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A study and research path for financial education

Un recorrido de estudio e investigación para la educación financiera

Un parcours d'étude et recherche pour l'éducation financière

Um percurso de estudo e pesquisa para a educação financeira

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Abstract

Financial Education is an increasingly widespread theme in national and international research, but it is still not very present in Brazilian schools, mainly public ones. In this sense, this article indicates how to deal with financial education in teacher education, with guidelines adopted for basic education. This research is based on the theory of didactical situations and the anthropological theory of the didactic. The latter provides a theoretical-methodological framework for a training device called study and research path. This device makes it possible to build collective responses to problematic educational issues, such as school financial education. The research was developed in a teacher education course for the initial years of schooling. The objective was to outline guidelines for a financial literacy model that meets the citizen's social literacy needs and the needs of Brazilian educational laws. The study process produced a collective response with guidelines for a model to develop financial literacy in teacher training, with the potential for adaptation to basic school. In this sense, modeling, via a didactical situation, must mobilize knowledge from financial education in a context that

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involves simulations of budgets, consumption, entrepreneurship, sustainability and/or investments, which require adequate and conscious decision-making regarding financial management.

Keywords: Mathematics education, Financial education, Anthropological theory of the didactic.

Resumen

La Educación Financiera es un tema cada vez más difundido en las investigaciones nacionales e internacionales, pero todavía poco presente en las escuelas brasileñas, principalmente públicas. En ese sentido, este artículo busca contribuir con indicaciones de cómo abordar la Educación Financiera, en la formación de docentes, con lineamientos adoptados para la educación básica. Esta investigación se fundamenta en la Teoría de las Situaciones Didácticas y la Teoría Antropológica de lo Didáctico, este último proporciona un marco teórico-metodológico articulado a un dispositivo formativo denominado Recorrido de Estudio e Investigación. Este dispositivo posibilita construir respuestas colectivas a problemáticas del área de la educación, en este caso, la Educación Financiera Escolar. La investigación se desarrolló en un curso de formación docente para los primeros años de escolaridad. El objetivo fue delinear directrices para un modelo de educación financiera que atienda las necesidades de alfabetización social de los ciudadanos y las necesidades de las leyes educativas brasileñas. A partir del proceso de estudio, se produjo una respuesta colectiva con lineamientos de un modelo para desarrollar la educación financiera en la formación docente, con potencial de adaptación a la escuela básica, en ese sentido, la modelación, a través de una situación didáctica, debe movilizar saberes desde la Educación Financiera en un contexto que involucra simulaciones de presupuestos, consumo, emprendimiento, sustentabilidad y/o inversiones, que requieren una adecuada y consciente toma de decisiones en materia de gestión financiera.

Palabras clave: Educación Matemática, Educación Financiera, Teoría Antropológica de lo Didáctico.

Résumé

L'éducation financière est un sujet de plus en plus répandu dans la recherche nationale et internationale, mais elle est encore peu présente dans les écoles brésiliennes, principalement publiques. Cet article cherche à apporter des indications sur la manière d'aborder l'éducation financière, dans la formation des enseignants, avec des lignes directrices étendues à l'éducation de base. Cette recherche s'appuie sur la Théorie des Situations Didactiques et la Théorie Anthropologique du Didactique, ce dernier fournit un cadre théorique et méthodologique articulé à un dispositif de formation appelé Parcours d'Étude et Recherche. Ce dispositif permet de construire des réponses collectives à des problématiques en matière d'éducation, a savoir l'éducation financière à l'école. La recherche a été développée dans un cours de formation des enseignants pour les premières années de scolarité. L'objectif était de définir des lignes directrices pour un modèle de littératie financière qui réponde aux besoins de littératie sociale des citoyens et aux exigences des lois brésiliennes sur l'éducation. A partir du processus d'étude, une réponse collective a été produite avec des lignes directrices d`un modèle de développement de la littératie financière dans la formation des enseignants, avec un potentiel d'adaptation à l'école primaire. En ce sens, la modélisation, via une situation didactique, doit mobiliser les connaissances de l'éducation financière en un contexte qui implique des simulations de budgets, de consommation, d'entrepreneuriat, de durabilité et/ou d'investissements, qui nécessitent une prise de décision adéquate et consciente en matière de gestion financière.

Mots-clés : enseignement des mathématiques, éducation financière, théorie anthropologique du didactique.

Resumo

A Educação Financeira é um tema cada vez mais difundindo em pesquisas de âmbito nacional e internacional, mas ainda pouco presente em escolas brasileiras, principalmente, públicas. Nesse sentido, busca-se, com este artigo, contribuir com indicativos de como lidar com a Educação Financeira, na formação de professores, com diretrizes estendidas à escola básica. Nossa pesquisa fundamenta-se na Teoria das Situações Didáticas e na Teoria Antropológica do Didático, que fornece um quadro teórico-metodológico articulado a um dispositivo de formação denominado Percurso de Estudo e Pesquisa. Tal dispositivo possibilita a construção de respostas coletivas a questões problemáticas da área da educação, nesse caso, a Educação Financeira Escolar. A pesquisa foi desenvolvida em um curso de formação de professores para os anos iniciais de escolarização. O objetivo foi traçar diretrizes para um modelo de

letramento financeiro que atenda às necessidades sociais de letramento do cidadão e às exigências das leis educacionais brasileiras. A partir do processo de estudo, produziu-se uma resposta coletiva com diretrizes de um modelo para desenvolver o letramento financeiro na formação de professores, com potencial para adaptação para a escola básica. Nesse sentido, a modelagem, via situação didática, deve mobilizar saberes da Educação Financeira, num contexto que enrede simulações de orçamentos, consumo, empreendedorismo, sustentabilidade e/ou investimentos, que requeiram tomadas de decisões adequadas e conscientes, no que se refere ao gerenciamento financeiro.

***Palavra-chave*:** Educação Matemática, Educação Financeira, Teoria Antropológica do Didático.

A study and research path for financial education

To contribute to citizen financial education, as reported by Silva et al. (2014), the Organization for Economic Cooperation and Development[4] (OECD) member countries instituted the *Financial Education* Project in 2003-2004. In 2005, a document was created based on the research report of that project[5], with practical suggestions for governments. Based on the OECD document, Brazil adopts the following definition:

> Financial Education is the process through which individuals and societies improve their understanding of financial concepts and products so that, with clear information, training, and guidance, they acquire the values and skills necessary to become aware of opportunities and risks involved, to make well-informed choices, know where to look for help, adopt actions that improve well-being. In this way, it contributes to forming responsible individuals and societies committed to the future. (OECD, 2019, p. 68)

Thus, the government prepared the Brazilian financial education strategy per the OECD recommendations. The private financial institutions were the first to discuss the topic, emphasizing financial products and services.

Although Brazil is only a guest member of the OECD, the country has begun to take slow but effective steps towards an educational policy within the scope of financial education. Thus, a proposal for teaching financial education has begun to take shape for discussion and implementation in Brazilian schools.

In Brazil, the Committee for Regulation and Supervision of Financial, Capital, Insurance, Pension, and Capitalization Markets (Comitê de Regulação e Fiscalização dos Mercados Financeiros, de Capitais, de Seguros, de Previdência e Capitalização - COREMEC)[6], was the body responsible for creating effective financial education policy by creating a working group (WG), COREMEC– GT. Thus, the National Strategy for Financial Education (Estratégia Nacional de Educação Financeira - ENEF) was instituted.

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[4] Organization for Economic Cooperation and Development (OECD) is an international organization that represents 34 member countries, which enable them to exchange information of political interest to enhance economic growth and collaborate to the development of participating countries. Brazil, however, is not a member: it is a guest country of this organization, which has its headquarters in Paris, France.

[5] Called *Recomendações sobre Princípios e Boas Prática de Educação Financeira e Conscientização*. [Recommendations on Principles and Best Practices of Financial Education and Awareness.]

