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Alternative praxeological model for the identification of primary numbers

Modelo praxeológico alternativo para la identificación de números primarios

Modèle praxéologique alternatif pour l'identification des nombres primaires

Modelo praxeológico alternativo para a identificação de números primos

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Abstract

This article aims to present an Alternative Praxeological Model for the Identification of Prime Numbers in any given range of numbers. The study carried out is based on the Anthropological Didactics Theory (ADT). The alternative model refers to the creation of two formulas that when combined generate prime numbers in any range of numbers, built from Basic Algebra reference models and from the concepts of Number Theory. It is expected that the model built will be applied in schools of basic education and in undergraduate courses that deal with Mathematics at various levels.

Keywords: Prime numbers, Prime generating formulas, Mathematics education, Anthropological didactic theory.

Resumen

Este artículo tiene como objetivo presentar un Modelo Praxeológico Alternativo para la Identificación de Números Primos en cualquier rango de números. El estudio realizado se

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basa en la Teoría Antropológica de lo Didáctico. El modelo alternativo se refiere a la creación de dos fórmulas que combinadas generan números primos en cualquier rango de números, construidas a partir de modelos de referencia del álgebra básica y conceptos de la teoría de números. Se espera que el modelo construido sea aplicado en las escuelas de educación básica y en los cursos de pregrado que traten Matemáticas en varios niveles.

Palavras-chave: Números primos, Fórmulas generadoras de primos, Educación matemática, Teoría antropológica de lo didáctico.

Résumé

Cet article vise à présenter un modèle praxéologique alternatif pour l'identification des nombres premiers dans n'importe quelle plage de nombres. L'étude menée est basée sur la Théorie Anthropologique du Didactique. Le modèle alternatif fait référence à la création de deux formules qui, combinées, génèrent des nombres premiers dans n'importe quelle plage de nombres, construits à partir de modèles de référence de l'algèbre de base et de concepts de la théorie des nombres. Il est prévu que le modèle construit sera appliqué dans les écoles d'enseignement de base et dans les cours de premier cycle traitant des mathématiques à différents niveaux.

Mots-clés: Nombres premiers, Formules génératrices de nombres premiers, Enseignement des mathématiques, Théorie anthropologique du didactique.

Resumo

Este artigo tem por objetivo apresentar um modelo praxeológico alternativo para identificação dos números primos em um intervalo qualquer de números. O estudo fundamenta-se na Teoria Antropológica do Didático (ADT). O modelo alternativo refere-se à criação de duas fórmulas que, combinadas, são geradoras de números primos em qualquer intervalo de número. Tais fórmulas foram construídas a partir de modelos de referência da álgebra básica e de conceitos da teoria dos números. Prospecta-se que o modelo construído seja aplicado nas escolas de educação básica e nos cursos de licenciatura que lidam com matemática, em variados níveis.

Palavras-chave: Números primos, Fórmulas geradoras de primos, Educação matemática, Teoria antropológica do didático.

Alternative praxeological model for identifying prime numbers

The main theme of this article is prime numbers, which are dealt with most emphatically in the 6th year of elementary school, in the study of natural numbers, and are then applied in the processes of factoring and decomposition into prime numbers.

Despite their importance, prime numbers are not a specific object of study in mathematics. They form part of the initial study of natural numbers: divisibility and come together with the study of multiples and divisors. Later, in higher education, their study is included in the subject of number theory, in the degree in mathematics and in the bachelor's degree in mathematics.

In elementary school, the study of prime numbers begins in the 5th grade in some schools and continues in the 6th grade with natural numbers. However, students only study prime numbers between 1 and 100, and sometimes in the range between 1 and 50. Thus, 6th graders are not given the opportunity to learn about larger prime numbers, such as those with three digits. This is a restrictive condition imposed by the method known as the Sieve of Eratosthenes (276 - 194 BC). We are referring to the most widespread method for identifying primes in elementary school textbooks.

Although the subject of number theory is part of the curriculum of mathematics degree courses, there is great difficulty on the part of students in studying prime numbers and, more particularly, in relating this study to basic school (Oliveira & Fonseca, 2017).

According to Oliveira and Fonseca (2017), number theory offers a consistent algebraic formalization for the arithmetic of integers. Significantly, future professors who are interested in the subject perceive the essence of mathematics in it.

Fonseca (2015, p. 58) emphasizes that

> Number theory topics such as factoring, divisors, multiples, and congruence provide the means to develop and solidify mathematical thinking, to develop an enriched appreciation of number structure, especially with regard to identifying and recognizing patterns, as well as to formulate and test conjectures, to foster an understanding of principles and their proofs, and to justify the truth of theorems in disciplined and reasoned ways.

Zazkis and Campbell (1996, 2006) and Zazkis and Liljedahl (2004) announced the need to establish a place for number theory in mathematics education research. In this sense, Zazkis and Campbell (2006) point out that number theory, despite representing the essence of mathematics, has not received due attention in educational research, as only occasional studies are evidenced.

In this sense, we have immersed ourselves in this subject in order to propose an expansion of the study of prime numbers, based on the presentation of an Alternative Praxeological Model (APM) for identifying prime numbers in any interval of numbers. In this model proposal, we seek to add another method for use in elementary school, in addition to the well-known Sieve of Eratosthenes.

Bibliographic study

In order to draw up an overview that would allow us to differentiate our proposal from others that have already been developed in Brazil, we carried out a study in the Catalog of Theses and Dissertations (CTD) of the Coordination for the Improvement of Higher Education Personnel (CAPES), initially restricting the search to theses in the area of Mathematics Education. To do this, we used the following search terms: *divisibility*, *number theory*, *arithmetic*, *prime numbers*. In the first search, we found only three theses with titles that indicated the subject of our study. By reading the titles and abstracts, we selected them for a first analysis (Table 1).

Table 1.

Theses dealing with Number Theory and Prime Numbers

Authors	Thesis title	Program/institution/year
RESENDE, Marilene Ribeiro	**Re-signifying the Discipline of Number Theory in the Training of Undergraduate Mathematics Professors.**	(Doctorate in Mathematics Education). Pontifícia Universidade Católica de São Paulo. São Paulo, 2007.
BARBOSA, Gabriela dos Santos	The Fundamental Theorem of Arithmetic: Games and Problems with 6th grade students.	(Doctorate in Mathematics Education). Pontifícia Universidade Católica de São Paulo. São Paulo, 2008.
FONSECA, Rubens Vilhena	Prime Numbers and the Fundamental Theorem of Arithmetic: An Investigation among Undergraduate Mathematics Students	(Doctorate in Mathematics Education). Pontifícia Universidade Católica de São Paulo. São Paulo, 2015.

We identified that the theory of numbers received attention in these three doctoral theses, in the area of Mathematics Education. However, only Fonseca's thesis (2015) presented prime numbers as its main theme, so we proceeded to read this research in full, which highlights, among other things, the scarcity of research related to Number Theory in the area of Mathematics Education. In fact, until the end of 2015, in our country, there were only two theses published on this topic: Rezende's (2007), and the following year, Barbosa's thesis (2008), but since then, several studies have appeared on the topic of divisibility, mainly dissertations from the Professional Masters in Mathematics Teaching in the National Network - PROFMAT[4] - and Mathematics Education.

In his concluding remarks, Fonseca (2015, p. 148) emphasizes that:

> The results revealed the need for a broader mastery on the part of the undergraduates with regard to issues related to understanding the topics at hand; specifically, difficulties were evidenced regarding working with certain numerical representations of prime numbers, and especially in relation to the concepts of prime numbers and the fundamental theorem of arithmetic.

The problem announced by Fonseca (2015) reverberates into the classroom, right from elementary school, as it reveals difficulties in understanding prime numbers among future mathematics professors (undergraduate students).

So, in order to delve deeper into this problem, we carried out an investigation in the Catalog of Theses and Dissertations (CTD-CAPES), now at master's level, using the same search parameters as in the theses. In this way, we found nineteen dissertations (Table 2) from the Master's Programs in Mathematics Education and Professional Master's Programs in Mathematics Teaching, mainly those of PROFMAT.

Table 2.

Dissertations on Prime Numbers

	Title/Program	**Author**	**Objective**
01	Gaussian prime numbers for secondary school. PROFMAT, 2014.	Cybele Verde Aragão de Almeida	Introduce a special category of numbers: Gauss's prime numbers.
02	Goldbach's Conjecture and Mathematical Intuition. PROFMAT, 2018.	Carolina da Silva Bitencourt	Its aim is to stimulate mathematical intuition in elementary school students to infer Goldbach's Conjecture, by means of a written activity and the use of an auxiliary manipulative material.

[4] The National Network Professional Master's Degree in Mathematics (PROFMAT) is a semi-presential master's degree program in the field of Mathematics, offered nationwide. It is formed by a network of higher education institutions, in the context of the Open University of Brazil/Coordination for the Improvement of Higher Education Personnel (Capes), and coordinated by the Brazilian Mathematical Society (SBM), with the support of the National Institute of Pure and Applied Mathematics (Impa). Available at <http://www.profmat-sbm.org.br/dissertacoes/>

03	Algorithms for Factorization and Primality as a Didactic Tool for Teaching Mathematics. PROFMAT, 2015.	Jean Peixoto Campos	It presents a proposal for teaching mathematics with the help of the development and implementation of algorithms for factoring and primality of positive integers.
04	Prime Numbers and the Fundamental Theorem of Arithmetic in the Sixth Year of Primary School. PROFMAT.	Fernando Ramires Carvalho	Present the content that a basic education professor must master in order to develop and apply divisibility and prime numbers in a 6th grade class.
05	Prime Numbers and Cryptography: from the Relationship with Education to the RSA System. PROFMAT, 2013.	Kelly Cristina Santos Alexandre de Lima Daineze	This work aims to establish a discussion on the concepts involving cryptography, through its application of prime numbers, and the possible relationships with education.
06	Prime Numbers and Divisibility: Study of Properties PROFMAT, 2013.	Cristina Helena Bovo Batista Dias	The aim of this work is to present the fundamentals of divisibility, to study the properties of prime numbers, their associations with factorials and arithmetic progressions, and to present some results equivalent to Goldbach's Conjecture.
07	The Study of the Teaching of Prime Numbers in Basic Education. PROFMAT, 2016.	Djalma Gomes de Farias	In this work, we will highlight some of the main properties of prime numbers, emphasizing, whenever possible, in addition to their applications in the rest of your school life, their historical context, in order to show their importance and the motivations of some of the important mathematicians who dedicated themselves to their study.
08	Prime Numbers and Bertrand's Postulate PROFMAT, 2014.	Antônio Eudes Ferreira	Show a study of prime numbers and a generalization of Bertrand's Postulate.
09	Title: Prime Numbers: An Educational Approach UFAM, 2015.	Edson Ribeiro Machado	To take a new approach to the construction of prime numbers, the way of looking at natural numbers, and the differentiation of composite numbers from natural numbers with the aim of improving our primary and secondary school students.
10	Prime Numbers PROFMAT, 2013.	José Cleiton Rodrigues Padilha	Introduce a special category of integers: prime numbers.

11	Prime Numbers and Primality Tests UNICAMP Campinas, 2014.	Glaucia Innocencio de Jesus Paulo Paiva.	Describe and study some primality tests, such as the Fermat Test, the Lucas-Lehmer Test, the Miller-Rabin Test and the AKS algorithm.
12	Prime Numbers and Goldbach's Conjecture. PROFMAT, 2017.	Andressa de Lima Pereira.	It studies prime numbers, looking at their role in the history of mathematics and verifying their importance for cryptography. It also discusses the progress made in the study of Goldbach's Conjecture.
13	The Riemann Zeta Function and Prime Numbers. UNICAMP, 2016.	Daniel Petravicius.	This work shows and demonstrates known results of the Riemann Zeta Function and its relationship with prime numbers
14	Prime Numbers: The Atoms of Numbers. PROFMAT, 2015.	Márcio Dominicali Rigoti.	This is a study of prime numbers that goes through basic results such as the infinitude of prime numbers and the TFA, Wilson's Theorem and consequently the prime generating function.
15	The Prime Numbers PROFMAT, 2014.	Cediglês Lima dos Santos.	The purpose of the text is not to find new prime numbers, the essence of the study is to try to explain the organization of these numbers and how they are presented and to approach them in a simple and special way, like a kind of super-interesting magazine about primes.
16	Prime Numbers: Applications and Primality PROFMAT, 2015.	Vagner Caceres Soares.	Basic concepts are introduced for understanding later studies, the definition of primes and applications of prime numbers.
17	Prime Numbers and Cryptography Professional Master's Degree UNICAMP, 2014.	André Vinícius Spina.	It introduces us to number theory by looking at the RSA and Diffie-Helman cryptographic methods, in which we can see situations in which they are efficient.

We selected the dissertations referenced in Table 2 with the initial idea of reading only the abstracts, but we saw the need for greater depth. We then went on to read each dissertation in full, noting down the points we considered relevant to the study of prime numbers.

