EMP Educação Matemática Pesquisa
ISSN 1983-3156
Qualis A1
http://dx.doi.org/10.23925/1983-3156.2024v26i2p229-255

Updates and production in mathematics education: the report of inventions from the meeting of future pedagogues with the act of defining space

Actualizaciones y producción en educación matemática: el relato de las invenciones a partir del encuentro de los futuros pedagogos con el acto de definir el espacio

Actualisations et production dans l'enseignement des mathématiques : le constat des inventions de la rencontre des futurs pédagogues avec l'acte de définir l'espace

Atualizações e produção em educação matemática: o relato das invenções a partir do encontro de futuros pedagogos com o ato de definir o espaço

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Abstract

The intention of the study is to monitor how processes of reflection on the act of defining space together with undergraduate students in Pedagogy come into operation in the composition of ways of thinking about the teaching of mathematics. Thus, the definitione device was developed to provoke thought production and inventions. As a result, discussions with the group started on the definition and conceptualization processes in the field of Mathematics, as well as on the production of curricula and quality education based on Mathematics Education. It is concluded that the debate of Mathematics Education in the Pedagogy courses needs to be more inventive and provocative, allowing updates and creations of the students.

Keywords: Mathematics education, Pedagogy, Space.

Resumen

La intención del estudio es acompañar cómo se operan procesos de reflexión sobre el acto de definir el espacio junto a los estudiantes de Licenciatura en Pedagogía en la composición de modos de pensar la educación matemática. Así, se desarrolló un dispositivo para provocar producciones de pensamiento. Como resultado, se iniciaron discusiones sobre los procesos de definición y conceptualización en el campo de las Matemáticas, así como la producción de currículos y una educación de calidad basada en la Educación Matemática. Se concluye que el debate de Educación Matemática en los cursos de Pedagogía necesita ser más inventivo y provocador, posibilitando actualizaciones y creaciones.

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Palabras clave: Educación matemática, Pedagogía, Espacio.

Résumé

L'intention de l'étude est de suivre comment les processus de réflexion sur l'acte de définir l'espace avec les étudiants de premier cycle en pédagogie s'opèrent dans la composition des modes de pensée sur l'enseignement des mathématiques. Ainsi, un dispositif a été développé pour provoquer des productions de pensée. En conséquence, des discussions ont commencé sur les processus de définition et de conceptualisation dans le domaine des mathématiques, ainsi que sur la production de programmes d'études et d'une éducation de qualité basée sur l'enseignement des mathématiques. Il est conclu que le débat sur l'enseignement des mathématiques dans les cours de pédagogie doit être plus inventif et provocateur, permettant des mises à jour et des créations.

Mots-clés : Education mathématiques, Pédagogie, Espace.

Resumo

A intenção do estudo é acompanhar como processos de reflexão sobre o ato de definir o espaço junto com os licenciandos em Pedagogia entram em funcionamento na composição de modos de pensar a educação matemática. Assim, elaborou-se um dispositivo para provocar produções de pensamento. Como resultado, entrou-se em funcionamento discussões sobre os processos de definição e conceituação no campo da Matemática, bem como a produção de currículos e de uma educação de qualidade a partir da Educação Matemática. Conclui-se que o debate de Educação Matemática nos cursos de Pedagogia necessita ser mais inventivos e provocativos, possibilitando atualizações e criações.

Palavras-chave: Educação matemática, Pedagogia, Espaço.

Updates and production in math education: the report of inventions from the meeting of future pedagogues with the act of defining space

The act of defining and the use of definitions are two elements that are found in the daily practices of mathematicians and math teachers. The term itself comes from the Latin definitione, understood as exposing with precision. According to the philosophy dictionary Abbagnano (1999, p. 237), the definition refers to the "declaration of essence-significance", in other words, any answer given to the question "what?" can be considered the definition of a thing2. However, we must consider at this point the inclusion of the term precision. To be precise can be categorized as having rigor and being as close as possible to exactness, to the essence3 of things.

Based on Fiorentini and Lorenzato (2009), I can imagine that labor processes that involve mathematic definitions occur differently between mathematicians and math educators since the object each of these groups works with is also different. In the initial education of the mathematical pedagogical educator, the topic leads to a return to educational practices experienced in Elementary Education, which can then invoke concerns, insecurities or, still, traditional conceptions regarding the teaching of the subject (AUTHOR, year).

Therefore, for the debate with the group, we created the device definitione as an element that can trigger inventions and productions in Math Education based on the act of defining. With this perspective in mind, the aim of this research was to observe how processes of reflecting on the act of defining space with undergraduate Pedagogy students work to compose ways of thinking math education.

Based on what has been mentioned, there are two pertinent points for discussion. The first is the presence of the term space in the paragraph above: its insertion occurs with the transversality of the concept, placing it as the reason for exercises in thinking. The second point we need to highlight is that we prioritize the search for invention and production in Math Education and that we do not necessarily conform our actions to practices that aim at teaching how to define or how to teach based on definitions.

[2] Proposals analyzed based on theorizations of the stoics.
[3] We do not intend to polemicize the term essence here.

To develop this process, a priori we experienced two moments in the research that are described in detail below. Initially, we problematized via historicity the conceptualizations of space throughout the history of humanity and, secondly, we built a toolbox to assemble the encounter of Pedagogy undergraduates and the act of defining space. After these debates, we set on the descriptive process of this research.

Problematizing is necessary

In the field of Math Education in general, the concept of space is found in the field of psychology, connected to the processes of appropriation by the subject of this concept (AUTHOR, year). Hence, we observe there is an increasing number of studies about topological, projective and Euclidian relations in the process of acquiring the notion of space (Piaget & Inhelder, 1993); learning levels (Hiele, 1986); geometric visualizations (Gutiérrez, 1991) etc.

Beyond studies in Psychology and Math Education, the concept of space is presented as a historical construct, carrying along with it significations of different types of knowledge produced by humanity. From the historical perspective, we can find studies on "mathematical revolutions" by Mlodinow (2005), the stylistic schools of Art, as proposed by Ostrower (1998), and the convergence between Art and Mathematics by Omission (Moraes, 2018), among others.

These studies propose that space takes on different forms throughout history and resonates in different areas of knowledge. To a certain extent, it belongs to what we can call the first field of production. A historical moment in which it became possible to think about it in a certain way and not in another. A model of ordering knowledge that could trigger directions and forms in different disciplinary fields.