[6] COREMEC, whose purpose was to promote the coordination and improvement of the performance of federal public administration entities that regulate and supervise activities related to the public collection of popular savings. Art. 2nd COREMEC, which has an advisory nature, will be composed of: I - the President of the Central Bank of Brazil – BCB and a Director of that Autarchy; II - by the President of the Securities and Exchange Commission - CVM and by a Director of that Autarchy; III - by the Secretary of the National Supplementary Pension Superintendence - PREVIC, of the Ministry of Social Security, and by a Director of that Secretariat; and IV - by the Superintendent of the Superintendence of Private Insurance - SUSEP and by a Director of that Superintendence. (http://www.planalto.gov.br).

At that time, four government bodies were part of the WG: the Central Bank of Brazil (Banco Central do Brasil – BCB, responsible for the financial education of adults); Securities and Exchange Commission (Comissão de Valores Mobiliários – CVM, responsible for the financial education in schools and for moving forward, aiming at the development, within the scope of a pedagogical support group, of didactic situations to be used in a pilot project); National Supplementary Pension Superintendence (Superintendência Nacional de Previdência Complementary – PREVIC); and the Private Insurance Superintendence (Superintendência de Seguros Privados – SUSEP) (Brasil, 2011).

During its work towards implementing a Brazilian national strategy for financial education, the WG received support from non-governmental initiatives, such as representative entities of the financial market, which promoted and collaborated with their experiences related to the work developed in the financial education area. Institutions such as ANCOR[7], ANBIMA[8], FEBRABAN[9] and others were part of this collaboration with the COREMEC WG. In the creation of the National Strategy for Financial Education (Estratégia Nacional de Educação Financeira), some of these institutions became part of the National Committee for Financial Education (Comitê Nacional de Educação Financeira) (Brasil, 2011).

From the curricular point of view, there was already evidence of financial education in mathematics teaching in basic school, as provided in the National Curriculum Parameters (Parâmetros Curriculares Nacionais - PCN), in the approach to cross-cutting themes.

The notion of transversality permeates the inclusion of social issues in the school curriculum. From this perspective, the articulations between school subjects and mathematical knowledge application in various contexts of local, regional, national, and international realities advance.

In the approach of the National Common Core Curriculum (Base Nacional Comum Curricular - BNCC), financial education is also defined as a cross-cutting theme but focused on a contextual approach in which content must be related, such as percentage, interest, and amount; social practices such as buying, selling, saving, etc. Such imbrication presupposes the need to make citizens financially literate. Thus, greater visibility of this context is needed in teaching. In other words, we must place financial education beyond mathematical knowledge.

[7]ANCOR -Associação Nacional das Corretoras de Valores Câmbio e Mercadorias Âncora Cambial (National Association of Securities Exchange and Commodity Brokers).
[8]ANBIMA - Associação Brasileira das Entidades dos Mercados Financeiro e de Capitais (Brazilian Association of Financial and Capital Market Entities).
[9] FEBRABAN - Federação Brasileira de Bancos (Brazilian Federation of Banks).

In addition to the indications in official documents, we must pay attention to the participation of financial institutions in disseminating notions of economics in schools. Banks have been in charge of preparing the financial education proposal in several countries and Brazil. This fact is a point of concern since the materials for school use are prepared by professionals linked to such institutions. Thus, they aim to prepare students to be good consumers of banking products and future "financially educated" investors.

On the other hand, research on mathematics education contributes to developing school practices within the scope of financial education, which favors the overlap between formalized school knowledge and everyday knowledge.

In this sense, this research aims to outline guidelines for a financial literacy model that meets the social literacy needs of citizens and the requirements of Brazilian educational laws.

We used two theoretical frameworks to achieve this goal: the theory of didactic situations (TDS) and the anthropological theory of didactics (ATD). The ATD has a formative device, the study and research path (SRP), which we will use in the study process with students from an undergraduate teacher education course training prospective teachers to teach the initial years of schooling.

Bibliographical study

Teixeira (2017) postulates that financial education should be developed within the scope of school mathematics from a perspective of contextualization, as announced in Brasil (2017). In addition, the author points out that the topic is not limited to problem solving and has the potential to create situations for debates, reflections, and decision-making on financial issues. In his research, the author found the need to address, in the initial and continuing teacher education in the early years, the principles of financial education that help in teaching practices in this context.

Campos et al. (2015) reinforce the assertion that financial education must permeate mathematics teaching from the early years, from a contextualization perspective, based on the students' reality. The authors assert that mathematics education can contribute to the teaching practices of financial education, based on research, regarding "[...] problem solving and mathematical modeling, accompanied by aspects related to the use of ICT in the classroom". (Campos et al., 2015, p. 575). In addition, in this context, the practices can help citizens' formation by allowing the exploration of "social problems arising from the mismanagement of personal finances". (Campos et al., 2015, p. 575).

From this perspective, Dantas et al. (2017) analyzed the interactions of 4th-grade students with teachers and knowledge, focusing on financial situations and the environment. They used the theory of didactic situations (TSD)[10] to "identify things that have a price and things that do not have a price but have value, stimulating the difference between money and happiness" (p. 64). They started by discussing: "What is the money for?" (p. 64).

They used, as a context of the a-didactic situation,

> [...] the resource of children's fables, as it is a good way for children to learn economic and financial concepts playfully. Students were asked to read the fable "The Grasshopper and the Ant" to reflect and discuss work, savings and economics ... After reading the fable, the children said that the cicada made a mistake for not having saved food and that it was very lazy. The children said that the teachings of the fable are: we should listen to our parents' advice, children should study to get a good job, to be someone in life, and children should not just play, but also study. If you do not study, you will not get a good job. They also said that the Ant saved food for the future and that we should also save for the future; if you spend everything you have on "silly objects", no money will be left. They said that some people spend money to "show themselves off" by having expensive parties, etc. They also mentioned saving water for the future. (Dantas et al., p. 64-65)

The authors showed that the proposed situation enabled the engagement of the students, who:

> Questioned, sought strategies to resolve issues, defended their arguments, associated work with money, and reflected on the proposed themes, such as participation in the household budget, shopping lists, child entrepreneurship, or even linking school activity with social practice. (Dantas et al., p. 68)

These authors highlighted the importance of discussing financial education in schools and emphasized the possibility of adding discussions about finance to issues such as the environment. Despite the announced focus, the authors could have emphasized the discussions about the environment, as the theme appears in the results in a specific way when they mention the care with water.

Raschen (2016, p. 61) uses the TDS, emphasizing:

> Financial education ... is a fertile field for applying the theory of didactic situations, because: 1. It is a theme that touches the students' realities, being easily applied in everyday life. 2. There is a wide range of possible devices since we can use problem situations, games, cards, or even computational resources to learn. 3. The students have some notions of money, whether by handling it, experiences at home or news on television or the internet, contributing to the situation of the action being qualified.

[10] See Brousseau (1996b).

Thus, the author uses as an adidatic *milieu*[11] the electronic spreadsheet, modeling the activities based on the notion of devolution and the moments of action, formulation, validation, and institutionalization. To implement the proposal, we idealized two activities to highlight the contribution that mathematics can offer to the development of financial education: the first addressed notions of economics via news published in the media in order to:

> Encourage the reading of economic news, clarify economic concepts, use mathematical knowledge to obtain additional information to the texts, and develop the ability to synthesize and understand, even if in a general way, the causes and consequences of some current economic situations. (Raschen, 2016 p. 62).

In the second activity, the author used the Electronic Spreadsheet to address financial problems, seeking to "understand how savings work, realize the advantages of, instead of paying in installments, saving money and buying in cash and, finally, understanding the calculations of the Price and SAC systems". (Raschen, 2016, p. 62-63).