As a general analysis of the dissertations, we found that most of the research approached the main themes of number theory, starting the presentation with a historical analysis. The development of number theory was emphasized, using formal and systematic language, with the presentation of conjectures, theorems and their proofs. In addition, the main works of the most prominent mathematicians of the past were presented, as well as the propositions, conjectures and theorems with their respective demonstrations, related to prime numbers. These works presented the most diverse attempts by mathematicians to find a formula for generating primes and some primality tests.

The main theme of these dissertations is prime numbers, but the authors present the main themes of Number Theory, starting with divisibility, with their definitions, properties and theorems. In addition to these topics, they present some open problems involving prime numbers, such as the Riemann problem and Goldbach's conjecture.

There is a focus on primality tests, such as Eratosthenes' sieve, Fermat's little theorem, Wilson's theorem, Fermat's test, Lucas-Lehmer's test, Miller-Rabin's test, the AKS algorithm and Carmichael's numbers. In other words, most of the papers presented a primality test and, in addition, some attempts to create prime generating formulas, without, however, presenting any that generated all the prime numbers or all the prime numbers in any given interval. The most important formulas generate some primes, such as those of Fermat and Mersennhe.

We also found that research indicates that, by following school curricula and restricting themselves to the use of textbooks, professors do not address topics involving divisibility in the grades after 6th grade, since most students arrive at high school without knowledge of the basics of prime numbers.

Given these problems, the aim of this study is to present an Alternative Praxeological Model, with two formulas that, when combined, generate all the prime numbers in any interval of numbers, to be used as an application in the teaching of prime numbers, in order to encourage the appropriation of conceptual notions of this subject.

Models: introductory discussions

This study considers the integration of pedagogy and mathematics. According to Gascón (2001, 2014), Delagado (2006) and Florensa et al. (2020), this integration takes place by questioning and broadening what is considered mathematical in the popular model of mathematics, which is a dominant epistemological model in the institutions where mathematical knowledge is handled, including university institutions and the community that produces mathematical knowledge.

This model, particularly used in bachelor's degrees in mathematics, is also present in undergraduate courses in mathematics, particularly in the subject of number theory. Thurston (1994) cited by Delgado (2006, p. 23) characterizes this model in the following terms:

> 1. Mathematicians start from some fundamental mathematical structures and a collection of "given" axioms that characterize these structures;
> 2. Regarding these structures, there are important and varied questions that can be expressed through formal mathematical propositions;
> 3. The task of mathematicians is to find a series of deductions that link the axioms with these prepositions or with their negation;
> 4. To give a reason for the origin of problematic questions, speculation is added as an important and supplementary ingredient to this model. Speculation consists of making conjectures, suggesting questions, making intelligent assumptions and developing heuristic arguments about what is credible. This leads to the definition-speculation-theorem-proof (DSTP) model.

According to this author, the *popular model* is definitely a naive and simplistic way of interpreting mathematical knowledge. Thurston (1994) therefore proposes the development of an alternative epistemological model which emphasizes that the work of the mathematician consists of advancing human understanding of mathematics and improving the communication of this understanding.

Gascón (2001, 2003), Delagado (2006) and Florensa et al. (2020) point out that there has been progress towards changing the mathematical model, with the origin of the epistemological program that began to have mathematical activity as its primary object of study, as an alternative to didactic analysis. The first step in this direction was taken historically by the *Theory of Didactic Situations* (TDS), which integrated the *mathematical* and the *pedagogical*, modeling mathematical knowledge and the *conditions of its use in school situations in an inseparable way.*

However, as the program developed, it was realized that it was not possible to properly interpret *school mathematical activity* without considering the phenomena related to the *school reconstruction of mathematics*, which originated in the very institution that produces mathematical knowledge. To deal with these issues, the phenomenon of *didactic transposition* (Chevallard, 1985) was used, which triggered the *Anthropological Theory of the Didactic* (ATD) (Chevallard, 2009).

Anthropological theory of the didactic (ATD)

ADT assumes that mathematical knowledge is constructed as a response to the study of problem situations, thus appearing as the result (or product) of a study process. This process, as an activity that leads to the construction (or reconstruction) of mathematical knowledge, is part of mathematical activity.

Mathematical knowledge is thus organized into two levels. The first level refers to the practice that is carried out, the praxis or knowledge, in other words, the types of problems or tasks that are studied and the techniques that are constructed and used to approach them. The second level refers to the descriptive, organizing and justifying part of the activity: this is logos, or simply knowledge. This level includes the descriptions and explanations that are drawn up to make the techniques intelligible, i.e. the technological discourse and the theory that gives meaning to the problems formulated and makes it possible to base and interpret the technological descriptions and demonstrations as a form of justification.

According to Chevallard (1999), the postulate of ADT goes beyond the particularized vision of the social world: it is admitted that every regularly performed human activity can be described with a single model, which is summed up with the word praxeology (Table 3).

Table 3.

Notion of Praxeology

Praxeology			
Practical-technical block (knowledge) [T/**τ**]		Technological-theoretical block [θ/Θ]	
Task types	Technique	Technology	Theory
At the root of the word praxeology are the joint notions of task t and types of task T. Thus, when a task t is part of task types T, we write t Є T (Chevallard, 1999).	Let T be a given type of task; a praxeology relative to T requires (in principle) a way of doing the tasks (t Є T): a given way of doing, t, is henceforth called a technique (**τ**).	Technology is understood and generally referred to, asθ as a rational discourse - *logos* - about the technique **- τ**, a discourse whose primary objective is to rationally justify the *technique* **τ**, in order to ensure that it makes it possible to carry out the tasks of type T, that is, to accomplish what is intended.	In turn, technological discourse contains more or less explicit affirmations of what can be thought. We then move on to a higher level of justification-explanation-production of *theory*, Θ, which takes up, in relation to technology, the role that the latter plays in relation to technique.

Around a type of task T, there is, in principle, a triple formation: a technique **τ** (at least), a technology of t (θ) and a theory of θ (Θ). The set, indicated by [T/**τ** /θ/Θ] constitutes a punctual praxeology, that is, it is a praxeology relative to a single type of task (T). In these terms, a praxeology or praxeological organization is therefore made up of a practical-technical block [T/**τ**] and a technological-theoretical block [θ/Θ].

According to Chevallard (1999), praxeological organizations in educational institutions can be described and analyzed in a dynamic whose minimum model is called a *punctual praxeological organization* [T/**τ**/θ/Θ], characterized by involving a single type of task T. In this dynamic, combining various specific praxeologies around a given technique results in the *local praxeological organization* [Ti/**τ**i/θ/Θ]. Increasing the complexity, several local praxeological organizations are combined around a given technology θ, giving rise to the *regional praxeological organization* [Tij/**τ**ij/ θj/Θ]. The process takes place at a level of increasing complexity (Bon, 2011), where the most complex level groups *regional praxeological organizations* around a theoretical discourse (Θ), which justifies the technology (θ), revealing the theory (Θ) that prevails in the praxeological block. In this respect, the praxeological complex revealed is a *global praxeological organization* [Tikj/**τ**ijk/ θjk/Θk].

In view of the process described above, supported by Delgado (2006), Matos et al. (2020, p. 465) postulate that the unit of analysis of didactic procedures

> must contain a school Didactic Organization (DO) that makes it possible to construct, at the very least, a relatively complete Local Mathematical Organization (LMO). This LMO can be artificially reconstructed in the school institution as the result of a process of progressive expansion and completion of specific praxeologies and intermediate praxeologies. These praxeologies (punctual and intermediate) are successively generated by the evolutionary development of the problem questions and types of tasks associated with the DO that will be the raison d'être of the reconstructed LMO (Matos et al., 2020, p. 465).

The notion of praxeology thus appears as a generic notion that should be studied in depth, especially through the empirical study and analysis of the observation data collected, which characterizes the way of doing and thinking configured in mathematical and didactic organizations.

In this dynamic, we will try to establish, albeit provisionally, an alternative praxeological model for identifying prime numbers in any number interval.

Mathematical organization MO_θ and didactic organization DO_θ

The types of objects considered will be of two classes: given a mathematical subject of study θ, we will consider successively: a) the mathematical reality that can be constructed in a math class where the subject θ is studied; b) the way in which this mathematical reality can be constructed, that is, the way in which the study of the subject θ can be carried out. The first object - *the mathematical reality that...* - is a mathematical praxeology or Mathematical Organization, which is called MO_θ. The second object - *the way that...* - is what will be called didactic organization, which will be indicated, in an analogous way, by DO_θ. Thus, the research mainly concerns two types of tasks (T): describing and analyzing the mathematical organization MO_θ, which can be constructed in a mathematics class where the subject θ is studied (T_1); describing and analyzing the didactic organization DO_θ which can be put into practice in a mathematics class where the subject θ is studied (T_2).

We will now say that *doing mathematics* consists of putting a mathematical praxeology into practice in order to carry out a certain type of task, and that studying mathematics consists of constructing or reconstructing certain elements of a mathematical praxeology in order to respond to a certain type of problem situation.

According to Delagado (2006, p. 32),

> [...] Mathematical Organization corresponds to the conception of mathematical work as the study of types of problems or problematic tasks. However, this is not the only aspect of mathematical work. In fact, the mathematician not only aspires to construct good problems and solve them, but also to characterize, delimit and even classify problems into "types of problems", to understand, describe and characterize the techniques used to solve them to the point of controlling and standardizing their use, to establish the conditions under which they work or are no longer applied and, ultimately, to build solid and effective arguments that support the validity of their ways of proceeding.

To analyze a mathematical organization, the first step is to construct or at least sketch, from the theoretical-technological elements introduced so far, a technique t for describing and analyzing a mathematical organization MO_θ (Chevallard, 1999).

As an example, we will now consider a simple kind of task T_1, choosing the topic θ = div of division of integers: T_{div}:: *describe and analyze the organization* MO_θ*, which can be constructed in a class where the topic of division of integers is studied.*

However, Chevallard (1999) emphasizes here that care must be taken to carefully distinguish this task from the task called t_{tdiv},, of describing and analyzing the corresponding didactic organization: T_{tdiv} *describe and analyze the didactic organization* $DO_\theta = \delta MO_{div}$ *that can be put into practice in a class where the topic of division of integers will be studied.*
Next, the mathematical organization that we must determine, MO_{div}, *is a priori a* local organization that can contain various types of tasks, for example, T_q*: given two positive integers a and b, b > 0, calculate the quotient of a by b.* The purpose of the study, in this case, would then be to specify a technique ôq to carry out tasks of type T_q.

Organizations can be structured into Epistemological Reference Models (ERMs), which underlie the way praxeologies are done and thought about, in particular school praxeologies.

Announcement of an epistemological reference model for determining prime numbers

In this section, we present a Mathematical Organization (MO) around the identification of prime numbers, which will serve as an Epistemological Reference Model (ERM).

Every ERM should be developed on the basis of wise mathematical organizations that epistemologically legitimize the process of teaching such an MO (Gascón, 2014). However, according to Delagado (2006, p. 55, our translation),

> [...] the very elaboration of the ERM is at the same time a tool to distance itself from the wise institution by allowing didactic research to make explicit its own point of view on the mathematical content at stake in the didactic processes that are planned, implemented, analyzed and evaluated. In particular, ERM must take into account the historical evolution of wise MO, but without cloning them.

In addition, Delagado (2006) and Gascón (2014) emphasize that an ERM is a tool for analyzing specific teaching processes; its construction must also consider the restrictions that come from those school institutions in which the OM in question is designated as the MO to be taught.

It is important to emphasize that the ERM we will describe here is a didactic proposal that will be subject to future adjustments and modifications as research emerges in this area and on this topic for the improvement of mathematics teaching. Chevallard's theory of didactic transposition (1985, 1999) shows us that there is no privileged, absolute system from which to observe and analyze the institutional life (both intra- and inter-institutional) of mathematical organizations.

For Delagado (2006), making explicit the ERM underlying the design of a study process makes it possible to better interpret the didactic activity that is being carried out (in other words, the didactic tasks that are formulated and the didactic techniques - study aids - that are used to carry out these tasks) and, more importantly, provides explicit criteria for modifying the DO, depending on the relationship between the response provided at any given moment by the study community and the direction of development defined by the ERM which, of course, can also be modified.

The school reconstruction of a mathematical organization

Let's take the following question as a starting point: *how can we identify the prime numbers in any interval of numbers (a finite sequence of natural numbers or positive integers) using two formulas based on an existing technique?*

We can answer this question in a simple way, i.e. to calculate prime numbers, the best way is to use the techniques already established institutionally. However, we can also cite the case of institutions that don't have any technique for identifying all the prime numbers in any interval of numbers, using a formula or that, even if they do have a technique, use it naturally, without questioning its relevance or the reason that explains its effectiveness.

However, if you don't have a technique to solve the problem, the question becomes problematic and, in trying to solve it, you generate a type of non-routine task designated by the letter T. Solving it will require the development of a set of mathematical techniques t, which, in turn, must be described, explained and justified through a mathematical discourse called technological-theoretical and which is designated with the notation [θ/Θ].

