

<http://dx.doi.org/10.23925/1983-3156.2024v26i1p259-282>

O estudo de aula no Brasil e a formação inicial de professores de matemática: Uma revisão de literatura

The lesson study and the initial education of mathematics teachers in Brazil: A literature review

El estudio de clases y la formación inicial de profesores de matemática en Brasil: Una revisión de la literatura

L'étude en classe et la formation initiale des professeurs de mathématiques au Brésil : Une revue de la littérature

Roselene Alves Amâncio¹
Universidade Federal de Minas Gerais
Doutorada em Educação
<https://orcid.org/0000-0001-9118-528X>

Samira Zaidan²
Universidade Federal de Minas Gerais
Doutorada em Educação
<https://orcid.org/0000-0001-7163-5546>

Resumo

O estudo de aula é um processo de formação docente que visa a melhorar a qualidade do ensino e da aprendizagem por meio de um trabalho coletivo e reflexivo que envolve a definição do objetivo da aula, estudos relacionados ao tópico a ser ensinado, planejamento detalhado da aula, condução da aula por um dos participantes e análise após a sua realização. Esse processo se originou no Japão ao final do século XIX e tem sido desenvolvido em vários países com docentes em serviço e/ou em formação inicial, passando por adaptações. Neste texto, apresentamos um panorama das produções científicas relativas ao estudo de aula, realizadas no âmbito da formação inicial de professores de matemática, até o ano de 2021. Nossa intenção é evidenciar possibilidades e desafios para o desenvolvimento desse processo formativo, bem como, suas contribuições. As pesquisas analisadas mostram que o estudo de aula pode propiciar várias contribuições para a formação de futuros professores de matemática, principalmente em relação à articulação entre teoria e prática e, também, favorecer a aproximação entre universidade e escola básica.

¹ roseleneamancio@ufmg.br

² saamira@fae.ufmg.br

Palavras-chave: Estudo de aula, Formação inicial de professores, Revisão de literatura.

Abstract

Lesson study is a teacher education process that seeks to improve the quality of teaching and learning through collective and reflective work, which involves defining the objective of the class, studies related to the topic to be taught, detailed lesson planning, class conducting by one of the participants and analysis after its completion, which culminates in the improvement of the planning. This process originated in Japan at the end of the 19th century and has been developed in several countries, with adaptations, with in-service and/or pre-service teachers undergoing adaptations. In this text, we present an overview of the scientific productions related to the lesson study developed within the scope of the education of pre-service mathematics teachers, which took place in Brazil until 2021. We intend to highlight possibilities and challenges for developing this formative process and their contributions. The analyzed surveys show that the lesson study can impact prospective mathematics teachers' education, mainly the connection between theory and practice, favoring an approximation between the university and basic school.

Keywords: Lesson Study, Pre-service teacher training, Literature review.

Resumen

El Estudio de Clases es un proceso de formación docente que tiene como objetivo mejorar la calidad de la enseñanza y el aprendizaje a través del trabajo colectivo y reflexivo que implica definir el objetivo de la clase, estudios relacionados con el tema a impartir, planificación detallada de lecciones, conducción de la clase por uno de los participantes y el análisis posterior a su realización. Este proceso se originó en Japón a fines del siglo XIX y se ha desarrollado en varios países con docentes en servicio y/o en formación inicial, sufriendo adaptaciones. En este texto presentamos un panorama de las producciones científicas relacionadas con el Estudio de Clases, realizadas en el ámbito de la formación inicial de profesores de Matemática, hasta el año 2021. Nuestra intención es resaltar posibilidades y desafíos para el desarrollo de este proceso formativo, así como sus aportes. Las encuestas analizadas muestran que el Estudio de Clases puede traer varios aportes a la formación de futuros profesores de Matemática, principalmente en relación a la articulación entre teoría y práctica y, también, para favorecer la aproximación entre universidad y escuela básica.

Palabras clave: Estudio de clases, Formación inicial del profesorado, Revisión de literatura.

Résumé

L'étude en classe est un processus de formation des enseignants qui vise à améliorer la qualité de l'enseignement et de l'apprentissage par un travail collectif et réflexif qui consiste à définir l'objectif de la classe, des études liées au sujet à enseigner, une planification détaillée, la conduite de la classe par l'un des les participants et l'analyse après son achèvement. Ce procédé est né au Japon à la fin du XIXe siècle et s'est développé dans plusieurs pays avec des enseignants en activité et/ou en formation initiale, en cours d'adaptation. Dans ce texte, nous présentons un aperçu des productions scientifiques liées à l'étude en classe, réalisées dans le cadre de la formation initiale des enseignants de mathématiques, jusqu'à l'année 2021. Notre intention est de mettre en évidence les possibilités et les défis pour le développement de ce processus formatif, ainsi que leurs apports. Les enquêtes analysées montrent que l'étude en classe peut apporter plusieurs contributions à la formation des futurs enseignants de mathématiques, principalement en ce qui concerne l'articulation entre la théorie et la pratique et, également, pour favoriser le rapprochement entre l'université et l'école fondamentale.

Mots-clés : Étude en classe, Formation initiale des enseignants, Revue de la littérature

Lesson study and the initial education of mathematics teachers in Brazil: a literature review

Mathematics degree courses in Brazil are usually little connected with the reality of basic education schools, as argued by Moreira and David (2005), Fiorentini and A. T. C. C. Oliveira (2013), and Moreira and Ferreira (2013). Fürkötter and Morelatti (2007) highlight a pivotal issue in teacher education courses: the lack of connection between theory and practice. The Bulletin produced by the partnership between the Brazilian Society of Mathematics Education and the Brazilian Society of Mathematics (SBEM/SBM, 2013) underscores the need to develop strategies to rethink mathematics teachers' undergraduate courses, including the reorganization of the entire course—all subjects and activities—to train the professionals to work in basic education.