The author concludes that

> [...] financial mathematics, ... Cartesian plane, ... linear and exponential functions, ... algebra and ... proportion (including percentage), are fundamental for the development of financial education in schools, skills such as the interpretation of graphics and tables, abstract and algebraic thinking, the ability to perceive numerical and/or algebraic relationships in different everyday situations and the ability to transform a result or a formula into an argument. (Raschen, 2016, p. 152)

The author points out the need for mathematical knowledge to develop financial education at school, emphasizing the contextual approach proclaimed by the BNCC (Brasil, 2017).

In turn, Trindade (2017) based his studies on the ATD to analyze mathematical organizations (MO) and didactic organizations (DO) of high school textbooks approved by the National Textbook Plan (PLND-2015).

Based on his analysis criteria, the author found that the collection addresses issues related to conscious consumption and ethical attitudes; contemplates social, natural, sustainable or economic phenomena; addresses reflections on awareness, relevance and importance of funding and investment reserves. However, only two books in the collection – one activity in each book – address financial planning, compromising the financial literacy of citizens who need to master their spending.

[11] See Brousseau (1996b).

Regarding the MO, the researcher found that there is no need for financial mathematics-specific knowledge to solve the tasks included in the analyzed collection, as the techniques are restricted to the application of algorithms. Therefore, we noticed the absence of mathematical modelling activity as a didactic resource for appropriating knowledge. About the DO, based on Gascón (2003), the author states that there is a classic approach (technical and theoretical axis) toward the teaching of mathematical objects that gives rise to financial education.

It is worth noting that, for Trindade (2017), the function object qualifies as a precursor to discussions regarding financial education in high schools. For him, the function theme favours discussions on consumption and ethics.

Hofmann and Moro (2012) postulate that financial education can articulate school and everyday knowledge. The authors point out:

> In societies like ours, where price seals a large part of exchange relations, and the market impersonally configures social relations, it is surprising that the absence of formal mechanisms for teaching elementary financial notions prevails. This negligence can compromise the development of socialization in one of the most salient dimensions of human experience: the economy as a practice. In a country where more and more children are exposed at an early age to contact with the economic universe, acting as consumers of products and services of the most varied kinds, forming and consolidating educational strategies that promote economic socialization guided by the integration between ME and FE are essential. (Hofmann & Moro, 2012, p. 52)

From this perspective, financial education emerges as an articulator of social practices and formal school education, which indicates, in our understanding, a reason for being[12] of financial education, in elementary school, as a foundation for developing practices with transversal themes in mathematics classes.

Trindade and Ferreira (2016, p. 11) highlight that the

> Financial education is directly related to critical education and financial mathematics for the formation of an autonomous student by contextualizing real-world situations and daily financial calculations, aiming at conscious consumption to avoid indebtedness, and elucidating the importance of financial education, which must be included in the school environment from the initial grades of elementary school.

Despite the indications of how to work with financial education in mathematics classes, we consider that the reason for it in elementary school mainly involves the formation of a *conscious* consumer before making choices of buying, selling, saving, etc. Based on

[12] The reason for the study of particular objects is the answers to problematic questions of the area in which they are inserted; that is, the reason responds to intradisciplinary and extradisciplinary questions. In the second case, it responds to circumscribed questions in social practices.

already established indicators, we sought to bring together the guidelines set out in surveys and official documents, in a model of financial education, with a focus on financial literacy that meets the need for forming critical and autonomous citizens. In particular, we will focus on teacher education, as they will be the main disseminators of financial literacy in schools.

Methodology

In this research, we used anthropological procedures for data production, which shows that it is qualitative research, as indicated by Fiorentini and Lorenzato (2006) and Borba and Araújo (2006). Thus, we used ethnographic technical resources of the most varied types, such as documentary research, observation techniques, field notes, collaborators' productions, audiovisual capture, and description. From a qualitative perspective, we used the theoretical and methodological framework of the anthropological theory of the didactic (ATD). In this sense, we assume a dynamic of continuing education conducted through a study and research path (SRP) (Chevallard, 2009a, 2009b, 2009c, 2009d).

Theoretical and methodological framework

The anthropological theory of the didactic (ATD), idealized by Chevallard,

> studies man in the face of mathematical knowledge, and more specifically, in the face of mathematical situations". The term "anthropological" in ATD situates mathematical activity against the set of human activities and social institutions (Almouloud, 2007, p. 111).

We base our research on this theoretical perspective, whose guiding notion is praxeology. The term praxeology has Greek origin from two components, *praxis* and *logos*, which, in their combination, summarize that any practice or know-how (*praxis*) is accompanied by a discourse or knowledge (*logos*).

The simplest praxeological structure, proposed in the ATD, called punctual, is denoted by [**T**/**τ**/**θ**/**Θ**], in which **T** is a type of task, **τ** is a technique, understood as the way of carrying out tasks **t** of the **T** type, **θ** is a technology, understood as a discourse that underlies technique **τ** and makes it comprehensible as a means to carry out **T**-type tasks and a theoretical component **Θ** that governs technology **θ**, providing descriptive, justificative, and generating elements of the other components of the praxeology (Chevallard 2009a; Bosch & Gascón, 2009).

Regarding the types of praxeologies, Bosch and Gascón point out that:

> Praxeologies are *punctual* when they are centered on a single type of task, usually associated with a small set of techniques such as: solving a first degree equation, simplifying fractions, calculating the perimeter of a circle or talking about derivatives as an elementary function. When the practical blocks are articulated around a common technological discourse, we start to have praxeologies *locations*, as are the "themes" in which we structure teaching: related functions, divisibility, similarity of figures, etc. And if local praxeologies are structured based on a theory, they form *regional* praxeologies which, in the case of school mathematics, are generally designated as "thematic blocks" or "sectors": functions, statistics, geometry, etc. (Bosch & Gascón, 2009, p. 92 – 93, emphasis added by the author, our translation)

According to the authors above, basic school textbooks generally present specific praxeologies with disjointed tasks. For the authors, these articulations are necessary to minimally establish a better relationship between the subjects who learn the notion to be learned. Such assertions give us guidelines for textbook analysis, i.e., praxeologies are configured as an analytical component of works (Almouloud, 2015).

From this perspective, we asked ourselves, initially, which mathematical or extra-mathematical questions answer each of the areas of financial education school activity. In searching for answers –even partial ones– to these questions, we sought to constitute a study and research path, investigating problematic issues that could generate new issues, based on tackling them.

Our bibliographical study leads us to infer that the dominant model of teaching for financial education proposed for Brazilian schools configured in textbooks is based on consumption, purchase, sale, economy, etc., even if it is declared that the intention is to form critical and reflective citizens. The paradigm that underlies financial education practices in Brazilian basic education schools –and even at a higher level– is the paradigm of visiting works (or monumentalization): the teacher presents and comments with their students as if they were presenting monuments. (Chevallard, 2008, 2009b, 2009c).

We infer that this is the case of bank interest rates, a topic not covered in depth, which does not allow students to have enough control to question them. To oppose this model, Chevallard (2008, 2009b, 2009c, 2009d) proposes a pedagogy of inquiry based on the paradigm of questioning the world - the student studies a question to which he must answer in a didactic process in which the teacher leads the student to find elements that make it possible to respond to that question. From this perspective, we can ask students to investigate what rates are used and how they vary from bank to bank, how this can affect/impact citizens' lives, and what techniques are used to calculate compound interest (which technology justifies them). The conduct thus described characterizes an investigation process.

Due to the several meanings of the notion of investigation, it is configured as problematic. It is addressed by the anthropological theory of the didactic (ATD) in an ecological study that seeks an answer to the problem of the conditions and restrictions of implementing such a process in math classes.