In this sense, the production of the concept of space in a specific society is linked to the cultural and social organizations of this society, stipulating it as "[…] a means by which the position of things becomes possible" (Merlau-Ponty, 1999, p. 328). In other words, the attempts to mark off the concept of space lead us more in the direction of what can be carried out through it than a definition that is based on immobility or the absence of practices.

In the attempt of encompassing the concept in his field of study, Certeau (1996) proposes the following differentiation between space and place:

> A place is the order (whatever that may be) in which elements are distributed in relations of coexistence. Therefore, here we find the exclusion of any possibility of two things being in the same place. This is the rule of "there is reigns"…There is space as long as vectors of directions, quantities of speed and the variable time are considered. Space is the crossing of mobiles… Space would be in relation to time in the same way the word when it is pronounced… In sum, space is a place in practice (Certeau, 1996, pp. 172-173).

The notion of space goes back to the dimension of existence in an inhabited place, established as such insofar as it is a possibility of producing social practices. Thus, its study is relevant in the action it allows, i.e., in the production of meanings that can be manifested from it. In this sense, Certeau says that "[…] it is the activity that qualifies space"; in other words, it is our way of appropriating it that would describe it. In this context, space is "a place where conflicts are expressed" and "the place of desire" (Lefebvre, 1991).

In carrying out any analysis of visible space, it will be considered that invisible spaces reign over it, inscribed by human activity, by the art of savoir-faire (knowing how to do it). This is an art that is built in the context of daily practices, gestures and narratives. Consequently, researching space coheres with emphasizing the multiplicity of ways of appropriating space in practice.

Therefore, to exercise a space will require from the user an act of recognition, an imposition of established mental plans and the rules of use space has over their body (types of knowledge). Its practices must be conditioned so they will not be seen as a deviation from the correct perception of the other about them.

Opening the toolbox

To create the research process, we produced a toolbox, considering what is found in Focault (2007).

A theory is like a toolbox. It has no relation to the significant… It is necessary that it fit, that it work. And not for itself. If there is nobody to use it, starting with the theorist

him/herself, who then stops being a theorist, then it is either worthless or the moment has not yet arrived (Focault, 2007, p.69).

Hence, our theoretical-methodological tool is based on the method of cartography proposed by Deleuze (1992). Our aim is to observe processes of inventing reality by the subjects of the research, those who "as they walk, chart their goals along the way" (Passos & Barros, 2009, p.17). Our wager was on the "experimentation of thought – a method that would not be applied but experienced and undertaken with an attitude" (Passos, Kastrup & Escossia, 2009, p.10).

The individuals involved in the research were twenty undergraduate students in Pedagogy at a private college in the state of Sao Paulo and who are taking the discipline *Content, Methodology and Practice in Teaching Mathematics*[4] in the sixth semester of the program.

Upon arrival, we created a device, the *definitione* workshop. In the research, we viewed the devices as "machines that make you see and speak" (Deleuze, 1992, p.155), and are able to update what is virtual[5] in the area of the workshop, producing ways of being and realities of speech. In its creation, we brought about some situations:

Defining space: we began by asking each student individually to define space, writing it down in their graph notebooks[6].

Exposing/exposing yourself: students present what they wrote and provocations/inventions about Math Education are touched upon.

Sample Space[7]: After the debate and some reading, a map of updates and group synthesis is developed.

To evince the processing that took place during the research, we prioritized the descriptive method instead of an interpretive one. In other words, we did not want to obtain mathematical data, which the students could perhaps create with the activities, but to map these encounters which, in estrangement, are able to potentialize the creation of meaning.

[4] This is the only discipline in the Pedagogy program directed to Math Education.
[5] According to Kastrup (2007, p.22), the virtual is updated by a principle of creation and differentiation. Hence, it is different from the possible, performed by a process of limitation and similarity.
[6] For research production, what is called a graph notebook became the field of recording.
[7] This term was chosen by the students and was created by joining the terms Mathematics and the object space.

Defining Space

First, I asked[8] them to create a definition. The definition of Space.

Me: I would like you now to define space (...) After agonizing looks and about ten minutes had gone by, complaints began.
Gaby Cinderella[9]: Oh, c'mon, Mr. M. ... this is hard... I can't do it.
Me: What's going on?
Gaby Cinderella: It's too hard ... To say what this thing is... There are so many...
(...) Complaints continue until Helena asks*:*
Helena: I think ... I think this doesn't fit in with our math proposal...
Me: Why not?
Helena: It's like it's destroying our math... I don't know!
Philosopher: It's cold.
Me: But isn't the definition required?
Philosopher: Probably, But, like, I think this is exactly what ruins math (...)
Noah: Can we do this at home?
Me: Sure, why not... (Laughter)

The work proposal was to problematize the logic of teaching definition-model-reproduction of a model (Zabala, 1998) found in many of the life stories of the Pedagogy students. My aim was to reproduce this logic in the classroom so we could reflect on it. This model holds the classroom hostage to a context in which the teacher is the knower, and the student has no knowledge whatsoever (Freire, 1991). Concomitant to this, I wanted to think with the group about how definitions in math were built and what was their role in this science (Zaslasvksy & Shir, 2005).

At the beginning of the proposal, the group realized the difficulties in building a definition and, consequently, as shall be seen below, they end up realizing that it is not cold (Philosopher), but that what we do with it in the classroom leads to it being perceived as such.

The difficulty in defining

During the following week in class, we went back to our discussion.

Me: How did the activity go, guys?
Luiza: Well, Mr. M... it was really hard... I tried to do it on my own like you said, but couldn't... I had to do research... Sorry.
Me: Tell me what was so hard?

[8] Due to a theoretical-methodological choice, the first-person singular was chosen in writing.
[9] Fictitious names were chosen by students.

Luiza: Well, it's like we really need to think hard ... (...) we need to base it on a body of work...(...) It won't come from nothing.
Me: What does it mean to have a body of work?
Luiza: Things don't just come out of nothing... I think we have to do something before that.. I don't know...
Me: Definitions have a past. (Laughter)

In her attempt to define, Luiza is beginning to realize that definitions are historical, not only as the production of specific humans, but as the production of humanity (D'Ambrosio, 2005). This entails the existence of a process that needs to be carried out before they find themselves inside a classroom and that there must be a high degree of certainty.