Branco and Ponte (2008) indicate how important it is that prospective teachers' education favors the construction of knowledge for teaching mathematics involving learning processes, curriculum guidelines, types of tasks, and classroom work.

Lesson study can enhance teachers' initial education because it is a collaborative and reflective process centered on teaching practice, as Murata (2011) affirms.

Lesson study originated in Japan at the end of the 19th century and is widely used in regular public education. Fujii (2014) explains that the lesson study started with the beginning of formal education in Japan. It is intrinsic to teaching and implemented in everyday school activities. According to Isoda and Ofos (2009), in the 1980s, it became known in the United States, and from the 1990s onwards, it began to be developed in other Western countries.

Lesson study is carried out by a group of teachers and/or pre-service teachers working together. Researchers, university teacher educators, and school managers can also participate. It begins with defining an objective for the investigation class, considering students' difficulties when learning the topic covered. Then, they conduct studies and plan a class collectively in detail. One of the participants teaches while the others observe. Soon after, participants meet to analyze the class and improve planning.

This investigation seeks to highlight possibilities and challenges for the development of a lesson study in mathematics teachers' initial education and identify contributions to this formative process.

Given the above, we conducted a literature review that included the analysis of scientific articles and doctoral and master's degree theses whose research addresses lesson studies in the context of mathematics teachers' initial education in Brazil, considering the objectives set and guided by the questions: "How has lesson study been developed in mathematics teachers' initial

education?” “What are the challenges to developing the lesson study in initial education?” and “How can mathematics study classes contribute to the education of prospective mathematics teachers?”

In the productions considered in our investigation, we found, addressing the formative processes, the Japanese expression *Jugyou Kenkyuu* (JK), translated into English as lesson study (LS) and *estudo de aula* (EA) in Portuguese).

The works analyzed show that the lesson study can contribute to prospective teachers’ education and favor the coordination between school and university, revealing different possibilities and challenges in its development.

The lesson study as a formative process

The lesson study is a collective and reflective process carried out by in-service and/or pre-service teachers whose primary focus is student learning (Ponte et al., 2006). The action consists of the stages in Figure 1.

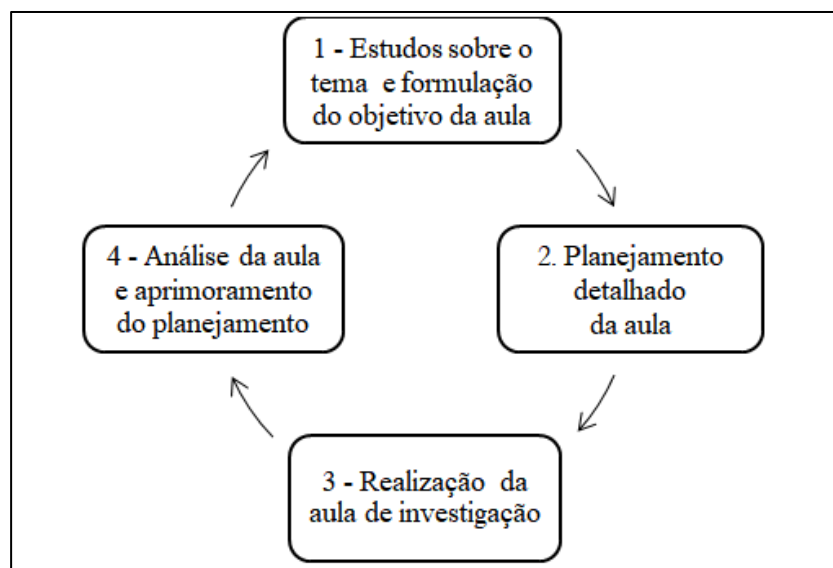


Figure 1.

Stages of the lesson study. Source: Adapted from Fujiji (2014)

The first stage aims to select and study the chosen topic and formulate the lesson objective. Fujii (2014) clarifies that the lesson study at school begins with teachers discussing educational goals and students’ current situation and then defining a specific objective for the lesson. The author explains that a chosen topic may be something that teachers find challenging to teach or a subject that, for students, may seem more difficult; or it may seem easy, but essential concepts derive from it, or the topic may be related to newly introduced content in the curriculum; and so on.

In the second stage, participants plan the class in detail. They select, adapt, or design tasks; select teaching support materials that can promote student learning; seek to anticipate possible strategies and students' doubts or errors; and prepare their interventions and the various moments of the class. Regarding the task types, Rich (2020) observes that although the problem-solving perspective has been the focus of the lesson study in several countries, in Portugal, emphasis has been placed on exploratory activities.

In the third stage, one of the participants teaches the class to a group of students while the others observe what the students are doing, how they solve problems, and the line of reasoning they use in discussions with their classmates or the teacher. Therefore, as Ponte et al. (2017) explained, students are at the center, i.e., the activity focuses on their strategies and difficulties, instead of the teacher's work.

In the fourth stage, the group meets to analyze the practice. At this time, participants discuss their observations with a focus on student learning. Given this, they can change the planning in areas they consider necessary to improve, such as statements of the proposed tasks, materials used, questions the teacher can ask, and treatment of students' doubts, among other aspects.

When developing the lesson study, the characteristics of each country's school culture, the vision of education, and the education of the participating teaching group must be taken into account. Regarding adapting the lesson study in other cultures, Rich (2020) says that its development outside of Japan requires some adjustments that consider the professional and cultural scenarios of each context. However, Fujii (2014) warns that, in specific countries, the strategy has been developed with some inaccurate aspects, highlighting the importance of keeping the principles of the formative process. In that regard, M. A. V. F. Souza (2022) states:

Planning, executing the plan, and reflecting on student learning results is simple, but, looking closer, it is a complex process that requires attention to many details. Adaptations of the Japanese LS to other school cultures are necessary and expected, but they cannot harm its essence. (M. A. V. F. Souza, 2022, p. 56).