In other words, if we consider the previous question to be a *problematic issue that needs to be studied* and which cannot be answered by simply providing information, then we need an *answer* based on the construction of a mathematical organization, in other words, we need a set of types of tasks or problems, techniques or procedures for solving these problems and definitions, properties and theorems that allow us to describe and justify the work carried out. As explained above, we will use the notation OM = [T/t/θ/Θ] to designate a mathematical organization generated by tasks of type T (Chevallard, 1999).

The fact that we consider a question like the one above means that when we call for the construction of an OM as a possible answer or solution, it doesn't mean that we should consider the elaboration of a single OM. In fact, in the process of studying such a question or developing possible answers, successive mathematical organizations usually come into play, each of which can correct, complement and/or develop the previous ones.

In this work, it will be possible to observe how the answers obtained from the above-mentioned question evolved, which can be expressed in terms of the progressive expansion of OMs. More importantly, we will see in what sense this expansion makes it possible to express the r*aisons d'être* of the alternative model for identifying prime numbers.

For Delagado (2006), studying a mathematical question at an educational institution I (including higher education institutions) consists of studying the mathematical organization that another institution I' proposes as an answer to that question. However, in order to study the organization built in I', it must be reconstructed in I, through a school reconstruction that is artificial, in the sense that it is not new. In other words, the organization must be transported or transposed from I' to I - what is known as the process of didactic transposition of a mathematical organization (Chevallard, 1985).

According to Delagado (2006, p. 58), a major type of didactic problem thus arises, so that our problem becomes a particular case:

> Given a question ***q*** that we want to be studied, in an educational institution **I**, how do we plan and manage the reconstruction process (which is a study process) in **I** of the Mathematical Organizations **MO = [T/t/θ/Θ]** that are given as the answer to this question in another institution **I'**?

In our specific teaching problem, the parameters q, I, MO and I' take the following values:

q = How can we identify prime numbers, firstly given any number and secondly a group of prime numbers in any interval of numbers, by means of a model that uses two formulas which combined identify all the prime numbers in the given interval?

I = Basic and higher education institutions.

MO = MOip = Mathematical Organization around the Identification of Prime Numbers (the alternative model for identifying prime numbers in any interval of numbers).

I' = Wise mathematical institution.

According to Delagado (2006, p. 58) it is necessary to point out that neither the initial question nor the Mathematical Organization or Mathematical Organizations to be constructed are entities that can be completely delimited a priori.

Models for identifying prime numbers

The classic model for identifying prime numbers in a range of numbers in elementary school is the Eratosthenes sieve, but this model, as presented, has a limitation, because if it is used to identify very large lists of prime numbers with "n" (the last number in the given range) getting bigger and bigger, in any given range, it becomes very laborious, requiring a large number of calculations. Therefore, in order to make the identification of prime numbers more efficient and more economical in calculations, we suggest:

1. That the concepts and properties that involve knowledge of prime numbers be emphasized and used to understand how these numbers work and their importance in the development of mathematics.

2. That ways of using the operations and properties be shown in order to facilitate the analysis and interpretation of the process of identifying prime numbers, and that this can be done with greater economy of calculation.

3. To provide new ways of understanding number sequences and using them in the process of identifying prime numbers, such as alternating sequences.

4. That the calculation for identifying prime and composite numbers in any given interval be based on elementary arithmetic and basic algebra, in mathematical language that is more accessible to our students.

5. That the study of prime numbers can lead to the recognition of patterns and an improvement in methods so that students and professors can realize the importance of this knowledge in the development of mathematics.

It seems reasonable to assume that the problem situations and associated tasks that will become OM_{ip}'s *raison d'être* in an institution should be drawn from these conditions.

These issues will be present throughout the process of this study. For this to be possible, certain intermediate MOs, in which these tasks are problematic, will have to be experienced in I and, therefore, their *raisons d'être* will become visible.

We will start from a model of development of the problematic issues in the process that leads us to the construction of MO_{ip} Here, we can outline this development, guided by three major categories of prime number identification (PNI). Subsequently, we will present them as useful techniques for answering certain problem questions and carrying out the mathematical tasks that will generate, respectively, several different mathematical organizations.

(1) Type I identification: individual prime number identification

In this first type, the number itself plays an important role. This involves identifying whether a given number a is prime or not. The technique is presented in mathematics textbooks for basic education and number theory (for higher education). From now on we'll call this technique tip, the initial technique for identifying prime numbers.

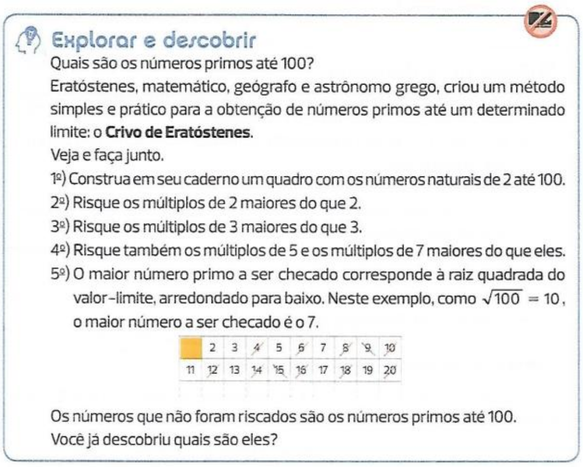
We will present the technique (tip) as it is found in a 6th grade textbook, according to Dante (2015, p. 240):

> To recognize whether a number a is prime, just go on dividing that number by the prime numbers starting from 2 smaller than $\sqrt{a}$, until one of these two factors occurs: (1) the division "gives" exact (in which case the number is not prime); (2) The quotient becomes equal to or smaller than the divisor without any division having "given" exact, then it is concluded that the number is prime.

However, as this number becomes larger, the identification process becomes more laborious, because the number of division operations that have to be done increases as the number of basic primes (for testing) also increases, depending on the range given. However, we can analyze this technique and see that we can make it more efficient, depending on the context in which it is applied.

(2) Type II identification: identifying groups of prime numbers

The technique presented in 6th grade textbooks on divisibility to identify prime numbers is the Eratosthenes sieve and we will identify this technique as TcE. Some mathematics textbooks use the interval from 01 to 100, listing all the numbers in this interval, as well as the numbers 2, 3, 5 and 7 to identify the prime numbers in this interval. Let's see what Dante (2015, p. 140) shows (Figure 1):


Explorar e descobrir

Quais são os números primos até 100?

Eratóstenes, matemático, geógrafo e astrônomo grego, criou um método simples e prático para a obtenção de números primos até um determinado limite: o **Crivo de Eratóstenes**.

Veja e faça junto.

1º) Construa em seu caderno um quadro com os números naturais de 2 até 100.

2º) Risque os múltiplos de 2 maiores do que 2.

3º) Risque os múltiplos de 3 maiores do que 3.

4º) Risque também os múltiplos de 5 e os múltiplos de 7 maiores do que eles.

5º) O maior número primo a ser checado corresponde à raiz quadrada do valor-limite, arredondado para baixo. Neste exemplo, como $\sqrt{100} = 10$, o maior número a ser checado é o 7.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Os números que não foram riscados são os números primos até 100.

Você já descobriu quais são eles?

Figure 1.

Identifying groups of prime numbers (Dante, 2015, p. 140)

As Figure 1 doesn't show the complete model, we have a task to do, which is to list all the numbers from 1 to 100, which can be done in tables or in the way it is presented in the sequence of natural numbers, because if the professor asks the students to write down the first hundred numbers, they could write the multiples of 2, 3, 5 and 7 except these numbers:

In this model indicates the task of crossing out the multiples of the basic primes, therefore the learner can calculate the multiples of each basic prime, such as 2 x 1 = 2, 2 x 2 = 4, ... 2 x 50 = 100; 3 x 1 = 3, 3 x 2 = 6, ..., 3 x 33 = 99; 5 x 1 = 5, 5 x 2 = 10, ..., 5 x 20 = 100; e 7 x 1 = 7, 7 x 2 = 14, ..., 7 x 14 = 98

However, if the student applies what they have learned previously, they will be able to use the divisibility criteria for numbers 2 and 5 and easily eliminate even numbers and numbers ending in 5. As for the divisibility criterion for number 3, if they already know how to identify multiples of three, they will also be able to identify them more easily. However, if the student has not yet acquired the ability to analyze on their own, and if the professor doesn't give them any indication, they will have to do a lot of calculations to obtain the multiples of the four basic primes, cross out those multiples and then identify the prime numbers, which will be those not crossed out.

This technique seems problematic to us. So some questions arise: is it possible to make it more efficient by avoiding a lot of calculations, since the number 2 is the only even prime number, so would it really be necessary to list the even numbers to calculate the multiples of 2 and then eliminate those numbers? Similarly, if we think about the number 5, which is the only prime number ending in 5, would it be necessary to list the numbers ending in 5 and then eliminate them? Based on these questions, other problems arise, such as: is it possible to create a technique using mathematical formulas to identify prime numbers in any range of numbers, in order to reduce the computation of algorithms? As a result of these questions, type III emerged, which is the method to be presented in this research.

(3) Type III identification: Alternative Method for Identifying Prime Numbers in Any Number Range (AMIP)

This method initially arose as a curiosity, in the search for other models for identifying primes, because it was found that only the sieve of Eratosthenes, created in approximately 200 BC, was used, and also because the formulas created so far by prominent mathematicians of past centuries only produce some primes, but not all of them. This is a method for identifying all the prime numbers in any range of numbers. In this type, the primes 2, 3 and 5 are exceptions - prime 2, because it is the only even prime; the number 3, because it generates most of the multiples of primes; and the number 5, because it is the only prime ending in 5.

In this way, two formulas were created which, when combined, produce all the prime numbers greater than 5. Formula 1 or the General Formula produces a universe of numbers in which all the prime numbers in the given interval are contained, but it also produces some compound numbers and, in order to specifically identify the compounds produced by the General Formula, Formula 2 was created, which generates these compound numbers, so that they can be eliminated, leaving only the prime numbers. The calculations with the formulas are made considering the endings of the numbers in 1, 3, 7 and 9 and, for each ending, two alternating sequences will be used. With the formulas, an economy of calculation has been sought. For example, for each interval of a thousand numbers, this formula produces only 266 or 268, just as for each interval of ten thousand numbers, the formula will produce only 2,666 or 2,668 numbers and all the primes are contained between them, as well as the multiples of the basic primes.

We propose a dynamic epistemological model guided by the evolutionary development of a succession of MOs, so that each new MO relatively expands and completes the different components of the previous one until it ends up in MO_{maip}. We will see that, in this succession, each MO finds its *raison d'être* in the limitations of the previous one (Delagado, 2006).

The choice of intermediate MO_s comes from a detailed analysis of the mathematical question q and the restrictions imposed by the fact that its answer must be reconstructed in a given institution.

This leads us to consider two major categories of Prime Number Identification: Identifying whether any number is prime or composite (MO_{ipc}), Identifying groups of prime numbers in any interval of numbers (MO_{igp}) - each of which gives rise to a new link in the succession of each OM, starting from a rudimentary initial MO (MO_{ip}), to end in (MO_{maip}): Identification of prime numbers by the alternative model: MO_{ip} => MO_{ipc} => MO_{igp} => MO_{maip}.

Individual identification of a prime number

We started with the idea of starting with the MO_{ip}, identifying any number individually to find out if it is prime or not, in order to see if the definitions of prime and composite numbers are in fact sufficient to solve problems such as: *check if the numbers, for example, 91 and 97 are prime.*

The type of task that generates the Initial Mathematical Organization (MO_{ip}) is associated with fairly elementary problem situations relating to the definitions of prime and composite numbers, as we can see in the following items:

(1) How are a prime number and a composite number defined in such a way that they can be identified unambiguously?

(2) How do you find out if any number is prime or not?

(3) How can you identify all the prime numbers in any given interval using mathematical formulas?

In order to answer these questions and produce an initial technique (t_i), we need *to use the definitions of prime and composite numbers and the Fundamental Theorem of Arithmetic.*

The two techniques presented so far answer questions (1) and (2), but not question (3). However, they will allow us to develop the initial technique (t_i) and move on to intermediate techniques that will be more effective, without losing their importance. When analyzing the two techniques, we see that the key to identifying prime numbers lies in the definition and the

TFA. The definition gives us an indication of what a prime number is, but not the process for identifying it in the set of natural numbers or positive integers.

If we consider that a positive integer a > 1 is composite, then a has more than two prime divisors and can be written as prime factors (TFA).

For example, the number 21 is a composite number because 3 x 7 = 21, so in addition to 1 and 21, there are also 3 and 7 as its divisors.

We call this primitive technique of identifying prime and composite numbers (ti) from the definitions of prime and composite numbers. Of course, this ti technique has strong limitations, as it only gives an indication based on the number of divisors of the numbers.