Me: Now I have a doubt to place before you... if definitions are historical on the one hand and, by themselves, they're cold... what do we do in Math Education?
Pink Butterfly: We don't use them... (Laughter)... Or we use them in moderation...
Luiza: We should put them into stories... We can't just not use them... They're a part of math... Like... I have to define what a triangle is with my students...
Pink Butterfly: (...) You're right, that's it. (...)
Ana: Mr. M., why do we always have to start by defining?
Me: Who said we do? (The doubtful looks come my way now).

The students realize they need to insert definitions in the field of math teaching. However, now the debate is about how to do it. The first suggestion is to historicize the definitions, which would be an approach aligned with the methodological proposals of Math History for Teaching (Miguel & Miroim, 2005). Another proposal that is beginning to come up is to start teaching using other paths that exclude definitions, which indicate methodologies such as the Resolution of Problems (Onuchie & Avellato, 2004).

Regardless of the teaching path that will be chosen, the question is what will be provocative right now. With doubt, the erasure of certainty, other teaching possibilities can come up (Moraes, 2018). In other words, in the gaps of the regulations and norms about what a classroom is, other forms of what teaching can be are visualized. And even, perhaps, the creation of what being a math teacher is (Moraes, 2018; Moraes, 2014).

Patterns and ways of defining

Me: Olivia, read your text.
Olivia: this is what I did (She reads).

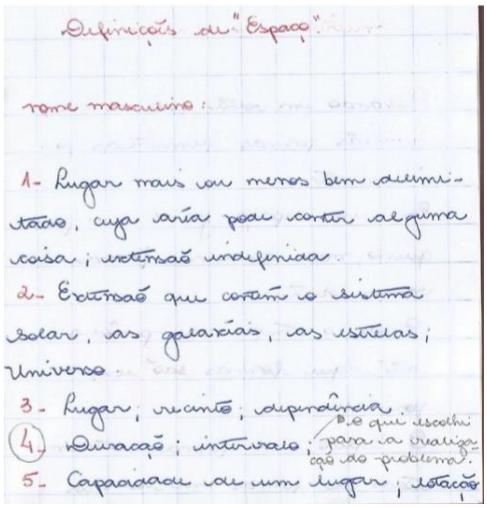

Figure 1[10].

Definition of Space (Olivia's graph notebook.

> The discussions begin to delve into the area of mathematical *Me: (...) Why did you choose to do it this way?*
> *Olivia: Because I thought a dictionary would, like, be appropriate… (...) Because there's a pattern in it… It's a type of text.*
> *Me: What do you mean, a type of text?*
> *Olivia: Because every text is written in a particular way ... We learned that in Portuguese Methodology ... And that is the way we write definitions.*
> *Me: And who said that they're written this way? (Laughter)*
> *Olivia: God did… (Laughter)*
> *Helena: It's a convention… Like, it was decided that this was the way…*
> *Me: Who decides?*
> *Philosopher: In Math, it's probably the mathematicians. Not everyone can speak. Like, only those who have the authority, as Foucault said11.*

conventions. The students emphasize that a definition is a mathematical convention (Knijnik, 2001). This demands certain forms of writing that are validated and thought about within the field of a discipline. As Philosopher says, based on Foucault (2007), these are validated by those who are authorized to speak, mathematicians themselves.

In this sense, I thought it was pertinent to tell the group about the role of Euclides in composing mathematics. With support from texts by Mlodnow (2005), I showed them his role

[10] Definition of "Space"
Masculine noun[TN].
1. A more or less well-delimited place whose area can contain something; indefinite extension.
2. Extension that contains the solar system, galaxies, stars; the Universe.
3. Place, enclosure, room.
4. Duration; interval (this is what I chose to solve the problem).
5. Capacity or a place, station.

[11] Foucault M. (2007). The Archeology of knowledge.

as the compiler of thought at a certain historical moment. In addition, I presented the issue of formalized language (Dahlberg, 1978) in the field of math as a work structure, a way of communicating in the field. About a month later, Helena sent me this image:

Figure 2[12].

Euclides by Olivia (Whatsapp).

The conversation went like this:

Helena: Poor Euclides. Look what they did to him… lol.
Me: What?
Helena: The look he has on his face, like he's in pain, after, after everything he did… lol.

Going on with the topic, I attempted to relate the conversation with Math Education:

Me: And how is this connected to Math Education? (Momentary silence).
Helena: Well, like… It's a language, Mr. M.… (…) a language that has to be learned by the student.
Me: But what is a language?
Olivia" Language is a form of expression.
Me: (…) Let's think about the connection between language, culture, and Math Education.
Helena: That's hard, Mr. M.
Philosopher: The thing is… Language is culture, if math is a language, it's culture too… Like, the language of math is a convention, but is also in movement… (…)
Gaby Cinderella: Mr. M., but now when I heard you talk, I imagined… We learned that a child acquires language, she doesn't know everything, but is always learning… Is that the same for Math Education, Mr. M.?

[12] Life in Greece in 300 B.C was really boring, so on this day I decided to use my mind to study the wonders of math =D. Come with me and hear a wonderful story.

Me: I always see it that way, Gaby Cinderella, there are conventions, the process of producing these conventions and the conventions we use… I think the worst scenario is to think that the conventions are math itself…
Philosopher: That's right, Mr. M. (showing excitement) … You just said what I wanted to say. It's as if we set down a done deal in place of the process. That's what we experienced… The teacher would always focus on the convention and forgot the math of life.
Me: So, we have to think if it is possible to produce texts, write and communicate in math…
Helena: If we could do that, it would be our math.

Throughout our discussions, I intended to create a discussion about the relation between language, culture and Math Education with the group. The inferences we picked up were that, although mathematical conventions mean stability, they are cultural and always in movement (D'Ambrosio, 2005).

The group also considered that, for our proposal on math, we need to move towards a movement that leads us to the mathematical expression, in which texts are produced, in which we can write and communicate with it. That would be our math.

Multiple definitions?