Regarding the development of the lesson study in prospective teachers' education, Elipane (2012) states that, in the Japanese educational system, national universities that offer teacher education courses are linked to laboratory schools called Fuzoku Schools. The author also explains that pre-service mathematics teachers' practicum in these schools incorporates fundamental elements of the lesson study that favor the construction of knowledge for

teaching mathematics and also provide an understanding of the social norms that involve a mathematics class.

Methodological aspects of this research

To meet the objectives of this investigation, we carried out qualitative research (Lüdke & André, 1986), descriptive and analytical (Fiorentini & Lorenzato, 2006), with state-of-the-art characteristics (Palanch & Freitas, 2015), to present an overview of scientific productions related to the lesson study developed within the scope of mathematics teachers' initial education in Brazil until 2021. We did not restrict the beginning of the data collection period, as productions in this field are still recent.

The empirical material was constituted in five stages. Initially, we searched the catalog of doctoral and master's degree theses (*Catálogo de Teses e Dissertações*) of the Coordination for the Improvement of Higher Education Personnel (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Capes), based on the descriptors “*estudo de aula*” and “lesson study.” After eliminating repetitions, thirty-nine productions between 2010 and 2021 remained. In the second stage, we read the titles of the works and keywords. Four works were discarded for not being related to that formative process, and another seven for being from different areas of mathematics education. In the third stage, we read the abstracts of the twenty-eight works to identify the productions that focused on initial teacher education. However, some abstracts did not contain that information, making reading parts of the texts necessary. Thus, we discarded twenty-three works aimed at continuing education. In this way, we selected five studies –two doctoral and three master's theses– focusing on the lesson study formative process in the context of mathematics teachers' initial education.

This initial survey verified a lack of research within the scope of postgraduate programs in the country in the period considered, addressing the lesson study in mathematics teachers' initial education. That said, we conducted a second survey of the leading Brazilian journals focused on mathematics education to select articles that also addressed empirical investigations using the lesson study in initial teacher education.

Thus, in the fourth stage, we accessed the website of each of the 19 journals we chose for our investigation to conduct a search procedure with the descriptors “*estudo de aula*” and “lesson study,” indicating the year 2021 as the closure period. In this process, 27 articles published between 2014 and 2021 were identified, as shown in Table 1.

Table 1.

Journals with the number of articles identified on lesson study

Journal	Number of articles identified
Amazônia – Revista de Educação em Ciências e Matemática	1
Bolema – Boletim de Educação Matemática	6
Boletim GEPEM	2
Educação Matemática Pesquisa	5
Jornal Internacional de Estudos em Educação Matemática	2
Perspectivas da Educação Matemática	5
Rencima – Revista de Ensino de Ciências e Matemática	1
RIPEM - Revista Internacional de Educação Matemática	1
Vidya	3
Zetetiké	1

In addition to the ten journals shown in Table 1, we also searched nine other journals that did not return articles: Educação Matemática em Revista; Ensino de Matemática em Debate; Revista de Educação, Ciências e Matemática; REMATEC – Revista de Matemática, Ensino e Cultura; Revista de Educação Matemática; REVEMAT – Revista Eletrônica de Educação Matemática; Revista Paranaense de Educação Matemática; Revista Sergipana de Matemática e Educação Matemática; Tangram – Revista de Educação Matemática.

In the fifth stage, we read the titles, keywords, and abstracts to identify works in which a lesson study was carried out in Brazil in the context of initial teacher education. However, some needed reading part of the text to identify this information. In this way, we discarded twelve works that referred to research carried out abroad and ten related to the teacher continuing education, leaving five works. Even so, two referred to research already selected in our first survey in the Capes theses catalog. Therefore, we downlisted three articles for our analysis.

The productions that constituted the empirical material of our study are shown in Table 2.

Table 2.

Title, authors, and type of production of the works analyzed

Title	Authors/year	Production type
A metodologia da Lesson study na formação de professores: uma experiência com licenciandos de matemática [The lesson study methodology in teacher education: an experience with mathematics undergraduates]	Coelho, 2014	Master's thesis
Formação de professores para aula de resolução de problemas a partir de um Lesson Study: contribuições, constrangimentos e desafios [Teacher education for problem solving classes based on a lesson study: contributions, constraints, and challenges]	Küster, 2020	Master's thesis
(Res)significações de saberes por licenciandos que vivenciam estudo de aula sobre distância entre dois pontos [(Re)signification of knowledge by undergraduate students who experience lesson study on distance between two points]	M. A. Oliveira, 2020	Master's thesis
Contribuições da Jugyou Kenkyuu e da engenharia didática para a formação e o desenvolvimento profissional de professores de matemática no âmbito do estágio curricular supervisionado [Contributions of Jugyou Kenkyuu and didactic engineering to the education and professional development of mathematics teachers within the scope of the supervised practicum]	Silva, 2020	Doctoral thesis
Estudo de aula de matemática com robótica educacional na formação inicial do professor de matemática [Mathematics lesson study with educational robotics in initial mathematics teacher education]	C. F. Souza, 2021	Doctoral thesis
A 'Glocal' Lesson Study: the case of pedagogical practices in mathematics	Rincón & Fiorentini, 2017	Article
Metodologia Lesson Study na Licenciatura em Matemática: possibilidade para a formação inicial [Lesson Study methodology in the mathematics teaching degree: possibility for initial education]	Carvalho, 2020	Article
Aprendizagens de Futuros Professores de Matemática em um Estágio Curricular Supervisionado em Processo de Lesson Study [Learning of prospective mathematics teachers in a supervise practicum in a lesson study process]	Neves & Fiorentini, 2021	Article

After constituting the empirical material, we read the articles in full, covering the guiding questions previously mentioned. Therefore, we prepared a file for each of the works,

considering the following categories that were defined a priori: (i) participants and development context; (ii) definition of the class theme; (iii) studies carried out; iv) lesson planning; v) execution of the class; (vi) class analysis; (vii) types of proposed tasks; (viii) challenges experienced; (ix) contributions.