> The didactics of the investigation have as its object of study the conditions and restrictions of the diffusion (and non-diffusion) of research praxeologies in society, including the study of research praxeologies themselves, the places where they will be mobilized ... It studies the quality of the praxeological equipment of the people who implement the research infrastructure on which they can rely. It addressed the following questions: What praxeological equipment helps investigate a particular problem? What is the required infrastructure? What are the tools and components of this infrastructure? To instrument the investigation, what techniques perform what types of tasks in the investigation? How effective is the investigation with respect to the tangible and intangible sets of conditions? (Ladage, 2016, p. 7)

From an anthropological perspective, research favors dynamic teaching, in which knowledge is not offered beforehand, as it usually happens in math classes. However, it emerges with the function of answering a problematic question that favors students' joining a study process. This dynamic was implemented in French secondary schools through a teaching sequence organized around study and research activities (SRA), which gave rise to the study and research path (SRP). This last device is similar to the first but extends the didactic time and the contents/themes worked on. (Chevallard, 2009c).

The premises of the pedagogy of the SRP are based, in part, on the theory of didactical situations (TDS), developed by Guy Brousseau, as in the case of the starting point for the study, from a *fundamental situation*[13], in the Broussenian sense (Chevallard, 2009c). This situation presents, as an epistemological requirement, a project to develop a mathematical infrastructure didactically adequate to the pedagogies of the SRA and the SRP.

Thus, we will have as a starting point a situation that favors students' joining a *study process* (Chevallard et al., 2001). From this perspective, the teacher/researcher must direct the process to bring out answers already established in the literature (responses approved $R^{\Diamond}$) and new responses instituted in the process ($R^{\blacktriangledown}$), even if partial and, sometimes, provisional.

The answer is constructive and assumes that to establish an SRP in the classroom, it is necessary, according to Chevallard (2009a, 2009b, 2009c), a *Herbartian didactic system* composed of a group of subjects (x), such as students, teachers, researchers, etc., who meet to study a matter of interest to them (Q) within the scope of a school activity, under the guidance of one or more director(s) of study (Y). In this way, the Herbartian system can be represented

[13] See Brousseau (1996b).

by the expression as follows: $S(X, Y, Q_0) \rightarrow R$. Among others, two instances of interest satisfy the notion of a *didactic system*: mathematics classes and research communities.

Thus, we will have the system $S(X, Y, Q_0) \rightarrow$ R. As previously described, the variable (X) will represent the group of 27 students (x will represent the individual student where x Є X) of the 4th semester, of the Integrated Degree in Sciences, Mathematics and Languages at the University Federal do Pará (training course for teachers to work in the initial years of schooling). We will have (Y) composed of the study director(s). In this case, the main author (y_1) of this research – excerpt from the doctoral thesis text and his supervisor (y_1), where y_n Є y.

The research was carried out over a semester (60h), divided into two moments and two sessions (from which we will extract some parts to present in this article). This work is justified because, in the course curriculum, the theme addressed includes studying financial education[14]. The path presents two great moments: the triggering of the study process via the fundamental situation and the study process after the situation. For this reason, at first, the research collaborators (X) will be indicated by D_n and, in the second moment, X_n (each component will be represented by x_n)**.** In addition, for the working group, the indication will be G_n.

In this sense, the dynamic that imposes itself in the classroom is the search for answers (even partial ones) to the questions posed:

> In this process of searching for answers, new questions are generated, possibly not formulated in previous stages, which may require the search and construction of new models that can advance the process. (Barquero et al., 2011, p. 24, our translation)

In this way, it breaks with the traditional atomized teaching of mathematics at school since, in this context, the objective is not to learn (or teach) specific concepts but to answer certain questions of interest to the study community (Barquero et al., 2011; Otero et al., 2013).

In this research, we sought to construct arguments that justify the presence of financial education in basic education, based on the notion of *reason for being*, conceived by the anthropological theory of the didactic (ATD). From this perspective, we sought to establish embryonically a reference model for the study of financial education in the initial teacher education course and to prospect that it is implemented in the classroom in elementary school.

[14] The course in question presents curricular organization in themes, instead of subjects. The theme in question was Linguagem e Conhecimento IV (Language and Knowledge IV).

Moments and sessions of the research

For didactic purposes, this stage of the research culminated with sequences of activities that fall under the epistemological basis of Chevallard (2009a, 2009b, 2009c) and Brousseau (1996a, 1996b). The activities were systematized in two moments: the first moment is the fundamental situation, and the second moment is divided into sessions. The starting point of the path was given through a *fundamental situation* (Brousseau, 1996a, 1996b). This first activity triggered the generative question Q_0 whose unfolding generated other questions Q_n.

First moment: Fundamental situation regarding the conception of the first enterprise

As a starting point for the financial education process of study, we resorted to the *fundamental situation*, proposed by Brousseau (1996a, 1996b). Such a situation should involve the student in an investigative activity that favors the formulation of hypotheses, conjecture tests, validations, construction of models of concepts and theories and, mainly, socialization of results among peers. Thus, Brousseau (1996a, 1996b) believes that the student is motivated to act as a researcher and, as stated by Chevallard et al. (2001), encourages them to join a study process.

It is also important to understand the role of knowledge in a didactic situation, which, in this case, to Brousseau (1996a), is to allow anticipation. So, for this, the teacher's role would be to enable students to act on the situation without interfering explicitly or conducting the process.

For Brousseau (1996a), the teacher's skill is essential in providing favorable situations so that, in this action, students appropriate knowledge to transform it into knowledge. Such a transformation enables the application of knowledge in various contexts in which it is required, i.e., knowledge is the rational use of knowledge. In this sense, a minimal approximation of the collaborators' (undergrad students) realities was outlined in this first activity.

The requirement of this moment of the work should follow the format that involves arithmetic calculations and the use of technology –calculator, internet access– in a practical and utilitarian way, so that it could raise a possible unfolding, as required in the SRP. Therefore, we chose the theme *My First Venture* as the *fundamental situation* to have the class join us in a study process related to financial education.

My First Venture:

The situation followed these conditions:

- *Activity name*: Construction of the first venture*;*
- *Chronology:* The activity was developed in three sessions, i.e., three meetings of 150 minutes each (three 50-minute classes in a row on the same day).
- *Proposal and methodological procedures*: The collaborators were divided into four teams. Each group had a specific activity to solve, starting from the question, "*How* to *conceive a first venture?*" The situation was intermediated by the fictitious use of financial resources, in particular, *the money*, in addition to the nuances involved in building any basic development. For example, managing incoming and outgoing cash flow, obtaining money to start an enterprise, expenses, profits and other results for using mathematical knowledge in this practice. The challenge was to initiate and manage, for a certain period, the various proposals that were arranged as follows:

1st Team: Structuring a "Haberdashery";

2nd Team: Organizing a "Class graduation party";

3rd Team: Organizing a "Birthday party";

4th Team: Structuring a small "Stationery shop".

The students were responsible for choosing and forming the teams and constructing the project proposal. After these first organizations, the students defined their choices, in general, according to their experience or their affinity with the enterprise, as exposed by student D_2.

> *... our choice was a relatively easy activity, which we are used to doing sometimes at home. A budget on what we would spend to hold an event.*

The situation leads us to what Brousseau (1996a) calls *devolution* in ATD. This is a fundamental condition and constituent of the acceptance of the student, who starts to assume responsibility in search of a solution to the questions posed. The student also begins to understand the teacher's proposal in a situation that can be addressed based on acquired knowledge. Therefore, when they get the devolution, the proposed situation becomes a problem for the student.

Below is the outline of the proposals for the construction of the first venture, with more details:

1st Team: Organizing a "*Haberdashery*"

This undertaking had as a forecast the construction of a haberdashery, like those that sell various products.