The Mathematical Organization for identifying whether any number is prime or composite (MO_{ipc})

The t_{ipc} technique uses the procedure and knowledge that underpin the study of prime numbers and that are necessary for the application of techniques to identify these numbers. This t_{ipc} technique, used to identify whether any number is prime or composite, can be considered an evolution of the initial t_i technique, generating a new MO_{ipc} organization, developed around the identification of prime numbers of type I, whose main quality is to identify whether any number is prime or composite, without restrictions. The types of tasks generated by this MO_{ipc} are associated with the problematic issues described for MO_{ip} that did not find a satisfactorily complete answer in MO_{ip}. According to the two techniques already presented, it can be seen that in the model shown in Figure 1, there is no indication of how to find out if 2, 3, 5 and 7 are prime numbers, it just says, "cross out the multiples of 2, except 2", and the same goes for 3, 5 and 7.

According to Delagado (2006), in each new organization, new questions can be formulated, such as (4), (5), (6) and (7) and, even so, we can see that not all of them will be fully answered by this OM_{ipc}.

(1) How are a prime number and a composite number defined in such a way that they can be identified unambiguously?

(2) How do you find out if any number is prime or not?

(3) How do you calculate the multiples of basic prime numbers?

(4) How do you identify the first prime numbers using partition-multiplication-division?

(5) Is it necessary to calculate all the divisors of a number to find out if it is prime or composite?

(6) How do you apply the divisibility criteria for prime numbers?

(7) How can we identify groups of prime numbers in a given interval with greater economy in writing and calculations?

The new t_{ipc} technique also uses definitions and theorems, so the tasks associated with questions (1) and (2), which had already been solved by ti, are also fully solved by tipc. The task associated with question 3, which was not solved by ti, will be answered by the new technique, based on the solution found for questions (4), (5), (6) and (7).

So far we can say that MO_{ipc} provides a technique for identifying whether any number is prime or not. Based on the definition of prime number and the fundamental theorem of arithmetic, you can answer question (1) and (2), but not question (3), as it doesn't show you how to do the calculations. On the other hand, ti could answer questions (1) and (2), but it proved to be limited, depending on other mathematical concepts and improving the technique so that there was no ambiguity in the result, so the tasks associated with question (3) did not find a satisfactory answer.

Furthermore, as it depended on a method for identifying the first prime numbers, regardless of the framework already defined, the technique could not be applied without knowing how to identify the first prime numbers, which only happened with the new OM_{ipc} organization, from which more problematic questions (4), (5), (6), and (7) arose, relating to the arithmetic operations that support testing the primality of any number.

Below, we present some elements that justify the use of some techniques associated with tipc:

The technique (tpmd) of identifying the equal parts in addition, the factors in multiplication and relating them to the terms in division, enabled the development of the technique of identifying the first prime numbers, through the association of partitioning, multiplication and division.

This technique was not presented in any textbook, according to research by Souza (2018) on divisibility in 22 6th grade math textbooks from 1984 to 2016. The technique arose from the observation that when presenting the Sieve of Eratosthenes, the first basic primes are used without saying how and why these numbers were classified as primes. Table 1 shows the technique (tpmd), in the first column are the first natural numbers (from 1 to 10) and in the second column, the representation by the operations of addition, multiplication and division. We want to show that the addition of equal parts will lead to multiplication and, consequently, division, obtaining the divisors of that number. We use partitioning to show the possible sums of this number with equal parts, because if there are other possibilities than the first, just by adding the number 1, then it will be a composite number, since the various sums of equal parts of a number lead to the product of various possibilities of two different factors and, later with exact division, to the divisors of that number. So, from the basic and fundamental operations of arithmetic, corroborated by the definition of prime and composite numbers, we can recognize that the numbers 2, 3, 5 and 7 have only two divisors and are therefore called prime.

Table 4.

Partition, multiplication and division technique (t_{pmd}) to identify the first prime numbers

	Technique for identifying the first prime numbers
1	1 => 1 x 1 = 1, 1 ÷ 1 = 1 => Therefore, there is only one divisor, 1 itself (which is neither prime nor composite).
2	1 + 1 = 2 => 2 x 1 = 2 => 2 ÷ 1 = 2, 2 ÷ 2 = 1 => 2 dv = {1 e 2}. It is therefore a prime number.
3	1 + 1 + 1 = 3, 1 + 2 = 3 => 3 x 1 = 3 => 3 ÷ 1 = 3, 3 ÷ 3 = 1 => 2 dv = {1 e 3}. It is therefore a prime number.
4	1 + 1 + 1 + 1 = 4, 2 + 2 = 4, 1 + 3 = 4 => 4 x 1 = 4, 2 x 2 = 4 => 4 ÷ 1 = 4, 4 ÷ 4 = 1, 4 ÷ 2 = 2 => 3 dv = {1, 2 e 4}. It is therefore a composite number.
5	1 + 1 + 1 + 1 + 1 = 5, 1 + 2 + 2 = 5, 2 + 3 = 5, 1 + 4 = 5 => 5 x 1 = 5 => 5 ÷ 1 = 5, 5 ÷ 5 = 1 => 2 dv = {1 e 5}. It is therefore a prime number..
6	1 + 1 + 1 + 1 + 1 + 1 = 6, 2 + 2 + 2 = 6, 3 + 3 = 6, 2 + 4 = 6, 1 + 5 = 6 => 6 x 1 = 6, 2 x 3 = 3 x 2 = 6 => 6 ÷ 1 = 6, 6 ÷ 6 = 1, 6 ÷ 2 = 3, 6 ÷ 3 = 2 => 4 dv = {1, 2, 3 e 6}. It is therefore a composite number.
7	1 + 1 + 1 + 1 + 1 + 1 + 1 = 7, 1 + 2 + 2 + 2 = 7, 1 + 3 + 3 = 7, 3 + 4 = 7, 2 + 5 = 7, 1 + 6 = 7 => 7 x 1 = 7 => 7 ÷ 1 = 7, 7 ÷ 7 = 1 => 2 dv = {1 e 7}. It is therefore a prime number.
8	1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8, 2 + 2 + 2 + 2 = 8, 4 + 4 = 8, 3 + 5, 2 + 6, 1 + 7 = 8 => 8 x 1 = 8, 2 x 4 = 4 x 2 = 8 => 8 ÷ 1 = 8, 8 ÷ 8 = 1, 8 ÷ 2 = 4, 8 ÷ 4 = 2 => 4 dv = {1, 2, 4 e 8}. It is therefore a composite number.
9	1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9, 1 + 2 + 2 + 2 + 2= 4, 3 + 3 + 3 = 9, 1 + 4 + 4, 4 + 5, 3 + 6, 2 + 7, 1 + 8 = 9 => 9 x 1 = 9, 3 x 3 = 9 => 9 ÷ 1 = 9, 9 ÷ 9 = 1, 9 ÷ 3 = 3 => 3 dv = {1, 3 e 9}. It is therefore a composite number.
10	1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10, 2 + 2 + 2 + 2 + 2= 10, 1 + 3 + 3 + 3 = 10, 2 + 4 + 4 = 10, 5 + 5 = 10, 4 + 6 = 10, 3 + 7 = 10, 2 + 8 = 10, 1 + 9 = 10 => 10 x 1 = 10, 2 x 5 = 5 x 2 = 10 => 10 ÷ 1 = 10, 10 ÷ 10 = 1, 10 ÷ 2 = 5, 10 ÷ 5 = 2 => 4 dv = {1, 2, 5 e 10}. It is therefore a composite number.

- The technique of calculating the divisors and the number of divisors is justified because it is necessary in the process of identifying prime and composite numbers, as well as using the divisibility criteria of the numbers 2, 3 and 5, which save calculations.
- The division technique is justified because it is used to define and establish the limit of calculations to test the primality of any number.

The key theoretical element is the TFA, as it shows that any composite number can be written as a single factorization of prime factors (except in order) and the conclusion that what differentiates prime numbers from composite numbers is the number of divisors it contains.

We can say that the initial technique (t_i) needed to be complemented by other techniques in order to provide answers to the problematic questions, giving rise to new MOs around the knowledge that underpins the study of prime numbers, supporting the application of the new t_{pmd} technique around the identification of the first prime numbers (MO_{ipc}).

This new t_{pmd} technique emerged with the new method for identifying prime numbers, which we will present later, as a process, since none of the textbooks researched showed how to identify the first four prime numbers.

To sum up, we can consider that the main objective or raison d'être of the study of the identification of prime numbers, type 1, is the representation of prime and composite numbers in such a way that there is no doubt about the identification of each of them or what differentiates them. It is known that what differentiates a composite number from a prime number is the number of its divisors and even if a prime number has the fewest divisors, with the exception of one, it is still not easy to confirm that a very large, odd number (for example, greater than 50,000), by the known method, is a prime number. So, of course, by mathematical induction, we can say that a number in the form 2n + 1 is odd, but not that it is prime.

The importance of prime numbers in the construction of other numbers is obvious, because it follows that prime numbers are the generators of all natural numbers. In fact, that's what the TFA says.

We saw the need to vary and complement the initial technique in order to provide a satisfactory answer to all the problematic questions that arose, thus: $t_i \Rightarrow t_{ip} \Rightarrow t_{ipmd} \Rightarrow t_{ipc}$.

We have to t_i is a technique that uses definitions and theorems (such as TFA) to differentiate between prime and composite numbers. tip is the technique for identifying whether a number a is prime, using the square root of a ($\sqrt{a}$) as an indication of the limit of the use of basic primes for testing, according to the model already presented. However, it was realized that question (7) cannot be answered, because it requires the evolution of existing techniques for identifying a larger list of prime numbers, so that they can be identified at increasingly larger intervals.

The TFA, which states that all numbers can be written as prime factors, gives us a clue that the other primes can only be found through the primes themselves, i.e. the basic primes.

With the intention of further expanding the possibility of identifying prime numbers, since they are the builders of the other numbers, the foundation of their existence. However, the tipc technique, applied to test any number, is not sufficient to identify a group of primes in any number interval. For now, with the knowledge of the first basic primes 2, 3, 5 and 7, you can identify all the primes among the first hundred numbers. However, we want to increase the efficiency of the process and the economy of the numbers used, and thus answer question (7). In this way, we are considering a new direction in the evolution of the tipc technique, which is characterized by expanding the scope of identifying prime numbers, now, in groupings of numbers, based on the sieve of Eratosthenes (t_{cE} technique), which has already been presented.

$$t_i \rightarrow t_{ipc} \rightarrow T_{cE}$$

The new technique uses all the theoretical foundations of the technique that used tipc to identify one number at a time, but now we're going to apply it to identifying groups of primes with the variation of relating fewer numbers (50% less).

According to Santos (2007, p. 12), if n is not prime, then n necessarily has a prime factor less than or equal to √n, and furthermore the square root of *n* gives the limit for using basic primes in the primality test.

The new t_{cE}' technique will answer question (7), albeit in a limited way, only for the interval from 01 to 100, by applying a variation of the Eratosthenes sieve to identify the prime numbers that are, for example, in the interval from 1 to 100, using only the odd numbers ending in 1, 3, 7 and 9 since in the (t_{cE}) technique all the numbers from 1 to 100 were used. So we have the following tasks:

- Define a given interval (a, b). In this case: from 1 to 100, making a table with only the odd numbers ending in 1, 3, 7 and 9.
- Calculate the limit of basic primes using pb ≤ √b; with b = 100, calculate the square root of b: √100=10, the root sets the limit for the basic primes.
- List the basic primes smaller than 10: 2, 3, 5 and 7 (already identified by t_{ipmd}).

Due to the previous techniques, it was found that 2 and 5 are exceptions, since 2 is the only even number that is prime, and 5 is the only prime number ending in 5 (since all the others ending in 5 are divisible by 5). These two numbers will not be used in the calculations. Therefore, the primes 3 and 7 will be used to calculate their multiples.

Calculate the multiples of 3 and 7. The multiples of 3 will be identified by the color blue and the multiples of 7, starting from the square of 7, will be identified by the color green. The number 1 is neither prime nor composite and will be identified by the color yellow and excluded, as shown in Table 5.

Table 5.

Sieve of odd numbers ending in 1, 3, 7 and 9

1	**3**	**7**	9
11	**13**	**17**	**19**
21	**23**	27	**29**
31	33	**37**	39
41	**43**	**47**	49
51	**53**	57	**59**
61	63	**67**	69
71	**73**	77	**79**
81	**83**	87	**89**
91	93	**97**	99

- List the numbers that have not been identified (they are blank); we have included the numbers 2 and 5 in the final list of prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

It can be seen that in order to identify the prime numbers in the open interval from 01 to 100, according to the technique ($t_{cE'}$), only two basic primes were needed.