Participants and development contexts

Coelho (2014) adapted the lesson study for five Mathematics Didactics II classes at the Mathematics Institute of the UFRJ, whose professor was supervised by the author.

Twenty-three undergraduate students attending the subject Pedagogical Practices in Mathematics at Unicamp Mathematics Institute participated in the research by Rincón and Fiorentini (2017). They voluntarily organized into six groups so that at least one undergraduate in each group should be a teacher, a prospective teacher, or a scholarship holder in a public institution or private school. The first author was the subject professor acting as a teacher educator and researcher.

In Silva (2020), the formative process was experienced in connection with a subject of the supervised practicum at a mathematics teaching degree course at a public university in Pernambuco. The process was developed with six pre-service teachers willing to participate in the research, with a basic education teacher who acted as a university practicum teacher educator and the researcher who also played the role of an educator. For the development of the formative process, they considered elements of the lesson study and didactic engineering.

Küster's (2020) research involved seven students from a mathematics teaching degree course at a Brazilian public university during classes of Initiation to the Supervised Teaching Practice with the presence of two research professors –one is the author of the master's thesis, and the other, the subject professor– and continued in the following semester, when the prospective teachers were taking the supervised practicum, also counting on the participation of the teacher educator responsible for the practicum subject.

M. A. Oliveira (2020) worked with six academics studying for a teaching degree in mathematics at the Instituto Federal do Acre, Campus Cruzeiro do Sul, and accepted the invitation to participate in a study group. The researcher also acted as a teacher educator, coordinating the study group.

Carvalho's research (2020) adapted the lesson study for twelve undergraduate students studying the subject Exploratory Teaching in Mathematics Classes, an optional subject offered by the mathematics degree course at the Mathematics Institute of the Federal University of Alagoas. The researcher was the subject teacher.

Souza (2021) was developed in a supervised practicum and a project called ROBOMAT with sixteen students from the mathematics teaching degree course at the Federal University of Goiás enrolled in the Supervised Teaching Practice I subject. The researcher and the professor of this discipline acted as teacher educators. The pre-service teachers formed six subgroups, each developing one to three lesson study cycles, totaling twelve classes.

In research by Neves and Fiorentini (2021), a hybrid version of the lesson study included elements specific to this proposal and the collaborative model of professional development used by the Saturday Group with narrative analysis. The first author worked as a teacher educator for the practicum in the mathematics teaching degree course at the University of Brasília (UnB). Seventeen prospective teachers participated in the research. They composed the four subgroups with the respective practicum teacher educator. In this article, the authors focus on the experience lived by one of these subgroups.

When focusing on the context in which the studies selected in our investigation were developed, we found that one was carried out in a study group coordinated by the researcher (M. A. Oliveira, 2020), four were carried out within the scope of the supervised practicum (Küster, 2020; Silva, 2020; C. F. Souza, 2021; Neves & Fiorentini, 2021) and three in subjects of the mathematics degree not directly linked to the practicum (Coelho, 2014; Rincón & Fiorentini, 2017; Carvalho, 2020).

In the investigations in a practicum context, the teacher educators of the university actively participated in the lesson study. Regarding the basic education teachers who acted as pre-service teacher educators, Silva (2020) and Neves and Fiorentini (2021) reported several moments when they actively participated in lesson planning and analysis. They were also present during class conduction. C. F. Souza (2021) mentions that the teacher educator followed the activities put into practice and participated in the moments of class analysis with the group. Küster (2020) does not mention the practicum teacher educator.

In research carried out in postgraduate programs, the authors were not in-service or pre-service (or prospective) teachers; however, in four of them (Coelho, 2014; Küster, 2020; Silva, 2020; C. F. Souza, 2021), those teachers performed the role of a teacher educator, cooperating with the faculty. M. A. Oliveira (2020) acted as the sole teacher educator and researcher. In the other investigations, the faculty conducted the lesson study themselves (Rincón & Fiorentini, 2017; Carvalho, 2020; Neves & Fiorentini, 2021).

A point of convergence in the works analyzed is that at least one of the authors was both a researcher and teacher educator, as expected, as one of the principles of the lesson study is

the collaboration between participants. As Desgagné (2007) states, in collaborative research, the researcher plays the double role of a teacher educator and researcher.

Choosing the themes of the classes

During our investigation, we found that in the studies by Rincón and Fiorentini (2017), Carvalho (2020), and Neves and Fiorentini (2021), the prospective teachers were free to choose the themes of the research classes. In Souza (2021), each team of prospective teachers, using educational robotics as a tool, researched, chose, and built a robot and identified the mathematical concepts that would be covered during the class with this robot.

In four studies, researchers chose or guided the choice of themes. Coelho (2014) defined the class theme for each of the two groups, formed by mathematics degree undergraduates, because he considered that the students did not have experience and knowledge of the curriculum; in addition, choosing the topic could be time-consuming. Silva (2020) proposed that the topic be related to magnitudes and measurements and consider the 7th-grade school curriculum; therefore, the classes covered length, area, and perimeter, as the time would not be enough to work with volume, even though it was foreseen in the school curriculum. In M. A. Oliveira's (2020) work, the theme "distance between two points" was chosen by the researcher. Küster (2020) made three problems involving varied themes available so prospective teachers could analyze them and pick one to address in the research class.

Studies performed

The analyzed studies are shown in Table 3.

Table 3.

Studies on each work analyzed.