2nd Team: Organizing a "*Class graduation party*";

This task aimed at generating an endeavour to organize a *graduation party*. According to D_3, "*The task was to plan the party without leaving out any details*". Thus, the group was concerned with the assembly and details of the activity (assisted by searches on internet sites with information that helped them), according to D_3's report:

> *We made a list of how many graduates there would be, how many guests each student would bring and what the graduates wanted to have at their party. After that, we estimated the value of each thing chosen: buffet, decor, reception hall, technical team, DJ, among other expenses. With this estimated value in hand, we calculated how much the final value would be for each graduate. The next task was to determine how the students would raise capital to hold the party.*

The important instrument present in the report shows the involvement and the search for solutions to the problem, in a concern with the details present in this type of endeavour. The group realized how important the activity and the central objective of this work were, according to D_3's report :

> *At the end of this work, we could see that we used financial education when we had the idea of investing and undertaking since we decided that the graduation committee would hold raffles and parties to raise funds to invest and profit.*

3rd Team: Organizing a *Birthday party*

The undertaking of a birthday party had extra motivation, mainly because group members already had experience organizing birthdays. In this aspect of the experience, D_2 states the following:

> *We chose to budget for a birthday party and we certainly had to carry out a price survey of the materials that would be needed for it, as we had no idea of values.*

4th Team: Organizing a *Birthday party*

The objective of the team that chose this theme was to set up a small stationery shop and think about its basic structure for its creation and maintenance. So, according to D_4:

> *We opted for buying and selling school supplies: notebooks, pens, pencils, erasers, rulers, colored pencils and other stationery. After choosing the probable enterprise, my study group and I constructed a table consisting of product, quantity, unit value, total purchase value, total sale value, and profit of all items to be purchased in bulk.*

So, the *fundamental situation* led the students to observe, during this construction movement, the need for a knowledge tool, still little experienced in the classroom, which is very present in our lives, the *financial organization*. The students perceived this tool during the development of the task:

> *And, after researching the values, we applied the idea of saving, that is, spending only the necessary, and certainly we choose the cheapest product ..... think about how to budget for parties, purchases and sales, to achieve it from a perspective of financial education.* **(***D_2*)

> *Organization (what and how much we will buy and need) → Budget (the total cost) → Search (search for better prices) → Control (knowing the minimum quantity and how the cost will be covered) → Goal → Investment (with which we will pay the expense) → Consciousness/Awareness (all these "ideas" put into practice).* **(***D_5***)**

> *With guidance from teachers to organize the party... we realized how useful it is to be financially educated, as this gives us a better quality of life when we are aware of how important planning is for finances.* **(***D_3***)**

> *On the subject proposed in the classroom, setting up a shop or haberdashery and other types of sale, my group opted for a school supplies shop. We made a spreadsheet with the prices of materials, the unit value, the amount purchased in quantities such as, for example, a box of pencils and the profit we would have with that sale. Financial mathematics comes in to make this relationship, addressing and raising issues, showing the importance of studying financial mathematics in conjunction with financial education.* **(***D_6***)**

In this way, the first activity (fundamental situation) fulfilled the objective of inserting students in a study process focused on financial education, as we can see in D_1's speech:

> *Something that also caused us apprehension was whether our small business would bring us profit or loss. I must remind you that this was a suggested activity, so that we could later delve into the theme of financial education; however, we did the task without knowing what the topic was about.*

In some students' speeches, we found recognition of financial education as necessary to face the problem related to "*My First Venture*". Thus, we conclude that it was satisfactory to use a *fundamental situation* as a starting point for the development of the SRP, mainly because it presents the potential for the emergence of questions to continue the study process, as evidenced by student D_7's questioning:

> *[...] So I even asked the teacher a question about teaching, why we are studying financial education in integrated education...the reason and why to study financial education in the early years.*

Question D_7 posed was assumed as the triggering question for the rest of the SRP sessions, that is, the generating question Q_0, which started to be studied later, as the path unfolded, to find a satisfactory answer $R^{\blacktriangledown}$ that complements answers already established by the literature $R^{\Diamond}$.

Second moment: Study of the generating question (Q_0) and its offshoots Q_n

Session 1: Study of scientific articles on the subject of financial education

The search for the answer $R^{\blacktriangledown}$, provoked by the system, led study directors to institute a *milieu* suitable for finding already established answers ($R^{\Diamond}$) in *works* (O) that discuss the theme of financial education, disseminated in the literature, to continue the study process that aimed to answer *why and for what purpose children should study financial education in the early years*. Furthermore, *the "how to study*" is present in this process.

The organization of the dynamics of the study process was structured as follows: collaborating students were organized into groups (G_n) and were instructed to proceed with the reading, discussion/socialization of scientific texts (O_1, O_2, …, O_7), as shown in Chart 1. In this sense, they sought questions(Q_n) and answers ($R_n^{\Diamond}$) in the referred works, i.e., what questions were evidenced/raised in works and what answers they presented to the problems posed. The works studied were made available to all students in the class, with enough time for reading and subsequent socialization.

Table 1.

Organization of study dynamics

GROUP G_n	O_1, O_2, …, O_7	$R_1^{\Diamond}$, $R_2^{\Diamond}$, …, $R_n^{\Diamond}$
G_1 (x_1, x_2, x_3 and x_4)	O_1: *Paradigma da Educação Financeira no Brasil* **[**Paradigm of financial education in Brazil] (Savoia et al., 2007). *Q_1:* How important is financial education in Brazil? Is there an ideal model?	*$R_1^{\Diamond}$***:** *We offered a brief overview by stating the relevance of financial education in the school context and a quick literary review as a theoretical contribution to implementing some financial education programs in the public or private sphere. We ended by comparing the "backwardness" of Brazilian education in approaching this new area of knowledge compared to other countries.*
G_2 (x_5, x_6 e x_7)	O_2: *Educação Financeira: como educar seus filhos* [Financial education: How to educate your children] (D'Aquino, 2008)	*$R_2^{\Diamond}$***:** *The importance of financial education in the social context is highlighted, more specifically, in corporate business, and how bad or good administration influences the national economy.*

	Q_2: How can financial education influence a country's economy and, in particular, citizens?	
G_3 (x_8, x_9, x_{10} e x_{11})	O_3: *A matemática financeira na educação básica e sua importância para a formação de um cidadão consciente* [Financial mathematics in basic education and its importance for the formation of a conscious citizen] (Farias, 2013). Q_3: How is the relationship of financial education in Brazilian education, who promotes it in teacher education?	$R_3^{\lozenge}$**:** *The importance and purpose of financial education are revealed against the official education documents, such as the National Curriculum Parameters (PCN) It is noteworthy that this topic does not receive due attention in the classroom. It also showed possible methodological strategies to help the basic education teacher.*
G_4 (x_{12}, x_{13}, x_{14}, x_{15} e x_{16})	O_4: *Educação Financeira nas séries iniciais: Saberes Docentes* [Financial education in the early grades: Teaching knowledge] (Teixeira, 2017). Q_4: What are the competencies and skills to teach financial education? And the subject responsible for it?	$R_4^{\lozenge}$**:** *It is explained that financial education is essential from the early years because it is important to learn to save and to know how to manage finances correctly from an early age. However, it turns out that the teachers did not master the topic accordingly.*
G_5 (x_{17}, x_{18}, x_{19} e x_{20})	O_5: *Investigação sobre as contribuições da matemática para o desenvolvimento da Educação Financeira na escola* [Research on the contributions of mathematics to the development of financial education at school] (Raschen, 2016). Q_5: Only in the teaching of mathematics can financial education be developed?	$R_5^{\lozenge}$**:** *It is verified what contributions mathematics can offer to the development of financial education at school. The subject of the origin and its developments in Brazil is discussed, with the analysis of academic works and the creation of the National Strategy for Financial Education (Estratégia Nacional de Educação Financeira - ENEF) and bringing up the relationship between critical mathematics and financial education.*
G_6 (x_{21}, x_{22}, x_{23} and x_{24})	O_6: *Educação Financeira Infantil: Como o incentivo a essa prática pode auxiliar na formação de adultos financeiramente mais conscientes* [Children's financial education: How encouraging this practice can help in the formation of more financially aware adults] (Kassardjian, 2013) Q_6: What can our course teach? Are we being prepared to teach financial education?	$R_6^{\lozenge}$**:** *This article shows the importance of talking about savings and spending with children, so that they learn the value of money from an early age. In this way, they may be more aware adults in the future. In addition, it talks about the influence of the media on children and the appealing role that the media has with the children's audience, saying that it is important to have something,*