Ana Cristina: Sorry, Mr. M., I think I made a mistake…
Me: What do you mean?
Ana Cristina: I wrote a bunch of stuff about space…
Me: Read it… (Laughter)… Let's see…
Ana Cristina: (her reading)

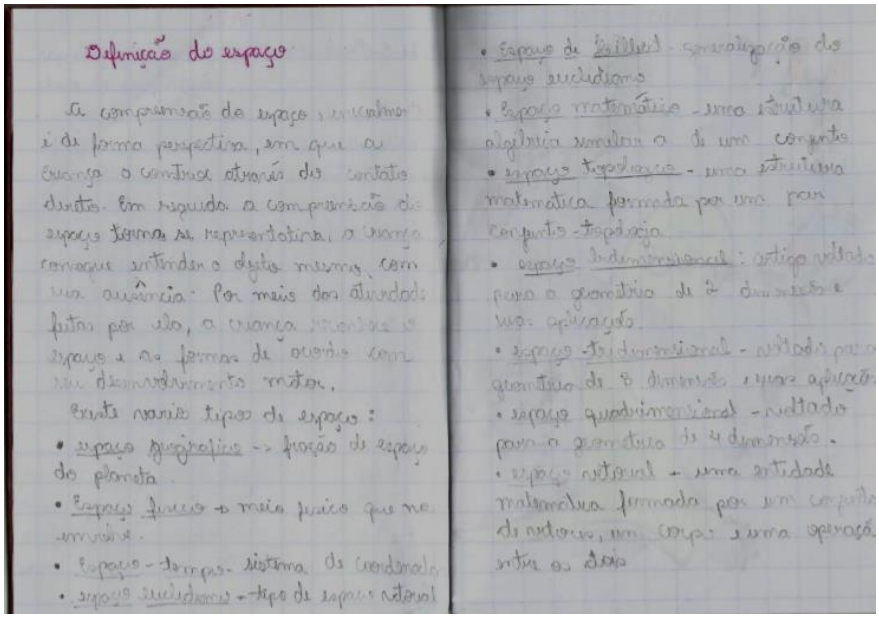

Figure 3[13].

Defining Space – Ana Cristina

Me: Why do you think this is wrong?
Ana Cristina: Because I wrote so much… And a definition is only one thing, right?
Me: To a certain extent, a definition is closed around one thing only … But we can think about something else: the concept.
Ana Cristina: Ok… but what's the difference?
Me: a definition[14] is something that's more rigid… Concept is more fluid. It's hard at times to think of a definition for a specific concept. But when we know a concept, we can do things with it without necessarily defining what it is… We even know when someone is not using it the right way… (…)
Ana Cristina: So, our matecarte[15], our math is related to concepts and not definitions…
Me: I guess… (kind of in doubt).

[13] Definition of space
Initially, understanding space is done perceptively; the child builds it through direct contact. After that, understanding space become representative – the child can actually understand the object itself, with its absence. With the activities a child carries out, he or she recognizes space and forms, according to his/her motor development.
There are several types of space:

- Geographical space – as a fraction of the space on the planet.
- Physical space – the physical environment that surrounds us.
- Space-time – a system of coordinates.
- Euclidean space – a type of vectorial space.
- Hillbert's Space – a generalization of Euclidean space.
- Mathematical space – an algebraic structure similar to a set.
- Topological space – a mathematical structure formed by a topological-set pair.
- Bidimensional space – directed towards the geometry of 2 dimensions and their application.
- Tridimensional space – directed to the geometry of 3 dimensions and its application.
- Quadrimensional space - directed to the geometry of 4 dimensions and its application.
- Vectorial space – a mathematical entity formed by a set of vectors, a body and an operation between them.

[14] Laudares, 2013.
[15] In a previous class, the students had decided that our math would be called matecarte.

Ana: It is, Mr. M. ... It's like I'm being watchful so it's experimented, like... It doesn't really matter to me in terms of the final product, but in the type of actions...
Angelina: That's that thing about the process and the product Ms. Sueli[16] *taught us, about the types of content*[17]*? (She looks at Philosopher)*
Philosopher: I guess so... (looks at me, who did not understand what was said).
Philosopher: It's like this, Mr. M., we discussed that more important than the product is the process in planning. So, we thought it might be better to think about math than definition.
Me: I see... And what about content?
Angelina: There are three contents, Mr. M. There are those related to the procedure, those related to attitude and those related to concepts. Our math brings the three together, if you think about it... A way of life.

During the debate, what it comes down to is that we are thinking about math produced by means of concepts, not definitions. The group seems to want to move away from the rigidness they see in definitions towards concepts which are, according to Furtado and Cabral (2011), information that we expect the educator to know and that is placed in movement in a network of relations with other information that has already been acquired.

Although we had not discarded the need for definitions, we decided that our praxis would have to "[…] start with the conceptual understanding so the student would reach satisfactory levels of generalizations and abstraction, and then formulate the definition" (Laudares, 2013).

Connected to this process, the students brought up the information discussed in Didactics, based on studies by Zabala (1998), in which, to build our math, we would need to think about it in a conceptual, procedural and attitudinal dimension. Only in this way would our math be considered a form of life.

A concept is never alone

Philosopher: Can I read mine, Mr. M.?

Me: Go ahead. (Reading is done).

[16] The Educational Planning teacher.
[17] Conteúdos Conceituais, Procedimentais e Atitudinais (Conceptual, Procedural and Atitude Contents) (Zabala, 1998).

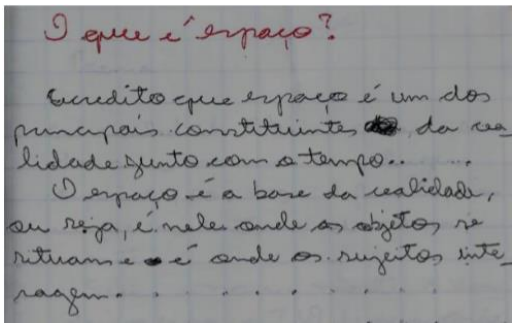

Figure 4[18].

Definition of Space – Philosopher

Philosopher: I wrote that. (Moment of silence).
Philosopher: So, I thought a lot… I don't think we can talk about space without connecting it to other stuff, with time. Since we were talking about things with concepts… Actually, I think space is where things happen… Even concepts.
Me: ok…
Philosopher: Just ok? You're not going to ask me anything? (Laughter).
Me: Take it easy, I'm thinking… (Laughter)… Ok. If space is where things happen, does that mean it doesn't change?
Philosopher: I have to think about that now… (Laughter)… I have an idea… Space doesn't change, reality changes.
Norberto: I think this talk is baseless… Is it necessary?
Philosopher: What?
Norberto: Well, it doesn't matter what came first, the chicken or the egg. What really matters is that one thing interferes with the other… If one of them changes, so does the other.
Ducarmo: We are what we are because of how people act towards us… and (Philosopher interrupts).
Philosopher: Jeez, Ducarmo, that really has nothing to do with… If you don't have a…
(I decide to intervene)
Me: I think we have to listen to Ducarmo, let her speak.
Ducarmo: It's like this… We behave according to how people behave towards us. Space must be like that too. If reality changes, space does as well.
Me: That's really interesting… (…)
Norberto: To me, space is used in everything…
Me: What do you mean?
Norberto: In everything… It's immense. It is connected to our internal life, to the galaxies, to daily life ... It's kind of everything really. (Reads his concept of space)

[18] What is space?
I believe space is one of the main components of reality, along with time…
Space is the basis for reality, in other words, it is in space where objects are found, and it is where subjects interact.