Authors/year	Studies on Lesson Study	Consultation of curricular documents	Analysis of materials on the class topic	Other studies
Coelho, 2014				x
Küster, 2020	x			
M. A. Oliveira, 2020	x		x	x
Silva, 2020		X	x	
C. F. Souza, 2021	x			x
Rincón & Fiorentini, 2017	x	X	x	x
Carvalho, 2020	x			x
Neves & Fiorentini, 2021		X	x	

Regarding carrying out studies to support lesson planning, Ponte et al. (2017) explain that participants in a lesson study consult curriculum guidelines, analyze teaching materials, make the most accurate diagnosis possible of their students' difficulties, and seek to know the results of research on students' problems and teaching strategies related to the selected topic. In our investigation, we found that, in some works, some studies were not about lesson study or the topic of the research class (indicated in the last column of Table 3), such as characteristics of a student-centered class, exploratory activities, and educational robotics.

Lesson planning

One of the advantages of the lesson study formative process is the collaborative planning that goes beyond the sections generally considered in lesson plans, being prepared with a focus on student learning and anticipating possible doubts, errors, and resolution strategies, in addition to the teacher's actions at various moments in the class.

The collaborative planning of the lesson study method differs from other planning not exactly in the sessions described by didactics authors –objectives, goals, content, procedures, resources, and assessment, for example– but, above all, in the ingredients regarding the *modus operandi* carried out by the group of teachers (Gaigher et al., 2017, p. 56).

In four studies (Küster, 2020; Carvalho, 2020; Neves & Fiorentini, 2021; C. F. Souza, 2021), the plans included several elements of the lesson study.

In Küster's research (2020), the lesson study approach was introduced and discussed after the undergraduates outlined the first planning of a class in which a problem would be posed. Subsequently, they carried out new planning aided by the researchers based on the same problem as the first. In this situation, the prospective teachers reworked the problem statement, thought of possible ways to solve it, ways of questioning students to guide them in interpreting and solving the problem, and sought to anticipate potential errors. Therefore, the second plan was prepared in detail.

In Carvalho (2020), when preparing the research class planning, they discussed students' possible questions and answers, the use of teaching resources to favor problem resolution and possibilities for prospective teachers' intervention.

According to Neves and Fiorentini (2021), class planning was designed in such a way that:

Each subgroup studies, discusses, and prepares the research class plan containing the task (preferably exploratory-investigative) to be developed and a detailed description of the ways of organizing the classroom and time, anticipating students' potential doubts or difficulties and predicting mediational strategies to be adopted by the prospective teachers. (Neves & Fiorentini, 2021, p. 11)

In C. F. Souza (2021), class planning was discussed with the entire group, who sought to analyze the structure of the classes, the coherence in the sequence of these classes, evaluation of both the mathematical and the robotics content that would be worked on in each class, the proposed tasks and their order in each class, and predict students' difficulties for each task and the time required for completion.

In the other works, some plans did not include elements foreseen in the lesson study. In Coelho's (2014) research, only one of the subgroups sought to anticipate students' questions and answers and the teacher's actions. The other subgroup prepared the plan in a very incomplete way. On the other hand, in Rincón and Fiorentini's (2017), some subgroups did very detailed planning. Still, one of the subgroups only listed what they would do in class without considering the different possibilities of student responses and actions for each

situation. In the lesson plan that appears as an appendix to M. A. Oliveira's production (2020), there is more information about the first moment of the class, which was planned to be developed in an expository-dialogue manner. After that, students should take up solving the textbook tasks; however, they merely cited the items to be solved in the book. Silva's work (2020) included five double classes according to the course determination for the practicum. The first classes were discussed in detail. However, the last one was planned less in advance, and there was no preparation to think about the possible answers and challenges of each problem. Although the researcher, supervisor, and educator requested it, the pre-service teachers did not prepare written plans. Thus, the outlines were transformed into slides that were used during classes.

Regarding the way planning occurred, in research by Rincón and Fiorentini (2017), Silva (2020), Neves and Fiorentini (2021), and C. F. Souza (2021), the prospective teachers collectively developed the first draft of the plan, which they changed after discussions held with the educators and other participants. In Carvalho (2020), Küster (2020), and M. A. Oliveira (2020), planning was carried out jointly by all participants from the beginning. In Coelho's (2014) investigation, the researcher and the subject teacher chose not to interfere with the plans the undergraduates constructed nor carry out any a posteriori review, just clarifying the questions asked by the prospective teachers.

Class execution

In investigations with contexts unrelated to the undergraduate practicum, the research class is offered alternatives. In research by Coelho (2014), one of the subgroups of undergraduates acted as teachers, and the other group as basic education students. Afterwards, the roles were reversed. M.A. Oliveira (2020) created a study group to develop the formative study, and the class was imparted in a grade where the researcher himself taught. In Carvalho (2020), the research class was held in a grade where one of the pre-service teachers played the part of the regular teacher. In Rincón and Fiorentini (2017), undergraduates organized themselves into teams, and the classes were imparted in public or private schools where at least one worked as an in-service or pre-service teacher or a monitor.

A strategy used by M. A. Oliveira (2020), Küster (2020), and Neves and Fiorentini (2021) was to propose that one of the students teach the planned class to the other colleagues, enabling new reflections on the details of the class and changes to the plan. Furthermore, in the process Rincón and Fiorentini (2017) developed, two subgroups made a second application,

including their reflections on the first class and the contributions and suggestions of their fellow undergraduates. In Silva's (2020) research, the pre-service teachers conducted classes in pairs.

Class analysis

The researchers organized meetings in the eight works examined to analyze the classes. However, in M. A. Oliveira (2020), the meeting was intended to analyze the formative process experienced, and the class analysis was not initially planned –mainly the reflection on the negative points– so a meeting was held on the points to be improved in the classroom ten months after its execution. Silva's work (2020) included five double classes, but analysis meetings were only held for the first three.

