeling and suggests that the dynamic process of production of signs is what favors attribution of meaning to mathematical objects in mathematical modeling.

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