Note that in tcE (Sieve of Eratosthenes) it was necessary to list all the numbers in the given interval (from 1 to 100) beforehand and calculate the multiples of all the basic primes (2, 3, 5 and 7), which makes the process quite laborious. However, in most elementary school mathematics textbooks, specifically those for 6th grade, the model is presented in a very restricted way, and it is necessary for the professor to show the students some expansion of the model used. As presented by the t_{Pmd} and t_{cE}' techniques.

The aim of this technique is to show a shorter way of using the Sieve of Eratosthenes, since the students already have prior knowledge of basic arithmetic and are able to distinguish odd numbers from even numbers, as well as prime numbers from composite numbers, and apply the divisibility criteria for at least the numbers 2, 3 and 5.

So, with our proposal for an alternative model, of which Table 4 is a part, it will be very useful for students to see how the number of basic primes can increase and is cumulative. The question may arise as to what this cumulative increase looks like. To answer this, let's take an example: based on the interval from 1 to 100, in the $T_{cE'}$ technique, only two primes were used for the same number of numbers. Table 4 shows that in order to identify the primes in the interval 100 to 200, we need to include the prime numbers 11 and 13 in the set of basic primes. If we consider a wider range from 1 to 1000, then we need to include more prime numbers. So, to identify the prime numbers from 1 to 1000, using the $t_{cE'}$ technique, the number of basic prime numbers for calculating multiples will be 9 primes (3, 7, 11, 13, 17, 19, 23, 29 and 31).

Table 6 shows the basic primes needed to calculate multiples or compounds in the intervals from 100 to 100 numbers up to 1000. We can see that the same basic prime numbers will be used to calculate from 1 to 1000, or from 100 to 1000, or from 200 to 1000 and so on up to 900 to 1000. Even if, in some intervals, the number of basic prime numbers doesn't increase, as in the 300-400 and 400-500 intervals, the same basic prime numbers will be used. For the next interval, 500-600, we see that only one prime number has increased and it will be the same for the intervals 600-700 and 700-800. Considering small intervals such as one hundred numbers, the number of basic prime numbers may or may not increase and this is due to the irregularity in the distribution of prime numbers in the sequence of natural numbers.

Table 6.

Number of basic primes in the range 01 to 1,000.

Interval	**Basic cousins**	**Quantity**
01–100	3 e 7	02
100–200	3, 7, 11 e 13	04
200–300	3, 7, 11, 13 e 17	05
300–400	3, 7, 11, 13, 17 e 19	06
400–500	3, 7, 11, 13, 17 e 19	06

500–600	3,7, 11, 13, 17, 19 e 23	07
600–700	3, 7, 11, 13, 17, 19 e 23	07
700–800	3, 7, 11, 13, 17, 19 e 23	07
800–900	3, 7, 11, 13, 17, 19, 23 e 29	08
900–1000	3, 7, 11, 13, 17, 19, 23, 29 e 31	09

Note that by applying the new $t_{cE'}$ technique, the numbers have been reduced by less than 50% when applying the restriction, without any loss, because at the end of the process, the prime numbers 2 and 5 will be computed.

However, even with the reduction in calculations by applying this new technique, we realize that following these steps, the process will become too laborious to identify large prime numbers or larger intervals of numbers. This raises another problematic question:

(8) How can we identify groups of primes in increasingly larger intervals more economically and effectively in the process, with another method (using formulas)?

To answer this question, new MOs are needed.

Technological-theoretical elements of the identification of prime numbers by the sieve of eratosthenes

From the development of this study and, as a consequence of the definition of primes and compounds, as well as the TFA, we can say that the MO_{cE} provides us with a t_{cE} technique for identifying groups of prime numbers, although, as shown, it is still limited by the basic primes for a range of numbers up to 100, but once the prime numbers in this range have been identified, the technique only encounters restrictions for increasingly larger ranges of numbers because it is a very laborious process. However, the extension from tcE to tcE' saves time in writing down the numbers and calculating the multiples of the basic primes, since the set of numbers has been reduced to work only with the set of odd numbers, using the four endings.

Therefore, we can conclude that this new representation of the $t_{cE'}$ technique makes it possible to perform arithmetic multiplication and division operations with basic primes more economically and efficiently. Regarding Eratosthenes' primality sieve, Fonseca (2016, p. 53) emphasizes

> In the presentation of prime numbers, a common question is to determine all the prime numbers up to a certain limit. It's a simple procedure, shown in elementary school. We

should also point out the relevance, in higher education, not only of determining a certain number of primes using the sieves, but also the fundamental question of discussing the primality of odd positive integers as a didactic work and problem-solving strategy, in the context of didactic situations.

In order to be able to work didactically with the identification of groups of prime numbers in increasingly wide intervals, as a result of everything that has already been explained in this study, the need has arisen to expand the technique of identifying a group of primes in any interval of numbers.

The alternative method for identifying groups of prime numbers in any interval of numbers

The limitations of the mathematical activity perceived in the Mathematical Organization MO_{cE} make it clear that this MO_{cE} needs to be expanded, in other words, a new one needs to be built, which we call MO_{maip}, to carry out the mathematical tasks that could not be performed in the MO_{cE}. We have a new problem question as the generating task for MO_{maip}:

(8) How can we identify groups of primes in increasingly larger intervals with greater economy and efficiency of the process, with another method (using formulas)?

In order to meet the new mathematical organization (MO_{maip}), we created a method, with its own processes and two formulas which, when combined, generate all the prime numbers in any interval of numbers.

Mathematical organization around the alternative method for identifying groups of prime numbers in any number interval

The identification of prime numbers of type 4 (using the alternative method of identifying groups of prime numbers in any number interval) is a technique for identifying groups of primes, t_{maip}, which makes it possible to give a definitive answer to question (3) and solve task (8).

Prior to the $t_{cE'}$ technique, a more economical process for identifying groups of primes was presented, using odd numbers with four endings. However, our aim with MO_{maip} is to further reduce this set of numbers and reliably solve the tasks associated with question (8), based on the proposed alternative method, with the creation of two formulas: a general formula (1) and a formula for generating prime compounds (2), as well as a well-defined process for applying the method.

For the development of this MO_{maip}, some elements and definitions will be used from now on for the initial application of the t_{maip} technique:

- The basic prime numbers 2, 3 and 5 are exceptions. Multiples of 3 are not part of the numbers that make up formulas 1 and 2.
- The set of odd numbers has been reduced to use only those ending in 1, 3, 7 and 9, identified by the unit digits, henceforth represented by: T_1, T_3, T_7, T_9.
- Use of two formulas: the general formula (F1) produces all the numbers in the range given as a base (primes and prime compounds); the second (F2) produces the prime compounds contained in the numbers generated by F1.
- Use of two (alternating) sequences for each of the four endings.
- The multiples of the basic primes $m(p_b)$ are called prime compounds (Č) because they belong to the universe of numbers generated by the general formula and will be used to identify the primes in the given interval.

To identify the prime numbers, it is necessary to calculate the multiples of the basic primes, but not all the multiples, only those restricted to the given interval and generated by formula 1, ending in 1, 3, 7 and 9, represented as follows: $p_b . n = m(p_b) = Č$, where "n" the second multiplication factor will have the same endings (n_1, n_3, n_7, n_9).

To make it easier to calculate the multiples of the basic primes, we have produced the arrangements of possible combinations with the four endings, as shown in Table 7:

Table 7.

Possible combinations for calculating m(pb) multiples.

Terminação 1	**Terminação 3**	**Terminação 7**	**Terminação 9**
$P_{b1} . n_1 = Č_1$	$P_{b1} . n_3 = Č_3$	$P_{b1} . n_7 = Č_7$	$P_{b1} . n_9 = Č_9$
$P_{b3} . n_7 = Č_1$	$P_{b3} . n_1 = Č_3$	$P_{b3} . n_9 = Č_7$	$P_{b3} . n_3 = Č_9$
$P_{b7} . n_3 = Č_1$	$P_{b7} . n_9 = Č_3$	$P_{b7} . n_1 = Č_7$	$P_{b7} . n_7 = Č_9$
$P_{b9} . n_9 = Č_1$	$P_{b9} . n_7 = Č_3$	$P_{b9} . n_3 = Č_7$	$P_{b9} . n_1 = Č_9$

The indices of p and n are the terminations (1, 3, 7 and 9). Where P = basic prime number and n Є N*, and Č = compound prime, at endings 1, 3, 7 and 9.

To determine the second factor of the basic primes of the two sequences of each termination, given a range of numbers (a, b), the smallest number in the given range (a) is used as a reference for the factor *n*, where $n > (a / P_b)$. For example, given a range from 500 to 1,000, we use the smallest number in the range, 500, and divide this number by the basic primes; the quotient is the number that gives the reference for the second factor that will start the calculations of the two sequences of each termination, being: $P_b \cdot n_{1,2} > 500$.

This will calculate the multiples of the basic primes that will start the two sequences of each ending.

Example: To calculate the 2nd factor of $P_b = 7 \Rightarrow 500 / 7 = 71.4$

Therefore, 71.4 is the benchmark for choosing the second factor "n", where $n \neq m(3)$, so $n_{1,2} > 71$ for the four endings.

When $(a / p_b) < P_b$, we use the concept of first divisibility, with the basic prime itself as a reference, so $n_{1,2} \geq P_b$, if $n = P_b$, implies $(P_b)^2$.

Concept of first divisibility

If one number is a multiple of another, for example: 77 = 7 x 11, 77 is said to be a multiple of 7 and 11, but first it is a multiple of 7 and 77 is said to be divisible by 7. In other words, 77 being the product of two primes has 4 divisors: 1, 7, 11 and 77. But there are numbers that only have three divisors: the unit, a prime and the number itself. These numbers are the squares of primes (P^2), like the number 49, which is divisible by itself, by 7 and by 1. These are the numbers that determine prime divisibility. In the range from 1 to 100, in the example given earlier, using the sieve of Eratosthenes, you can see this fact when you choose to calculate the multiples of 7 from its square, i.e. 7 x 7, because if you did 7 x 5 or 7 x 3, the products already existed when you first multiplied by 3 (3 x 7) or by 5 (5 x 7).

This is an important concept for reducing the calculation of multiples of basic primes and avoiding calculations of repeated numbers. When $P_b < \sqrt{b}$, and the quotient of $(a / P_b) = n$ becomes smaller than P_b, then the products of P_b by $n_{1,2}$ will be numbers already produced by the basic primes smaller than the P_b in reference. So, to avoid this situation, the basic prime that will produce the multiples will be the reference for the second factor (n), i.e. $n \geq P_b$, if $n = P_b$, then the product will be the perfect square of P_b. Therefore, the square of P is a benchmark for producing prime multiples of P or numbers that are first divisible by P. Hence the name prime divisibility.

The perfect squares of prime numbers ending in 1, 3, 7 and 9 have endings 1 and 9, so the perfect squares of basic primes ≥ 7 (in this context) will only appear in sequences with endings 1 and 9.

The formulas used to apply the tmaip technique

In order to identify a group of prime numbers in any interval of numbers, two formulas were created: the first, called the general formula or formula (1), produces the universe of numbers (primes and prime compounds) in the given interval (a, b); the second formula (2) produces all the multiples of the basic primes in the given interval (a, b), called prime compounds (Č), because they are contained in the universe of numbers produced by formula (1).

$$\textit{General formula: } 30n + \alpha_{1,2} = P \textit{ or } \check{C} \quad (1)$$

The number "α" is any odd number ending in 1, 3, 7 and 9, for the two sequences, being distinct and sequential numbers ≠ m(3); where n = 0, 1, 2, 3, ... and n Є Z+

At the end of the calculations with the General Formula, we will join the two sequences, forming a single sequence for each ending.

The second formula is designed to generate only the multiples of the basic primes (Č) that are contained in the universe of numbers generated by formula 1.

$$rn + m(p_b)_{1,2} = \check{C} \quad (2)$$

Where "r" is the ratio and, r = 30pb, with n = 0, 1, 2, 3, ... and n Є Z+; m(pb)1, 2, any multiple of the basic primes, being two distinct and sequential numbers, for each of the sequences, in the four endings and different from a multiple of 3 ($m(p_b) \neq m(3)$). At the end of the calculations for each $m(p_b)$ we list the primate compounds (Č)in a single sequence per ending.

As formula (2) produces Č which is contained in formula (1), we have that:

$$(30n + \alpha_{1,2}) - (rn + m(p_b)_{1,2}) = P$$

At the beginning of our studies, when we were trying to improve this technique of identifying groups of prime numbers in any given interval, when we did the first test for the interval from 01 to 1,000, we observed that the multiples of three were equivalent to 56.8% of the compound numbers produced, in other words, while the basic primes (7, 11, 13, 17, 19, 23, 29 and 31) produced 43.2% together, in a total of 101 multiples, the number 3 alone produced 133 multiples. Therefore, removing the calculation of multiples of 3 was a decision that allowed the two formulas to be improved and the process of applying the method to be developed.