We also observed that the class analysis meeting included points to be changed in class planning in research by Carvalho (2020), Küster (2020), Silva (2020), and C. F. Souza (2021). According to Coelho (2014), the reconstruction of the lesson plan was thought of but not done due to lack of time.

Proposed tasks

In their works, Rincón and Fiorentini (2017), Neves and Fiorentini (2021), and C. F. Souza (2021) propose exploratory-investigative tasks. Carvalho (2020) and Küster (2020) posed problems. According to M. A. Oliveira (2020), the pre-service teachers carefully selected challenging tasks from the textbook, yet the class was conducted in an expository and not very dynamic. According to Silva (2020), not all proposed tasks were challenging for students, although sometimes, attention to activity selection was highlighted during the meetings. Furthermore, the pre-service teachers seemed to value definitions as a baseline for carrying out tasks, thus getting away from a class centered on solving problems.

Some works express that it was essential to rely on previous experiences (Neves & Fiorentini, 2021) or carry out studies (Rincón & Fiorentini, 2017; Carvalho, 2020) about these types of tasks so that the prospective teachers could plan classes better. Some prospective teachers experienced challenges conducting classes with tasks requiring the teacher to foster students' active participation, exchanging ideas, and sharing resolution strategies, unlike classes based on exercises. In this sense, C. F. Souza (2021) demonstrates that the pre-service teachers were instructed to develop open-ended tasks, which enabled mathematical investigation; however, as the exploratory approach was something entirely new for them –used as they were to traditional teaching– some had difficulties in acting as mediators of the teaching-learning process by following this approach.

Therefore, on the one hand, we conclude that the lesson study can enable pre-service mathematics teachers to plan, execute, and analyze classes with challenging tasks (problem solving, exploratory tasks). On the other hand, it shows that initial education courses must allow undergraduates to experience classes that include different types of tasks in the curriculum components from a theoretical and practical point of view.

Challenges

In some works, authors cited difficulties experienced during the lesson study. The first we highlight is time because it is a collaborative teaching and formative process that requires a protracted development period. Coelho (2014) made changes so that this process could be developed in just five meetings, stating that the lesson plans were not reviewed by the researcher or the subject teacher nor discussed with the entire class before being executed; the author also noted that although the classes were analyzed, the lesson plans were not enhanced due to the short time available. Carvalho (2020) explains that some phases of the lesson study were changed because the time for developing the process was limited to eight meetings (classes) in a subject. In the study by Silva (2020), the undergraduates taught five double classes; however, some were not prepared in advance, which did not allow them to reflect on the students' possible answers and difficulties in solving each problem, such as the most appropriate words for a given activity. The author also notes that the pre-service teachers would have liked to have carried out an individual activity at the end of the sequence to understand students' learning better. Still, again, time prevented this from happening.

Some authors cited difficulties anticipating students' potential doubts, errors, and resolution strategies. According to Küster (2020), the resolution strategies present in the planning were not sufficient to help all students when solving the problem. Rincón and Fiorentini's (2017) work revealed prospective teachers' difficulties anticipating students' doubts and errors. Not all groups prepared detailed plans, and some situations in classes were not predicted. C. F. Souza (2021) explains that anticipating doubts and surveying students' prior knowledge was challenging. They used the curriculum to try to understand what the students had already studied up to that point. However, this examination proved insufficient for understanding the students' prior knowledge and difficulties. This knowledge expanded as they interacted with students, observing them in learning situations.

Another point to consider is that, in some works, according to Coelho (2014), M. A. Oliveira (2020), and C. F. Souza (2021), although the prospective teachers planned classes with challenging activities, they found it hard to promote an environment in which students have an

active participation. Such an ambiance enables the sharing of strategies and the communication of ideas. Some classes were expository and focused on procedures. Regarding this situation, C. F. Souza (2021) clarifies that the exploratory approach was completely new for the pre-service teachers, who were used to traditional teaching.

Some works also mention that problem solving and exploratory tasks were new for basic education students. In Carvalho's research (2020), most students could not solve the problem posed and when analyzing the class, the prospective teachers concluded that the graph paper handed out to students did not favor problem solving. Thus, they reflected that using another resource, such as Frac-Soma, would have been more appropriate. According to Silva (2020), some basic education students had difficulties carrying out the proposed tasks during classes due to a lack of basic mathematical knowledge and resistance to being more active, as they were used to traditional teaching. Then, they planned actions to contribute to the greater involvement of less participative students.

Another challenge highlighted concerned the availability to participate in the formative process. Silva (2020) mentions how hard it was to find students and professionals willing to partake in the research; another challenge the researcher presented was bringing together research participants at various times. M. A. Oliveira (2020) considers that some undergraduates could not adjust to the dates established for the meetings, jeopardizing the progress of the activities.

Regarding prospective teachers' feelings, Silva (2020) found, as already predicted, that they were anxious and nervous because it was their first class and being participants in doctoral research. Coelho (2014) observes that the pre-service teacher who led the group 1 class seems to have felt judged and adopted a defensive stance when analyzing the class. The pre-service teacher who led the group 2 class said he felt nervous during class execution as he knew more knowledgeable people were evaluating him. According to Carvalho (2020), the prospective teacher was anxious when he welcomed his colleagues and teacher to school, which the author considered natural because they would observe him when conducting the class.

Other challenges experienced by the participants were also mentioned. According to Küster (2020), the student who led the class had difficulty relating specific mathematics to students' familiar topics and mathematical thoughts. Coelho (2014) reports that he experienced challenges when having to play both the role of researcher and educator. According to C. F. Souza (2021), in the first class, the prospective teachers faced difficulties and constraints caused by several situations, which were discussed, and new agreements were established so that the other classes could take place in a more appropriate way. Rincón and Fiorentini (2017) state

that a challenge still to be overcome at the time the research took place is that there must be cooperation between the scholars from the Institute of Mathematics and the Faculty of Pedagogy at the State University of Campinas so that lesson study may continue to be developed in the mathematics course.