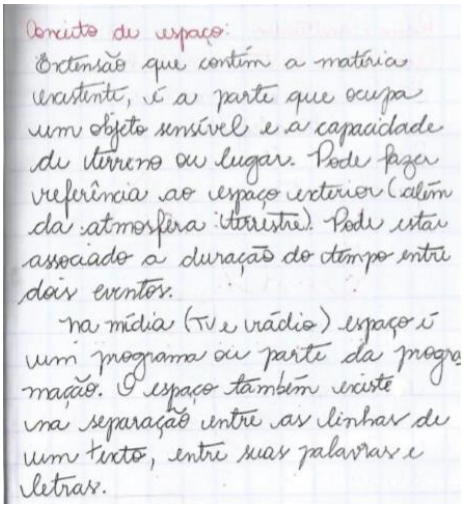

Figure 5.

Definition of space – Norberto

Cecilia: You know I was thinking, Mr. M.?
Me: What?
Cecilia: The relation between definition and space… (…) Concept is not alone. You saw what was said about TV, reality and the text… (…) A definition is one thing and concept is bigger.
Bianca: I was thinking about that too… Even while reading what I picked up on the internet (Laughter). Sorry, Mr. M., I saw that there is like matter, extension. Things I also don't know how to define… (Laughter). But we start putting things together.
Me: So read what you wrote.
Bianca: (reading)

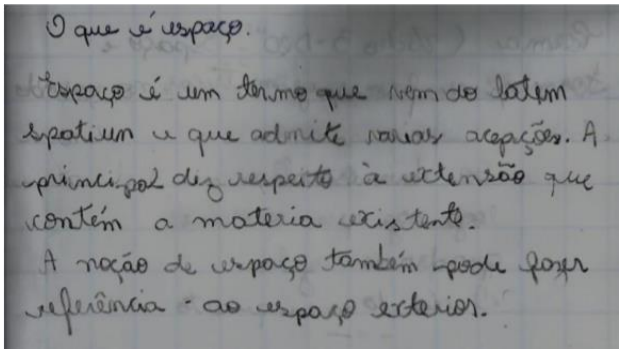

Figure 6[19]

Definition of space – Bianca

Me: Let's create a Sample Space for space, ok? And we'll see how it comes out…

The group decided that, with the concept of space, other issues would be attached:

[19] What space is
Space is a term from the Latin, spatium, and includes several meanings. The most important one is related to extension, which contains existing matter.
The idea of space can also refer to external space.

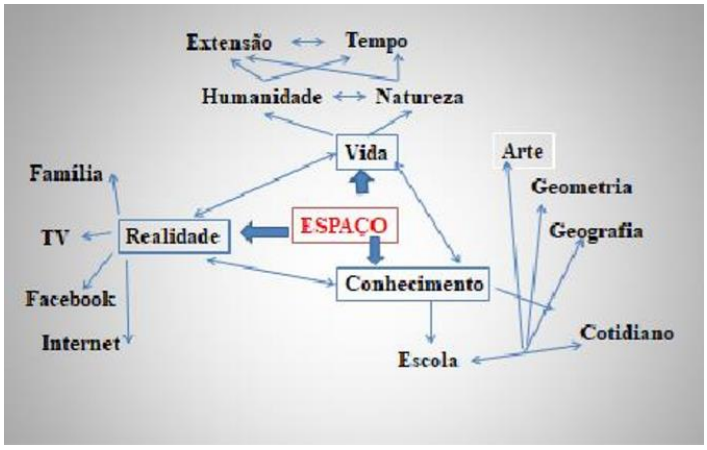

Figure 7[20].

Sample Space of Space

The main topics related to the concept of space were Reality, Knowledge and Life. Reality was chosen because the group concluded that it needs space to occur (Philosopher), but it changes. Today, we live in a cybercultural[21] world in which there is a world inside the computer (Norberto) that can reduce the space between us all and transform private space into something almost impossible to experience (Marsha).

Knowledge is also transformed by space. In home space, in daily life, it is one thing. In school space, it is another[22] (Joy). Space shapes the type of knowledge. *Unfortunately, it seems to me that knowledge is not able to change space* (Marsha). At this point, we discussed issues related to Paulo Freire, especially his concept of Banking Education[23] (Freire, 1991). When we observe Paulo Freire's concept, *we can see [school] still saying that one space is poor, the other is rich* (Joy), that one has knowledge while the other does not.

We also discussed the segmentation of the concept of space in school. *Space is divided into disciplines, there is space in Geography, Art …* (Brida), *geometry* (Me). I used this

[20] Words: Extension, Time, Humanity, Nature, Family, Life, Art, TV, Reality, SPACE, Geometry, Geography, Facebook, Knowledge, Internet, School, Daily life.
[21] Cyberculture is "[…] group of techniques (material and intellectual), practices, attitudes, ways of thinking and values that develop along with the growth of cyberspace." Cyberspace "[…] is the new means of communication that arises from the worldwide interconnection of computers" (Levy, 1999, p.17).
[22] A similar discussion can be found in Carraher, Schliemann & Carraher (1993).
[23] According to Freire (1991), Banking Education is the educational model that is reduced to depositing knowledge by the teacher into the student. It is based on memorization instead of critical, problematizing thought.

moment to show them that disciplinary construction is itself a historical production attempting to divide reality into slices to better apprehend it.[24]

The last element that is considered essential by the group is Life. Life is the space in which we produce ourselves (Noah/Philosopher). Throughout time, we build our living space. It is an extension of ourselves. *Only a good relationship between humanity and nature would make this space nice for all* (Noah).

> *Me: And the question that begs to be asked, what does all this have to do with Math Education[25]?*
> *Brida: Mr. M., it's… If we don't create a kind of math that is different from what we experienced, nothing will ever change.*
> *Noah: We can't think of Math Education as something that isn't part of our lives…*
> *Brida: With everything divided into disciplines...*
> *Me: Oh, really? Why not? (Moment of silence).*
> *Joy: Because things are not divided, they're all together ... (...) everything is all together…*
> *Philosopher: Concepts are interconnected, reality is one thing… Not several.*

Concept and action

> *Me: Let's move forward with our discussion on definitions!*
> *Pink Butterfly: But haven't we finished?...*
> *Me: Let's see if there is anything else we can use for Math Education (...)*
> *Angelina: This is what I wrote, (She reads)*

[24] Fazenda, 2010.