Following the procedure of the previous techniques, T_{eE} and $T_{cE'}$, when using the Eratosthenes sieve according to Dante (2015, p. 140), the first 100 numbers were written down and asked to calculate the multiples of the four basic primes and then cross them out. The proposed variation of the T_{cE} technique is to reduce the calculations and writing, with the sieve appearing only for odd numbers ending in 1, 3, 7 and 9 with the $T_{cE'}$ technique. So, in the same way, our proposal for the alternative method is to further reduce calculations by using mathematical formulas to identify prime numbers. For this reason, we will first apply it to the interval from 01 to 100, to show that there has been a reduction in calculations.

Procedures for applying the two formulas

Step 1: Choose the interval (a, b) for identifying groups of primes;

Step 2: Determine the limit of basic primes from the square root of the largest number in the given interval (b), i.e. $p_b \leq \sqrt{b}$, if the root of b is not an integer, use only the integer part as a reference.

Step 3: Apply formula 1 to find the universe of numbers;

Step 4: Apply formula 2 to calculate all the prime compounds in the given range and at the end of the calculations list them in a single sequence for each ending;

Step 5: List the four sequences of numbers produced by formula 1 to eliminate the prime compounds produced by formula 2:

Step 6: List all the prime numbers in the given interval.

Applying the t_{maip} technique - identifying groups of primes in the open interval from 01 to 100

Step 1: Choose the interval (1, 100) to identify groups of primes;

Step 2: Determine the limit of the basic primes, starting from the square root of the largest number in the given interval (100), i.e. $p_b \leq \sqrt{100} = 10$. Therefore, the prime smaller than 10 is the number 7.

Step 3: Apply formula 1: $(30n + \alpha_{1,2})$ in two sequences for the four endings, as shown in tables 8 and 9:

Table 8.

Application of the General Formula in Terminations 1 and 3 within the range of 1 to 100

$30n + \alpha_{1, 2}$ **Being $\alpha \neq m(3)$, n = 0, 1, 2, 3, ... e n Є Z_+**			
T_1		T_3	
S_1	S_2	S_1	S_2
30x0 + 11 = 11	30x0 + 31 = 31	30x0 + 13 = 13	30x0 + 23 = 23
30x1 + 11 = 41	30x1 + 31 = 61	30x1 + 13 = 43	30x1 + 23 = 53

30x2 + 11 = 71	30x2 + 31 = 91	30x2 + 13 = 73	30x2 + 23 = 83

Table 9.

Application of the General Formula in Terminations 7 and 9 within the range of 500 to 1,000

30n + $\alpha_{1,2}$ being $\alpha \neq M(3)$, n = (0, 1, 2, 3, ...) e n Є Z_+			
T_7		T_9	
S_1	S_2	S_1	S_2
30x0 + 7 = 7	30x0 + 17 = 17	30x0 + 19 = 19	30x0 + 29 = 29
30x1 + 7 = 37	30x1 + 17 = 47	30x1 + 19 = 49	30x1 + 29 = 59
30x2 + 7 = 67	30x2 + 17 = 77	30x2 + 19 = 79	30x2 + 29 = 89
30x3 + 7 = 97			

Following we'll put together the two sequences for each ending:

- Ending 1: 11, 31, 41, 61, 71 and 91.
- Ending 3: 13, 23, 43, 53, 73 and 83.
- Ending 7: 7, 17, 37, 47, 67, 77 and 97.
- Ending 9: 19, 29, 49, 59, 79 and 89.

Total numbers: 25

Step 4: Apply the formula 2 ($rn + m(p_b)_{1,\ 2}$) to calculate all the prime compounds in the given interval, in two sequences for each of the four endings.

In the second step we obtained the prime number 7 as the smallest prime before 10, the square root of the largest number in the given interval. As 2, 3 and 5 are exceptions, let's calculate the compound primes of 7 only.

First we calculate the 1st and 2nd multiple of the two sequences for each ending, and the ratio of each basic prime (r = 30p) to then apply to the formulas.

- Calculating the ratio for the basic prime number 7: 30 x 7 = 210
- Ending 1: S_1: 7 x 13 = 91 S2: 7 x 23 = 161 (exceeds the range)

Applying to Formula 2: 210 x 0 + 91 = 91

- Ending 3: S1: 7 x 19 = 133 (exceeds the range), so no need to calculate S2 and does not apply in Formula 2.
- Ending 7: S1: 7 x 11 = 77 S2: 7 x 31 = 217 (exceeds the range and does not apply)
- Applying to Formula 2: 210 x 0 + 77 = 77

- Ending 9: S1: 7 x 7 = 49 S2: 7 x 17 = 119 (exceeds the range and does not apply)

Therefore, the multiples of 7 for each ending are as follows: T_1: 9_1, T_3: don’t have, T_7: 77, T_9: 49.

Step 5: List the four sequences of numbers produced by Formula 1 to eliminate the prime compounds produced by Formula 2, crossing them out:

- Ending 1: 11, 31, 41, 61, 71 and 91.
- Ending 3: 13, 23, 43, 53, 73 and 83.
- Ending 7: 7, 17, 37, 47, 67, 77 and 97.
- Ending 9: 19, 29, 49, 59, 79 and 89.

Step 6: List all the prime numbers in the given range.

- Ending 1: 11, 31, 41, 61 and 71.
- Ending 3: 13, 23, 43, 53, 73 and 83.
- Ending 7: 7, 17, 37, 47, 67 and 97.
- Ending 9: 19, 29, 59, 79 and 89.

In a single sequence, we will present the list of the 25 prime numbers in the range 1 to 100, now including the three primes that are the exceptions: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

Analyzing this new technique (t_{maip}), we can conclude that there was a considerable reduction in the calculation of multiples compared to the techniques presented previously (t_{cE} and t_{cE}'), because we only calculated with a basic prime number. It also confirms our claim that all the prime numbers (with the exception of 2, 3 and 5) and the prime compounds found by Formula 2 were all produced by Formula 1.

We therefore conclude that, in fact, by applying the two formulas we have found all the prime numbers in the given interval,

F1 – F2 = P, i.e., $(30n + \alpha_{1,2}) - (rn + m(p_b)_{1,2}) = \boldsymbol{P}$

In order to claim that this method can be applied to any given range of numbers, we're going to prove that the two formulas used are valid, using the formulas from the first given range, in the way they were used at each end.

Here, we have presented the validity of a particular case, which cannot guarantee that it will be valid for any range of numbers, even if we can say that we have tested it up to the range of one hundred thousand numbers.

According to Sominski (1996, p. 13):

> If we have a proposition that is valid in several particular cases and it is impossible to analyze all the cases, when can we say that this proposition is valid in general? Sometimes the answer is found by applying a special argument, known as the method of complete or perfect mathematical induction.

This method is based on the principle of mathematical induction which consists of the following: a proposition is valid for every natural number n if: 1º) it is valid for n =1 and 2º) from its validity for any natural number n = k we deduce its validity for n = k + 1 (Sominski, 1996, 13).

So let's test the validity of the two formulas using the method of mathematical induction. First, let's test formula 1.

To use the General Formula or Formula 1 we have the proposition that this formula produces the universe of odd numbers, at endings 1, 3, 7 and 9, with the condition that the number α must be different from multiples of 3, so for any interval of numbers in which it will be applied, the choice of this number must take this condition into account, so, for example, for ending 1, we don't choose the number 21, because it is a multiple of 3 (3|21) for the 2nd sequence, just as we chose the number 3 for the 1st sequence.

Let's apply the method to 30n + 11, where n = 0 => 30 x 0 +11 = 11

1º) The proposition is valid for n = 0

2º) Suppose the proposition is valid for n = k, i.e. 30k + 11

$A_k = 30k + 30$

$$U_{k+1} = A_k + 11 = (30k + 30) + 11 = 30k + 41 =$$
$$= 30(k+1) + 11$$

As you can see, the formula for the 1st sequence ending in 1 is valid for n = k and n = k+1.

Let's check it for the 2nd sequence with 30n + 31, where n = 0 => 30 x 0 +31 = 31

1º) The proposition is valid for n = 0

2º) Suppose the proposition is valid for n = k, i.e. 30k + 31

$A_k = 30k + 30$

$$U_{k+1} = A_k + 31 = (30k + 30) + 31 = 30k + 61 =$$
$$= 30(k+1) + 31$$

Therefore, we can conclude that if the general formula is valid for the two sequences ending in 1, it will be valid for the two sequences ending in 3, 7 and 9.

Now let's check the validity of Formula 2 $(rn + m(p_b)_{1,2})$ for p = 7, r = 30x7, i.e. r = 210, where the 1st multiple of 7 is 91, because 7 x 13 = 91 and for the 2nd sequence the 2nd multiple of 7 is 161, because 7 x 23 = 161. So for the 1st sequence we have the formula: 210n + 91 and for the 2nd sequence we have: 210n + 161.

The first proposition is that 210n + 91 is divisible by 7.

1º) The proposition is valid for n = 0

2º) Suppose that the proposition is valid for n = k, i.e. that

$A_k = 210k + 210$

$A_{k+1} = A_k + 91 = (210k + 210) + 91 =$

$= 210k + 301$

$= 210(k+1) + 91$

Since A_{k+1} is the sum of two parts, both divisible by 7, then A_{kt1} is divisible by 7.

The second proposition is that 210n + 161 is divisible by 7.

1º) The proposition is valid for n = 0

2º) suppose that the proposition is valid for n = k, i.e. that

$A_k = 210k + 210$

$A_{k+1} = A_k + 161 = (210k + 210) + 161 =$

$= 210k + 371$

$= 210(k+1) + 161$

Since A_{k+1} is the sum of two parts, both divisible by 7, then A_{kt1} is divisible by 7.

Therefore, if formula 2 is valid for the basic prime number 7, then we can infer that it is valid for any basic prime number in the range given by the application of the t_{maip} technique

So, to better visualize the process in any number range, let's apply the method to the range of 500 to 1000 numbers.

Procedures for applying the two general formulas (1) and formula (2)

Step 1: Determine the given interval (a, b): 500 to 1000.

Step 2: Apply formula 1 ($30n + \alpha_{1,2}$) to generate the universe of numbers in the range 500 to 1000.

- Choosing the number "α" for the two sequences at each end, where α > 500 and α ≠ m(3):T_1: (S_1: α = 511, S_2: α = 521); T_3: (S_1: α = 503, S_2: α = 523); T_7: (S_1: α = 517, S_2: α = 527); T_9: (S_1: α = 509, S_2: α = 529).

Table 10.

Application of the General Formula in Terminations 1 and 3 within the range of 500 to 1,000

$30n + \alpha_{1,2}$

Being α ≠ m(3), n = 0, 1, 2, 3, ... e n Є Z_+			
T_1		T_3	
S_1	S_2	S_1	S_2
30x0 + 511 = 511	30x0 + 521 = 521	30x0 + 503 = 503	30x0 + 523 = 523
30x1 + 511 = 541	30x1 + 521 = 551	30x1 + 503 = 533	30x1 + 523 = 553
30x2 + 511 = 571	30x2 + 521 = 581	30x2 + 503 = 563	30x2 + 523 = 583
30x3 + 511 = 601	30x3 + 521 = 611	30x3 + 503 = 593	30x3 + 523 = 613
30x4 + 511 = 631	30x4 + 521 = 641	30x4 + 503 = 623	30x4 + 523 = 643
30x5 + 511 = 661	30x5 + 521 = 671	30x5 + 503 = 653	30x5 + 523 = 673
30x6 + 511 = 691	30x6 + 521 = 701	30x6 + 503 = 683	30x6 + 523 = 703
30x7 + 511 = 721	30x7 + 521 = 731	30x7 + 503 = 713	30x7 + 523 = 733
30x8 + 511 = 751	30x8 + 521 = 761	30x8 + 503 = 743	30x8 + 523 = 763
30x9 + 511 = 781	30x9 + 521 = 791	30x9 + 503 = 773	30x9 + 523 = 793
30x10 + 511 = 811	30x10 + 521 = 821	30x10 + 503 = 803	30x10 + 523 = 823
30x11 + 511 = 841	30x11 + 521 = 851	30x11 + 503 = 833	30x11 + 523 = 853
30x12 + 511 = 871	30x12 + 521 = 881	30x12 + 503 = 863	30x12 + 523 = 883
30x13 + 511 = 901	30x13 + 521 = 911	30x13 + 503 = 893	30x13 + 523 = 913
30x14 + 511 = 931	30x14 + 521 = 941	30x14 + 503 = 923	30x14 + 523 = 943
30x15 + 511 = 961	30x15 + 521 = 971	30x15 + 503 = 953	30x15 + 523 = 973
30x16 + 511 = 991		30x16 + 503 = 983	

Table 11.