Lesson Study Contributions

The research analyzed presents several contributions from the lesson study formative process developed within the scope of mathematics teachers' initial education.

Concerning knowledge, the lesson study enabled the establishment of a connection between mathematical knowledge and pedagogical knowledge (Coelho, 2014); it was relevant to the didactic-pedagogical and mathematical education of prospective teachers (Silva, 2020); favored the expansion of knowledge about teaching mathematics and the ability to identify students' difficulties (Neves & Fiorentini, 2021); contributed to expanding knowledge for teaching, highlighting ways to approach class content with high school students and reflection (M. A. Oliveira, 2020); made it possible to identify school content and school mathematics; identify educational needs based on other classroom reading (Rincón & Fiorentini, 2017); provided the opportunity to build or deepen knowledge of some mathematical content for teaching, students' mathematical thinking, and how they learn (C. F. Souza, 2021)

Specifically regarding lesson planning, Coelho (2014) states that most undergraduates did not know how to construct it. The author says that, although they did their lesson plans superficially and informally, the process experienced contributed to reducing this gap. Neves and Fiorentini (2021) emphasize that elaborating the lesson plan and its refinement resulting from the discussions held in the subgroup with the supervisor and the large group was especially important for the prospective teachers' learning. According to M. A. Oliveira (2020), when planning the lesson, the pre-service teachers reflected on the difference in how content is worked on in academic and school mathematics. According to Küster (2020), the development of quality planning made it possible to reduce unpredictability during class, giving confidence to the pre-service teacher who led the class, as he felt prepared to help the students and was not surprised by the questions they asked.

In all the analyzed works, Coelho (2014), Rincón and Fiorentini (2017), Carvalho (2020), Küster (2020), Neves and Fiorentini (2020), M. A. Oliveira (2020), Silva (2020), and C. F. Souza (2021), the authors highlighted the importance of collective work for prospective teachers' education.

Regarding the types of tasks proposed, some authors also highlight, as contributions from the lesson study, the opportunity for prospective teachers to reflect on the characteristics of challenging tasks (Coelho, 2014), plan and conduct classes with exploratory tasks (Rincón & Fiorentini, 2017), and develop exploratory tasks and conduct student-centered classes (C. F. Souza, 2021).

Regarding the theoretical and practical dimensions of mathematics teaching, Silva (2020) highlights that they were interconnected at various moments in the formative process. C. F. Souza (2021) notes that the reflective and collaborative nature of the lesson study helped prospective teachers' reflection and practice. Neves and Fiorentini (2021) observe that the undergraduates considered the split between theory and practice in the degree course detrimental to their professional development.

Our first intention was to identify the contributions of the lesson study to the mathematics teachers' education. However, we also found that this formative process favored the rapprochement between basic school and university. Neves and Fiorentini (2001) consider that the lesson study carried out in the context of the mathematics teaching degree supervised practicum contributed to a greater rapprochement between the university and basic schools and created an opportunity for teacher educators, basic school teachers, and prospective teachers to work collectively and, sometimes, collaboratively. Silva (2020) observes that the collaboration between the teacher educator, supervisor, and prospective teachers made a difference in the undergraduates' learning.

Thus, the analyses of the works selected for our investigation show that lesson study made different contributions to the prospective teachers' education and favored the rapprochement of teachers from basic schools and university professors. However, they indicate a greater demand for time and continuity.

Final considerations

To answer our guiding questions, “How has lesson study been developed in mathematics teachers’ initial education?” “What are the challenges to developing lesson study strategy in initial education?” and “What are the contributions of lesson study to prospective mathematics teachers’ education?”, we undertook bibliographical research on scientific articles and master’s degree and doctoral theses.

We concluded that the works indicate some closeness to and distancing from the principles of the lesson study. In some works, the class theme was defined collectively; in others, it was indicated by the researchers. Regarding studies to support planning, not all processes included consulting curriculum documents, text-reading, and analysis of materials on the class topic. Some works were aimed at the characteristics of the lesson study and other topics related to mathematics teaching. In all the studies analyzed, planning was collective, and, in some, it was carried out in more detail, with anticipation of students’ potential doubts, answers, errors, and resolution strategies, besides prospective teachers’ actions. However, in other studies, lesson plans were incomplete. In all investigations, at least one of the prospective teachers led the class, even though in Coelho (2014), the classes were taught to the other undergraduates. Another convergent point was the proposition of challenging tasks (problems or exploratory activities). Furthermore, in almost all works, there were meetings to analyze the classes, but some did not include planning improvements.

Several authors reported challenges experienced in the development of the lesson study, among which we highlight the time that should be spent in carrying out all the planned steps, mainly for studies and detailed lesson planning; participants’ difficulties in anticipating students’ possible doubts, errors, and resolution strategies; difficulties in conducting classes with challenging tasks. However, we must consider that the lesson study approach was a new experience for prospective teachers and almost all professors/researchers.

The research analyzed in this text shows the great potential of the lesson study, adapted to the reality of each context, with emphasis on its aspect of strengthening the relationship between theory and practice, of treating mathematical content in approaches in which students exercise the central role, enabling significant development of knowledge for teaching

mathematics and benefiting the rapprochement between universities and basic schools, in a collaborative process.

Therefore, we understand that much could contribute to mathematics teachers' initial education if, throughout the degree course, prospective teachers participated in several lesson studies so that the steps could be worked on well.

We hope the research reported in this text contributes to the debate about how the lesson study formative process can be developed to assist prospective mathematics teachers' education and overcoming challenges to their development.