		rather than being somebody or constructing something, influencing them to become, consumerists from an early age.
G_7 (x_{25}, x_{26}, and x_{27})	O_7: *Educação Financeira na Escola: a matemática e as relações pedagógicas na vida dos alunos dos anos iniciais* [Financial education at school: Mathematics and pedagogical relationships in the lives of students in the early years] (Silva, 2016). *Q_7:* What are the didactic materials used and the fundamental methodologies to apply in financial education in the early years?	*$R_7^{\Diamond}$***:** *The teacher who teaches financial education in schools and the teacher's ideas to improve the way children relate to knowledge. It also highlights that the responsibility for the child's learning comes from both the school and the family.*

We highlighted several issues in the texts studied, such as how financial education is neglected in the Brazilian educational system, generated by the treatment it receives compared to other countries, as presented by the collaborators of *G_n:*

> *[...] so, bringing these problems to the classroom, encouraging this child from an early age to have this vision, planning what they will spend next year, what they can spend... so, what caught my attention, too, was the text that [said that], concerning the university, there was no program to train university students that could draw their attention in this regard (...) (x_2)*

It is important to emphasize the connotation given by *x_2*, with a tone of concern when stating that there are still few studies in the financial education area, mainly at universities responsible for training teachers, based on observation of their degree courses. His reflection is corroborated by *x_{17}*'s speech:

> *I realized, by reading the article and the presentation,... that Brazil is far behind compared to other countries, as far as financial education is concerned..., this new area of knowledge.*

We see with *x_{17}* that it is a topic that is strongly constituted as an agenda in developed countries –especially those that are part of the OECD – and in Brazil, where financial education is expanding its horizons, being increasingly reported in the conventional media (radio, newspapers and television) and social (Internet). Moreover, this theme has proved to be essential in terms of its application in everyday life, therefore, necessary in times of economic crisis.

In O_4, it is clear the understanding of the need for studies of financial education in schools from an early age, as stated by *x_7*:

> Children and adolescents become victims of appealing advertisements and soon come into contact with the use of money, and parents, because they think that this type of education is transmitted by the school, do not offer their children any help, and according to the author, the consequence of this is the generalized unpreparedness of the population and the subsequent financial failure.

In the first session, we saw a set of answers ($R^{\Diamond}$) disseminated in scientific research on the presence of financial education at school. The study brought an overview that indicates the need to include this theme since the early years. It indicates the guidelines in some Brazilian documents about the approach of financial education in schools and the need for better preparation of prospective and in-service teachers to deal with the topic in an adequate way, in the sense of acting in the formation of literate citizens financially. This study provoked another question (Q_2) by student x_{19}: *"Where can one find (didactic) material on financial education?"*

Session 2: Textbooks as works that guide the dissemination of financial education in schools

During the search for answers to Q_0 and Q_1, a third question emerged, Q_2, allowing us to discuss how specific didactic materials (books) and/or incorporated into other subjects approach financial education.

To advance the study process, in the effective search for the answer to question Q_2, the directors of studies (y_1 and y_2) proposed studies and analyses of mathematics textbooks for the early years that could/should address financial education in some way (directly or indirectly).

When choosing textbooks, some teams used books available in the sectoral library of the college to which they belonged, and other groups brought their books, as they already worked as teachers in private schools; others were made available by the director y_1, because they were works of great circulation in public and private schools in the metropolitan region of Belém do Pará. It is essential to highlight that the analysis of textbooks is part of the guidelines in the pedagogical project of the course in which we developed this investigation.

The book collections had the following distribution by groups.

Table 2.

Books to be reviewed

Groups	Level	Works
G_4 and G_5	1st grade	O_4: *Saber Matemática* [Knowing Mathematics] (Smole et al., 2013)
		O_5: *Construindo e Aprendendo Matemática* [Building and learning mathematics] (Noronha &

		Soares, 2018)
G_2 and G_3	2nd grade	O_2: *Coleção Fazendo e Compreendendo: Matemática* [Collection Doing and understanding: Mathematics] (Sanchez, 2010)
		O_3: *Construindo e Aprendendo Matemática* [Building and learning mathematics] (Noronha & Soares, 2018)
G_1 and G_6	3rd grade	O_1: *Projeto Buriti Matemática* [Buriti Project Mathematics] (Gay, 2011)
		O_6: *Matemática do Sistema de Ensino Sucesso* [Mathematics of the Sucesso Teaching System] (Noronha & Soares, 2018)
G_7	4th grade	O_7: *Saber Matemática* [Knowing Mathematics] (Smole et al., 2013)

After the organization (Chart 2), question Q_2 provoked the emergence of a sub-question Q_{21} on how to analyze the books, as student x_{15} says

So, what criteria are we are going to use to analyze the textbooks?

The needs of criteria for the analysis of textbooks were fundamental to guide the subjects in search of such an answer. Given the above, the ATD was used to guide the process, specifically, the guidelines listed in Almouloud (2015).

To understand how the criteria work as an analysis tool within the theoretical framework of the ATD, we had to insert the temporary director y_3 (researcher specialist in textbook analysis using the ATD). Thus, we instituted a new didactic system in the SRP, in the form of a workshop, *on evaluative criteria in the praxeological analysis of textbooks*, lasting four hours/class.

Thus, one more work was inserted into the study process. In this sense, Almouloud (2015, pp. 16-17), quoting Chevallard (1999), points to a minimum structure as criteria that can be considered for the analysis of teaching materials, more specifically textbooks, with the praxeological approach of the ATD:

1. For the assessment of task types (T), Chevallard suggests the following criteria:

a) **Identification criteria**: verify that the types of tasks are clearly and well-identified;

b) **Criterion of reasons for being**: check if the reasons for being of the types of tasks are explicit or if, on the contrary, these types of tasks appear without valid reasons;

c) Relevance criterion: verify whether the types of tasks considered represent the most frequently encountered mathematical situations and whether they are relevant in view of the students' mathematical needs.

2. To evaluate the techniques (τ):

The assessment of the techniques relies on the same criteria discussed in the assessment of task types. In addition, one must answer the following questions:

a) Are the proposed techniques actually elaborated or just sketched?

b) Are they easy to use?

c) Is its importance satisfactory?

d) Is their reliability acceptable, given their employment conditions?

e) Are they sufficiently intelligible?

3. Regarding the technological-theoretical block (θ): We can make similar observations regarding the technological-theoretical block. Thus, given a statement, is the problem of its justification only posed or is it tacitly considered pertinent, evident, natural, or still well-known?

a) Are the forms of justification used close to mathematically valid justifications?

b) Are they adapted to the problem posed?

c) Are the arguments used scientifically valid? The technological output of a given activity can be exploited to produce new techniques for solving new tasks.

Following the study process, we move on to the analysis of textbooks based on the praxeological criteria of the ATD. Because they are extensive, in this article, we will discuss excerpts that represent the data produced as a whole since the content of the analyses is similar.