[25] The answer to this can be found in Ubiratan D'Ambrosio (2005, 106): "But how do you connect trinomial factoring with Peace?" It is likely these same individuals are in the habit of teaching trinomial factoring by giving as an example the trajectory of a cannon projectile. But I am sure they do not say – or even suggest – that this beautiful mathematical instrument, trinomial factoring, gives certain individuals – professional sharpshooters who were probably the best in their math class – the capacity to fire a deadly bomb form a cannon onto a population of human beings, of flesh and blood, with emotions and desires, and kill them, destroying their homes and temples, destroying nearby trees and animals, polluting lakes or rivers around them. And when then return from their mission, they will calmly receive praise and awards. The implicit message is learn trinomial factoring and you will be able to do all that. Only those who do well in math will have the necessary theoretical basis to aim cannons at populations."

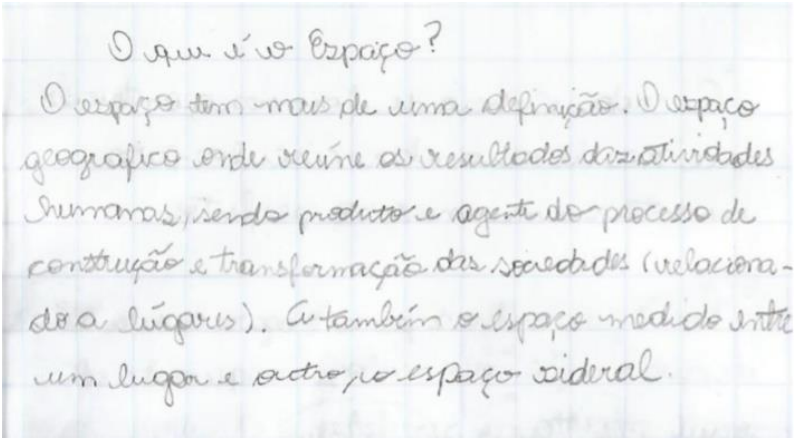

Figure 8[26].

Definintion of Space – Angelina

Angelina: I wrote that because, out of everything I read, it seems to be more like what I think.
Me: What do you mean?
Angelina: First, I imagined the space in my room. I do things in there. Then, I read this (points to the graph notebook) and thought, this is what really meant something. It is how I see that space includes a person's activity (...) You can measue it, for example. (...)
Pink Butterfly: What I did was say that space has features. Like (she begins to read)

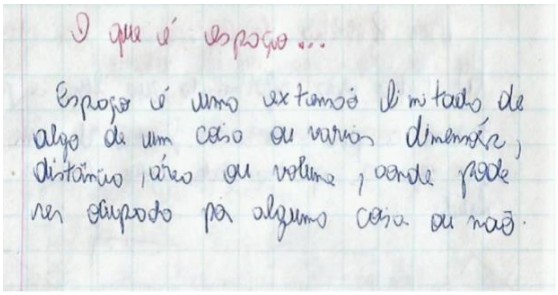

Figure 9[27].

Definition of Space – Pink Butterfly

Pink Butterfly:I don't know if it... Because, like, to change it around, we have to measure it, like Angelina said.
Ducarmo: It's as if it only exists if we can change it around.
Me: Could we think about that for other concepts we were taught in school?
Ducarmo: I think so, Mr. M. School doesn't think it's important, but I think we should shuffle concepts around to know them better.

[26] What is Space? Space has more than one definition. Geographical space is where the results of human activity come together, the product and agent of the process of construction and transformation of society (related to places). It is also the space measured between one place and another in outer space.

[27] What is space…

Space is the limited extension of something, of a thing or several dimensions, distance, area or volume. It can be occupied by something or not.

Me: When I think about it, I agree, Ducarmo.
Pink Butterfly: Even if it is all set, we should try. (...) We might even be able to mke a new connection. Can you imagine, Mr. M., us as mathematicians!

Our group begins to see that understanding a concept is a matter of action. In other words, we can operate a concept, make it something that can be used with our activity. This is such an act of experimentation that the group is even able to see themselves as mathematicians. This would go from one point to another, such as those lined up by Carvalho (2007):

1. Clearly mark off where the concept is applied, also referring to where it does not.
2. Relate the concept with other concepts, extending it to broader situations.
3. Show several examples and counterexamples where the concept is applied or not.
4. Apply the concept to other situations, solving new problems and, consequently, creating new concepts (Carvalho, 2007, p.1)

In this sense, Ducarmo expresses the view that school has forgotten to stir up its concepts. Perhaps, due to the awe with which concepts have been treated within school, or the view that we have come to the end of a thought, a structure in which a subject's action is constrained, favoring memorization, was created. Maybe the illusion that math is an act of pure abstraction has reached such a level that, besides retreating, the very activity of math was made smaller.

It's always great to have someone help you think

Samantha Noway: Mr. M., what I wrote down is really connected to what you've been saying. (she reads).

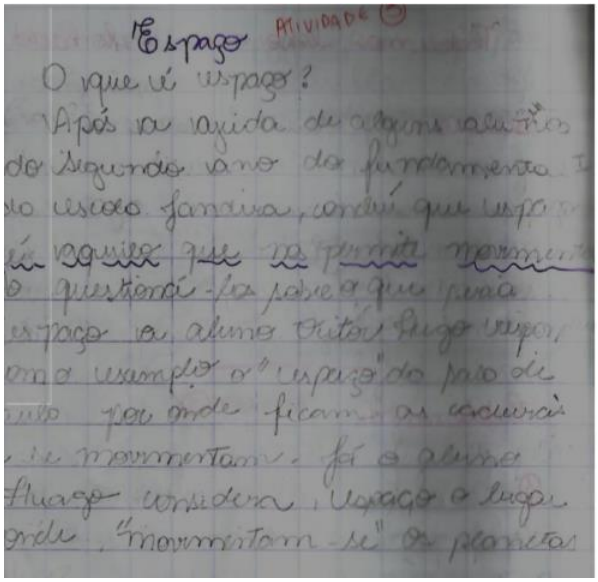

Figure 10[28].