References

- Branco, N., & Ponte, J. P. (2008). Um estudo de integração de recursos multimédia na formação inicial de professores do 2.º ciclo do ensino básico. In Ponte, J. P. *Práticas Profissionais dos Professores de Matemática*. Lisboa. (pp. 515 – 536). <https://repositorio.ul.pt/handle/10451/15310>
- Carvalho, M. (2020). Metodologia Lesson Study na Licenciatura em Matemática: possibilidade para a formação inicial. *Boletim GEPEM*, (77), 1-13. <http://costalima.ufrj.br/index.php/gepem/article/view/421/981>
- Coelho, F. G. (2014). *A metodologia da Lesson study na formação de professores: uma experiência com licenciandos de matemática*. (Dissertação de Mestrado em Ensino de Matemática). http://www.pg.im.ufrj.br/pemat/MSc%2060_Fellipe%20Gomes%20Coelho.pdf
- Elipane, L. E. (2012). *Integrating the essential elements of lesson study in pre-service mathematics teacher education* (Doctoral dissertation, Københavns universitet). https://www.researchgate.net/profile/Levi-Elipane-3/publication/323856354_Integrating_the_Essential_Elements_of_Lesson_Study_in_Pre-service_Mathematics_Teacher_Education/links/6235a073d545b7729403564a/Integrating-the-Essential-Elements-of-Lesson-Study-in-Pre-service-Mathematics-Teacher-Education.pdf
- Fiorentini, D., & Lorenzato, S. (2006). *Investigação em educação matemática: percursos teóricos e metodológicos*. Autores Associados.
- Fiorentini, D., & Oliveira, A. T. C. C. (2013). O lugar das matemáticas na Licenciatura em Matemática: que matemáticas e que práticas formativas?. *Bolema: Boletim de Educação Matemática*, 27(47), 917-938. <https://www.scielo.br/j/bolema/a/99f8nsJSh8K9KMpbGrg8BrP/?format=html&lang=pt>
- Fujii, T. (2014). Implementing Japanese lesson study in foreign countries: misconceptions revealed. *Mathematics Teacher Education and Development*, 16(1), 65-83. <https://eric.ed.gov/?id=EJ1046666>

- Gaigher, V. R., Souza, M. A. V. F., & Wrobel, J. S. (2017). Planejamentos colaborativos e reflexivos de aulas baseadas em resolução de problemas verbais de matemática. *VIDYA*, 37(1), 51-73. <https://periodicos.ufn.edu.br/index.php/VIDYA/article/view/1929>
- Isoda, M., & Olfos, R. (2009). *El Enfoque de Resolución de Problemas: en la enseñanza de la matemática a partir del estudio de clases*. Ediciones Universitarias de Valparaíso. <https://ade.edugem.gob.mx/handle/acervodigitaledu/49512>
- Küster, J. S. (2020). *Formação de professores para aula de resolução de problemas a partir de um Lesson Study: contribuições, constrangimentos e desafios*. (Dissertação de Mestrado Profissional em Educação em Ciências e Matemática, Instituto Federal de Educação, Ciência e Tecnologia do Espírito Santo). <https://repositorio.ifes.edu.br/handle/123456789/1450>
- Murata, A. (2011). Introduction: conceptual overview of lesson study. In HART, L., ALSTON, A., & MURATA, A. *Lesson study research and practice in mathematics education*. (pp. 1-12). Springer. https://link.springer.com/chapter/10.1007/978-90-481-9941-9_1
- Neves, R., & Fiorentini, D. (2021). Aprendizagens de Futuros Professores de Matemática em um Estágio Curricular Supervisionado em Processo de Lesson Study. *Perspectivas da Educação Matemática*, 14 (34), 1-30. <https://desafioonline.ufms.br/index.php/pedmat/article/view/12676>
- Oliveira, M. A. (2020). *(Res)significações de saberes por licenciandos que vivenciam estudo de aula sobre distância entre dois pontos*. (Dissertação de Mestrado Profissional em Ensino de Ciências e Matemática). <http://www2.ufac.br/mpecim/menu/dissertacoes/turma-2018/dissertacao-michael-araujo-de-M. A. Oliveira.pdf>
- Palanch, W., & Freitas, A. V. (2015). Estado da Arte Como Metodologia de Trabalho Científico na Área de Educação Matemática: Possibilidades e Limitações. *Perspectivas da Educação Matemática*, 8(18). <https://desafioonline.ufms.br/index.php/pedmat/article/view/867>
- Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2017). A adaptação dos estudos de aula ao contexto português. *Anais do Seminário de Investigação em Educação Matemática* (pp. 129-141). https://repositorio.ipv.pt/bitstream/10400.19/4557/1/AtasXXVIIIISIEMFINAL_domipdf#page=139
- Rincón, J. P. A., & Fiorentini, D. (2017). A ‘glocal’lesson study: The case of pedagogical practices in mathematics. *Revista Internacional de Pesquisa em Educação Matemática - RIPEM*, 7(2), 24-44. <http://sbemrevista.kinghost.net/revista/index.php/ripem/article/view/1230>
- Silva, A. D. R. M. (2020). *Contribuições da Jugyou Kenkyuu e da engenharia didática para a formação e o desenvolvimento profissional de professores de matemática no âmbito do estágio curricular supervisionado*. (Tese de Doutorado em Educação Matemática e Tecnológica, Universidade Federal de Pernambuco). <https://repositorio.ufpe.br/handle/123456789/40028>
- Souza, C. F. (2021). *Estudo de aula de matemática com robótica educacional na formação inicial do professor de matemática*. (Tese de Doutorado em Educação, Universidade Federal de Uberlândia). <https://repositorio.ufu.br/handle/123456789/32933>

Souza, M. A. V. F. (2022). Lesson Study Sem Fronteiras: limitações, desafios e algumas soluções de implementação. In Neves, R., & Fiorentini, D. *Seminário internacional de Lesson Study no ensino de matemática*. (pp. 49-57) Editora do Instituto Federal de Educação, Ciência e Tecnologia do Espírito Santo. <https://repositorio.ifes.edu.br/handle/123456789/1540>