After discussions on the components of theory (praxeology) and how to use it as a tool for analyzing works, x_2 says:

> Reflecting on what would be the mathematical components found in math books from 1st to 5th grades that would be related to financial education... percentage, interest, monetary system, discount, time, entrepreneurship, buying, selling, etc. When analyzing the task and the technique described within these components, we need to pay attention to: What is the purpose of the task? What tasks lead us to reflect on mathematical needs? Do the techniques and tasks work with the concept of profit and loss? The tasks refer to a choice.

x_4 stated that:

> [...] This activity was extremely important to "clear" our thoughts in relation to the research we were going to carry out, to better establish the objectives, objects, assessment methods, techniques and social practices that are contained in the books.

In turn, x_{15} says:

> [...] We learned that to analyze textbooks, we first need to consult the official documents (PCN and BNCC), stipulate criteria, and check whether the tasks presented there meet what they recommend.

It is important to emphasize that the given guidelines guided the analyses, serving as a basis and not as a prescription to be followed, as the study will show. Our reasoning is evident with the insertion of the component assistance with the technique and, in the second moment of the analysis of *critical responses*, *the works* indicated by $\mathcal{G}$.

Before starting the analysis, group $_{G1}$ systematizes:

> x_3: My team immediately catalogued and selected tasks that would be relevant to the analysis. Our first step was to look through the entire book, looking for activities that were related to some notion of financial education, the focus of the research. After that, we discussed and selected the tasks, marked the pages, wrote down in a notebook what that task was or could be related to. Then, following the guidelines, we carried out a more detailed analysis, using each one's classification, which involved analyzing how the author works with financial education and how he could work using that same task obeying the BNCC recommendation.

Thus, the analyses follow (Figure 1)

Figure 1.

Image of work O_5 captured by group G_5

Below is the explanation given by x_{17} about the task of the work O:

This task works with estimates: the first question asks the student to estimate how much each product is worth and then to link the product to each bill that the student thinks equals its price. The technique applied in this case was the knowledge of cash values.
In the second task, Clara appears. She will go shopping. Clara intends to buy a doll that costs 40.00 and another blouse that costs 30.00. The technique applied is of knowledge, of cash values, and the operation is of summing quantities, as the student will have to analyze the banknotes and add them up to achieve the right amount to pay for the desired products.

According to the analysis pointed out above by the student x_{17} in terms of tasks and techniques, we have:

During the analysis, I could see that most of the activities involving the issue of money (reason for being) serve for the student to learn to count and understand specific amounts. Still, there is no concern with teaching financial education itself. The authors use money to teach addition and subtraction operations, nothing more.

Figure 2a.

Figure 2b.

Images of the work O_4 captured by group G_4

Next, the explanation x_7 gives on the task of the crafted work (Figures 2a and 2b):

Tasks are divided into Summary content (small text about); The history of money); Bundling and swapping; Applying what we learn; Representing numbers; Exploring the calculator. The techniques used for development are aided by: Golden material; Game “Ten count one”,; basic operations.

For the analyses, some groups, such as G_2, said that they built charts to facilitate the framing and understanding of the activity. In terms of clipping, we present two of the paintings produced. It is worth noting that some groups included a component called assistance with the technique (Table 3), referring to O_2's analysis.

Table. 3

Analysis of Unit 1 of the work O_2, carried out by G_2

Work	Sample Tasks	Technique	Assistance with the technique	Technological-theoretical block	The reason for being
O_2	TASKS 1, 2, and 3 - Brazilian Monetary System banknotes from R$ 1.00 to R$ 100.00 are presented. The task proposes to the student the idea of buying and changing.	The tasks propose knowledge of numbers up to 100, counting, quantity, notions of addition (join, add), and subtraction (separate, remove).	Use the calculator to perform small operations; illustration of activities and concrete material. Highlight banknotes, support material.	Basic arithmetic.	Monetary transactions – save, buy, sell, and change.

On the analysis of the book (O_2), x_{11} declares that:

> The topic of financial education was found in the part of the book under the title Monetary System. It is worth mentioning that this part of the book has few activities, as only five sheets are dedicated to the monetary system, and only one sheet slightly addresses financial education, as the idea of saving money to be able to buy something in the future is obvious. This book rarely approaches the issue, as most activities use money only to add and subtract, leading to learning only operations and not the core of financial education.

Next, x_7 explains the task of the analyzed work (Table 4):

> Calculate the remainder and the difference between two numbers; solve problems with questions such as: "How much is left?", "How much more?", "What is the difference? Mentally calculating the results of subtractions.

Table. 4

Analysis of Unit 3 of the work O_2, carried out by G_2

Work	Sample Task Types	Technique	Assistance with the technique	Technological-theoretical block	The reason for being
O_2	Observe the plates in the image, associating prices with the value you have; tasks involving buying and selling and the idea of cost.	Operations of additions and subtractions to know how much you will spend and how much you will have left. Use the Brazilian monetary system; Arrange numerals in ascending order.	Highlight banknotes, support material.	Basic arithmetic.	Money manipulation to work on basic operations.

In general, the analyses of the works promote the following reflections:

> X_7: The purpose of talking about financial education in the early years of basic education is that children acquire critical thinking about money, consumption, and the use of credit cards. The financial education field is not prioritized; therefore, the teacher has little preparation to play the role of a financial educator. It is not the teachers' fault. It is a failure of the education system that does not prioritize this subject. Our analyses showed that the books deal much more with financial mathematics and its contents (interest, loan, expense, values) than financial education. They use the money to work on mathematical subjects such as fundamental operations and not awareness of the use of money, planning, investments, and savings.

The aspects of the analysis of financial education in the analyzed mathematics books indicated that there is greater interest in the works in the application for the context of use as a mathematics tool for the practical life of the student, as we can see in the comment made by x_{18}, referring to the completion of his group, G_5.

> [...] The book does use financial education much. It does bring some activities with money, though. However, the idea behind it is not to know how to manipulate or save, but just to teach how important it is to know how to deal with money, since it is part of the mathematics content. Therefore, I see that there is no concern about this teaching, since money only appears to add, subtract, multiply, or divide.

The development of the SRP, in particular, the works read, and the discussions held led the collaborators to assume a perspective of financial education that enabled them to criticize the textbooks with inferences, such as those of the student x_{24}:

> [...] we found financial mathematics, but at no time did we see the financial education approach in content or proposed activities [...] We concluded that there is a big difference between financial mathematics and financial education. I understood that financial education addresses much more than mathematics itself, school and money. It addresses a whole life, customs, habits, and individual behavior.

After this first part of the analysis of the works (in terms of praxeologies), the critical analysis follows (Table 5), in which we highlight the insertion of the component $Ꞓ_{Gn}$ as a *critical response* given by the group to the analyzed work.

Table. 5

Critical responses to the works

Groups	Works	Critical Answers - Ꞓ
G_1	O_1: Matemática do Projeto Buriti, 3º ano $_{Q1}$: To analyze how these authors (books) work with the notions of financial education in mathematics textbooks from the first to the fourth grades of elementary school to discover the techniques and tasks under the theoretical light of the ATD.	$Ꞓ_{G1}$: Although the author of the analyzed work uses activities involving money, little has addressed financial education and its impacts on an individual's life. We found that some tasks can be used, reformulated, or even added to emphasize what was implied in financial education, thus stimulating the student's reflection without losing the mathematical content.
G_2	O_2: Coleção Fazendo e Compreendendo: Matemática, 2º ano Q_2: To analyze how these authors (books) work with the notions of financial education in mathematics textbooks from the first to the fourth grades of elementary schoo, to discover the techniques and tasks under the theoretical light of the ATD.	$Ꞓ_{G2}$: the books deal much more with financial mathematics and its contents (interest, loan, expenses, amounts) than with financial education. They use money to work on mathematical subjects such as fundamental operations instead of awareness of the use of money, planning, investments, and savings. investment and saving.
G_3	O_3: Construindo e Aprendendo, 2º ano Q_3: To analyze how these authors (books) work with the notions of financial education in mathematics textbooks from the first to the fourth grades of elementary school to discover the techniques and tasks under the theoretical light of the ATD.	$Ꞓ_{G3}$: Although the authors of the book work a lot on the concept of money, purchases, sales, discounts, installments, tasks, and techniques are out of context about financial education, and in need of contextual reassessment.
G_4	O_4: Saber Matemática, 1º ano Q_4: To analyze how these authors (books) work with the notions of financial education in mathematics textbooks from the 1st to the fourth grades of elementary school to discover the techniques and tasks under the theoretical light of the ATD.	$Ꞓ_{G4}$:...the issue of money and other systems involving tasks, technique and technological elements, such as purchases, sales, change, is introduced to provide, from this, the formation of a more critical individual. Although financial education is not explicit in textbooks, the teacher can introduce it based on the knowledge they have acquired, use of these resources in their activities in the classroom