Definition of Space – Samantha Noway

Me: That's cool. You asked your students to answer?
Samanta Noway: You should have seen how happy they were when I told them they were helping me with a college assignment.
Me: And what did they say?
Smantha Noway: That we're stupid. That it's an easy assignment. (Laughter)
Me: How was it working with them?
Samantha Noway: It was cute seeing them discuss it. Then I told them it was a math assignment and they said math is something you do by yourself. This wasn't math.
Me: Really?
Samantha Noway: Because their teacher always tells them "when you're doing math, you have to be quiet, you can't talk to anyone." She says, "you need to concentrate, because math is done by yourself."
Me: what do you think?
Smantha Noway: I don't think so.
Me: So how do you do it?
Smantha Noway: Don't ask me such a hard question. (Laughter)
Katyane: You do it by talking, right? Like we just did here. Like you did (referring to Samantha Noway) with the kids.

[28] Space

What is space?

After getting help from some students in second grade at the Jandira Elementary School, I came to the conclusion that space is what allows us to move. When I asked them what space is, a student answered, as an example, "space …. Where the chairs are and are moved. Another student said that "space is where planets move around".

Me: That's what I think too. Mathematics of action needs some conversation. Listening, reaching an agreement. Not agreeing with the solution to a problem. It should be connected to where we live, we should discuss things, solve the group's problems. But you can't do it alone. Those who decide on their own are more like dictators. (Laughter).
Samantha Noway: And what do we do with exactness?
Me: I always thought, well, there's this idea of exact math, but there's also the issue of how to use exact math. There's the issue of what is just, what is right, of respecting what others think. The daily problems make us think about this, and thinking of how math works to help us with this. (A discussion about daily school problems begins).
Noah: Mr. M., I was almost tearing up my sheet and writing another definition… Not one thing you guys said fit into what I wanted. So, I think this is my chance. (Reads).

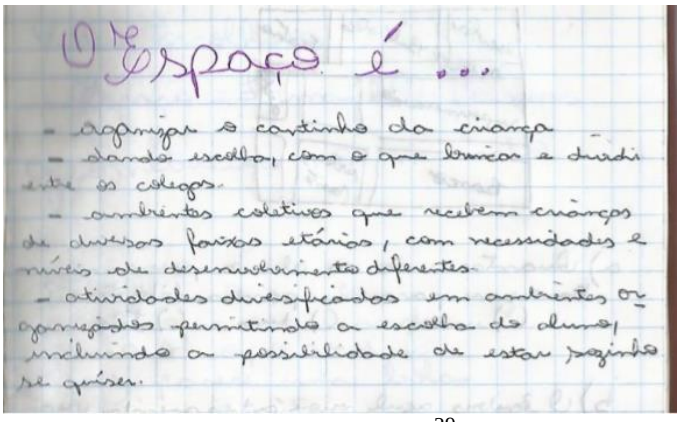
O Espaço é ...
- organizar o cantinho da criança
- dando escolha, com o que brincar e dividir entre os colegas.
- ambientes coletivos que recebem crianças de diversas faixas etárias, com necessidades e níveis de desenvolvimento diferentes.
- atividades diversificadas em ambientes organizados permitindo a escolha do aluno, incluindo a possibilidade de estar sozinho se quiser.

Figure 11[29].

Definition of Space – Noah

Me: Why was this your chance?
Noah: Because I wrote what I live. I thought about my life. What I am experiencing with Childhood Education. We think of every little corner with love, every space for the kids. And then we let them try it out. (…) I am all about the corners now. (…)
Me: If you had to think about all this for Math Education, what would you say?
Noah: I don't know, Mr. M. I see how much what we experience is important for us. And school doesn't say that. I think maybe concepts should start from there. We have to live stuff, concepts have to have that connection with our lives. (…) So, I'm teaching addition, and this boy is living in poverty and sees stuff adding up in other people's lives and not in his? And we're not supposed to ever talk about that? It's a lot!

[29] Space is

- Organizing a child's corner.
- Giving choice as to what they can play with and what to share with classmates.
- Collective environments that welcome children of different ages, with different needs and levels of development.
- Different activities in organized environments that allow children to choose, with the possibility of being alone, if this is what they want.

At this point, the discussions brought up were related to individuality in learning and the social role of math taught in school (Moraes, 2018). Talking about the process of deciding the solution to problems was put into action as an essential element in the practice of math (Moraes, 2018), considering also that these practices are valid when they allow us to discuss the reality experienced by our students, as well as the reflections about it.

Updates and Final Consideration. From the readings and the Sample Space

From the readings to a Sample Space

We defined we would make our Sample Space. To build it, two readings had to be done[30], problematizing the documents based on what we developed in class with the definitions referring to space. The text would be a starting point for our sample Space.

On building our Sample Space

Class had hardly started and I hear the first comment:

Noah: I hated these documents.
Me: Lord, why… From the start?
Noah: When I read the PCN, I actually liked it. As for the Basis…
Me: What's wrong with it?
Noah: Mr. M., it's actually kind of interesting in the beginning when it's talking about theory. After that, it's a disaster. It really breaks our backs… (…) When will I ever be able to propose math while thinking about all that with the kids? (Referring to the objects of knowledge of the BNCC). (…)
Bianca: We were talking about that here, Mr. M. The cool thing is that both include all that citizenship stuff, of forming a citizen. But then, when it comes to the crunch, the Basis includes thousands of contents. Like… it's the teacher who has to go through the whole textbook.
Olivia: Oh, but there's more. In the Basis, I thought what I found on integrating topics[31] *was really cool. The PCN doesn't have that.*
Me: Actually, the PCN refers to transversal topics.
Philosopher: That's actually stupid in both of them, Mr. M.
Me: What?
Philosopher: These topics… Damn… (…) They're just for show. If it were for real, it would all be together. Not separated. To focus on content. If it were for real, it would be all together. Where you find it in the document doesn't say much either, does it, Mr. M.?

[30] Brazil (1998). Ministry of Education/Department of Middle and Technological Education. National Curricular Parameters (PCN): First Years of Elementary Education – Mathematics. Brasilia, MEC/SEMTEC.
With the reading from pages 51 to 61, in which the following questions are discussed: selection; blocks and organization; and the evaluation of math content for the first years of elementary school.
Brazil (2015). Ministry of Education. National Basis of Common Curriculum (BNCC): preliminary document. Department of Elementary Education. Brasilia.
[31] Brazil, 2015.