Application of the General Formula in Terminations 7 and 9 within the range of 500 to 1,000

30n + $\alpha_{1,2}$ being α ≠ M(3), n = (0, 1, 2, 3, ...) e n Є Z_+			
T_7		T_9	
S_1	S_2	S_1	S_2
30x0 + 517 = 517	30x0 + 527 = 527	30x0 + 509 = 509	30x0 + 529 = 529

30x1 + 517 = 547	30x1 + 527 = 557	30x1 + 509 = 539	30x1 + 529 = 559
30x2 + 517 = 577	30x2 + 527 = 587	30x2 + 509 = 569	30x2 + 529 = 589
30x3 + 517 = 607	30x3 + 527 = 617	30x3 + 509 = 599	30x3 + 529 = 619
30x4 + 517 = 637	30x4 + 527 = 647	30x4 + 509 = 629	30x4 + 529 = 649
30x5 + 517 = 667	30x5 + 527 = 677	30x5 + 509 = 659	30x5 + 529 = 679
30x6 + 517 = 697	30x6 + 527 = 707	30x6 + 509 = 689	30x6 + 529 = 709
30x7 + 517 = 727	30x7 + 527 = 737	30x7 + 509 = 719	30x7 + 529 = 739
30x8 + 517 = 757	30x8 + 527 = 767	30x8 + 509 = 749	30x8 + 529 = 769
30x9 + 517 = 787	30x9 + 527 = 797	30x9 + 509 = 779	30x9 + 529 = 799
30x10 + 517 = 817	30x10 + 527 = 827	30x10 + 509 = 809	30x10 + 529 = 829
30x11 + 517 = 847	30x11 + 527 = 857	30x11 + 509 = 839	30x11 + 529 = 859
30x12 + 517 = 877	30x12 + 527 = 887	30x12 + 509 = 869	30x12 + 529 = 889
30x13 + 517 = 907	30x13 + 527 = 917	30x13 + 509 = 899	30x13 + 529 = 919
30x14 + 517 = 937	30x14 + 527 = 947	30x14 + 509 = 929	30x14 + 529 = 949
30x15 + 517 = 967	30x15 + 527 = 977	30x15 + 509 = 959	30x15 + 529 = 979
30x16 + 517 = 97		30x16 + 509 = 989	

The general formula (Tables 10 and 11) produced 132 numbers. Among these numbers, there are prime numbers and prime compounds. By calculating the production of prime compounds using formula 2, all the primes generated by formula 1 will be identified.

Step 3: Determine the limit of basic primes to apply formula 2, which generates the prime compounds, by calculating the square root of the largest number in the given interval:

$\sqrt{1000} = 31.6 \Rightarrow p_b \leq 31.6 \Rightarrow p_b = 31$

Therefore, the basic primes to apply in F2 are: 7, 11, 13, 17, 19, 23, 29 and 31.

Step 4: Apply formula 2, using two alternating sequences for each termination:

$rn + m(p_b)_{1,2} + = \check{C}$

Where the ratio $r = 30p_b$, n = 0, 1, 2, 3, ... with $n \in Z+$ and $m(p_b)$, any multiple of the basic primes that starts the given interval for each of the two sequences of each termination

1) Calculating multiples of 7, r: 30 x 7 = 210 (Table 12)

- Combinations for the calculations: T_1:

T_1: (p_7 x n_3), T_3: (p_7 x n_9), T_7: (p_7 x n_1), T_9 = (p_7 x n_7)

- Calculating m(7) to start the two sequences: 500 / 7 = 71.4, for n > 71.

T_1, S_1 = 7 x 73 = 511, T_1, S_2 = 7 x 83 = 581

T_3, S_1 = 7 x 79 = 553, T_3, S_2 = 7 x 89 = 623

T_7, S_1 = 7 x 91 = 637, T_7, S_2 = 7 x 101 = 707

T_9, S_1 = 7 x 77 = 539, T_9, S_2 = 7 x 97 = 679

Table 12.

Generation of primate compounds m(7) by Formula 2

210n + m(7)$_{1, 2}$			
Termination 1		Termination 3	
S_1	S_2	S_1	S_2
210 x 0 + 511= 511	210 x 0 + 581= 581	210 x 0 + 553= 553	210 x 0 + 623= 623
210 x 1 + 511= 721	210 x 1 + 581= 791	210 x 1 + 553= 763	210 x 1 + 623= 833
210 x 2 + 511= 931		210 x 2 + 553= 973	
Termination 7		Termination 9	
S_1	S_2	S_1	S_2
210 x 0 + 637= 637	210 x 0 + 707= 707	210 x 0 + 539= 539	210 x 0 + 679 = 679
210 x 1 + 637= 847	210 x 1 + 707= 917	210 x 1 + 539= 749	210 x 1 + 679 = 889
		210 x 2 + 539= 959	
Total of m(7) = 19			

2) Calculation of multiples of 11: r: 30 x 11 = 330 (Table 13)

- Combinations for the calculations:

T_1: (p_1 x n_1), T_3: (p_1 x n_3), T_7: (p_1 x n_7), T_9: (p_1 x n_9)

- Calculating m(11) to start the two sequences: 500 / 11 = 45.4, for n > 45.

T_1, S_1 = 11 x 61 = 671, T_1, S_2 = 11 x 71 = 781,

T_3, S_1 = 11 x 53 = 583, T_3, S_2 = 11 x 73 = 803

T_7, S_1 = 11 x 47 = 517, T_7, S_2 = 11 x 67 = 737

T_9, S_1 = 11 x 49 = 539, T_9, S_2 = 11 x 59 = 649

Table 13.

Generation of primate compounds m(11) by Formula 2

330n + m(11)$_{1, 2}$			
Termination 1		Termination 3	
S_1	S_2	S_1	S_2
330 x 0 + 671= 671	330 x 0 + 781= 781	330 x 0 + 583= 583	330 x 0 + 803= 803
		330 x 1 + 583= 913	

Termination 7		Termination 9	
S_1	S_2	S_1	S_2
330 x 0 + 517= 517	330 x 0 + 737= 737	330 x 0 + 539= 539	330 x 0 + 649 = 649
330 x 1 + 517= 847		330 x 1 + 539= 869	330 x 1 + 649 = 979
Total of m(11) = 12			

3) Calculating multiples of 13, with r: 30 x 13 = 390 (Table 14)

- Combinations for the calculations:

T_1: (p_3 x n_7), T_3: (p_3 x n_1), T_7: (p_3 x n_9), T_9: (p_3 x n_3)

- Calculating m(13) to start the two sequences: 500 / 13 = 38.4, for n > 38.

T_1, S_1 = 13 x 47 = 611, T_1, S_2 = 13 x 67 = 871

T_3, S_1 = 13 x 41 = 533, T_3, S_2 = 13 x 61 = 793

T_7, S_1 = 13 x 49 = 637, T_7, S_2 = 13 x 59 = 767

T_9, S_1 = 13 x 43 = 559, T_9, S_2 = 13 x 53 = 689

Table 14.

Generation of primate compounds m(13) by Formula 2

390n + m(13)$_{1, 2}$			
Termination 1		Termination 3	
S_1	S_2	S_1	S_2
390 x 0 + 611= 611	390 x 0 + 871= 871	390 x 0 + 533= 533	390 x 0 + 793= 793
		390 x 1 + 533= 923	
Termination 7		Termination 9	
S_1	S_2	S_1	S_2
390 x 0 + 637= 637	390 x 0 + 767= 767	390 x 0 + 559= 559	390 x 0 + 649 = 689
		390 x 1 + 559= 949	
Total of m(13) = 10			

4) Calculating multiples of 17, with r: 30 x 17 = 510 (Table 15)

- Combinations for the calculations:

T_1: (p_7 x n_3), T_3: (p_7 x n_9), T_7: (p_7 x n_1), T_9: (p_7 x n_7)

- Calculating m(17) to start the two sequences: 500 / 17 = 29.4, for n > 29.

  T_1, S_1 = 17 x 43 = 731, T_1, S_2 = 17 x 53 = 901

T_3, S_1 = 17 x 49 = 833, T_3, S_2 = 17 x 59 = 1003*

T_7, S_1 = 17 x 31 = 527, T_7, S_2 = 17 x 41 = 697

T_9, S_1 = 17 x 37 = 629, T_9, S_2 = 17 x 47 = 799

Note: (p x n) > 1000, (*) exceeds the range, not valid.

Table 15.

Generation of primate compounds m(17) by Formula 2

510n + m(17)$_{1,2}$			
Termination 1		Termination 3	
S_1	S_2	S_1	S_2 (*)
510 x 0 + 731= 731	510 x 0 + 901= 901	510 x 0 + 833= 833	
Termination 7		Termination 9	
S_1	S_2	S_1	S_2
510 x 0 + 527= 527	510 x 0 + 697= 697	510 x 0 + 629= 629	510 x 0 + 649 = 799
Total of m(17) = 07			

5) Calculating multiples of 19, with r: 30 x 19 = 570 (Table 16)

- Combinations for the calculations:

T_1: (p_9 x n_9), T_3: (p_9 x n_7), T_7: (p_9 x n_3), T_9: (p_9 x n_1)

- Calculating m(17) to start the two sequences: 500 / 19 = 26.3, for n > 26.

T_1, S_1 = 19 x 29 = 551, T_1, S_2 = 19 x 49 = 931

T_3, S_1 = 19 x 37 = 703, T_3, S_2 = 19 x 47 = 893

T_7, S_1 = 19 x 43 = 817, T_7, S_2 = 19 x 53 = 1007*

T_9, S_1 = 19 x 31 = 589, T_9, S_2 = 19 x 41 = 779

Note: (p x n) > 1000, (*) exceeds the range, not valid.

Table 16.

Generation of primate compounds (19) by Formula 2

570n + m(19)$_{1,2}$			
Termination 1		Termination 3	
S_1	S_2	S_1	S_2
570 x 0 + 551= 551	570 x 0 + 931= 931	570 x 0 + 703= 703	570 x 0 + 893= 893
Termination 7		Termination 9	
S_1	S_2 (*)	S_1	S_2
570 x 0 + 817= 817		570 x 0 + 589= 589	570 x 0 + 779 = 779
Total of m(19) = 07			

6) Calculating multiples of 23, with r: 30 x 23 = 690 (Table 17)

- Combinations for the calculations:

T_1: (p_3 x n_7), T_3: (p_3 x n_1), T_7: (p_3 x n_9), T_9: (p_3 x n_3)

- Calculating m(23) to start the two sequences: 500 / 13 = 21.7 for n > 21.

T_1, S_1 = 23 x 37 = 851, T_1, S_2 = 23 x 47 = 1081*

T_3, S_1 = 23 x 31 = 713, T_3, S_2 = 23 x 41 = 943

T_7, S_1 = 23 x 29 = 667, T_7, S_2 = 23 x 49 = 1127*

T_9, S_1 = 23 x 23 = 529, T_9, S_2 = 23 x 43 = 989

Note: (p x n) > 1000, (*) exceeds the range, not valid.

Table 17.

Generation of primate compounds m(23) by Formula 2

690n + m(23)$_{1, 2}$			
Termination 1		Termination 3	
S_1	S_2 (*)	S_1	S_2
690 x 0 + 851= 851		690 x 0 + 713= 713	690 x 0 + 943= 943
Termination 7		Termination 9	
S_1	S_2 (*)	S_1	S_2
690 x 0 + 667= 667		690 x 0 + 529= 529	690 x 0 + 989 = 989
Total of m(23) = 06			

7) Calculating multiples of 29, with r: 30 x 29 = 870 (Table 18)

- Combinations for the calculations:

T_1: (p_9 x n_9), T_3: (p_9 x n_7), T_7: (p_9 x n_3), T_9: (p_9 x n_1)

- Calculating m(29) to start the two sequences: 500 / 29 = 17.3.

Note that 17, 3 < 29, so n ≥ 29.

For n ≥ 29. T_1, S_1 = 29 x 29 = 841, T_1, S_2 = 29 x 49 = 1.421*

T_3, S_1 = 29 x 37 = 1073*, T_3, S_2 = 29 x 47 = 1363*

T_7, S_1 = 29 x 43 = 1247, T_7, S_2 = 29 x 53 = 1537

T_9, S_1 = 29 x 31 = 899, T_9, S_2 = 29 x 41 = 1189

Note: (pn) > 1000, (*) exceeds the range, not valid.

Table 18.

Generation of primate compounds m(29) by Formula 2

870n + m(29)$_{1, 2}$			
Termination 1		Termination 3	
S_1	S_2 (*)	S_1 (*)	S_2 (*)
870 x 0 + 841= 841			
Termination 7		Termination 9	
S_1 (*)	S_2 (*)	S_1	S_2 (*)
		870 x 0 + 899= 899	
Total of M(29) = 02			

8) Calculating multiples of 31: r: 30 x 31 = 930 (Table 19)

- Combinations for the calculations:

T_1: (p_1 x n_1), T_3: (p_1 x n_3), T_7: (p_1 x n_7), T_9: (p_1 x n_9)

- Calculating m(31) to start the two sequences: 500 / 11 = 16.1.