		and, based on this knowledge, work with their students on these concepts, instigating them to think, to reflect, because we need to be financially literate and educate conscious citizens.
G_5	O_5: Construindo e Aprendendo Matemática, 1º ano. Q_5: To analyze how these authors (books) work with the notions of financial education in mathematics textbooks from the first to the fourth grades of elementary school to discover the techniques and tasks under the theoretical light of the ATD.	R_{G5}: The book does use financial education much. It does bring some activities with money, though. However, the idea behind it is not to know how to manipulate or save, but just to teach how important it is to know how to deal with money, since it is part of the mathematics content. Therefore, I see that there is no concern about this teaching, since money only appears to add, subtract, multiply, or divide.
G_6	O_6: Livro de Matemática do Sistema de Ensino Sucesso, 3º ano **Q_6:** To analyze how these authors (books) work with the notions of financial education in mathematics textbooks from the first to the fourth grades of elementary school to discover the techniques and tasks under the theoretical light of the ATD.	R_{G6}: The traditional approach, starting with the summary, which is not contextualized as it is in the current book. We found financial mathematics, but at no time did we approach financial education, either in terms of content or proposed activities.
G_7	O_7: Saber Matemática, 4º ano. Q_7: To analyze how these authors (books) work with the notions of financial education in mathematics textbooks from the first to the fourth grades of elementary school to discover the techniques and tasks under the theoretical light of the ATD.	R_{G7}: The book deals with the issues very superficially, as it lacks encouragement for students' reflection; the questions are directive, leading the student to solve simple calculations, without greater reflection about the questions.... The idea is to work in a more appropriate way to stimulate students' reflection, formulating questions with a more worked context. With just a few simple changes in the context of the questions, it is possible to contemplate financial education for the initial years.

The study process, instituted through the didactic system, favored the dynamics of questions and answers, as proclaimed in the SRP. Regarding the praxeologies on financial education, we found that most of the works present tasks related to the recognition of bills and coins, buying and selling, and primarily, exploring basic operations in the context of financial education since the first grade of elementary school I.

Although the works explore ideas related to the history of money, profit and loss, they do not take a more global approach that gives meaning to the theme. As the analyzed books are the teachers' versions, we could identify that the guidelines for developing tasks related to money –recognition, purchase, sale, and profit– propose to explore counting and basic operations. In addition, the study at the SRP points out that financial mathematics has assumed the role of financial education when the first should be an instrument for the second.

It is worth noting that the materials used to support the resolution of the tasks, such as a calculator and golden material, were characterized by the collaborators of this research as an aid to the technique (AJ). In this way, a new component was inferred in the classic constitution of the Chevallarian praxeology. Regarding the Herbatian system, the group also contributed to the nomenclature of the critical response to the work (G).

Thus, the study process and the composition of the listed responses give evidence of response R♥ (praxeologies), constituting a reference model that favors specific work with elements of financial education (as indicated in research and legislation), enabling financial literacy, which will be summarized below in the concluding part of this article.

Conclusion: A Discussion about the SRP

The directors of this investigation provoked a study process from the *fundamental situation*, *My First Venture*. This situation enabled the research collaborators to apprehend some essential principles of *financial organization*. This first activity encouraged the formulation of various questions, the confrontation of which led to the emergence of knowledge necessary for the search for the answer(s) to the generating question, posed proactively by a collaborator: Q_0: *Why and for what purpose should children study financial education in the early years?*

The generative question Q_0 provoked in the collaborators the desire to better understand the object (financial education) since, until that moment, the relationship of the collaborators with this object was incipient.

Thus, the study directors structured a *milieu* specific *M*, with works *O* (scientific articles, PCN and BNCC, textbooks, etc.), whose readings and discussions between collaborators and directors revealed approved answers ($R^{\blacklozenge}$) culturally by the institutions that produce such works, in terms of teaching and learning notions of financial education at school. This study made it possible to identify, collect, apprehend, reflect, and question the works (*O*), culminating in producing the response $R^{\heartsuit}$ circumscribed in praxeologies that structure a financial literacy model.

To structure the model, in this discussion, we add indicative discourses on how to promote financial education in schools, concomitantly with the promotion of financial literacy, to work on competencies and skills necessary for this literacy.

Our studies reveal that the financial education that is desired for schools is involved in a complex network of knowledge. To carry out this task, teachers must have the opportunity in their initial or continuing education to take courses or topics (as in the case of our research) specific to financial education since it is a field of study with its peculiarities and concepts.

In this sense, in line with the BNCC, studies highlight the need for contextual work in financial education (Teixeira, 2017; Campos et al., 2015). From this perspective, financial education will require students to develop skills and/or competencies in consumer decision-making (Ferreira, 2007, 2008), entrepreneurship, financial planning, and investments (Branchini, 2013; Trindade, 2017). To add skills and competencies required from financially educated citizens, students must be committed to sustainability and the environment (Perissé, 2014; Maturana & Rezepka, 2000; Dantas et al., 2017). Thus, it is important to know/recognize notions of bioeconomics (Fumagalli, 2010), ethnoeconomics (Rodrigues, 2018) and ecological economics (Cavalcanti, 2010; Ontibeller et al., 2012).

Furthermore, Coutinho and Teixeira (2015, p. 4), when discussing the dependence of financial education on financial literacy, highlight that citizens must face, at school, situations in which they must "identify, understand, interpret, create and use new technologies in contexts related to the treatment of problems involving planning and management of personal finances". This point of view is corroborated by Raschen (2016), who highlights the need to use new technologies and mathematical concepts as tools for financial literacy.

In this sense, the study we carried out collectively forwards a prototype of a model for financial literacy. Although the research was developed within the scope of higher education, the indications that we envision can be disseminated at any level of education that deals with financial education.

In this way, we emphasize that the fundamental situation was imposed as a central point to trigger the study process. This situation can be configured differently according to the theoretical ideas underlying it. In this research, we chose the fundamental situation, postulated by Guy Brousseau. In any case, we understand that the situation is the starting point that should trigger the study process.

Taking the situation as the excellence of the model configured as the main answer of this research ($R^{\blacktriangledown}$), we postulate that the situation should enhance the mobilization of financial education knowledge in financial literacy. Therefore, it must be a context that involves

simulations of budgets, consumption, entrepreneurship, sustainability and/or investments, which require adequate and conscious decision-making regarding financial management. The situation should also bring to light the need to evaluate, plan, and understand financial concepts –linked, for example, to personal expenses and taxes– to idealize investments (personal or micro-enterprises), which lead to the identification of possible financial risks in investments and consequent returns to short and long term.

We also emphasize that this situation needs mobilizing knowledge of basic mathematics, notions of financial mathematics, economics, financial documents – such as those relating to taxes and the physical-financial budget – and, above all, to some extent, the use of new technologies should be added, so that one can have the necessary understanding of tools that help in the formation of critical and autonomous citizens.

The ideas announced here make up a model project for financial literacy very broadly, lacking applications that envision the specificities of various levels of education. By doing it, we can validate and/or adjust the guidelines listed here.

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