Me: Yes. They're thought through.
Philosopher: So, they propose we be focused, but they don't give you any space for it. All for show… (…)
Me: What did you think of the blocks?
Norberto: Even then, the PCN is more put together. (…) Look at the name: "Space and Form" (referring to the PCN). Look at this one: "Geometry" (referring to the BNCC). (…) Imagine how many cool, different things can be found between space and form. And geometry, it's just that math that says "the square of the hypotenuse is equal to the sum of the sides"[32] It's just bland.
Me: Bland?
Norberto: Bland because it could be different.
Me: Now what, Joe?[33] You're going to become teachers with this curriculum…
Cecilia: Mr. M., tell me, do we have to follow the PCN or the Basis?
Me: After it's sanctioned, the curriculum from different places must align with the Basis.
Cecilia: And what do you think?
Me: Who me?
Cecilia: Yeah, you.
Me: Oh, Lord… (my moment of silence).
Cecilia: What is our obligation as a math educator?
Me: To offer quality education.
Cecilia: In other words, the quality you mentioned before[34]. (…)
Brida: We should follow what we think is important…That the main focus is the formation of a citizen… We won't be going against what is documented. It's there… the formation of a citizen. Math should be directed to that.

The students considered the proposal of both curricula contradictory – the PCN and the BNCC – because they believed that they do not allow space for the formation of a citizen to occur, in the terms used in their own philosophies. They also questioned the separate occurrence of integrating topics (BNCC) and transversal topics (PCN) as if they were things you should do if there was time left (Norberto) and not priorities in education.

To a certain extent, I backed off when they wanted me to be present and judge the value of the documents. When that happened, I saw it as the act of a researcher who steps back from the subject of the research. Today, I am inclined to regard this behavior as cowardly. After all, I too was a member of the group. I thought that, as a teacher, I could not express opinions, but

[32] Transcribed from what an Elementary school student says.
[33] An expression taken from the poem "Jose" by Carlos Drummond de Andrade.
[34] The quality education I described to the students was based on the aim of education developed by Beatriz D'Ambrosio (2015, p. 2), "[…] to support, encourage and create opportunities for all children, young people and adults to reach their human potential."

I do not believe that any longer[35]. I believe we should uphold ethics as our guiding principle, even if it is contrary to others.

I can see in the group, on the other hand, the signs of a teaching insubordination (Lopes & D'Ambrosio, 2015). They want to think about quality education. That is their priority. As for the curricular proposal, they want to come up with actions that are aligned with it, but that also allows them to carry out a math education that they believe is able to fulfill quality education.

With all this in mind, we decided to wrap up our Sample Space. It had this format:

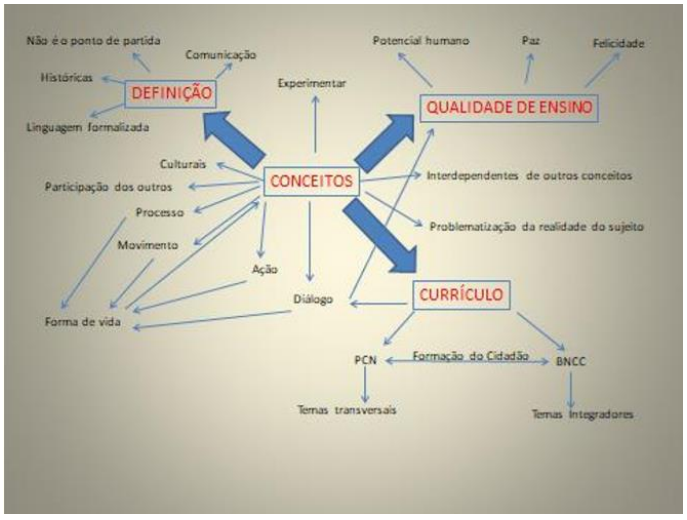

Figure. 12[36].

Sample Space – Concept

In our analysis in regard to our work as math educators, four points stood out:

a) Definition. Definitions are historical constructs (Luiza/me) represented by formal language (Dahlberg, 1978). From them, as conventions (Helena), there is the possibility that there be the communication of academic math (Philosopher). As for our role, as math educators, we should historicize them (Luiza) or not make them a starting point for teaching (Ana/me).

[35] I might not accept the pedagogical concepts of this or that author, and I must tell the students why I am opposed to them, but I cannot, with my criticism, lie" (Freire, 2004, p.18.).

[36] It is not a starting point: Communication, Experimenting, Human Potential, Peace, Happiness, DEFINITION, TEACHING QUALITY, Historical, Formal Language, Cultural, CONCEPTS, Interdependent of other concepts, Participation of others, Process, Movement, Problematizing the subject's reality, Action, Dialog, Way of life, CURRICULUM, PCN, Forming a citizen, BNCC, Transversal topics, Integrating topics

b) Quality of teaching. Teaching quality must be the focus of a teacher's activity. We are the ones who should consider what teaching quality is. As an answer to the students' questions about the topic, my role was to understand and encourage human potential (D"Ambrosio, 2015) . This led the students to associate this direction with an education for peace (Norberto), based on their readings of D'Ambrosio (2005) for the development of a happy and ethical subject (Noah).

c) The Curriculum. When analyzing the current curriculum, the National Curriculum Parameters (PCN – Brazil, 1998) and the National Basis of a Common Curriculum – the preliminary document (BNCC – Brazil, 2015), the directions indicated the formation of a citizen, as seen in the documents (Bianca). They also saw that this principle is not very relevant in the proposals for action (Olivia), since the group believes the separation between Transversal Topics (Brazil, 1998) and Integrating Topics (Brazil, 2015) from the field of the Disciplines themselves does not emphasize more humanizing action. This is something that, with the excess of work content, would probably be neglected (Philosopher). Furthermore, the group considered that the path that should be adopted for the use of a curriculum for quality education is that of communication (communicating with the subjects (Noah), communicating with the problems found in local reality (Norberto), communication among teachers (Noah)).

Concepts. Another point that was extensively debated brought up the relations created around mathematical concepts in Math Education. According to the group, these demand that those involved have an attitude of problematizing reality (Norberto) which is done through communication, experimentation, action and movement. Its potential for development occurs in its relation to other concepts (Cecilia) withing cultural spaces (Noah) and with the participation of others (Samantha Noway) as individuals who debate and wish to reach a consensus regarding it. The group also pointed out that this process gives math teaching life, a life that is always in transformation and for quality teaching.

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