Note that 16.1 < 31, so the first divisibility criterion applies. For n ≥ 31:

T_1, S_1 = 31 x 31 = 961, T_1, S_2 = 31 x 41 = 1271*

T_3, S_1 = 31 x 43 = 1.333*, T_3, S_2 = 31 x 53 = 1.643*

T_7, S_1 = 31 x 37 = 1.147*, T_7, S_2 = 31 x 47 = 1.457*)

T_9, S_1 = 31 x 49 = 1.519*, T_9, S_2 = 31 x 59 = 1.829*

Note: (p x n) > 1000, (*) exceeds the range, is not valid.

In this case, the calculation of m(31) only applies in Formula 2 to the first sequence of termination 1, because for the other terminations the p.n value exceeds the given range.

Ending: S1: 930 x 0 + 961= 9611

Step 5: we list the prime composite numbers of the given range in single sequence in the four terminations and identify them in the numbers produced by formula 1.

1) The unique sequence of the prime compounds m(7) in the four terminations:

Termination 1: 511, 581, 721, 791 and 931 Ending 3: 553, 623, 763, 833 and 973.

Termination 7: 637, 707, 847 and 917 Ending 9: 539, 679, 749, 889 and 959.

2) The unique sequence of the primate compounds m(11) in the four endings:

Termination 1: 671 and 781 Termination 3: 583, 803 and 913.

Termination 7: 517, 737 and 847_r Termination 9: 539_r, 649, 869 and 979.

Note: they have two repeated primate compounds (Čr), as they appear in m(7).

3) The unique sequence of m(13) primate compounds in the four endings:

Termination 1: 611 and 871 Termination 3: 533, 793 and 923.

Termination 7: 637_r, and 767 Termination 9: 559, 689 and 949.

Note: there is a repeated primate compound (Čr), as it appears in m(7).

4) The unique sequence of m(17) primate compounds in the four endings:

Termination 1: 731 and 901 Termination 3: 833_r

Termination 7: 527, and 797 Termination 9: 629 and 799.

Note: there is a repeated primate compound (Čr), as it appears in m(7).

5) The unique sequence of the primate compounds m(19) in the four terminations:

Termination 1: 551 and 931_r Termination 3: 703 and 893

Termination 7: 817 Termination 9: 589 and 779.

Note: there is a repeated primate compound (Čr), as it appears in m(7).

6) The unique sequence of the primate compounds m(23) in the four terminations:

Termination 1: 851 Termination 3: 713 and 943

Termination 7: 667 Termination 9: 529 and 989.

7) The unique sequence of the primate compounds m(29) in the four terminations:

Termination 1: 841 Termination 3: 0 Termination 7: 0 Termination 9: 899.

8) The unique sequence of the primate compounds m(31) in the four terminations:

Termination 1: 961 Termination 3:0 Termination 7: 0 Termination 9: 0

The total number of primate compounds generated was 64, but we removed the five repeated compounds (C - Čr) because they don't appear more than once in the general formula. This leaves 59 primate compounds to be eliminated.

- Let's use Table 19 and 20, which contain the numbers produced by the general formula, to identify the primate compounds produced by formula 2.

Table 19.

Result of calculations with the two formulas (F1 - F2)

$(30n + \alpha_{1,2}) – (rn + m(p_b)_{1,2}) = P$			
Termination 1		Termination 3	
Sequence 1	Sequence 2	Sequence 1	Sequence 2
30x0 + 511 = 511-M(7)	30x0 + 521 = 521	30x0 + 503 = 503	30x0 + 523 = 523
30x1 + 511 = 541	30x1 + 521 = 551-M(19)	30x1 + 503 = 533-M(13)	30x1 + 523 = 553-M(7)
30x2 + 511 = 571	30x2 + 521 = 581-M(7)	30x2 + 503 = 563	30x2 + 523 = 583-M(11)
30x3 + 511 = 601	30x3 + 521 = 611-M(13)	30x3 + 503 = 593	30x3 + 523 = 613
30x4 + 511 = 631	30x4 + 521 = 641	30x4 + 503 = 623-M(7)	30x4 + 523 = 643
30x5 + 511 = 661	30x5 + 521 = 671-M(11)	30x5 + 503 = 653	30x5 + 523 = 673
30x6 + 511 = 691	30x6 + 521 = 701	30x6 + 503 = 683	30x6 + 523 = 703-M(19)
30x7 + 511 = 721-M(7)	30x7 + 521 = 731-M(17)	30x7 + 503 = 713-M(23)	30x7 + 523 = 733
30x8 + 511 = 751	30x8 + 521 = 761	30x8 + 503 = 743	30x8 + 523 = 763-M(7)
30x9 + 511 = 781-M(11)	30x9 + 521 = 791-M(7)	30x9 + 503 = 773	30x9 + 523 = 793-M(13)
30x10 + 511 = 811		30x10 + 503 = 803-M(11)	30x10 + 523 = 823

30x11+511= 841-M(29)	30x10 + 521 = 821	30x11 + 503 = 833-M(7)	30x11 + 523 = 853
30x12+511= 871-M(13)	30x11 + 521 = 851-M(23)	30x12 + 503 = 863	30x12 + 523 = 883
30x13+511= 901-M(17)	30x12 + 521 = 881	30x13 + 503 = 893-M(19)	30x13+523= 913-M(11)
30x14 + 511 = 931-M(7)	30x13 + 521 = 911	30x14 + 503 = 923-M(13)	30x14 + 523 = 943-M(23)
30x15+511= 961-M(31)	30x14 + 521 = 941	30x15 + 503 = 953	30x15 + 523 = 973M(7)
30x16 + 511 = 991	30x15 + 521 = 971	30x16 + 503 = 983	

Table 20.

Final result of calculations with the two formulas (F1 - F2)

$(30n + \alpha_{1,2}) - (rn + m(p_b)_{1,2}) = P$

Termination 7		Termination 9	
Sequence 1	Sequence 2	Sequence 1	Sequence 2
30x0 + 517 = 517-M(11)	30x0 + 527 = 527-M(17)	30x0 + 509 = 509	30x0 + 529 = 529-M(23)
30x1 + 517 = 547	30x1 + 527 = 557	30x1 + 509 = 539-M(7)	30x1 + 529 = 559-M(13)
30x2 + 517 = 577	30x2 + 527 = 587	30x2 + 509 = 569	30x2 + 529 = 589-M(19)
30x3 + 517 = 607	30x3 + 527 = 617	30x3 + 509 = 599	30x3 + 529 = 619
30x4 + 517 = 637-M(7)	30x4 + 527 = 647	30x4 + 509 = 629-M(17)	30x4 + 529 = 649-M(11)
30x5 + 517 = 667-M(23)	30x5 + 527 = 677	30x5 + 509 = 659	30x5 + 529 = 679-M(7)
30x6 + 517 = 697-M(17)	30x6 + 527 = 707-M(7)	30x6 + 509 = 689-M(13)	30x6 + 529 = 709
30x7 + 517 = 727	30x7 + 527 = 737-M(11)	30x7 + 509 = 719	30x7 + 529 = 739
30x8 + 517 = 757	30x8 + 527 = 767-M(13)	30x8 + 509 = 749-M(7)	30x8 + 529 = 769
30x9 + 517 = 787	30x9 + 527 = 797	30x9 + 509 = 779-M(19)	30x9 + 529 = 799-M(17)
30x10 + 517 = 817-M(19)	30x10 + 527 = 827	30x10 + 509 = 809	

30x11 + 517 = 847-M(7)	30x11 + 527 = 857	30x11 + 509 = 839	30x10 + 529 = 829
30x12 + 517 = 877	30x12 + 527 = 887	30x12 + 509 = 869-M(11)	30x11 + 529 = 859
30x13 + 517 = 907	30x13 + 527 = 917-M(7)	30x13 + 509 = 899-M(29)	30x12 + 529 = 889-M(7)
30x14 + 517 = 937	30x14 + 527 = 947	30x14 + 509 = 929	30x13 + 529 = 919
30x15 + 517 = 967	30x15 + 527 = 977	30x15 + 509 = 959-M(7)	30x14 + 529 = 949-M(13)
30x16 + 517 = 997		30x16 + 509 = 989-M(23)	30x15 + 529 = 979-M(11)

As we can see we have identified the prime compounds, which are multiples of the basic primes within the given range. Thus, by eliminating these prime compounds, only the prime numbers will remain, i.e. by applying the two formulas, we will have only the prime numbers as the result of the whole process.

$(30n + \alpha_{1,2}) - (rn + m(p_b)_{1,2}) = P$

The following are all the prime numbers in the range 500 to 1000 using the technique

Step 6: list the numbers that were not identified by formula 2. These numbers are all the prime numbers in the range 500 to 1000 (Table 21).

Table 21.

Numbers that were not identified by formula 2

Prime numbers in the range 500 to 1000											
503	509	521	523	541	547	557	563	569	571	577	587
593	599	601	607	613	617	619	631	641	643	647	653
659	661	673	677	683	691	701	709	719	727	733	739
743	751	757	761	769	773	787	797	809	811	821	823
827	829	839	853	857	859	863	877	881	883	887	907
911	919	929	937	941	947	953	967	971	977	983	991
997											

Calculating the difference between the numbers produced by the two formulas gives the number of prime numbers (F1 - F2 = P). Formula 1 produced 132 numbers and Formula 2 produced 59 prime compounds. Therefore, 132 - 64 = 73 prime numbers.

Therefore, with the T_{maip} technique, combining the two formulas made it possible to generate all the prime numbers in the given range from 500 to 1000.

Comparison of techniques for identifying groups of primes

The t_{maip} technique came about when the difficulties in using the tcE technique for identifying primes using the Eratosthenes sieve were realized, when it came to applying it to ranges greater than 100. With the idea of improving the technique, we developed an alternative method of identifying groups of primes in any number interval. The initial aim was to create a formula to generate primes, but we realized that just one formula wouldn't be enough to achieve the idea, which was made possible by creating the second formula and, above all, applying it in two (alternating) sequences in both formulas. As a result of the process used to apply the two formulas, a great reduction in numbers and calculations was achieved. However, compared to the t_{cE} technique, it is more reliable, as the process is well defined.

Using formula 1, it is possible to project the total number of numbers generated within the range of every thousand numbers, which was initially tested up to 10,000 numbers. From then on, the two formulas were tested for the range of 1 to 10,000, using the Excel program to make it easier to compute the data, so that any student who doesn't understand programming could do it. The general formula produced a total of 2,668 odd numbers, of which 1,443 are composite and 1,226 are prime numbers. Adding the three primes that are exceptions in this method (2, 3 and 5), the total number of primes within the range 1 to 10,000 was 1,229.

We could present another application of our method, with other ranges of numbers, but it would exceed the number of pages required by the rules for writing this work.

Conclusion

In this research, we propose an alternative praxeological model for identifying groups of primes, within any range of numbers, to fill the gap in teaching the recognition of primes greater than 100.

In this sense, we started with the first 100 numbers to identify the first 25 prime numbers, presenting an initial and traditional technique in mathematics textbooks, with some variations until we reached the T_{maip} technique - our method - with initial application within the range of 1 to 100 numbers and, later, we presented the process for the range of 500 to 1000 numbers.

The T_{maip} technique came about when we realized how difficult it was to use the tcE technique to identify primes using the Eratosthenes sieve, when it came to applying it to ranges greater than 100. We therefore developed an alternative method for identifying groups of primes in any number interval. The initial aim was to create a formula to generate primes, but we realized that just one formula wouldn't be enough to make the idea a reality, which was made possible by creating the second formula and, above all, by applying both formulas in two (alternating) sequences.

As a result of the process used to apply the two formulas, we have achieved a significant reduction in calculations, as we can see when we compare the initial technique (tcE) (Eratosthenes' sieve) - which uses all 100 numbers and calculates all the multiples of the primes (2, 3, 5 and 7) - and our proposal, because when we used our Tmaip technique with Formula 1, we generated 25 numbers and only used the basic prime 7, which only calculates three multiples (49, 77 and 91). In addition, we extend the identification of prime numbers by applying the technique within the range of 500 to 1000 numbers.

Formula 1 allows us to project the total number of numbers generated within the range of 1,000 to 1,000, which was initially tested up to 10,000 numbers. From then on, the two formulas were tested for the range of 1 to 10,000, using the Excel program to make it easier to compute the data. The general formula produced a total of 2,668 odd numbers, of which 1,443 were composite numbers and 1,226 were prime numbers. Adding the three primes that are exceptions in this method (2, 3 and 5), the total number of primes within the range of 1 to 10,000 was 1,229 numbers.

In these terms, we present an alternative praxeological model for identifying groups of primes for any number interval, in order to expand knowledge or even reinvigorate existing knowledge about prime numbers. Our intention was not just to provide another calculation algorithm with formulas, but a proposal to help in the understanding of prime numbers and their dissemination in schools.

We hope that this study will help mathematics teaching professionals to reflect on the issues raised here and to improve the current state of teaching. It is also hoped that the subject of prime numbers will have a prominent place in teaching due to its importance in the development of mathematics itself, both in the mathematics that is taught in classrooms and in research in pure and applied mathematics, as well as in its application in other areas of knowledge.

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